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Power-Law Behaviour Evaluation from Foreign Exchange Market Data Using a Wavelet Transform Method

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Abstract: Numerous studies in the literature have shown that the dynamics of many time series including observations in foreign exchange markets exhibit scaling behaviours. A simple new statistical approach, derived from the concept of the continuous wavelet transform correlation function (WTCF), is proposed for the evaluation of power-law properties from observed data. The new method reveals that foreign exchange rates obey power-laws and thus belong to the class of self-similarity processes.

Keywords: Wavelet transform, correlation function, foreign exchange rates, power law.

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1. Introduction

The wavelet transform [1,2] provides a powerful tool for analyzing and synthesizing signals. The wavelet transform has the property of localisation both in time and frequency. In wavelet analysis, the scale that can be used to look at data at different resolution levels plays a special role, because wavelet algorithms process data at different scales or resolutions. At a coarse resolution level, one would notice gross features. Similarly, at a fine resolution level, one would get detailed features. This enables us to see both the 'forest' and the 'trees', so to speak, and makes wavelets very useful [3] for data modelling and analysis in diverse fields including dynamical systems modelling [4-12], as well as random signal processing and analysis in for example statistical self-similarity detection and fractal property characterization [13-34].

Numerous studies in the literature have shown that the dynamics of many time series in foreign exchange markets exhibit scaling behaviours [35-45]. For example, Muller et al. [35] and Guillaume1et al. [38] have shown that the mean absolute price changes over certain time intervals for foreign exchange rates obey scaling laws. Recently, Xu and Gencay [44] have shown that US dollar to Deutschemark (USD-DEM) returns present scaling and multifractal properties.

The objective of this paper is to introduce a simple new wavelet transform based approach to detect and evaluate the fractal self-similarity properties from observed time series. The new method involves the calculation of a continuous wavelet transform correlation function (WTCF), which plays a key role in linking the time-domain data with the associated scaling law properties that are explicitly presented by the wavelet scale (revolution) parameter. The introduction of the wavelet transform correlation function (WTCF) here is not original; it can be viewed as an extension of the commonly used second order moment of the associated transforms such as the wavelet cross-transform or cross-coherence/correlation [16-21]. However, unlike most existing wavelet based methods where the continuous wavelet transform is adapted and developed to estimate the Hurst exponent for given fractal signals or where the standard dyadic discrete wavelet transform is used to estimate the associated scaling exponents, the proposed approach here uses the wavelet transform correlation function to directly calculate the power-law exponent parameter that is related to but different from the Hurst exponent.

2. The Wavelet Transform Correlation Method

2.1 The wavelet transform

Let *f*(*t*) be a square integrable function defined in $L^2(R)$. For a given *mother wavelet* ψ, the continuous wavelet transform (CWT) of the function *f*(*t*) is defined as [1,2]

$$W_f^{\psi}(b,a) = \int_{-\infty}^{+\infty} f(t)\overline{\psi_{b,a}(t)}dt = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)\overline{\psi\left(\frac{t-b}{a}\right)}dt \tag{1}$$

where $\psi_{b,a}(t) = a^{-1/2}\psi((t-b)/a)$, $a \in R^{+}$ and $b \in R$ are the dilation (scale) and translation (shift) parameters, respectively. The over-bar above the function $\psi(\cdot)$ indicates the complex conjugate. In order to guarantee (1) is invertible so that f can be reconstructed from W_f^{ψ}, the following admissible condition is required

$$C_{\psi} = \int_0^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty \tag{2}$$

where $\hat{\psi}$ is the Fourier transform of the function ψ.

For a stochastic process *f*(*t*), the wavelet transform $W_f^{\psi}(b,a)$ can be viewed as a random field on the upper (positive) half plane. For a given scale parameter *a*, the transform $W_f^{\psi}(b,a)$ contains a piece of information of the original process at this given scale. Extensive research has been done in recent years to exploit the wavelet transform to analyze and determine the characteristics of random or fractal processes [24-34, 46-53].

2.2 The wavelet transform correlation function

Let $x(t)$ be a wide-sense (weak-sense) stationary random process that is square integrable in $L^2(R)$. For a chosen wavelet ψ, the *wavelet transform correlation function* (WTCF) of the signal $x(t)$, with respect to the locations b_1 and b_2 at scales a_1 and a_2, is defined as below

$$\begin{aligned}\Phi_{x,x}^{\psi}(b_1,b_2;a_1,a_2) &= E[W_x^{\psi}(b_1,a_1)\overline{W_x^{\psi}(b_2,a_2)}] \\ &= \frac{1}{\sqrt{a_1 a_2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[x(\tau_1)\overline{x(\tau_2)}]\,\psi\left(\frac{\tau_1 - b_1}{a_1}\right)\overline{\psi\left(\frac{\tau_2 - b_2}{a_2}\right)} d\tau_1 d\tau_2\end{aligned} \tag{3}$$

Note that $R_{x,x}(\tau_1,\tau_2) = E[x(\tau_1)\overline{x(\tau_2)}]$ is the correlation function of $x(t)$. Using the property that $R_{x,x}(\tau_1,\tau_2) = R_{x,x}(\tau_2-\tau_1,0) = R_x(\tau_2-\tau_1) = E[x(t)\overline{x(t+\tau_2-\tau_1)}]$, it can be derived from equation (3) that

$$\begin{aligned}\Phi_{x,x}^{\psi}(b_1,b_2;a_1,a_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{x,x}(\tau_1,\tau_2)\,\psi_{b_1,a_1}(\tau_1)\overline{\psi_{b_2,a_2}(\tau_2)} d\tau_1 d\tau_2 \\ &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} R_x(\tau_2-\tau_1)\psi_{b_1,a_1}(\tau_1) d\tau_1\right]\overline{\psi_{b_2,a_2}(\tau_2)} d\tau_2 \\ &= \int_{-\infty}^{\infty}\left[(R_x * \psi_{b_1,a_1}(\tau_2)\right]\overline{\psi_{b_2,a_2}(\tau_2)} d\tau_2\end{aligned} \tag{4}$$

where the symbol "$*$" indicates the convolution of two functions.

Assume that the power spectrum $P_x(\omega)$ of the signal $x(t)$ exits. From Parseval's theorem, which states that the inner product of two functions is equal to the inner product of the Fourier transforms of the two individual functions, as well as the convolution theorem that states that under suitable conditions the Fourier transform of a convolution is the pointwise product of the Fourier transforms of the two individual functions, it can then be further derived that

$$\Phi_{x,x}^{\psi}(b_1, b_2; a_1, a_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P_x(\omega)\hat{\psi}_{b_1,a_1}(\omega)\overline{\hat{\psi}_{b_2,a_2}(\omega)}d\omega$$

$$= \frac{\sqrt{a_1 a_2}}{2\pi}\int_{-\infty}^{\infty} P_x(\omega)\hat{\psi}(a_1\omega)\overline{\hat{\psi}(a_2\omega)}e^{-i(b_1-b_2)\omega}d\omega \tag{5}$$

This shows that for the wide-sense stationary process $x(t)$, the wavelet transform correlation function $\Phi_{x,x}^{\psi}(b_1, b_2; a_1, a_2)$, with respect to the locations b_1 and b_2 at given scales a_1 and a_2, is a function of b_1 and b_2 only through the difference $(b_2 - b_1)$. Here, the relationship between the time-domain signal and the frequency-domain behaviour, presented by the spectra of the signal and the associated wavelet transform correlation, is derived by means of the Parseval's theorem and the convolution theorem.

2.3 The power-law case

As a special case of the wavelet transform correlation function, the *wavelet transform autocorrelation function* (WTAF) of the signal $x(t)$, at scale a, can be calculated from (5) by letting $a_1 = a_2 = a$ and $b_2 = b_1 = b$, that is,

$$\Phi_x^{\psi}(a) = E[|W_x^{\psi}(b,a)|^2] = \frac{a}{2\pi}\int_{-\infty}^{\infty} P_x(\omega)\,|\hat{\psi}(a\omega)|^2 d\omega \tag{6}$$

Now assume that the dynamics of the process $x(t)$ exhibits a power-law behaviour, that is, the power spectral density of the process has a power-law dependence in frequency as given below

$$P_x(\omega) \propto |\omega|^{-\beta} \tag{7}$$

It can then be obtained from (6) that

$$\Phi_x^{\psi}(a) = \frac{a}{2\pi}\int_{-\infty}^{\infty} \frac{\left|\hat{\psi}(a\omega)\right|^2}{\left|\omega\right|^{\beta}}d\omega = \frac{a^{\beta}}{2\pi}\int_{-\infty}^{\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{\left|\omega\right|^{\beta}}d\omega = Ca^{\beta} \tag{8}$$

where $C = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\omega\right|^{-\beta}\left|\hat{\psi}(\omega)\right|^2 d\omega < \infty$. Equation (8) suggests that for a power-law signal $x(t)$ that obeys the power-law given by (7), the wavelet transform autocorrelation function $\Phi_x^{\psi}(a)$ also obeys a power-law with respect to the wavelet scale parameter a, and the value of the scaling exponent is exactly the same as in the original power-law presentation but with an opposite symbol, that is $-\beta$ in (7) becomes $+\beta$ in (8). Therefore, the new introduced formulas (8) can be used to estimate the power-law exponent of the signal $x(t)$. Note that the relationship between the power-law exponent β, the Hurst exponent H, and the fractal dimension D is given by $\beta = 2H + 1 = 5 - 2D$. For a self-affine process, $0 \le H \le 1$, $1 \le D \le 2$ and $1 < \beta < 3$; for a Brownian motion, $H = 0.5$, $D = 1.5$ and $\beta = 2$.

3. Results for foreign exchange rates

Monthly average dollar exchange rates, taken from the Federal Reserve Bank of St. Louis for a selection of twenty countries, were considered in this study, and these are shown in Table 1. Monthly average exchange rates are of more interest than daily exchange rates for at least four groups of investors [33]: program traders, investors who follow deterministic rules, investors who routinely accept exposure approximately one month or longer, and currency hedgers.

The proposed wavelet algorithm was used to analyze the twenty datasets. The calculation procedure is as below:

- For each dataset, apply the continuous wavelet transform to calculate the wavelet coefficient $W_x^{\psi}(b,a)$, where four different types of Daubechies' wavelets of order 3, 6, 12 and 20 were respectively used, and the objective is to inspect whether the choice of wavelet basis would significantly affect the associated calculation results. The scale parameter a was allowed to vary from 1 to 16, and the shift parameter b was allowed to vary from 1 to N (N is the data length).
- For each single value of the scale (or resolution level) parameter a, calculate the expectation $\Phi_x^{\psi}(a) = E[|W_x^{\psi}(b,a)|^2] = \mathrm{var}[W_x^{\psi}(b,a)] + <W_x^{\psi}(b,a)>^2$, where 'var' indicates calculating the variance and '< >' indicates taking the average of the relevant signal; $\Phi_x^{\psi}(a)$ is a function of the scale parameter a.
- Plot the graph of $\log_2[\Phi_x^{\psi}(a)]$ (vertical axis) versus $\log_2(a)$ (horizontal axis).
- Calculate the slope of the plot formed by $\log_2[\Phi_x^{\psi}(a)]$ and $\log_2(a)$; the value of the slope can be viewed as an estimate of the power-law exponent β.

The graphs of power-law exponent, calculated by using the Daubechies' wavelet of order 20, for the twenty datasets are shown in Figure 1, where graphs are displayed, from left to right and from top to bottom, in the order that is exactly the same as in Table 1. Values of the power-law exponent β, calculated by using the four Daubechies' wavelets, are given in Table 1, where the last two columns present the associated mean and standard deviation. For convenience of visual inspection and comparison, the values of the power-law exponent β calculated by using the four types of wavelets are also shown in a histogram, see Figure 2. For visualization purpose, the mean and the standard deviation of the power-law parameter produced by the four types of wavelets are shown in Figure 3 and Figure 4 respectively. The above calculation procedure was also performed over some daily average dollar exchange rates for some countries and it has been observed that relevant results are very similar to those that were obtained for the associated monthly average cases.

[**Table 1 is here**]

[**Fig.1 is here**]

[**Fig.2 is here**]

[**Fig.3 is here**]

[**Fig.4 is here**]

Note that in the above calculation the original datasets were directly used to test and evaluate the power-law properties of the foreign exchange rates; no data pre-processing procedure has been performed. Data normalization for example mean-removal might very slightly affect the estimation results.

From Table 1 and Figures 2 and 3, the following observations can be obtained:

- The choice of wavelet basis affects the estimate of the power-law exponent, but the effect is not significant. In other words, different wavelet bases would lead to slightly different estimates for the power-law parameter.
- Among the twenty datasets considered here, the power-law parameter corresponding to the foreign currency exchange rate of Greece against the U.S. is the largest, with β =2.8873 ('db20') and β=2.9148 (mean), which is followed by the rate of Portugal against the U.S., with β =2.6922 ('db20') and β=2.6345 (mean). From fractal Brownian motion theory, these would suggest that there exist stronger persistence (or long-range positive correlation) in the two exchange rates compared with others, meaning that if the amplitude of such rates increases in an interval over a time horizon, then it is more likely to continue the increasing trend in the period immediately following.
- The power-law parameter β for many of the countries considered here is just slightly greater than 2, meaning that there exists weak persistence in these time series.
- There are some cases for example Switzerland where the corresponding power-law parameter β is very close to (or even less than) 2. This means that there exists no correlation or very weak long-range correlation in the relevant time series. This would suggest that the process exhibits some stochastic behaviour possessed by the Brownian motion.

4. Discussions and Conclusions

The proposed wavelet transform correlation analysis method can be used to detect and evaluate the fractal scaling-law behaviours from observed time series. The proposed method has several advantages, for example, it is not necessary for this method to use a large number of observations to obtain accurate estimates of the power-law exponent; unlike traditional power spectral density estimation methods which require data smoothing (windowing) and which are sensitive to the window 'shapes', the new method does not need any windowing techniques. Moreover, this new non-parametric method can be performed speedily and efficiently using existing tools for continuous wavelet transform calculation in Matlab. The presented results have shown that the foreign exchange rates, for the twenty countries considered, exhibit power-laws and thus belong to the class of fractal self-similarity processes.

It should be pointed out that the analysis result here is not our final destination along this research direction, but rather this is only one preliminary stage and the presented result will be used as a basis for the next step. For example, we plan to extend and adapt the recently developed nonlinear system and identification methods and algorithms including some wavelet based dynamical modelling approaches [7-12, 54] to forecast foreign exchange rates. One direct application of the result here is to aid the selection and determination of model variables using the value of the power-law parameter β . It is known from Taken's embedding theorem [55] that in order to sufficiently characterize a dynamical process, the embedding dimension (number of model variables) should be $d \geq 2D+1$ where D is the relevant fractal dimension. Using the embedding theorem and the result here, along with other variable selection methods [56], we can determine the best model variables used for constructing dynamical models that are suitable for predicting individual foreign exchange rates. It is also believed that the reported result has another potential application, that is, it can be used to aid the determination of wavelet scale parameters if dynamical multiscale or multiresolution wavelet models [7-12], which have been proved to be very effective for dynamical system modelling, are to be employed to model and forecast foreign exchange rates, where wavelet models for different individual exchange rates will require different wavelet scale parameters.

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Table 1 The power-law exponents estimated using the wavelet transform correlation function for the monthly average dollar exchange rates of twenty countries. The data came from the Federal Reserve Bank of St. Louis.

Country	Observation period of the rates (dd/mm/yy)	Data length	Lowest and highest rates	Power-law exponent β					
				db3	db6	db12	db20	Mean	Std
Austria	01/01/1971—01/12/2001	372	9.72 / 25.873	2.0809	2.0824	2.0302	2.0113	2.0512	0.0360
Belgium	01/01/1971—01/12/2001	372	27.96 / 66.31	2.1676	2.1620	2.1376	2.1235	2.1477	0.0207
Brazil	01/01/1995—01/12/2007	156	0.84 /3.80	2.4512	2.4261	2.4671	2.4642	2.4522	0.0187
Canada	01/01/1971—01/12/2007	444	0.96/1.60	2.1239	2.1189	2.0722	2.0544	2.0923	0.0344
Denmark	01/01/1971—01/12/2007	444	5.08 /11.81	2.1212	2.1105	2.0751	2.0630	2.0924	0.0278
Finland	01/01/1971—01/12/2001	372	3.49/6.96	2.2180	2.1976	2.2365	2.2413	2.2233	0.0199
France	01/01/1971—01/12/2001	372	4.00 /10.09	2.2443	2.2300	2.2414	2.2331	2.2372	0.0068
Germany	01/01/1971—01/12/2001	372	1.38/3.64	2.0874	2.0851	2.0329	2.0137	2.0548	0.0372
Greece	01/01/1981—01/12/2000	237	53.18 /398.29	2.8976	2.9332	2.9410	2.8873	2.9148	0.0263
India	01/01/1973—01/12/2007	420	7.27 /49.02	2.5134	2.4704	2.5803	2.5634	2.5319	0.0499
Italy	01/01/1971—01/12/2001	372	265.26/2271	2.4575	2.4152	2.5316	2.5347	2.4848	0.0585
Japan	01/01/1971—01/12/2007	444	83.69 /358.02	2.1381	2.1433	2.0618	2.0193	2.0906	0.0604
Mexico	01/01/1993—01/12/2007	170	3.108 /11.52	2.2763	2.2678	2.3543	2.3566	2.3138	0.0483
Netherlands	01/01/1971—01/12/2001	372	1.55 / 3.74	2.1222	2.1223	2.0764	2.0589	2.0949	0.0323
Norway	01/01/1971—01/12/2007	444	4.82 /9.47	2.1324	2.1314	2.1075	2.1079	2.1198	0.0140
Portugal	01/01/1973—01/12/2001	348	22.41 /235.17	2.5917	2.5436	2.7104	2.6922	2.6345	0.0800
Spain	01/01/1973—01/12/2001	348	55.8 /195.17	2.3453	2.3066	2.4219	2.4333	2.3768	0.0610
Sweden	01/01/1971—01/12/2007	444	3.92 /10.79	2.2080	2.2109	2.2329	2.2444	2.2241	0.0175
Switzerland	01/01/1971—01/12/2007	444	1.12/4.31	2.0154	1.9670	1.9592	1.9271	1.9672	0.0365
U.K.	01/01/1971—01/12/2007	444	1.09/2.62	2.0963	2.0973	2.0479	2.0470	2.0721	0.0285
All the rates (except the last one) are defined to be the relevant currency against the US dollar. The last one is defined to be Pound against Dollar.									

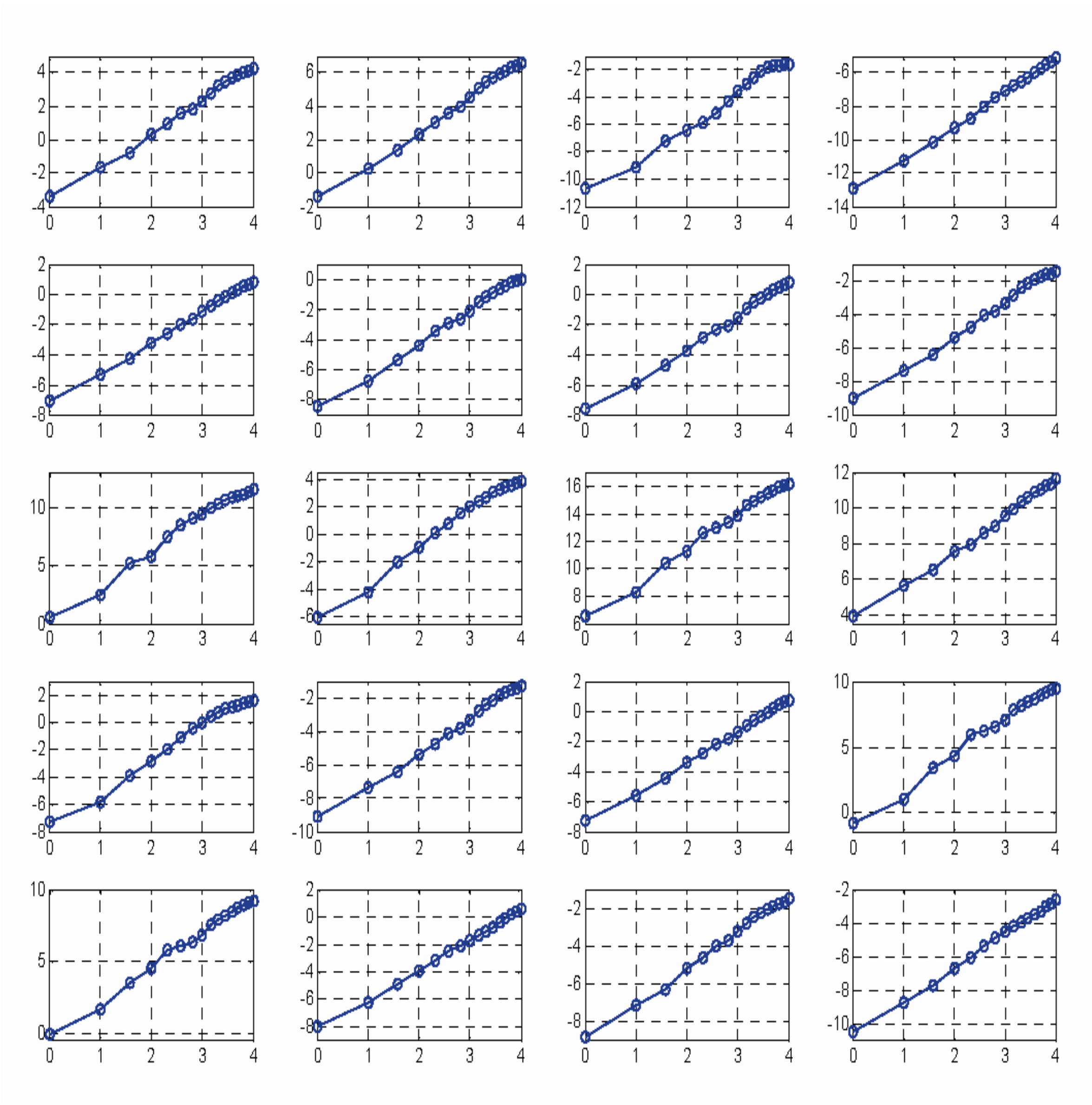

Figure 1 Graphs of the wavelet transform correlation function defined by (8) for the twenty datasets of foreign exchange rates listed in Table 1 where Daubechies' wavelet of order 20 was used. From left to right and from top to bottom, these are displayed in order that is exactly the same as in Table 1. In these graphs, the vertical axis is $\log_2[\Phi_x^{\psi}(a)]$ and the horizontal axis is $\log_2(a)$.

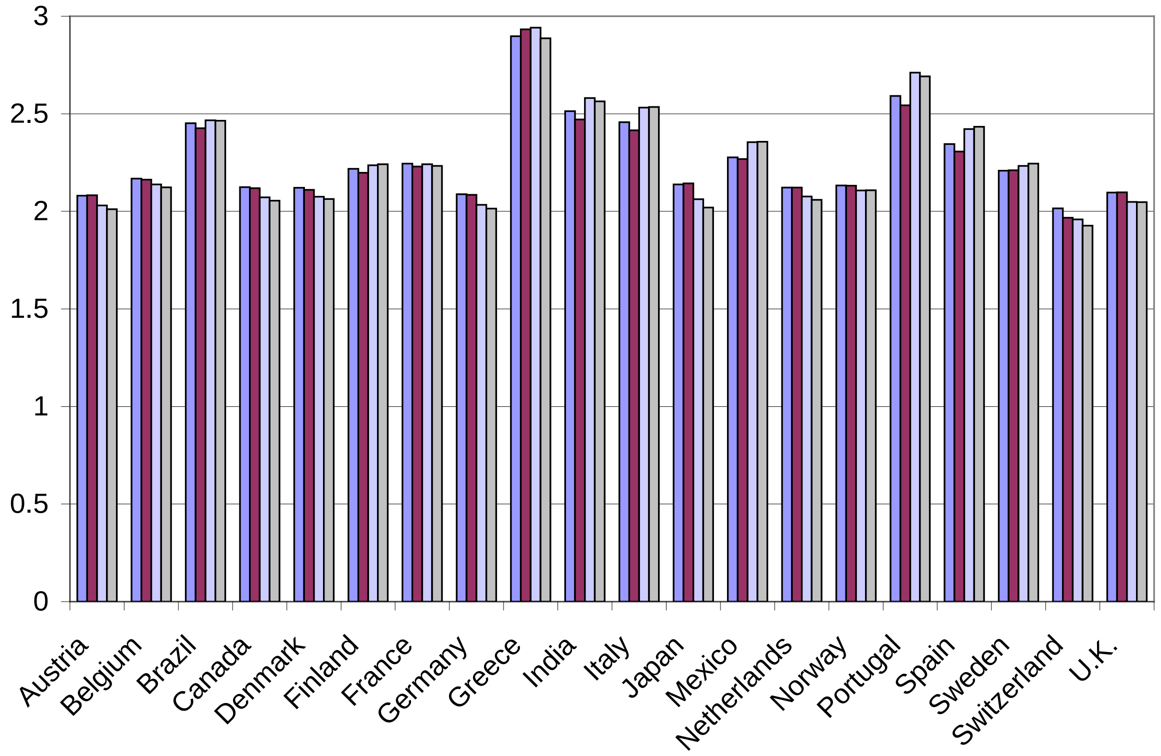

Figure 2 Values of the power-law exponent β calculated by using Daubechies' wavelets of order 3, 6, 12, and 20, respectively, for each of the exchange rates of the twenty countries.

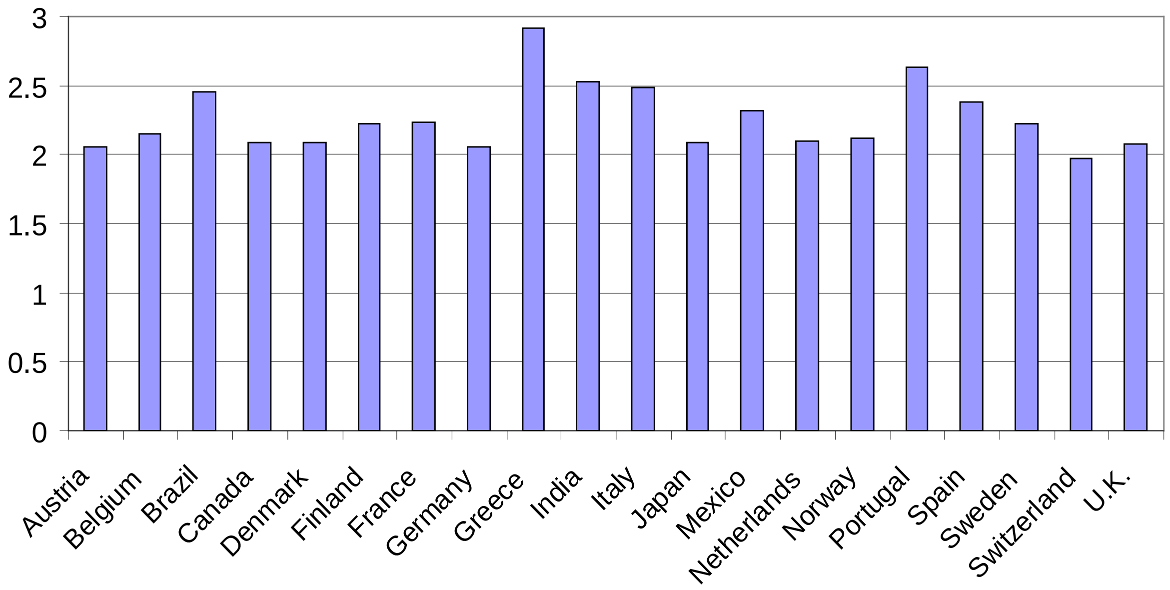

Figure 3 The average of the power-law exponent β calculated from the four types of Daubechies' wavelets for each of the exchange rates of the twenty countries.

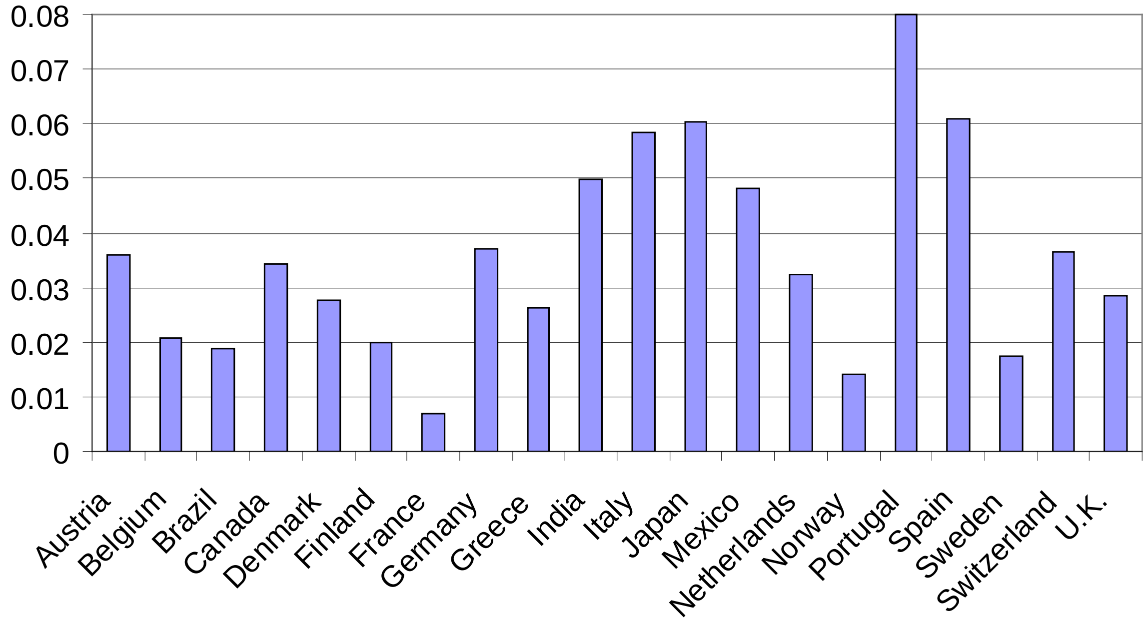

Figure 4 The standard deviation of the power-law exponent β calculated from the four types of Daubechies' wavelets for each of the exchange rates of the twenty countries.