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Reconciling yield stability with international fisheries agencies precautionary preferences: the role of non constant discount factors in age structured models
Reconciling yield stability with international fisheries agencies precautionary preferences: the role of non constant discount factors in age structured models

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Summary: International fisheries agencies recommend exploitation paths that satisfy two features. First, for precautionary reasons exploitation paths should avoid high fishing mortality in those fisheries where the biomass is depleted to a degree that jeopardise the stock's capacity to produce the Maximum Sustainable Yield (MSY). Second, for economic and social reasons, captures should be as stable (smooth) as possible over time. In this article we show that a conflict between these two interests may occur when seeking for optimal exploitation paths using age structured bioeconomic approach. Our results show that this conflict be overtaken by using non constant discount factors that value future stocks considering their relative intertemporal scarcity.

Keywords: Fisheries management, optimization in age-structured models, non-constant discount factor, utility function.

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Introduction

Fisheries agencies have a preference for stable exploitation paths with low annual intervariability on captures and that implement precautionary reductions of the fishing mortality in those fisheries where the biomass is below its Maximum Sustaniable Yield (MSY). The objective of this article is to show that stable and non-overfishing exploitation paths can be the optimal response for age-structured fisheries that have to restore the stock to levels that produce MSY.

It is commonly accepted among practitioners that exploitation paths should be stable over time without showing drastic changes in fishery mortalities. Penas (2007) argues that plans that cause substantial decreases in fishing possibilities from one year to the next will meet with fierce opposition from fishers and stakeholders. Dichmont et al. (2010) point out that closing a fishery could be optimal only if vessels had a viable alternative and fishers can cover fixed costs, and regaining skilled fishers when a fishery reopen is possible. Thus, long-term plans that recommend closing fisheries or that involve drastic capture reductions in the short-term are not realistic, even if those restrictions result in greater benefits in the long-run. This issue is reflected, for instance, in the development of the European multi-annual plans of fish stocks that set maximum limits on year-on-year TAC variations of 15%, to provide stability in the fishery industry (Penas, 2007).

On the other hand, in real practice fisheries agencies boost the reduction rather than the increase of the fishing mortality when stock size is below that which produces MSY level. For instance the National Oceanic and Atmospheric Administration (NOAA) for tracking the status of US fish stocks penalises “overfishing” stocks, defined as those stocks whose fishing mortality is above the rate that produces MSY (NOAA, 2015).

In our analysis the exploitation trajectories are the solutions of a bioeconomic model in which a regulator maximizes the present value of the utility associated to the profits considering that the fish population is age-structured.

Age-structured population models have been the centrepiece of fisheries management for long time. From Baranov’s seminal article (1918) to subsequent developments by Beverton and Holt (1957), Ricker (1975) or Shepherd (1982), there has been a vast amount of studies showing the advances of adopting age-structure approach. In real practice, the vast majority of contemporary stock assessments that attempt to reconstruct population biomass for marine species are based on age-structured models (Punt et al., 2013). In fact, age composition is the common population structure used in Virtual Population Analysis for fish stock assessment (Lassen and Medley, 2000).

During years many economists have considered that models with explicit age structure were very convenient for practical management problems but intractable from the analytical point of view when studying optimal harvesting decisions. This has changed recently and an increasing number of authors argue that age-structured models are more appropriated than biomass models to best reflect the complexity of fish stocks (Wilen, 1985; Getz et al., 1985; Towsend, 1986; Clark, 1990; Quinn and Deriso, 1999; Hilborn and Walters, 2001, Walters and Martell, 2004). Moreover, some studies show that optimal harvesting decisions when age-structured information is taken into account may be different to those found when optimization is based on conventional biomass variables (e.g. Tahvonen, 2009; Skonhoft et al. 2012).

The analysis of optimal harvesting based on age-structured models has shown that in many circumstances the optimal solution takes the form of pulse fishing (Hannesson, 1975; Clark, 1976, 1990; Tahvonen, 2009; Steinshamn, 2011; Da Rocha et al., 2012a, 2013). This means that optimal exploitation paths consist of periodic cycles of fishing followed by fallow periods to enable stocks to recover. Therefore, pulse fishing represents a type of solution far away from the stable trajectories usually advocated by practitioners and regulators.

Given our interests in stable exploitation trajectories which are intrinsically continuous (against pulse fishing trajectories which are non-continuous), our analysis integrates the findings of the economics literature that links the concavity of the objective function and the continuity of the optimal trajectory solutions (Scarf 1959; Stokey et al. 1989). In fisheries economics this link between concavity and
continuity is established by Dawid and Kopel (1997, 1999) for biomass models. For age-structured models, Da Rocha et al. (2012c) show numerically that pulses are not longer a viable optimal solution by maximizing the logarithm of the catches instead of just the catches. This result is reinforced in Da Rocha et al. (2013) where it is shown analytically that the concavity properties of the management objective function are essential to remove optimal pulse fishing under imperfect selectivity. In the context of schooling fisheries, Tavhonen et al. (2013) also show that using nonlinear harvesting costs that guarantee the concavity of the manager’s objective function generates smooth optimal paths toward the steady state. On the contrary, a linear harvesting cost implies pulse fishing instead of smooth continuous harvesting.

This article continues with this research line of proposing concave functions to be maximized in the management problem as a way to guarantee stable exploitation paths. In particular, in our age-structured bioeconomic model managers maximize the present value of the utility associated to the profits of the fishery. In economics, a utility function is a formula that assigns a numerical score representing the satisfaction that an economic agent gets from consumption of a given basket of goods or services. In our context, the objective in the management problem is not to maximize net profits but the utility derived from those net profits. We consider a constant elasticity substitution (CES) utility function which is the most frequently used in intertemporal decision problems (Blanchard and Fischer, 1989) and which allow us here to represent the decisions over stability in profits when regulating commercial fisheries.

Even though the CES function has been extensively used in macro and micro economic analysis, it has hardly been used in fisheries economics. Some exceptions are Hoff (2004) and Quaas and Requate (2013) which apply a CES utility function in a static context to characterize the substitutability between the inputs of the production function of the Danish trawler fleet operating in the North Sea, and between the consumption of different species of fish, respectively. In our case, the CES utility function will be used to stress the substitutability between current and future harvesting in dynamic context. As far as we know, the CES utility function has not been used to represent the preferences of decision makers in charge of regulating commercial fisheries.

The CES utility function is also justified because it can be fully parameterized with the intertemporal elasticity of substitution (IES) in dynamic contexts. The IES is a positive parameter that expresses the degree of substitutability between current and future consumption (harvesting in our case). The closer to (farther from) zero the IES parameter is, the more substitute (complementary) current and future captures are. Decision makers that are more (less) willing to substitute between current and future harvests can be considered to have a lower (higher) desire for smooth exploitation paths. Consequently, the IES parameter represents the level of importance attributed to stability (smooth paths) in decision making and can be interpreted as the smoothness parameter.

Moreover, the conventional approach of maximizing profits or yield used to evaluate recovery plans (Gröger et al., 2007; Da Rocha et al., 2010; Da Rocha and Gutiérrez, 2011; Simons et al., 2014), multispecies stocks (Punt et al., 2011; Da Rocha et al., 2012b; Gourgue et al., 2013) or new management policies (Quaas et al. 2013) can be seen as a particular case of the methodology proposed here with the maximization of a CES utility function. This is because when the smoothness parameter tends to infinity, the CES utility function represents a situation where the discounted fishery profits (or yield) are maximized. Also when the smoothness parameter equal one, the logarithm of the profits or yield is maximized (Da Rocha et al., 2012c).

Our results show that including concavity in the management decision problem with a utility function and keeping constant the discount factor generates a non desirable response in the short term. Our numerical analysis illustrates that if the fishery is in the steady state situation (MSY) and there is a

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Following the economics terminology we use smoothness to refer to the level of variability in intertemporal paths. The lower the average of annual variability between captures in long-term management plans, the smoother we say the exploitation path is. Smoothness is also related with the term stability. In a broad sense, stability can be considered a synonymous of smoothness for intertemporal paths; although stability is a more precise mathematical concept applied to dynamic systems.
shock that affects negatively the biomass, then the optimal exploitation decision consists of increasing the fishing mortality in the short term approaching smoothly to the MSY in the long term. This means that the precautionary principle is not followed, as this optimal response promotes the increase of the fishing mortality and the fishery can be considered as having overfishing. Moreover, our analysis shows that this effect over the short run exploitation not only disappears but can be reverted when the present value is calculated using adequate non constant discount factors.

Constant discount factor is the common manner for accounting future events in present terms. For instance, when we take a constant discount factor of 0.95, we are assuming that economic agents are indifferent between receiving 100 Euros in year t or 105.26 Euros in year t+1, regardless of how far the future period t is. However, an increasing number of authors argue that the hypothesis of constant discount factor is systematically violated (e.g. Frederick et al., 2002; and Grijalba et al., 2014). The standard experiment used in the literature to show that discounting is not constant over time involves the comparing short-term decisions with long-term decisions, including questions such as: "Would you prefer 100 Euros today or 105 Euros tomorrow?" or "Would you prefer 100 Euros in one year or 105 Euros in one year and one day?" The constant discount hypothesis requires the same answer to both questions. However, a significant fraction of respondents choose the lesser amount today in the first question, but will gladly wait one extra day in a year in order to receive the higher amount in the second question. This result means that the discount factor used in today’s decision is lower that the discount factor employed in one year’s decision, supporting the idea that future events should be discounted at non constant and increasing factor (or decreasing rate) over time.

The use of non constant discount factors is not a novelty in the context of long-term environmental and natural resource problems issues (Carson and Tran, 2009; Gollier, 2012). Hyperbolic discounting that considers that discount rates decline hyperbolically as the discounted event is moved further away in time (Loewenstein and Prelec, 1992), has recently been considered in renewable resource management in climate change (Karp, 2005; Fujii and Karp, 2008; Karp and Tsur, 2011). In fisheries economics there are also some recent advances applying non constant discount factor in dynamic biomass management problems. Ducan et al. (2011) examine harvesting plans when the discount factor increases over time. They find that the planner reduces stock levels in the early stages (when the discount factor is low) and intends to compensate by allowing the stock level to recover later (when the discount factor will be higher). Such a plan may be feasible and optimal, provided that the planner remains committed throughout. Ekeland et al. (2015) analyze optimal exploitation of a fishery in an overlapping generation economy. In this case, non-constant discount factors emerge in the model because it is assumed that the current generation discounts the utility of future generations at a rate different from the rate used to discount their own utility. They find that if the current generation has some concern for the not-yet born, the equilibrium policy does not depend on the degree of that concern. Unlike these articles, we consider that the discount of a future event is proportional to the marginal utility of this future event relatively to the marginal utility of the event today; that is we discount the future considering the intertemporal scarcity of the resource.

The main contribution of this article is to show that both, stability and precaution, can be reconciled by using appropriated non constant discount rates that take into account the intertemporal scarcity of the resource. Applying our bioeconomic model to the European Southern Hake Stock, we find that a CES utility with a significant smoothness parameter together with a non constant discount rates generate optimal non overfishing exploitation paths 10 times smoother than those that result from the conventional approach.

\[ \beta = 1 / (1 + r) \]

3 Discount is frequently introduced into fishery economics using the discount rate, r, instead of a discount factor \( \beta \); the former is usually applied in continuous time frameworks, whereas the latter is more commonly used in discrete set ups.
Materials and Methods

The intertemporal allocation of harvesting is modelled using a constant elasticity substitution (CES) utility function (see Blanchard and Fischer, 1989, pp 44),

\[ U(\Pi) = \frac{\Pi^{1-\sigma} - 1}{1-\sigma}, \]

where \( \Pi \) represents an economic indicator (e.g. profits, or yield), the parameter \( \sigma \geq 0 \) represents the curvature of the utility function and the parameter \( \rho = 1/\sigma \geq 0 \) is the smoothness parameter representing the intertemporal elasticity substitution (IES) between profits at any two points in time. Different values for the smoothness parameter, \( \rho \), represents different degree of desire for smooth exploitation paths. In general, the closer to (farther from) zero the smoothness parameter \( \rho \) is the higher (lesser) the desire to capture intertemporal smoothing is.

In particular, when \( \sigma = 0 \) the utility coincides with the economic indicator. In this case the IES, \( \rho \), tends to infinity representing the case in which current and future harvests are perfect substitute goods. This means that managers are indifferent between having captures today and harvesting in the future because the utility does not depend on the temporal allocation of harvest, i.e. managers have a minimal desire for having smooth harvesting paths. By contrast, when \( \sigma \) tends to infinity, the IES, \( \rho \), tends to zero. In this case current and future harvests are perfect complementary goods which implies that utility increases when current and future harvests are extracted in exact proportions over time. This case represents a situation where managers have strong interest for having smooth harvesting paths. Finally notice that for the case in which \( \sigma = 1 \), the CES function becomes the logarithm function.

It is worth mentioning, that under specific conditions the CES utility function can be used to describe attitudes toward risk in uncertain settings. In those cases, the utility function is called Constant Relative Risk Aversion utility function and the parameter \( \sigma \) represents the coefficient of relative risk aversion. In this article we abstract from uncertain situations and focus on deterministic settings.

The dynamic management problem

Optimal management problem consists of selecting the fishing mortality path, \( \{F_t\} \), that maximises the present value of future utilities considering the stock dynamics implied by an age-structured population.

Formally, the function to be maximised in the management problem can be expressed as

\[ \sum_{t=0}^{\infty} \beta_t \frac{\Pi_t^{1-\sigma} - 1}{1-\sigma}. \]

(1)

We take the profits as the economic indicator; so \( \Pi_t = \sum_{a=1}^{A} pr_a y_{a,t} - C(F_t) \) represents the net profits associated with the whole harvest captured in year \( t \), with \( pr_a \) and \( y_{a,t} \) being the price and the yield of age \( a = 1,..A \), respectively, and \( C(F_t) \) is the total cost in year \( t \), which depends positively on fishing mortality, \( F_t \). Notice that this objective function indirectly could represent different economic indicators depending on the prices and costs considered. For instance, if \( pr_a \) has a value of 1 and the the total cost is zero, the objective function represents the present value of the utility of the yield in weight. When the total cost is zero and \( pr_a \) is not 1, the objective function coincides with the present
value of the utility of the gross profits (yield in value). When \( pr_a \) is assumed different than 1 and the total cost is fully considered the objective function represents the present value of the utility of net profits. The utility derived from future harvests in year \( t \) is discounted by a non-constant subjective factor, \( \beta_t \). We explain latter on how the discount is selected.

The stock dynamics is represented by \( N_{a,t+1} = e^{-\alpha_{a,t}} N_{a,t} \), where \( N_{a,t} \) and \( z_{a,t} \) are the size in numbers and the total annual mortality rate of age group \( a \) in year \( t \), respectively. The total mortality rate is decomposed into fishing mortality \( F \) and natural mortality \( M^a \), which may differ from one age to another. Formally, \( z_{a,t} = p_a F_t + M_a \), where \( p_a \) represents the selectivity parameter for age \( a \).

Captures for age \( a \) are given by the Baranov equation (1918), \( y_{a,t} = \frac{F_t}{z_{a,t}}(1 - e^{-\alpha_{a,t}}) N_{a,t} \).

For the sake of simplicity, we present the model for the case of three ages, \( A = 3 \), and constant recruitment: \( N_{t} = N_1 \) for any \( t \). In the Supplementary Material we prove that all the results derived with this simple model holds for the general case of \( A \) ages and considering a generic endogenous stock-recruitment relationship.

Notice that by backwards substitution \( N_{a,t} \) can be expressed as a function of recruitment: \( N_{a,t} = \phi_{a,t} N_{1,t-1} \), where \( \phi_{a,t} \) can be interpreted as the survival function that shows the probability of a recruit born in year \( t-(a-1) \) reaching age \( a \), for a given \( F \) path \( \{ F_t, F_{t-1}, F_{t-2}, \ldots F_{t-(a-1)} \} \). For the case three ages, the survival functions are

\[
\phi_{1,t} = 1, \\
\phi_{2,t} = e^{-p_1 F_t - M_t}, \\
\phi_{3,t} = e^{-p_1 F_t - M_t} e^{-p_2 F_t - M_t}.
\]

And the function to be maximised can be expressed as

\[
L = \sum_{t=0}^{\infty} \frac{\beta_t}{1-\sigma} \left\{ \left[ pr_1 y_1(F_t) \phi_{1,t} N_1 + pr_2 y_2(F_t) \phi_{2,t} N_2 + pr_3 y_3(F_t) \phi_{3,t} N_3 - C(F_t) \right]^{1-\sigma} - 1 \right\}.
\]

In order to obtain the first order condition of this maximization problem, we have to calculate \( \partial L / \partial F_t = 0 \). Notice that \( F_t \) only appears in the function \( L \) above in those sums multiplied by \( \beta_t \), \( \beta_{t+1} \) and \( \beta_{t+2} \). That is,

\[
L = \ldots + \frac{\beta_{t+1}}{1-\sigma} \left\{ \left[ pr_1 y_1(F_{t+1}) \phi_{1,t+1} N_1 + pr_2 y_2(F_{t+1}) \phi_{2,t+1} N_2 + pr_3 y_3(F_{t+1}) \phi_{3,t+1} N_3 - C(F_{t+1}) \right]^{1-\sigma} - 1 \right\} + \\
+ \frac{\beta_{t+2}}{1-\sigma} \left\{ \left[ pr_1 y_1(F_{t+2}) \phi_{1,t+2} N_1 + pr_2 y_2(F_{t+2}) \phi_{2,t+2} N_2 + pr_3 y_3(F_{t+2}) \phi_{3,t+2} N_3 - C(F_{t+2}) \right]^{1-\sigma} - 1 \right\} + \ldots
\]
To simplify the notation, note that the expressions between square brackets are just the net profits, $\Pi_t$, $\Pi_{t+1}$ and $\Pi_{t+2}$, therefore the first order condition of the maximization problem, $\partial L / \partial F_t = 0$, can be expressed as,

$$
\beta_1 \left[ pr_3 d y_3(F_t) \frac{d y_3(F_t)}{dF_t} \phi_{3,1} N_t + pr_2 d y_2(F_t) \frac{d y_2(F_t)}{dF_t} \phi_{2,1} N_t + pr_1 d y_1(F_t) \frac{d y_1(F_t)}{dF_t} \phi_{1,1} N_t - \frac{dC(F_t)}{dF_t} \right ]
$$

$$
+ \beta_{t+1} \left( \frac{\Pi_{t+1}}{\Pi_t} \right)^{\sigma} \left[ pr_3 y_3(F_{t+1})(-p_2) \phi_{3,1} N_{t+1} + pr_2 y_2(F_{t+1})(-p_1) \phi_{2,1} N_{t+1} \right ]
$$

$$
+ \beta_{t+2} \left( \frac{\Pi_{t+2}}{\Pi_t} \right)^{\sigma} \left[ pr_3 y_3(F_{t+2})(-p_1) \phi_{3,1} N_{t+2} \right ] = 0.
$$

Equation (2) reveals that the fishing mortality path that maximises the net present value of utility of the profits is the balance between two effects: (i) the instantaneous change in the current utility due to changes in current fishing mortality, when the age distribution of the population is constant over time (first line); and (ii) the changes in future harvests possibilities due to changes in the future age distribution of the fish population induced by changes in current fishing mortality (second and third lines).

Notice that the term $\beta_{t+k} \left( \frac{\Pi_{t+k}}{\Pi_t} \right)^{\sigma} = \beta_{t+k} \frac{U'(\Pi_{t+k})}{U'(\Pi_t)}$ multiplies all variables of year $t+k$ in the first order condition, (2)$^4$. This term can be interpreted as the value, in utility terms, of an additional euro payoff received in year $t+k$ given the value of one euro today. Note that since profits depend on the resource scarcity; the above expression represents the scarcity of the stock in the future relative to scarcity of the stock today.

Reference points and the dynamic management problem

Da Rocha and Gutiérrez (2011) prove that for the case of $\sigma = 0$ the reference point $F_{max}$ can be obtained as a particular case of the stationary solution of the maximisation problem (1) in which $pr_1 = 1$, $\beta_t = 1$ and $C = 0$. We show here that $F_{max}$ appears as the stationary solution under the same conditions regardless of the value of the parameter, $\sigma$ (and consequently of IES).

The stationary solution can be understood as the steady-state value of $F$ that is reached in the long term by the optimal path. Therefore if $F = F_t = F_{t+1}$, Equation (2) becomes

$$
\sum_{\sigma=1}^{3} pr_\sigma \frac{d y_\sigma(F_t)}{dF_t} \phi_\sigma(F_t) N_t - \frac{dC(F_t)}{dF_t} = pr_3 y_3(F_t) \phi_3(F_t) N_t \left[ \frac{\beta_{t+1}}{\beta_t} p_t \right ] + pr_2 y_2(F_t) \phi_2(F_t) N_t \left[ \frac{\beta_{t+2}}{\beta_t} p_t + \frac{\beta_{t+1}}{\beta_t} p_t \right ],
$$

where

$$
\phi_1 = 1,
$$

$$
\phi_2 = e^{-\gamma_F F_t - M_t},
$$

$$
\phi_3 = e^{-\gamma_F F_t - M_t} e^{-\gamma_F F_t - M_t},
$$

$^4$ In uncertain settings, this term is known as the stochastic discount factor and has been widely used by financial economists (Cochrane and Culp, 2003) to describe asset prices.
are the stationary survival functions.

Observe that for the case in which \( p_{r_a} = 1 \), \( \beta_t = I \) and \( C = 0 \), the above expression can be written, after some manipulation, as

\[
\sum_{a=1}^{3} \frac{d \gamma(F)}{dF} \frac{\phi_a(F)}{\phi_a(F)} + \sum_{a=1}^{3} \frac{d \phi_a(F)}{dF} = 0. \tag{3}
\]

Equation (3) represents the first order condition of an optimisation problem that maximises the stationary yield, \( L = \sum_{a=1}^{3} \gamma(F) \phi_a(F) N_a \), which by definition characterises \( F_{\text{max}} \) when recruitment is not constant and \( F_{\text{max}} \) when recruitment is constant.

As already mentioned above, the reference point that satisfies equation (3) does not depend on smoothness parameter, \( \rho = 1/\sigma \). Therefore it can be asserted without doubt that reference points do not depend on the specification of the parameters of the CES utility function used in the regulator’s decision problem.

The role of the discount factor and the smoothness of the optimal paths

In order to explore the exploitation trajectories, which depend on the discount factor and on the curvature parameter \( \sigma \) (i.e., on IES), we rewrite the first order condition (2) as

\[
\beta_t \left[ \frac{p_{r_3} \frac{d y_3(F)}{dF} N_{3,t} + p_{r_2} \frac{d y_2(F)}{dF} N_{2,t} + p_{r_1} \frac{d y_1(F)}{dF} N_{1,t} - dC(F)}{\frac{dC(F)}{dF}} \right] =
\]

\[
\frac{\beta_{t+1}}{\Pi_{t+1}} \left[ p_{r_3} p_{r_2} y_3(F_{t+1}) N_{3,t+1} + p_{r_2} y_2(F_{t+1}) N_{2,t+1} \right] + \frac{\beta_{t+2}}{\Pi_{t+2}} \left[ p_{r_1} y_1(F_{t+2}) N_{1,t+2} \right].
\]

For the case of a constant discount rate, \( \beta_t = \beta' \), it can be seen that there is no pattern about the annual growth rate of profits along the optimal path.

\[
= \beta \left( \frac{\Pi_{t+1}}{\Pi_t} \right)^{\sigma} \left[ p_{r_3} y_3(F_{t+1}) N_{3,t+1} + p_{r_2} y_2(F_{t+1}) N_{2,t+1} \right] + \beta'^{\sigma} \left( \frac{\Pi_{t+2}}{\Pi_t} \right)^{\sigma} \left[ p_{r_1} y_1(F_{t+2}) N_{1,t+2} \right]
\]

However, if the discount factor for the current year is selected as \( \beta_t = \beta' \) (that is 1 for \( t=0 \)) and for the future years such that \( \beta_{t+k} = \beta'^{k} \) for any \( k \geq 1 \), then the optimal exploitation path satisfies that

\[
\left( \frac{\Pi_{t+k}}{\Pi_t} \right)^{\sigma} = \frac{\beta^{\sigma} \left[ p_{r_3} y_3(F_{t+k}) N_{3,t+k} + p_{r_2} y_2(F_{t+k}) N_{2,t+k} \right]}{p_{r_3} \frac{d y_3(F)}{dF} N_{3,t} + p_{r_2} \frac{d y_2(F)}{dF} N_{2,t} + p_{r_1} \frac{d y_1(F)}{dF} N_{1,t} - dC(F)}
\]

The growth rate of the profits along the optimal path can be expressed as:
\[
\frac{\Pi_{t+1} - \Pi_t}{\Pi_t} = \left[ \beta \left[ \sum_{a=1}^{a} P_{\alpha} \gamma_{\alpha}(F_{t+1}) \frac{\partial N_{a,t+1}}{\partial F_t} + \beta^2 \sum_{a=1}^{a} P_{\alpha} \gamma_{\alpha}(F_{t+2}) \frac{\partial N_{a,t+2}}{\partial F_t} \right] \right] - 1.
\]

Notice that with this manner of including non constant discount factor, the growth rate of profits between any two consecutive periods depends on the specific age structure of the population. Inside the square brackets the equation contains information on the population, prices and fishing selectivity for each age group.

Observe also that, the lower is the smoothness parameter, \( \rho = 1/\sigma \), the lower the growth rate of the profits along the optimal path is. In particular when the manager’s desire for smoothness is maximum (i.e. \( \rho = 1/\sigma \to 0 \)) the growth rate of the profits along the optimal path is zero and the path is totally smooth.

To understand the nature of the discount factor we are applying, notice that the discount factors we are using discount future events to a higher (lower) factor that it would do with constant discounting whenever the profits increases (decreases) along the optimal path. To see this, observe that the path of temporal discount factors when the discount is constant is given by \( \{1, \beta, \beta^2, \ldots \} \); however the path of temporal discount factors when the discount is non constant discount factors is given by \( \{1, \beta \frac{u'(\Pi_{t+1})}{u'(\Pi_{t+1})}, \beta^2 \frac{u'(\Pi_{t+2})}{u'(\Pi_{t+2})}, \ldots \} \). Since the CES utility function is concave whenever the profits increases over time, we have that \( U'(\Pi_{t+1}) > U'(\Pi_{t+2}) > U'(\Pi_{t+3}) \). Therefore the term \( \frac{u'(\Pi_{t+1})}{u'(\Pi_{t+1})} \) are greater than one for any \( j > 1 \). And this implies that the factor applied for example to the 3 periods ahead profit with our non constant discount factor, \( \beta^3 \frac{u'(\Pi_{t+1})}{u'(\Pi_{t+1})} \), will be higher that with the constant discount factor, \( \beta^3 \).

**Case Study: European Southern Stock of Hake**

In order to assess the relevance of the use of CES utility functions in determining optimal exploitation paths, we apply the methods describe above to the European Southern Hake Stock (SHS). The Northern limit of this stock is the Spanish–French frontier and the Southern limit is the Straits of Gibraltar (see Figure 1). The SHS is a fishery managed with the advice of the International Council for the Exploitation of the Sea (ICES), and it includes all fisheries in subareas VIIIc and IXa.

Hake (*merluccius merluccius*) is caught in a mixed fishery by the Spanish and Portuguese fleets (trawls, gillnetters, longliners and artisanal fleets). Spain accounts for most of the landings. Total landings and discards were 14 573 t and 1,992 t, respectively, in 2012. Total catches in 2012 represented a 13% catch reduction with respect to 2011. The fishery is managed by TAC, effort control and technical measures. Technical measures applied to this stock include a minimum landing size of 27 cm, protected areas and minimum mesh size. These measures are set depending on areas and gears by several national regulations. The ICES advised, on the basis of the transition to the MSY approach, that landings for SHS in 2013 should be no more than 10 600 t. Nevertheless, the agreed TAC for SHS in 2013 was 14 144 t.

A Recovery Plan for the SHS was implemented in 2006 (CE 2166/2005). This plan aims to rebuild the stock to within safe biological limits by decreasing fishing mortality by a maximum of 10% a year with a TAC constrain of 15%. An SSB target (35 000 t) is not considered suitable under the new assessment model. This regulation includes effort management in addition to TAC measures. Since
2006, a 10% annual reduction in fishing days at sea has been applied to all vessels, although with some exceptions. In 2012, vessels that landed less than 5 t of hake in 2009 or 2010 were excluded. The effort from fishing trips which catch less than 3% hake is excluded from the regulation (ICES 2013).

Fernandez et al. (2010) show that SHS assessment depends on whether or not discards is included in the analysis. We therefore incorporate also discards in the application of our methodology. The parameterization for the age-structured population is the same as that conducted in the ICES Working Group on the Assessment of Southern Shelf Stocks of Hake, Monk, and Megrim (ICES WGHMM, 2015), as presented in Table 1.

As stock-recruitment we use a Shepherd relationship (1992) described as

\[ N_t = \frac{\alpha SSB}{1 + (SSB/K)^{\beta}}. \]

Time series of recruitment and spawning stock biomass (SSB) were used for the estimation. The estimation results imply a Ricker type stock-recruitment relationship with \( b = 2 \), \( \alpha = 9.96 \) and \( K = 31.540 \). We also tested a density-independent Beverton and Holt stock-recruitment relationship with \( b = 1 \), but it generated huge recruitments at SSB over the historically observed range. Given this apparent overestimation we decided not to present these results.

Under this parameterisation the \( F_{\text{msy}} \) for hake in this fishery is 0.54. When the constant discount factor of \( \beta = 0.95 \) is considered. \( F_{\text{msy}} \) becomes 0.58.

Despite of its importance the value of the IES parameter continues to be disputed among economists, largely because of the associated difficulties in its econometric estimation. Existing estimates of the IES rank from 0 to 1 in studies that use time series (Hall, 1988; Mulligan, 2002) and from 0.1 to 1.17 in articles based on micro-data (Dyan, 1993; Blundell et al., 1994). For this study considering the lack of specific studies, we selected a value of \( \sigma = 1 \) which represents a significant desire for smooth solutions.

**Results and Discussion**

To study the behaviour of the fishery and the optimal level of harvest in time, we simulated the system forward in time from a range of initial population conditions. We assume that the fishery initially is in the stationary values, \( N_{a,0} = N_{a,ss} \) and \( F_0 = F_{ss} \), where the subscript SS stands for steady state and refers to the MSY sustainability level. Then a random shock affects negatively the entire population moving away the fishery from the steady state. Technically a shock \( \varepsilon \) is drawn from a uniform distribution for the rank (0.5, 1), so \( N_{a,0} = N_{a,ss}(1-\varepsilon) \). This shock drives the initial SSB below its stationary value. From this point, we calculate the optimal exploitation path that drives the fishery back to the stationary values. We focus on the case in which the utility of the yield is the economic indicator to be maximised. This analysis is performed by running 10,000 simulations. Optimal paths are obtained using global numerical methods (see Da Rocha et al., 2012 for more details).

Three alternative scenarios are simulated:

- **Scenario I** (shown in black): No desire for smoothness and constant discount factor: \( \sigma = 0 \) and \( \beta = 0.95 \).
- **Scenario II** (blue): Significant desire for smoothness and constant discount factor: \( \sigma = 1 \) and \( \beta = 0.95 \).
Scenario III (red): Significant desire for smoothness and non constant discount factor: \( \sigma = 1, \beta_t = 0.95 \) and \( \beta_{t+k} = 0.95^{t+k} \left( \frac{\Pi_{t+k}}{\Pi_{t+1}} \right) \) for \( k > 1 \).

Smoothness of optimal trajectories

First of all the smoothness of the optimal trajectories under the three scenarios is analysed. Scenarios with significant desire for smoothness are expected to generate less oscillatory trajectories because they represent a situation where managers consider that current and future harvesting have a certain degree of complementarity. Figure 2 shows the optimal trajectories for fishing mortality and landings under the three scenarios for a single simulation. The solutions are presented in relative terms to the stationary values of fishing mortality and landings which are normalized to one. So the results can be interpreted as the percentage change of the variables with respect to their stationary values. These results illustrate that once the shock affects the initial population, landings drop by more than 20%. After that fishing mortality evolves optimally until the fishery retains its stationary values. Clearly, Scenarios II and III with significant desire for smoothness generate smoother paths towards the stationary value than the conventional Scenario I.

The results shown in Figure 2 are robust. Table 2 summarises the quantitative results for the 10,000 simulations run. For each scenario, it shows the net present yield and the mean and standard deviations of the growth rate for yield and effort along the optimal exploitation paths. The main findings are: i) Significant desire for smoothness with constant discount rate (Scenario II) is the scenario with the lowest fluctuations in yield and effort growth. The standard deviation for the growth rate of yield and effort in Scenario II is almost 10 times lower than in Scenario I, in which no desire for smoothness is considered. ii) All three scenarios generate similar yields: the highest net present yield appears in the conventional scenario where desire for smoothness is not included in the analysis; however this yield represents less than 1‰ more than the net present value in Scenario II. iii) The use of a non-constant discount factor does not greatly affect to the smoothness of the optimal trajectories. Scenario III, where significant desire for smoothness in combination with non discount rates is explored, offers very similar results to Scenario II with constant discount rates.

In summary we can state that the incorporation of CES utility function significantly reduces fluctuations in optimal paths toward stationary values.

Optimal trajectories and overfishing in the short run

It is crucial for managers to know how to address situations in which the stock of the fishery is below the SSB target. They should certainly not be indifferent, but should they adopt a precautionary policy reducing fishing mortality? Or should fishing mortality be increased to buffer yield and fishermen’s income also in the short run? Our simulations show that the right answer to these questions depends on how the desire for intertemporal smoothness in catches is modelled.

Figure 2 shows that the optimal behaviour of fishing mortality differs in the short run under the three scenarios analyzed in this simulation. In cases that there is a significant desire for smoothness and discount rates keep constant the optimal policy in the short run is to increase fishing mortality, leading to an increase in landings (see the blue line representing Scenario II). However this is not observed if the analysis does not include the desire for smoothness, or if non constant discount factors are considered (see the black and red lines representing scenarios I and III, respectively).

Figure 3 plots the combinations of fishing mortality and SSB along the optimal exploitation paths for the three scenarios. Values of these variables are shown relative to the stationary values which are represented by value one.
The top panel shows the trend over time of the combination F and SSB until the steady state values are reached for a particular simulation. The bottom panel shows the same information but incorporating the results of the 1,000 simulations run.

Figure 3 shows the robustness of the smoothness results. The preference for smoothness in the management plans (Scenarios II and III) reduces fluctuations along the optimal paths. It can be seen that when the objective of the manager is just to maximise the present value of future yield in weight, without including desire for smoothness (Scenario I) fishing mortality can be as much as 75% below the stationary value in some years and 5% above the stationary value in others. However when preferences for smoothness are considered (Scenarios II and III) fishing mortality fluctuates between 20% below and 10% above its stationary value.

In addition, Figure 3 illustrates the different relationships between fishing mortality and SSB under the three scenarios. The usual positive relationship between fishing mortality and SSB is observed in Scenarios I (black) and III (red): when the SSB is below its stationary value the optimal fishing mortality is reduced below the stationary value and vice versa. However this relationship is reversed in Scenario II (blue): for the early years with an SSB below the stationary value the optimal policy consists of increasing fishing mortality with respect to its stationary value.

The kind of paths that appear as optimal in Scenario II are not highly rated by fishery agencies, which promote fishing policies more in accordance with the 2002 World Summit on Sustainable Development WWSD requirement for restore stock to their MSY level as well as with the precautionary principle. For instance, NOAA classifies a stock as “overfished” if its biomass level is depleted to a degree that the stock's capacity to produce MSY is jeopardised. When the fishing mortality is above the rate that produces MSY the stock is classified as “overfishing”. Moreover in calculating the Fish Stock Sustainability Index NOAA scores stocks at 0.5 points if they are overfished and 1 point otherwise. If the stock is overfishing it is also scored at 0.5 points, and at 1 otherwise. So taking into account only these two characteristics, any stock can be classified in one of the four categories, each with different rating. Figure 4 summarises the four possible cases in an F-SSB graph.

In conclusion, in situations where the SSB is below its stationary value overfishing optimal paths emerge when significant desire for preferences is included in the management decision problem. A technical solution to avoid this type of optimal path is to consider appropriated non constant discount factor to value future harvests. This reflection allows obtaining as optimal smoother as well as non overfishing exploitation path towards the MSY.

**Conclusions**

The conventional approach of focusing on optimal harvesting solutions that maximise the present value of profits (or yield) can generate trajectories with large fluctuations in fishing mortality and captures over time. In many studies, the recommended optimal policies may even involve pulse fishing with fallow periods to recover the stocks. But this type of solution is typically found inadequate in practice.

It is well established that pulse fishing disappears as optimal harvesting solution when the function to be maximized in the fishery management problem is concave (Tahvonen, 2009; Da Rocha et al. 2012c, 2013; Tahvonen et al. 2015). Following with this idea we propose to include a CES utility function of fishing profits as the objective function to be maximized. The CES utility function has been profusely used in economics in intertemporal decision problems to analyze substitutability of consumption (harvest in our case) between periods. However it has barely used in fisheries economics.
The use of a CES function generates stable transitional paths towards the stationary solution, and the degree of smoothness depends on the value of the IES parameter selected. Our analytical and empirical results show that the value of this smoothness parameter does not affect the references points of the fishery but does affect the fluctuations along the optimal paths.

Moreover, our empirical results show that when the CES utility function is included in the management problem with a significant smoothness parameter and constant discount factors are employed, then if the biomass is below its stationary value the optimal exploitation decision consist of increasing the fishing mortality in the short term. This means that the optimal solution implies overfishing which in general it is not an acceptable strategy by fisheries agencies and regulators. Why do we obtain such counterintuitive result? Because the optimal solution is a forward solution; that is, the path that maximizes the objective function goes from the current period to the future, without taking into account the exploitation of the fishery in the past. Therefore when looking for a smooth forward path there is no restrictions in the selection of the initial mortality rate (higher o lower than past values). Our results also show that overfishing policies do not appear as optimal when adequate non constant discount factors that value the stock as a function of its relative intertemporal scarcity are included in the management problem. This ocurrns because the concavity of the utility function implies that the discount factor we use decreases (increases) over time as the fishing mortality increases (decreases).

In quantitative terms, we conclude that a combination of CES utility with a significant smoothness parameter and to non constant discount rates generate optimal stable and non overfishing exploitation paths 10 times smoother than those resulting from a conventional approach.

The theoretical results were obtained using constant price, which may be considered reasonable for species that admit freeze catches. However it would be appealing to extend the theoretical model to include demand functions that consider, for instance, that prices are isoelastic functions of the captures. This possibility was analyzed in Da Rocha et al. (2012a) where it was shown that when prices are sensitive to variation in catches, a solution with stable captures is even challenging (against pulse fishing solutions). Our guess is that this result would be reinforced with the introduction of the CES function on management problem. Finally, we acknowledge that the fishery problem is too complex and many other situations such as multi-species, multi-fleet or schooling fisheries should be taken into account to complete the analysis. Notwithstanding, we demonstrate the viability of pursing smooth exploitation paths by incorporating economic concepts such as the CES utility function in the management problem.

References
Figure 1: The Southern Stock of Hake includes ICES subareas VIIIc and IXa (in small square design).
<table>
<thead>
<tr>
<th>Age</th>
<th>Natural</th>
<th>Landings</th>
<th>Discards</th>
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<th>Weight (Kg)</th>
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</table>

Source: ICES Working Group on the Assessment of Southern Shelf Stocks of Hake, Monk, and Megrim (WGHMM, ICES 2015)
Figure 2: Optimal paths for the fishing mortality (upper panel) and landings (lower panel). The black line represents scenario 1 with no desire for smoothness and constant discount rates. The blue line represents scenario 2 with significant desire for smoothness and constant discount rates. The red line represents scenario 3, with significant desire for smoothness and non constant discount rates. The steady state value for F and landings are normalised to one.

Table 2: Comparisons for the smoothness of alternative scenarios in the SHS

<table>
<thead>
<tr>
<th></th>
<th>Non Desire for Smoothness ($\sigma = 0$)</th>
<th>Significant Desire for Smoothness ($\sigma = 1$)</th>
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<td></td>
<td>Constant Discount Rate</td>
<td>Constant Discount Rate</td>
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<td>Scenario I</td>
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<td>204,120</td>
</tr>
<tr>
<td>Net Present Yield (t)</td>
<td>Yield Growth rate (mean) (std)</td>
<td>Effort Growth Rate (mean) (std)</td>
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<tr>
<td></td>
<td>10.33 (0.3675)</td>
<td>7.43 (0.2651)</td>
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<td></td>
<td>2.31 (0.0378)</td>
<td>0.99 (0.0236)</td>
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<tr>
<td></td>
<td>3.13 (0.0378)</td>
<td>1.51 (0.0333)</td>
</tr>
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</table>

Growth rate refers to the percentage change in the variable from one year to the next. The mean and the standard deviations (std) are calculated for the 30 years of 1,000 simulations run.
Figure 3: Combinations of yield and SSB along optimal exploitation paths in the SHS. The top panel shows one simulation, the bottom panel shows 1,000 simulations. Variables are measured relative to their stationary values. The left-hand panels (black) represent Scenario I with no preferences for smoothness and constant discount rates. The central panels (blue) represent Scenario II with preferences for smoothness and constant discount rates. The right-hand panels (red) represent scenario III with no preferences for smoothness and non constant discount rates.
Figure 4: NOAA classifications for stocks considering their status related to the $F$ and SSB levels compatible with the MSY sustainability level.