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Case II: Formulation where the Piezo-Electrolytic Process is Regulated by a Feedback Control Law

The system model is modified by adding a feedback control law $U(t)$ to give Equation (1).

$$\dot{Q_a}(t) = m(V(t) + U(t)) + \Delta I(t)$$  \hspace{1cm} (1)

The derivative of the surface $\dot{s}(t)$ and the control law $U(t)$ are expressed as (2) and (3) respectively.

$$\dot{s}(t) = \lambda m(V(t) + U(t)) + \lambda\Delta I(t) - \lambda \dot{Q}_d(t)$$ \hspace{1cm} (2)

$$U(t) = -K_ds(t) - \dot{V} + \dot{\phi}\dot{Q}_d(t) - k^s sat\left(\frac{s}{\epsilon}\right)$$ \hspace{1cm} (3)

$V(t)$ and $\phi$ are unknown during the control design but are estimated as $\hat{V}$ and $\hat{\phi}$. The Lyapunov candidate $W$ used to obtain the adaptive laws while guaranteeing system stability is given as 4 and the derivative of $W$ along the system trajectory is expressed in (5).

$$W(t) = \frac{1}{2}\left(\frac{1}{m}s^2 + \frac{1}{\gamma}\phi^2 + \frac{1}{\Gamma}V^2\right)$$ \hspace{1cm} (4)

$$\dot{W}(t) = \frac{1}{m}s\dot{s} + \frac{1}{\gamma}\dot{\phi}\dot{\phi} + \frac{1}{\Gamma}\dot{V}\dot{V}$$ \hspace{1cm} (5)

Equation (2) and (3) are substituted into (5) to give (6).

$$\dot{W}(t) = -\lambda K_ds\epsilon s + \lambda s\epsilon\left(V(t) - \hat{V} - \dot{\phi}\dot{Q}_d(t) - k^s sat\left(\frac{s}{\epsilon}\right)\right) + \lambda s\epsilon\left(\frac{\Delta I(t)}{m} - \dot{\phi}\dot{Q}_d(t)\right)$$

$$+ \frac{1}{\gamma}\dot{\phi}\dot{\phi} + \frac{1}{\Gamma}\dot{V}\dot{V}$$ \hspace{1cm} (6)

From (6), the adaptive laws become (7) and (8).

$$\dot{\phi} = -\gamma \lambda \dot{Q}_d(t)s\epsilon$$ \hspace{1cm} (7)

$$\dot{V} = \lambda \Gamma s\epsilon$$ \hspace{1cm} (8)

Substituting the adaptive laws (7) and (8) into (6) gives the expression in (9).

$$\dot{W}(t) = -\lambda K_ds\epsilon s - \lambda k^s s\epsilon sat\left(\frac{s}{\epsilon}\right) + \frac{\Delta I(t)}{m}\lambda s\epsilon$$ \hspace{1cm} (9)

$$\dot{W}(t) = -\lambda K_ds\epsilon \left(s\epsilon + \epsilon sat\left(\frac{s}{\epsilon}\right)\right) - \lambda k^s s\epsilon sat\left(\frac{s}{\epsilon}\right) + \frac{\Delta I(t)}{m}\lambda s\epsilon$$ \hspace{1cm} (10)

When $|s| \leq \epsilon$, $|s\epsilon| = 0$ and (10) is zero (11).

$$\dot{W}(t) = 0 \hspace{1cm} \forall |s| \leq \epsilon$$ \hspace{1cm} (11)

When $|s| > \epsilon$, $|s\epsilon| = s\epsilon sat(s/\epsilon)$. By also taking into account $k^s \geq \rho/m_{min}$, (10) is expressed to give (14).

$$\dot{W}(t) = -\lambda K_ds^2\epsilon - (K_d\epsilon + k^s)\lambda |s\epsilon| + \frac{\Delta I(t)}{m}\lambda s\epsilon$$ \hspace{1cm} (12)
\[
\dot{W}(t) \leq -\lambda K_d s^2 - K_d \epsilon \lambda |s| - \left(k^* - \frac{I(t)}{m}\right) \lambda |s|
\] (13)

\[
\dot{W}(t) \leq -\lambda K_d s^2 \quad \forall |s| > \epsilon
\] (14)

Again, the above formulations in (11) and (14) indicate that \(s, \dot{\phi}\) and \(\ddot{V}\) are globally bounded. This also means that \(s(t)\) is bounded and the control design guarantees that the system trajectory will converge to the sliding mode.

**Rectified Sinusoidal Input**

![Graph showing rectified sinusoidal input](image)

**Figure 1.** When a sinusoidal load with an amplitude of 300N is applied to the self-healing material, the piezoelectric direct effect generates a sinusoidal voltage of amplitude 559mV. The resulting sinusoidal voltage is rectified and smoothed to create the dc voltage required for electrolysis.