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EFFECT OF RESIDUAL STRENGTH PARAMETERS ON FRC FLEXURAL CREEP: MULTIVARIATE ANALYSIS

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Abstract

This paper reports the multivariate analysis of experimental results from more than one hundred FRC prismatic specimens tested under sustained flexural loads for at least 90 days, collected from previously published sources. Principal Component Analysis was used to minimise the number of variables in the modelling process while compromising the minimum amount of information. The creep parameters analysed were the creep coefficients at 14, 30, and 90 days and the corresponding crack opening rates. They were related to the following factors: concrete compressive strength, residual load-bearing capacity in flexure, fibre material, and load. Multiple linear regression was used for the modelling of these relationships. Higher levels of flexural toughness were found to significantly reduce the variability of all creep parameters. Differences in fibre material were detected to introduce important differences in interaction with other factors, especially the load ratio, which was attributed to the association between the range of flexural toughness to be expected and the type of fibres used.

Keywords: fibre material, flexure, linear models, multivariate, residual strength, toughness.

1. INTRODUCTION

The major feature of fibre reinforced concrete (FRC) is its residual load-bearing capacity when cracked, in terms of flexural residual strength and flexural toughness [1].

FRC show considerable scatter in their flexural response [2], and in consequence their deformational behaviour when under sustained flexural loads shows high levels of variability, especially in its cracked state. A number of different factors affect creep of cracked FRC elements, relative to the composition and mechanical properties of the concrete matrix, fibres type and dosage, environmental conditions, loading conditions, or time, among other factors [3,4].

Previously published papers and reports concerned with the flexural response of cracked FRC elements under sustained loads contain important volumes of experimental results obtained from different FRC mixes produced with different types of fibres at different dosages. The authors of this paper have been working on the compilation of analysis of these experimental data [5]. This information gathered from different sources can be collectively analysed using a data mining approach in an attempt to draw conclusions that might be generalised beyond the particular limitations of any individual study.

2. OVERVIEW OF DATA AND METHODOLOGY

2.1 Summary of the information collected

This research was based on the creep test results corresponding to 118 FRC specimens, tested under sustained flexural loads for at least 90 days. For each of these cases, the information collected included parameters concerning the following aspects:

- Concrete matrix: maximum aggregate size, average compressive strength at 28 days (f_c, MPa).
- Fibres employed: material (glass, steel or synthetic), length, aspect ratio, and fibres content as volume fraction.
- Flexural residual strength: limit of proportionality f_L, and residual strength parameters f_{R1}, f_{R2}, f_{R3} and f_{R4} (MPa) as per the standard EN 14651 [6].
- Sustained load during the creep test typified by the residual capacity, or load ratio *IFa* [7].
- Creep coefficients at the ages of 14, 30, and 90 days [7].
- Crack opening rates (CORs) between 0 and 14 days, 14 and 30, and 30 and 90 days ($COR^{0\text{-}14}$, $COR^{14\text{-}30}$, and $COR^{30\text{-}90}$ respectively) [7].

Representative values for each of the variables are given in Table 1 for reference. Minimum, average and maximum values were intended to be descriptive of the entire set of data and were calculated as the 5%, 50%, and 95% percentiles. The percentage of missing values for each variable represents those cases for which complete information could not be obtained.

Table 1: Summary of the information in the database

	Minimum (5%-percentile)	Average	Maximum (95%-percentile)	Percentage of missing values
Fibres content, V_f (%)	0.47	0.58	0.95	0.0%
Fibre material	n.a.	n.a.	n.a.	0.0%
Fibre length (mm)	35.00	42.89	54.00	0.0%
Fibre slenderness	45.00	71.09	158.00	0.0%
Max. aggregate size (mm)	10.00	12.46	20.00	11.0%
Compr. strength f_c (MPa)	34.00	42.31	56.54	0.0%
Limit of prportionality f_L (MPa)	2.97	4.32	6.04	23.7%
Residual strength f_{R1} (MPa)	0.87	3.65	9.55	0.8%
Residual strength f_{R2} (MPa)	0.79	3.92	9.06	36.4%
Residual strength f_{R3} (MPa)	0.78	4.01	10.09	32.2%
Residual strength f_{R4} (MPa)	0.60	3.80	9.57	31.3%
Load ratio *IFa* (%)	36.90	66.21	89.56	0.0%
Creep coefficient at 14 days	0.09	0.70	1.93	0.8%
Creep coefficient at 30 days	0.11	0.91	2.19	0.0%
Creep coefficient at 90 days	0.31	1.26	3.11	2.5%
COR between 0-14 days	0.86	15.11	41.07	0.0%
COR between 14-30 days	0.20	3.58	10.15	0.0%
COR between 30-90 days	0.27	1.51	4.16	0.0%

2.2 Treatment of missing values

As the sources of these data did not always report the mix characteristics, their flexural response or their flexural creep behaviour in the same way, some of the values of interest had not been reported.

This is quite common in the context of data mining applications or multivariate statistics [8], and different strategies can be adopted to moderate the impact of missing cases on the reliability of the information that can be extracted from a set of data. In this case, the prevalence of missing values was relatively small and did not justify complete elimination of any cases. Instead, a multiple imputation by fully conditional specification [9] was done to simulate the missing values taking advantage of the multiple correlations between the different variables.

3. MULTIVARIATE DESCRIPTION OF FRC MECHANICAL PERFORMANCE

Six variables were considered as descriptors of the mechanical performance of each FRC mix: f_c, f_L, f_{R1}, f_{R2}, f_{R3} and f_{R4}, all of them expressed in MPa.

After some preliminary analyses, it was observed that this number of variables needed to be reduced in order to correlate the material's performance with the flexural creep parameters and the level of sustained load applied. A multivariate analysis was carried out in an effort to find a compromise between the reduction of the number of variables and the amount of information unaccounted for.

3.1 Bivariate correlations

The Pearson correlation coefficients [10] between any pair of these six variables are shown in Table 2. Very strong correlations were observed between the residual flexural strength parameters (values between 0.902 and 0.978). On the other hand, the average compressive strength of concrete was practically uncorrelated to these parameters. The relationship between any of these and the limit of proportionality was less clear, as it was found to be slightly correlated to the compressive strength and moderately to the residual flexural strength parameters.

Table 2: Correlation matrix

	f_c	f_L	f_{R1}	f_{R2}	f_{R3}	f_{R4}
f_c	(1.000)	0.310	0.055	0.090	0.066	0.076
f_L		(1.000)	0.654	0.633	0.598	0.572
f_{R1}			(1.000)	0.954	0.939	0.908
f_{R2}				(1.000)	0.949	0.902
f_{R3}					(1.000)	0.978
f_{R4}						(1.000)

3.2 Principal Component Analysis (PCA)

The structure of correlations in Table 2 proved that these six variables are not independent and therefore they could not be treated as such when modelling their effect on creep. Principal Component Analysis (PCA) was used to condense the information they describe into a reduced set of variables.

After these variables were centered and scaled to unit variance, principal components were extracted by singular value decomposition of the correlation matrix [11], and a Varimax rotation was applied [12]. The first three principal components were retained as sufficiently informative, explaining 97.11% of the total variance in the original variables.

Each one of the principal components is a linear combination of the original variables, and therefore they define new rotated axes PC1, PC2 and PC3. By plotting the weights of the original variables in these linear combinations against the new coordinate system defined by PC1, PC2, and PC3, the two biplots in Figure 2 were obtained.

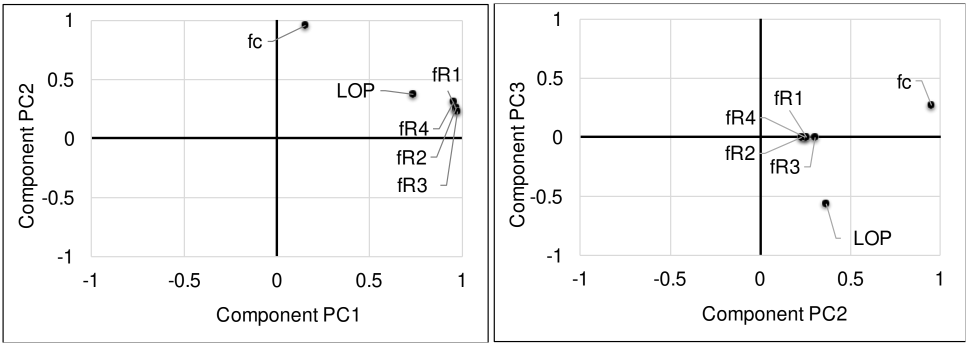

Figure 1: Biplots for interpretation of the principal components after the PCA.

The four residual flexural strength parameters formed a very clear cluster. This cluster defined the first component PC1, and therefore PC1 was interpreted as representative of the residual load-bearing capacity. Furthermore, as the distances between f_{R1}, f_{R2}, f_{R3} and f_{R4} were negligible, it followed that PC1 values were proportional to $f_{R1}+f_{R2}+f_{R3}+f_{R4}$,

Concrete compressive strength, on the other hand, was clearly apart from this cluster, consistently with what the bivariate correlations indicated. Therefore, the second component PC2 was defined by compressive strength alone.

These two components PC1 and PC2 described 89.70% of the total variance. The third component (PC3), accounted for only 7.41%, and had no clear interpretation, as neither the limit of proportionality nor any of the other variables were found to have a determining weight.

3.3 Redefinition of the variables characterising FRC mixes

The results of the PCA clearly indicate that the mechanical performance of FRC mixes as described by the six initial variables can be effectively reduced to only two variables while retaining nearly 90% of the total variance:

- Concrete compressive strength, f_c, in MPa.
- Toughness, defined as $T = f_{R1}+f_{R2}+f_{R3}+f_{R4}$, in MPa.

4. MULTIPLE REGRESSION ANALYSIS

4.1 Definition of regressors

As concluded in the previous section, two variables were retained as representative of the mechanical performance of the material: the average compressive strength at 28 days (f_c, in MPa), and the flexural toughness (T, in MPa).

The load ratio (IFa, in percentage) was considered to account for the sustained load applied.

The fibre material was also considered, as a binary variable (Mat): synthetic or non-synthetic, as synthetic fibres were found to be significantly different (p-value=0.00125).

4.2 Summary of the MLR models obtained

Multiple linear regression (MLR) was used to relate each of the creep parameters under consideration to the aforementioned regressors. Initially, models with different formulations including all variables, squared variables, second order interactions and some selected third order interactions were considered. Stepwise and best subsets regression procedures were applied to simplify these models by discarding those terms that were not statistically significant.

Table 3 summarises the MLR models as obtained after this sequential process. The terms that were identified as statistically significant (p-values up to 0.05) are marked with an asterisk. These models were used as a tool to obtain average estimates and trends for the creep coefficients, which allowed the study of these multivariate relationships as reported in the following sections. Furthermore, it is anticipated that future post-processing of these models can increase their predictive accuracy.

Table 3: Summary of the MLR models

	Creep coefficients			Crack opening ratios		
	14 days	30 days	90 days	0-14 days	14-30 days	30-90 days
IFa						*
IFa^2						*
f_c						
T					*	*
Mat	*	*	*			
$T \times IFa$	*					
$T \times fc$	*	*	*			*
$Mat \times IFa$	*	*	*	*	*	*
$Mat \times IFa^2$	*	*	*	*	*	*
$Mat \times T$	*	*	*			
$Mat \times f_c$			*	*	*	*
$Mat \times f_c^2$	*	*	*	*	*	*
$Mat \times IFa \times T$	*	*	*	*		
$Mat \times IFa \times f_c$	*	*	*	*	*	*
$Mat \times IFa^2 \times T$	*	*		*		
$Mat \times IFa^2 \times f_c$	*	*	*	*	*	*
R-squared	0.55	0.51	0.46	0.54	0.43	0.41

5. DISCUSSION OF THE MODELS FOR CREEP COEFFICIENTS

The MLR models summarised in Table 3 made it possible to plot response curves for the creep coefficients with respect to load ratio, concrete compressive strength and toughness. Furthermore, the differences introduced by the fibre material and their modifying effect on other variables were also scrutinised.

These aspects were examined through the contrastive effects plots for the MLR models obtained, using the open source package "visreg" in R [13]. In these plots the relative change in the creep coefficients is represented, instead of their predicted average values. This way the effect of any one variable could be analysed without assuming constant values for the other variables, which is an important advantage when interpreting complex models with many interactions.

5.1 Effect of concrete compressive strength

Variations in the creep coefficients with respect to concrete compressive strength are shown in Figure 2. Average trends as obtained from the MLR models are drawn with continuous lines, while dots correspond to individual data. Coloured bands represent the 90%-confidence band.

It was observed that higher compressive strength values tend to reduce the creep coefficients at 30 and 90 days, following a quadratic trend. However, this relationship between concrete compressive strength and changes in the creep coefficient was not detected to be significat at 14 days.

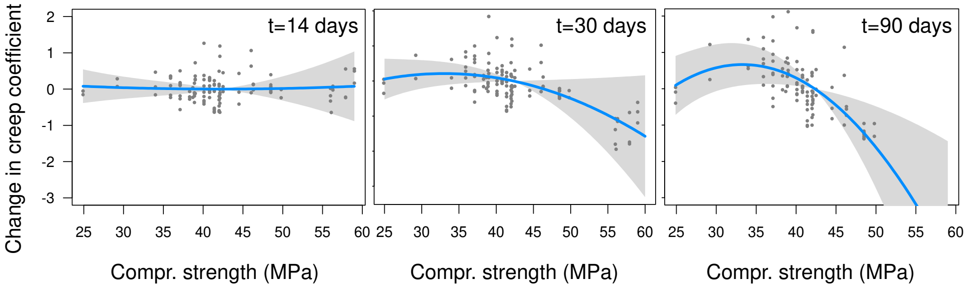

Figure 2: Effect of compressive strength on creep coefficients.

5.2 Effect of loading ratio and fibre material

Figure 3 shows the variation in the creep coefficients at 14, 30, and 90 days with respect to the load ratio applied during the creep tests. Higher load ratios implied higher creep coefficients at all ages. However, the effect of increasing the load ratio was less noticeable on the creep coefficient at 14 days: an increase of 10% in *IFa* led to an average increase of the creep coefficient in 0.2, whilst this increase was of 0.35 at 90 days.

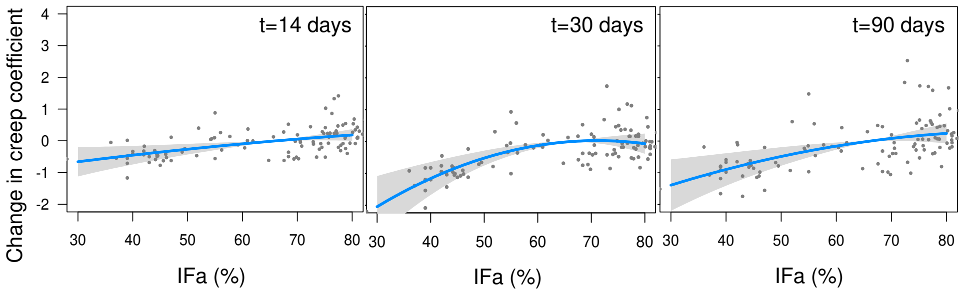

Figure 3: Effect of load ratio on creep coefficients.

These average trends were not significantly affected by the fibres length or aspect ratio. However, interesting differences were observed when the interaction between fibre material and load ratio was analysed in addition to the simple effect of increasing the load ratio. Figure 4 shows the contrastive effects plots with respect to the load ratio but distinguishing between synthetic and non-synthetic fibres.

At lower load ratios, creep coefficients were observed to be systematically higher in those cases where synthetic fibres had been used. The gap between them, however, was gradually reduced for increasing load ratios, and differences due to the fibre material were negligible for load ratios of 60% or higher. This pattern was consistent for all three ages considered.

However, the trends represented in Figure 4 can be misleading: it is important to emphasise that Figure 4 shows an association between the fibre material and higher creep coefficients, but this is not necessarily of a cause-effect nature. In fact, the analysis of variance on the squared residuals of these MLR models revealed that only one variable had a statistically significant effect on the variability of creep coefficients, and therefore on the differences represented in Figure 4. And it was not the fibre material, but the flexural toughness T (p-values = 0.056, 0.020, and 0.016 at 14, 30, and 90 days respectively). In consequence, the differences between synthetic and non-synthetic fibres in Figure 4 could not be directly attributed to the fibre material: the role played by the flexural toughness in these models and its interaction with the fibre material needed to be explored more closely, and that is precisely what is reported in the following section.

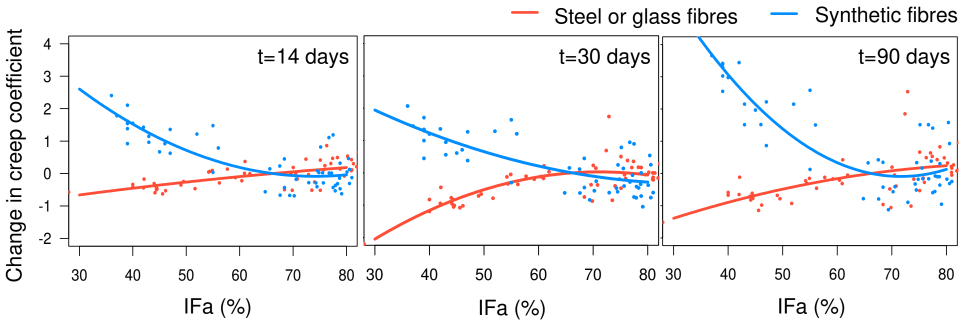

Figure 4: Effects of fibre material and load ratio on creep coefficients.

5.3 Effect of the flexural toughness

The effect of increasing flexural toughness on creep coefficients at 14, 30 and 90 days is shown in Figure 5. The same average trend was consistently observed at all ages: creep coefficients are reduced when the flexural toughness is increased, although this was less pronounced at the age of 14 days. However, it was observed that creep coefficients presented increasing variability with age, especially so for the lower toughness values, as indicated by the scatter of the dots in Figure 5. Enhancing the flexural toughness has therefore the advantage of reduced creep coefficient, with more stable and predictable values.

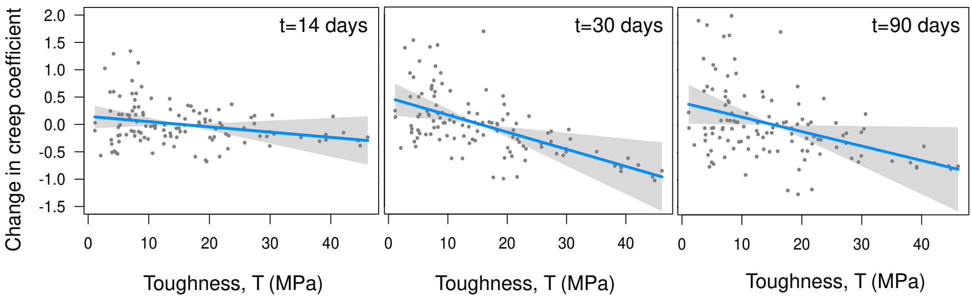

Figure 5: Effect of toughness on creep coefficients.

When the distinction between synthetic and non-synthetic fibres was accounted for, Figure 6 was obtained. It was observed that, as long as a similar level of flexural toughness is achieved, the choice between synthetic or non-synthetic fibres does not introduce a substantial difference in terms of creep coefficients. Therefore, Figure 6 confirms that the differences observed in Figure 4 cannot be directly attributed to the fibre material, because generally synthetic fibres are used in dosages that lead to considerably lower levels of flexural toughness when compared to elements with steel fibres. If this aspect is disregarded, cause-effect relationships can be misinterpreted, introducing a bias in favour of the steel fibres not sufficiently supported by evidence.

With respect to the variability of the creep coefficients, both Figures 5 and 6 confirm that their values are more scattered when lower levels of flexural toughness are considered.

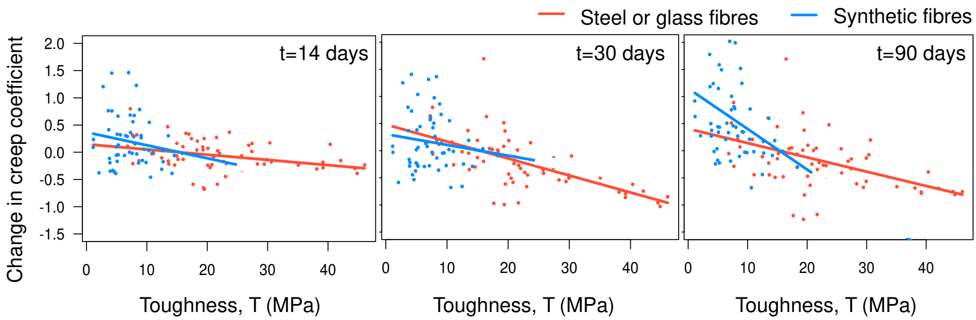

Figure 6: Interaction between fibre material and toughness on creep coefficients.

6. DISCUSSION OF THE MODELS FOR THE CRACK OPENING RATES

6.1 Effect of load ratio and fibre material

Figure 7 shows the effect of increasing the load ratio on the variation of crack opening rates for the three timespans considered, distinguishing between synthetic and non-synthetic fibres. The relationship between the crack opening rates and the load ratio was not significantly modified by the fibre material. Although more variability was detected for $COR^{0\text{-}14}$, this was significantly reduced afterwards and the increasing trend with respect to the load ratio was gradually moderated.

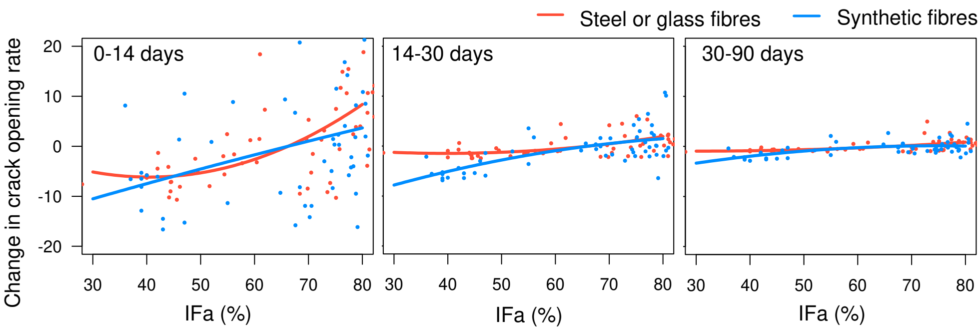

Figure 7: Effect of load ratio on the crack opening rates.

6.2 Effect of flexural toughness

In consistency with the findings reported for the creep coefficients, the effect of fibre material on the crack opening rates was examined in conjunction with that of flexural toughness. Figure 8 shows the average trends for the crack opening rates with respect to the flexural toughness. In terms of relative magnitude, the effect of flexural toughness on crack opening rates was more moderate than that of load ratio if Figures 7 and 8 are compared. It is also interesting to note that, if Figures 5 and 8 are compared, the effect of increasing flexural toughness on creep coefficients was more noticeable than on the crack opening rates.

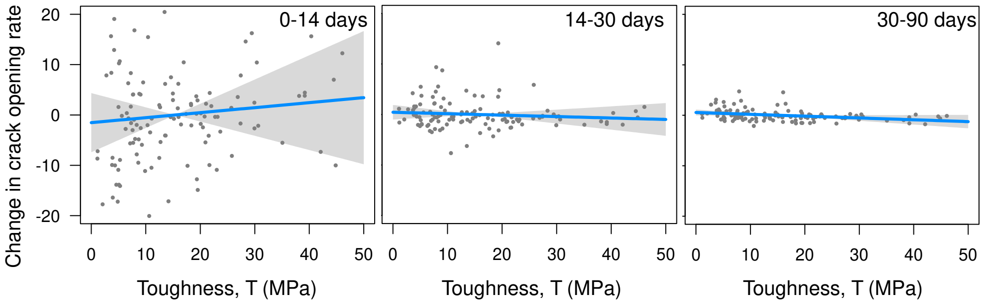

Figure 8: Effect of toughness on the crack opening rates.

COR$^{14\text{-}30}$ and *COR*$^{30\text{-}90}$ were detected to slightly decrease in average when flexural toughness values increased. On the other hand, *COR*$^{0\text{-}14}$ showed a considerably higher variability, and this was one of the reasons why the first effects plot in Figure 8 seems inconsistent with the other two.

Such inconsistency was resolved when the distinction between cases with synthetic fibres and non-synthetic fibres was considered, as shown in Figure 9. The fibre material introduced no substantial modification to the relationship between the flexural toughness and the crack opening rates after 14 days. However, with respect to *COR*$^{0\text{-}14}$, it was evidenced that data corresponding to synthetic fibres followed a different trend, thus the apparently excessive variability noticed in Figure 8. When the fibres used are synthetic, improvements in flexural toughness clearly reduce *COR*$^{0\text{-}14}$. After that age, the interaction between the fibre material and the flexural toughness is not significant in terms of crack opening rates.

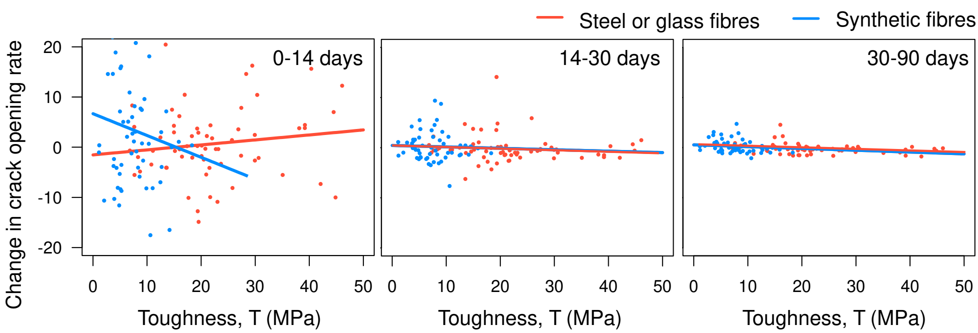

Figure 9: Effect of fibre material and toughness on the crack opening rates.

11. CONCLUSIONS

- The application of multivariate semi-empirical modelling techniques is a useful approach to advance in the understanding of flexural creep of cracked FRC elements. The collection of a representative number of data from different mix compositions, types of fibres, and subject to different sustained load levels will feed more comprehensive meta-analyses in the future.
- Three parameters have been detected to have a determining impact on the flexural creep response of cracked FRC sections: concrete compressive strength, the load ratio, and the flexural toughness defined as the sum of the residual flexural strength values f_{R1}, f_{R2}, f_{R3} and f_{R4}.
- Differences in fibre material do not play a direct determining role on the response of cracked FRC sections under sustained flexural loads. Rather, their influence is on the flexural toughness of the material, which in turn affects the creep response.
- A general trend towards stabilisation in time is observed and consistent for all the creep coefficients and crack opening rates analysed, even in those cases when the load ratio is high.

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