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An airborne acoustic method to determine the volumetric water content of unsaturated sands

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A new acoustic method to determine the volumetric water content of unsaturated sands

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Abstract

This paper presents an innovative experimental approach for simultaneous measurements of the suction head, volumetric water content and the acoustic admittance of unsaturated sands. Samples of unsaturated sands are tested under controlled laboratory conditions. Several types of sand with a wide range of uniform particle sizes are investigated. The reported experiments are based on a standard Buchner funnel setup and a standard acoustic impedance tube. It is a novel non-destructive, non-invasive technique that relates the key geotechnical parameters of sands such as volumetric water content, density and grain size distribution to the acoustic admittance and attenuation. The results show a very sensitive dependence of the acoustic admittance on the volumetric water content controlled by the value of suction head applied. Analysis of the obtained data demonstrates that the relationship between the volumetric water content and the real part of the surface admittance in the frequency range of 400 – 1200 Hz can be represented using a logarithmic equation. It is found that the coefficients in the proposed equation are directly related to the uniformity coefficient and the acoustic admittance of the dry sample, which can easily be measured or predicted for a broad range of sands. A validation exercise is conducted to examine the accuracy of the proposed equation using a sand sample with markedly different properties. The results of the validation exercise demonstrate that the proposed relations can be used to determine very accurately the volumetric water content within the porous specimen from the acoustical data. The error in

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the acoustically measured volumetric water content is found to be $\pm$ 2.0% over the full range of volumetric water contents ($0 \leq \theta \leq n$, where n is the sample porosity).

Key Words

Volumetric water content, Acoustic admittance, Measurements, Unsaturated Sand

Introduction

Accurate data for the volumetric water content/degree of water saturation in unsaturated soils is essential for a range of geotechnical and geoenvironmental engineering problems such as modelling of hydraulic hysteresis, coupling flow and stress-strain relations in unsaturated soils, rehabilitating a drainage system, migration of contaminants in unsaturated soils and designing a cost-effective remediation scheme etc. For example, recent proposed models for the mechanical behaviour of unsaturated soils incorporate coupling between the degree of saturation and stress-strain measurements (Wheeler et al. 2003; Gallipoli et al. 2003). Many models have been developed to predict the flow of water and contaminants in unsaturated zones (Bear 1979; Parker and Lenhard 1987; Fenwick and Blunt 1998). The key parameter in a majority of the existing models for unsaturated soils is the volumetric water content/degree of saturation and/or matric suction. For a successful application of the proposed models, the relationship between volumetric water content and matric suction head needs to be accurately measured.

A serious major limitation of a majority of the currently available measurement techniques is that they are either intrusive/destructive or their accuracy is not sufficient for the geotechnical applications. Here we ~~call~~ assume that a method is non-destructive if it does not require disturbance or removal of part of the sample while the test is in progress. A non-intrusive method means that no devices are inserted into sample. Current methods which are ~~N~~non-destructive and~~,~~ non-intrusive ~~methods, such as~~are mainly based on measuring the X-ray ~~attenuation~~ or~~and~~ gamma ray attenuation (Tidwell and Glass 1994; Lenhard et al. 1993). These methods have been developed and used to measure the volumetric water content or the degree of saturation which require ~~. These methods require~~ an expensive laboratory setup and accurate calibration is to be undertaken for specific experimental arrangement and soil type.

In addition, these~~y~~ only enable measurement of the volumetric water content or degree of saturation at a single point and while the water is stagnant. Recently, the synchrotron X-ray technique has been developed to overcome the difficulties in measuring water saturation during dynamic flow with short delay of seconds (Tuck et al. 1998; DiCarlo et al. 1997). However, only a relatively small area of less than 0.5 cm^2 can be tested at a particular time. In addition, the use of X- and gamma-ray attenuation techniques can pose a health hazard, especially if proper precautions are not undertaken.

Intrusive methods, such as time domain reflectometry and electrical conductivity methods are also used to measure volumetric water content by inserting a probe into the soil sample. The electrical conductivity method works by measuring the electrical conductivity of a medium, which is linked, with the amount of water present since solid particles are considered as a non-conductive medium. To enhance the electrical conductivity of the water a small quantity of sodium chloride is added. The electrical conductivity method has been proven to produce successful results (Leverett and Lewis 1941; Rhodes et al. 1976; Eckberg and Sunada 1984). However, non-availability of standard electrical conductivity probes hinders the widespread use of this technique. The time domain reflectometry technique is a recent non-destructive but~~and~~ intrusive technique for measuring volumetric water content using electromagnetic waves (Topp et al. 1980; Topp and Davis 1985). The measurements of the volumetric water content are based on the dielectric properties of the soil sample, which is a function of the amount of water in the soil. It should be noted that any concentration of air bubbles around the rods results in reduced accuracy. In addition, it is recommended to calibrate the equipment before use for a particular soil in order to eliminate the associated error.

It is reasonable to conclude that there is scope for the development of an alternative, accurate, simple, environmentally safe, non-invasive and inexpensive approach for measuring of the volumetric water content of sands. This paper presents for the first time a technique that enables simultaneous measurement of the volumetric water content under an increasing value of suction head using airborne sound waves which are able to penetrate the open pore structure of the sand sample.

Background: Propagation of sound waves in granular media

This section presents basic definitions and review of the fundamentals of sound propagation in granular media such as sands. Propagation of sound waves in granular media is complex. It involves elastic and kinetic energy exchange between the solid particles and the liquid as well as visco-thermal interactions. In granular media, the solid particles form a relatively rigid skeleton, which can be filled with various fluids. Figure 1 shows a schematic drawing of an unsaturated sand skeleton filled with water and air. The vibration of the skeleton, which an airborne sound wave incident on a sand sample can excite, is relatively small in amplitude because of the marked difference between the densities of air and skeleton material. In this case, the acoustic energy tends to enter and propagate through the interconnected pores between the particles in the form of an airborne compressional wave. Therefore, the proportion of the open, interconnected pores and their distribution and size are major material characteristics that influence the propagation of sound waves in a porous medium over the audio frequency range. It is generally accepted that the pores developed between the solid grains are related to the particle size distribution (Brooks and Corey 1964; Horoshenkov and Swift 2001). The presence of water in the pore spaces reduces the availability of air-filled pore channels for sound propagation as well as it affects the elastic properties of the unsaturated soils (Lambe and Whitman~~,~~ 1969). Two key acoustic parameters, which are the complex, frequency dependent characteristic impedance ($z_b = \frac{p}{u}$) and complex wavenumber ($k_b = \frac{\omega}{c_b}$), can be used to describe sound propagation in a granular medium and account for the amount of water trapped in the pores. Here p is the acoustic pressure in the material, u is the acoustic particle velocity, ω is the angular frequency equals to $2\pi f$ in which f is the frequency of sound in Hertz and c_b is the complex speed of sound in the material pores. The value of the complex wavenumber relates to the speed of the sound in the porous space ($\mathrm{Re}\, c_b$) and the rate of its attenuation $\mathrm{Im}\, c_b$, where Re and Im stand for the real and imaginary parts, respectively. The value of the characteristic impedance determines the ability of sound waves to enter the porous medium. In outdoor sound propagation the characteristic acoustic admittance (β_b) is often adopted and is defined as the inverse of the characteristic acoustic impedance $\beta_b = z_b^{-1}$.

If the thickness of a porous layer is less or comparable to the wavelength of sound and the acoustic attenuation in the layer is relatively low, then the reflection from the back of the porous layer (e.g. from the water table) is of importance. In this case, the characteristic admittance needs to be replaced with the surface admittance which takes into account the interaction between the incident and reflected acoustic waves in the layer (see chapter II in Zwikker and Kosten~~,~~ 1949). The surface acoustic admittance is defined as the ratio of the acoustic particle velocity to the acoustic pressure at the interface between the porous layer and the air half-space. In a particular case, when the back of the granular layer is bounded with an impervious, acoustically hard and reflective surface (e.g. water or a low-permeability ceramic), then the surface acoustic admittance can be obtained using the following equation:

$$\beta_s(\omega) = \beta_b(\omega)\tanh(-ik_b(\omega)d) \quad (1)$$

where $i = \sqrt{-1}$ and d is the thickness of a non-deformable porous layer. It should be noted that the value of the surface acoustic admittance is complex and is composed of two parts: the real and imaginary parts. The real part ($\mathrm{Re}\beta_s(\omega)$) is linked to the specific acoustic resistance and relates to the acoustic energy which is able to penetrate and propagate through the porous sample. The imaginary part ($\mathrm{Im}\beta_s(\omega)$) refers to the specific acoustic reactance and relates to the acoustic energy conserved in the sample.

A number of parameters including the type of soil, the moisture content and degree of compaction would influence the surface admittance of unsaturated soils. Attenborough (1985) and Marten et al. (1985) found that propagation of the sound wave is significantly affected by the soil type. These and other works illustrate that the acoustic surface admittance for the majority of sands and clays can be approximated by the value of the characteristic admittance, ($\beta_s(\omega) \cong \beta_b(\omega)$) because of the relatively high acoustic attenuation and the relatively high layer thickness typical for many types of natural sands for which $\tanh(-ik_b(\omega)d) \to 1$. It was suggested that soils could be classified as acoustically soft, moderate or hard materials (Attenborough 1985). For acoustically soft soil, the real part of the impedance is relatively independent of frequency and the imaginary part strongly decreases with frequency. Changes in the structure of the soil due to the compaction process and the presence of moisture could result in significant changes in the pore structure (Zhiqu et al. 2004) and acoustic velocity, which is determined by the real part of the complex wavenumber, $\mathrm{Re}\,k_b(\omega)$. This was basically due to the reduction in the volume of pores that are available for the sound wave to travel through. In this way the structure and distribution of pore sizes can be linked to the acoustic admittance so that the condition of soil can be determined remotely by exposing a sample to airborne sound. The method is likely to be limited to soils with median particle size approximately 150 μm and above. This limit is determined by the viscous boundary layer thickness which becomes comparable or greater than the maximum pore size, r_p, at the frequencies of sound considered in this work (50 – 1600 Hz). The thickness of the viscous boundary layer for sound propagation in a pore is given by $\delta_v = \sqrt{\frac{\eta_a}{\rho_a \omega}}$ (Chapter II in Zwikker and Kosten, 1949), where $\eta_a = 1.84 \times 10^{-5}$ kg m^{-1} s^{-1} is the dynamic viscosity of air, ρ_a = 1.24 kg/m^3 is the equilibrium density of air and $\omega = 2\pi f$ is the angular frequency of sound. It can be shown that for the maximum frequency of sound considered in this work (f = 1600 Hz), the viscous boundary layer thickness (δ_v) is $\cong 38$ μm, which determines the lower pore size limit. Below this limit, the value of the wavenumber of sound in soil (see equation (1)) becomes relatively large because of the asymptotic dependence $k_b \propto \sqrt{-i}\,\frac{\delta_v}{r_p}$, where $i = \sqrt{-1}$. In this case, the maximum penetration depth of sound into the soil sample, $\left(\mathrm{Re}\, k_b\right)^{-1}$, is considerably reduced and the attenuation of sound in the pores, $\mathrm{Im}\, k_b$, is considerably increased for the method to be of practical use in the adopted frequency range.

The effect of water content in porous ground on sound propagation has been considered in earlier studies (Dickinson and Doak 1970; Cramond and Don 1987; Johnson 2001). Despite the several attempts, this effect has not yet been properly characterised under controlled laboratory conditions throughout the possible range of volumetric water content/degree of water saturation. Dickinson and Doak (1970) and Zuckerwar (1983) reported that the measured flow resistivity of porous ground increases with moisture content. However, only limited data for the acoustic impedance were presented and the soil water was not accurately controlled. Shields et al. (2000) carried out an investigation for the effect of moisture content on compression and shear waves in granular materials. It was found that moisture content has a little effect on the wave velocities unless water reacts with the solid particles to produce a high viscous liquid. Sound speed and attenuation have been used greatly in investigations of unsaturated soils and gassy soils (Sills et al. 1991). However the results are far more complex and lead to inconsistent outcomes. It was found that sound velocity and attenuation of sound waves vary substantially with porosity, fluid content, temperature and pressure.

It is reasonable to conclude that so far no accurate acoustical measurements were achieved in a controlled laboratory environment in which the volumetric water content could be varied and measured accurately simultaneously with the acoustical properties. There is no evidence ~~In addition, neither~~that either the sound speed ~~n~~or flow resistivity can be used with confidence to determine the volumetric water content and density of a unsaturated sample of sand. However, some evidence ~~has recently been~~ obtained recently by the authors (Horoshenkov and Mohamed 2006) suggest that there is a clear link between the acoustic admittance ~~impedance~~ and the degree of water saturation. Thus, ~~in this paper the use of acoustic surface admittance is proposed as the primary acoustical characteristic that will be investigated.~~ we propose to ~~The~~ measure directly the acoustic surface admittance ~~is measured directly~~ and link it~~ed~~ to the volumetric water content in a sand sample which is determined simultaneously by independent means. ~~The acoustic surface admittance is an effective parameter in assessing sound absorption which is controlled by the size distribution of open pores within a granular sample, amount of water present in the sample and the sample porosity.~~

Sand samples

Eight different types of available silica sand were used in this paper for investigating the relationships between the acoustic admittance and volumetric water content. The samples were carefully selected so that a wide range of particle size distributions can be studied. Figures 2 and 3 show the particle size distribution of the selected sand samples. Table 1 presents summary of the physical properties of these samples. The median particle size (D_{50}) ranged between 300 to 880 μm. The uniformity coefficient varied from 1.2 to 3.0. The coefficient of curvature was around 1.0 for all samples. According to the data presented in Table 1 and based on the British Standard BS5930 (BS5930 1999), all samples would be classified as poorly graded sand. Three samples (2, 4 and 6) with markedly different mean particle size diameter and uniformity coefficient are selected for presentation. Microscopic images of samples 2, 4 and 6 are presented in Figure 4. Inspection of Figures 2-4 suggests that samples 2 and 4 have a narrow range of particle sizes whereas sample 6 consists of a relatively wide range of particle sizes. In addition, in all samples the majority of the sand particles are rounded to sub-rounded

Experimental Set-up

A special experimental facility was developed to measure simultaneously the acoustic properties of unsaturated sands as well as the volumetric water content. Figure 5 shows a schematic diagram of the developed experimental set-up. It is based on two standard pieces of equipment, namely a Buchner funnel and impedance tube. The Buchner funnel set-up has been used routinely in previous studies in geotechnical engineering to determine accurately the relationship between volumetric water content/degree of saturation and matric suction head (Mohamed and Sharma 2007; Sharma and Mohamed 2003; Leclaire et al. 1998).

Impedance tubes are available commercially (e.g. from Bruel and Kjaer (Denmark) or BSWA Technology Co., Ltd (China)). The impedance tube used in these experiments is a standard Bruel and Kjaer 100 mm round tube (Type BK 4206) designed to measure the acoustic admittance of material samples in the frequency range of 50 - 1600 Hz (BS 10534-2 2001). This device is provided and controlled with software, which is easy to setup and use on an ordinary PC and can be operated by a person with little training in acoustics. In addition, the standard impedance tube can easily be calibrated to provide reliable and consistent measurements of the acoustic properties of the sand sample. A typical duration of one acoustic measurement is under one minute~~within seconds~~. The acoustic equipment consists of a straight tube, sample holder attached to one end of the tube, two microphones and a loudspeaker attached to the other end of the tube (see Figure 5). The loudspeaker is driven by an amplifier and the resultant sound field in the tube is received by a pair of 1/4” microphones installed in the vicinity of the sample and separated by a 100mm base. The microphones are connected to the PULSE^{TM} data acquisition system. The function of the data acquisition system is to control the sound wave, to record the sound pressure inside the tube, analyse and present the recorded data.

The impedance tube works by sending a broadband, airborne sound wave from the loudspeaker (see Figure 5). The sound emitted by the loudspeaker propagates in the form of a plane wave within the impedance tube and a proportion of its energy is reflected by the sand surface. The rest~~A proportion~~ of the sound energy travels through the open pores of the sample of unsaturated sand and~~,~~ dissipates as explained in the previous section~~because of the viscous friction and thermal exchanges in material pores and reflects back from any inhomogeneities in the pores (see chapter II in Zwikker and Kosten, 1949)~~. The direct and reflected sound waves create a distinctive acoustic interference pattern inside the tube, which is then recorded by the two microphones and analysed according to the standard procedure (BS 10524-2 2001) so that the acoustic properties including the surface admittance can be determined. The variations of the air temperature in the tube and atmospheric pressure affect the acoustic impedance and thus they were monitored during the experiments to 0.1^{o}C and 100Pa, respectively. These data were then used to compensate for the related changes in the sound speed and air density as suggested in (BS 10534-2 2001).

Figure 5 also shows that the Buchner funnel is attached to the bottom of the impedance tube in order to facilitate simultaneous measurements of both volumetric water content and acoustic properties without disturbing the sand sample. The internal diameter of the Buchner funnel is identical to the internal diameter of the impedance tube, which is 100mm. A 50mm deep sample holder of the same diameter is machined in the top of the Buchner funnel so that it can be attached to the base of the impedance tube without creating an apparent change in the tube cross-section (see Figure 5). A flexible O-ring is used to seal any air gap between the surface of the impedance tube and the Buchner funnel. The Buchner funnel holder is then clamped securely to the tube using the two standard brackets.

The thickness of sand samples in the Buchner funnel in the reported experiments was always maintained at 50 mm. The thickness of samples was selected to: (i) guarantee that no strong acoustic reflection could be detected from the bottom of the Buchner funnel containing the sand specimen; (ii) to yield reliable results when the continuum approach is applied for determination of suction head and volumetric water content; and (iii) practically complete the water extraction experiment in a single day. Although the bottom of the Buchner funnel is perforated, its openings are not small enough to sustain high values of suction head. Therefore, a single layer of filter paper (Whatman 1541 110) having nominal pore sizes of

20μm was fitted to cover the whole cross section of the Buchner funnel (see Figure 5). The used filter paper can sustain suction head upto 750 mm, which is sufficient for the tested samples of sand. As a result, a continuous water phase between the water in the burette and that within the pores of the sample was maintained. The circumference of the filter paper was sealed properly with a thin layer of silicon sealant to prevent air passage. It was important to keep the air pressure inside the impedance tube equal to atmospheric as water was progressively extracted from the material pores. This was achieved by removing between the measurements the dummy microphone head from position 2 in the impedance tube to vent it to the atmosphere (see Figure 5).

Sample preparation and testing procedure

Fully saturated samples were initially prepared by pouring dry sand from a certain height into the Buchner funnel, which was partially filled with water. Since the dry sand was always poured into the water, it is unlikely that air would be trapped in the pore voids, thus resulting in fully saturated samples being obtained (Mohamed 2003). In addition, the stratification of sand layers was less likely to occur as (i) the tested samples are generally poorly graded with no fines < 75 μm (ii) sand particles were dropped continuously from the same height in a partially filled Buchner funnel with water and (iii) a similar water head (around 20 mm) was maintained above the sand surface by addition of water from the burette to ensure non-entrapment of air bubbles. As a result, limited chance exists for the smaller particles to become suspended in water and to form layers, which is confirmed by the visual observations. Upon filling the Buchner funnel with saturated sand, the sand surface was levelled off using a specially designed tool and any excess water and sand particles were carefully removed. This ensured that the volume of the prepared sand samples was known and kept relatively constant. Careful examination of the samples after completion of experiments indicated that the volume of the samples remains unchanged which suggests that the tested samples were non-deformable under the range of suction heads applied. This observation is in good agreement with earlier studies by Mohamed and Sharma (2007). The amount of dry sand that was used for the preparation of each sample was measured accurately to enable the calculation of dry density and material porosity. Since all samples were initially fully saturated with water, the initial volumetric water content is then equal to the porosity.

The Buchner funnel was then attached carefully to the impedance tube marking the start of the experiment. The acoustic surface admittance was initially measured at full saturation. Table 1 presents data for the acoustic admittance for all samples at full saturation. Then an incremental increase in suction head was applied by lowering the burette to a predetermined height and the outflow of water was measured. The acoustic surface admittance of the sample was then measured each time when the equilibrium of water within the sand sample was achieved. The point of equilibrium was determined when no more outflow of water from the burette was observed. This procedure was repeated by gradually increasing the applied suction head and monitoring the outflow of water. The experiments were continued until the minimum volumetric water content which is equivalent to the residual degree of saturation was reached, which was marked by no further draining of water with further increase in the value of matric suction head. Due to practical difficult in extracting water beyond the minimum volumetric water content, the surface acoustic admittance was measured for dry sand samples separately and also provided for all samples in Table 1.

Since the initial volume of water within the prepared samples and drained quantity of water after applying an incremental suction head are known, the volumetric water content can be determined at each stage (for further details see Mohamed 2003; Sharma and Mohamed

2003). It should be noted that in all reported experiments, no air was observed in the burette and connecting tubes indicating that a continuous water phase is maintained within the sand samples and in the burette.

Results and discussion

The relationships between the volumetric water content, uniformity coefficient and the acoustic surface admittance for samples of unsaturated sand are derived as follows:

Water retention curves

Water retention curves for sand samples 2, 4 and 6 are presented in Figure 6. The water retention curves were obtained starting from the volumetric water content at full saturation. In these experiments it was observed that some water drained immediately with slight increase in the matric suction. However, all samples remain almost saturated with water (θ close to *n;* where θ and n stand for volumetric water content and porosity respectively) up to reaching a particular value of suction head, which is often called the bubbling pressure head. In our experiments the bubbling pressure head ranged from 100 mm for sample 4, to 210 mm for sample 6. The bubbling pressure head was determined according to the proposed procedure detailed in (Bear 1979). With further increase in the matric suction head, the volumetric water content reaches almost a minimum value, which is often called irreducible water content and is corresponding to the residual degree of saturation. In these experiments the minimum value of volumetric water content varied from 3.0% to 6.0 %.

Acoustic properties

In this section, data for the measured real and imaginary parts of the normalised surface acoustic admittance ($\beta = \rho_0 c_0 \beta_s$) are presented in Figures 8 to 10 for samples 2, 4 and 6 respectively as a function of sound frequency. Here ρ_0 and c_0 represent the equilibrium density of air and sound speed in air, respectively. The values of ρ_0 and c_0 in the experiments were calculated in accordance with the procedure detailed in (BS 10534-1 2001) using the recorded temperature and atmospheric pressure data. Figures 7a to 9a show data for the real part of the normalised acoustic admittance, whereas Figures 7b to 9b present data for the imaginary part of the acoustic admittance. Lower values of the acoustic surface admittance relate to a higher proportion of the sound energy being reflected by the surface of the specimen. This acoustical characteristic can be used for quantifying the amount of pores that are open and interconnected to be able to conduct and absorb the energy of the incident sound wave.

Data presented in Figure 7 for sample 2, show that both the real and imaginary parts of the acoustic admittance vary with the volumetric water content. In general there is a reduction in the real part of the admittance of up to 15-fold as the volumetric water content increases from 0% to 40.2%. For volumetric water contents below 21% it is noticed that the real part of the surface admittance increases with frequency up to a frequency of 600 Hz and then reaches a relatively constant value for frequencies in the range of 600-1100Hz. However, for volumetric water contents above 21%, the real part of the acoustic admittance remains almost constant irrespective of the frequency. There is a proportional reduction in the value of the real part of the admittance for a given increase in the volumetric water content across the considered frequency range. This indicates that there is a clear dependence of the real part of the acoustic surface admittance on the volumetric water content.

Figure 7b shows the results of the imaginary part of the acoustic admittance for sample 2. It can be seen that there is a significant change in the imaginary part of the acoustic admittance from zero volumetric water content (dry sample) up to 40.2 % volumetric water content at full saturation. The imaginary part of the acoustic admittance increases at low frequencies, approaches a relatively constant value in the 600-1100 Hz range and then increases again in the higher frequency limit. The imaginary part is negative at very low frequencies which is attributed to the particle vibration (see chapter III~~,~~ in Zwikker and Kosten~~,~~ 1949) and water surface tension effects (Nagy and Nayfeh 1995). Careful inspection of Figure 7b demonstrates that there is almost 100% reduction in the imaginary part occurred when the volumetric water content increases from 0% to 5.6% (minimum volumetric water content). For volumetric water contents between 5% and 21%, the imaginary part varied with the change in volumetric water content at low frequency (400 – 500 Hz). For volumetric water contents above 21% the imaginary part shows no significant variation. Thus, it is difficult to draw a clearly defined relationship between the volumetric water content and the imaginary part of the acoustic admittance.

Data for the real and imaginary parts of the acoustic admittance for sample 4 are shown in Figures 8a and b. The results of the real part of the acoustic admittance are similar to those obtained for sample 2. The results for the imaginary part of the acoustic admittance for sample 4 are shown in Figure 8b. The results for the imaginary part are markedly different to that of sample 2. Unlike the data for sample 2, the imaginary part of the surface admittance stays positive across the considered frequency range. The imaginary part reaches a minimum between 300 and 400 Hz and then increases with the increased frequency of sound up to $f \leq$ 1300 Hz (see Figure 8b). Above that the imaginary part decreases with frequency. The imaginary part of the admittance of the dry sample is completely different to those of the wet sand (see Figure 8b).

The results for the real and imaginary parts of the acoustic admittance of sample 6 are shown in Figure 9. The data for sample 6 show that the greatest effect of the volumetric water content is obtained during the transition from the dry to the minimum volumetric water content (5.2%) state. This transition results in up to 400% reduction in the value of the acoustic surface admittance (see Figure 9a). It can be seen that there is a significant gap between the results for the imaginary part for the dry sample and those for the sample with the minimum volumetric water content. The behaviour of the imaginary part is complex and relatively independent of the volumetric water content for 5.2%< θ <35%. The patterns for the real and imaginary parts of the acoustic admittance for other tested samples are similar to those presented. The results of acoustic admittance depend upon the pore size distribution and porosity which are controlled by the properties of the porous specimen and its volumetric water content. Thus similar results can be obtained from repeated experiments as long as the sand samples have the same properties, porosity and volumetric water content.

The presented data demonstrate clearly that there is a direct influence of the volumetric water content on the real part of the surface admittance. Here an increase in the volumetric water content results in a decrease in the real part of the surface admittance across the considered frequency range. ~~However,~~The results for the imaginary part are more ambiguous and difficult to be related uniquely to the effect of volumetric water content. This is because the imaginary part of the acoustic surface admittance represents the acoustic reactance, i.e. the proportion of the acoustic energy conserved within the porous sample associated with the interference between the direct and reflected sound waves. On the contrary, the real part relates to the proportion of sound energy penetrated and dissipated in the porous sample so that we propose to focus our discussion on the behaviour of the real part of the surface admittance. This quantity is likely to be affected only by the proportion of the connected air-filled pores and their size distribution (Horoshenkov and Swift 2001).

Modelling

It is proposed to develop a method by which the volumetric water content can be predicted using the measured real part of the acoustic admittance. Firstly, we plot the results for the acoustic admittance against the volumetric water content. Figures 10 and 11 show the volumetric water content plotted against the values of the real part of the surface admittance for the eight tested samples. In these figures symbols represent the measured values of acoustic admittance at different volumetric water content. It can be noted that the values of the surface admittance are obtained at the frequency of 800 Hz. This frequency was selected as the median of the frequency range of 400-1200 Hz. In~~in~~ this frequency regime the real part of the admittance is likely to be controlled by the ratio of porosity over tortuosity ~~which~~ and it ~~the real part of the admittance~~ is relatively independent of the acoustic frequency (see expression (4) in Voronina and Horoshenkov, 2004). ~~(why)!!!!!~~ This assumption is valid for all the samples investigated in this work.

A close examination of the data presented in Figures 10 and 11 allows the following conclusions to be made: (i) the real part of the acoustic admittance increases with the decreasing volumetric water content; (ii) the dependence of the real part on the volumetric water content seems linear in a semi-log plot. As water is taken out of the sample, more pores become filled with air, connected and open to the incident sound wave. At higher values of volumetric water content, the available flow paths become more tortuous resulting in lower values of the acoustic admittance. An analysis of the data presented in Figures 10 and 11 for the relationships between the volumetric water content and the real part of the surface acoustic admittance suggests that this dependence is logarithmic and the following equation is proposed:

$$\theta = +a \ln\left(\mathrm{Re}(\beta_{800})\right) \quad b \tag{2}$$

where β_{800} is the real part of the acoustic admittance measured at a frequency of 800 Hz and *a* and *b* are coefficients whose values are determined using the least mean square fit. The best-fit lines are shown in Figures 10 and 11 by solid lines. Table 2 presents the values of the coefficients (*a* and *b*) for different sand samples as well as the corresponding values of R^2, which represents the coefficient of determination. It is the correlation between the measured data and the model (fit). The coefficient *b* relates to the value of the real part of the acoustic admittance of the dry sand which is controlled by the particle size and porosity of sand. The coefficient *a* relates to the sensitivity of the acoustic surface admittance to the volumetric water content. Figures 10 and 11 demonstrate that slope of the best-fit line (coefficient *a*) varies ~~greatly~~ with the change in the particle size distribution of the sand samples. Several characteristic grain diameters and coefficients were proposed to characterize the particle size distribution from different perspectives. Among these is the uniformity coefficient which is well-accepted parameter to describe how uniform a soil is. Generally for soils with a wide range of particles, a better particle arrangement can be achieved resulting in a reduced porosity and size of pores. Thus, in this investigation, it is suggested that the coefficient of uniformity can be utilised to account for the effect of particle size distribution on the observed relationships between volumetric water content and acoustic admittance for different sands. Careful inspection of Figures 10 and 11 and Table 2 suggests that as the uniformity of sand sample improves, the range of values of the real part of the acoustic admittance decreases proportionally. This is true since for sands with higher uniformity coefficients prepared with the same packing technique the material density is expected to be higher resulting in an increased number of particle contacts and a decreased size of pore channels (compare microscopic images for samples 4 and 6 in Figure 4). Figure 12 shows the relationship between the coefficient *a* (see Table 2) and the uniformity coefficient C_u (see Table 1) derived for the investigated specimens of sand. It can be seen that there is a linear dependence between the slope of equation (2) (coefficient *a*) and the uniformity coefficient. The relation between the coefficient *a* and uniformity coefficient can be described by the following best-fit line:

$$a(C_u) = 0.0284C_u + 0.0892. \qquad (3)$$

Equation (3) can be used to determine the coefficient *a* from the particle size distribution data for a sand sample which can be determined using a standard testing procedure. Then the coefficient *b* can be obtained using equation (2) and data on the real part of the acoustic admittance of the dry sand sample. It should be noted that for accurate prediction the coefficient *b* should be determined for a dry sample with density identical to that of the prepared sample under saturation. Because of difficulties in preparing samples with the same density, data for the real part of the acoustic admittance can be developed as a function of the dry density. These data can then be used to determine an accurate value for the coefficient *b*.

However, the volumetric water content is not the only parameter that has a significant effect on the acoustic properties. Sound waves propagate through the interconnected pores, in which the acoustic attenuation is a function of the size and distribution of the pore voids. The size of the pores formed between the solid grains is directly related to the material density or porosity. Porosity is a function of the mode of packing, particles shape, their arrangement and size distribution (Bear 1972; Lowe and Greenaway 2005). The effect of the density of sand is complex and it can be accounted for by using a suitable model for the acoustical properties of granular media (e.g. Horoshenkov and Swift 2001; Voronina and Horoshenkov 2004). A preliminary investigation has been carried out to quantify the effect of mode of packing and hence the effect of density/porosity on the measured real part of the acoustic surface admittance of dry samples. Dry samples with different densities have been prepared using sand raining technique. Figure 13 shows the results of dry density against the real part for sample 4. The results suggest that the real part of the acoustic admittance decreases almost linearly with the increase in the dry density. This phenomenon deserves a separate detailed study. Because only one mode of packing is adopted here and the shape of sand particles is almost similar (see Figure 4), changes in the sample porosity and pore size distribution can reasonably be attributed to the particle size distribution.

The obtained results show clearly that the developed method for the first time relates successfully the acoustic admittance with the volumetric water content of the sand. There is a strong experimental evidence that the acoustic admittance of the top layer of soil can be measured in-situ using one- or two-microphone methods (Embleton et al. 1983; Cramond and Don 1987; or ANSI S1.18 2004). It should be noted that the in-situ measurements of acoustic admittance is relatively cheap as it only requires a laptop equipped with a good quality sound card. Further work is planned to extend the proposed approach for determination of in-situ volumetric water content.

A validation exercise has been conducted to examine the accuracy of the developed procedure for the determination of the volumetric water content using the acoustic measurement of the surface admittance. Sample 9 with properties different to those attributed to Samples 1-8 was selected for this purpose. This material had the median particle diameter of 630 μm and its uniformity and curvature coefficients were 2.62 and 1.23, respectively as shown in Table 1. The porosity of sand sample was 40%. The coefficients *a* and *b* were determined according to the procedure detailed in the previous section. The values of these coefficients were found to be -0.1636 and -0.3611, respectively. Figure 14 shows the correlation between the determined volumetric water content and that derived from the acoustic surface admittance data. It should be noted that in Figure 14 maximum volumetric water content is 40% which is equivalent to full saturation. The comparison suggests that all the points are located along the 45^o line indicating that a very close agreement between the two methods was obtained for $\theta \leq 0.4$. The error in the results is within 1.7% up to the volumetric water content of 0.34 and 3.5 % for measurements near full saturation. The accuracy of the proposed method is limited at states close to full saturation because of the complexity of water distribution within the sand sample, low proportion of interconnected air-filled pores and low values of the real part of the acoustic surface admittance. Only samples of sand with uniformity coefficient between 1.2 and 3.0 are investigated.

Conclusions

A new experimental technique has been proposed, examined and validated for determination of volumetric water content of unsaturated sandy soils using acoustic measurement of the surface admittance. The proposed technique is non-destructive, non-invasive, quick and relatively inexpensive. It is found that the real part of the acoustic admittance is dependent upon the volumetric water content. The acoustic surface admittance is related to the size of the air-filled, interconnected pores and its proportion, which are functions of soil density and particle size distribution. This is then used to develop a relation between the volumetric water content and the measurable acoustic properties of sands. Specifically, a logarithmic relation (equation 2) is proposed to determine the volumetric water content using the acoustic surface admittance data, which can be obtained routinely using a standard impedance tube connected to a Buchner funnel. The coefficients of the proposed equation are directly related to the uniformity coefficient and the acoustic admittance of the dry sample, which can be obtained using standard testing. The proposed equations are developed based on data for poorly graded sand and further study is underway to investigate their applicability for well graded samples.

Finally, a validation study has been conducted to examine the accuracy of the proposed equations. The results of the validation exercise demonstrate that the volumetric water content can be measured with an accuracy of $\pm 2.0\%$.

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