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**Proceedings Paper:**

https://doi.org/10.1109/ULTSYM.1994.401721

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A STUDY OF THE TRANSMISSION OF ULTRASOUND ACROSS REAL ROUGH SOLID-SOLID INTERFACES

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INTRODUCTION

The authors are currently working on a project to improve the performance of rubber coupled ultrasonic transducers. This work has already enabled the construction of a rubber coupled wheel probe that operates at 5MHz [1, 2]. For the successful operation of such probes it is important to know the conditions that will give maximum coupling. To this end a study of solid-solid coupling was undertaken.

Reflection coefficients from solid-solid interfaces and the similar case of reflection from a layer of embedded pores have been studied by many authors [3-8]. Baik and Thompson [3] describe a spring model approximation to calculate reflection coefficients in the low frequency range, the spring constants being determined from the static stiffness of the solid-solid interface. Margetan et al [4] verified this type of model experimentally by measuring the reflection coefficient from artificially introduced defects of simple geometry and known size. Angel and Achenbach [5] have derived a closed form solution for the reflection coefficient from a regular array of equally sized defects which is valid at all frequencies, and have demonstrated that this model is in good agreement with the quasi-static approximation at low frequencies. Nagy [6] used both longitudinal and shear waves to classify interfacial imperfections and showed that by using both these wave types it is possible to gain additional information about the state of the interface.

Other authors have investigated the reflection coefficient from randomly rough surfaces under pressure. Haines [7] considered the reflection coefficients from rough solid-solid interfaces. He described a statistical model of the rough surface and calculated the true area of contact by considering the independent elastic, and elastic-plastic, contact of each asperity. From the true area of contact a spring model was used to calculate reflection coefficient. At high percentages of contact the neighbouring asperities interact, hence limiting this model to low percentages of contact. Krolikowski [8] attempted to measure contact parameters by measuring the reflection coefficient from the solid-solid interface by using a similar model of the contact to Haines.

This paper describes a link between the reflection coefficient from a rubber-solid interface, surface roughness and interfacial pressure. Rubber-solid contact provides an example of elastic contact and so an elastic contact model can be used. The surface profiles of the two substrates are measured and a numerical elastic contact model is then used to predict the size and distribution of the non-contacting regions (gaps) along the rubber-solid interface as a function of applied load. Measurements of the reflection coefficients at varying loads, together with the results from the numerical contact model then allow the variation of the reflection coefficient with changes in the contact conditions.
(e.g. percentage contact) to be determined. Interfacial spring constants have been calculated from the information provided by the numerical contact model and from simplifying assumptions about the interfacial geometry. These spring models are then used to calculate reflection coefficients which are then compared to those measured experimentally.

**REFLECTION COEFFICIENT MEASUREMENTS**

In a rubber coupled transducer system the reflection coefficient from the solid-rubber interface will determine how much energy is transmitted into the test structure. This reflection coefficient will vary with the applied load. This section describes the experiments used to measure this behaviour.

If the contact pressure is to be calculated from the applied load then the pressures across the contact region must be simple in form and repeatable. This is difficult to achieve when two flat plates contact as they must be perfectly aligned and perfectly flat in two axes [9]. If the plates are not flat or are misaligned, unknown pressure variations are developed across the contact region. With this situation it is impossible to determine the pressure corresponding to the measured reflection coefficient. These problems are made worse if either solid is soft, as a soft solid is less dimensionally stable. In order to surmount this problem a 2mm thick rubber sheet was bonded to a 12mm diameter solid cylinder. This rubber coated cylinder makes contact with an interchangeable perspex plate. Using this technique a line contact is made. Such a contact is only sensitive to misalignments in one plane and has a simple pressure distribution. Fig. 1 shows the experimental set-up used to measure the reflection coefficients. A 5MHz centre frequency focused transducer was placed in a water bath above a perspex plate. This water allows coupling between the transducer and the perspex and enables the transducer to be scanned over the perspex-rubber contact. The amplitude of the reflection from the perspex-rubber interface was compared with that from a perspex-air interface and hence the reflection coefficient was calculated. Fig. 2 shows the measured variation of reflection coefficient over the contact region for different applied loads in tests on a smooth perspex plate. It can be seen that at low loads the reflection coefficient profile is dumbbell shaped. Where no contact is made the reflection coefficient is equal to unity; as the contact pressure increases towards the centre of the cylinder, so more energy is transmitted into the rubber and the reflection coefficient is reduced. Once the contact pressure is sufficient to cause perfect contact, the reflection coefficient reaches a minimum value equal to the reflection coefficient calculated from the acoustic impedances of the rubber and perspex, as shown in Fig. 2.

![Diagram](image-url)

**Fig.1** Experimental set-up used to measure the perspex-rubber reflection coefficient.
Fig. 2 Measured rubber-perspex reflection coefficient under different loads for a smooth perspex plate.

These experiments were repeated for a number of perspex plates of different surface roughness. The varying degrees of roughness were created using different grades of emery paper. The surface profiles of both the solid and the rubber were measured using a stylus profilometer. This provides a discretised height map of the sample surface along a line. The line is then assumed to be representative of the roughness of the whole sample. The surface profiles were then statistically evaluated to find the roughness parameters. The roughness parameter used in this paper is the centre line average (abbreviated to CLA or Ra) which is the arithmetic mean of the departure of the profile from the centre line. The unroughened perspex had a roughness of 0.19μm. Coarser emery paper was used create perspex plates of increased roughness (CLA=0.47μm and 1.61μm). The reflection coefficients measured on the rougher plates had a similar form to those of Fig.2, but the loads needed to achieve a given level of reflection coefficient below unity were increased.

PREDICTION OF PRESSURE PROFILES

The aim of the experiments was to measure changes in reflection coefficient with pressure and so contact pressure had to be calculated across the contact zone. The geometry considered is the contact of a cylindrical layered body on a flat plate. Unfortunately there is no simple analytical solution for this case. However, Majers [10] presented an approximate iterative scheme which showed that the expected pressure distribution is parabolic. A finite element model of this geometry was constructed by the authors which also showed that the pressure distribution was parabolic. Both these models need inputs that are unknown, for example the coefficient of friction between the substrates. However, it is possible to reduce the need for these numerical models by measuring the contact width optically with a travelling microscope. The contact pressures can then be calculated as the load and contact width are known and the profile can be assumed to be parabolic.

The transducer, despite being focused, will not measure the reflection coefficient at a point but will give a weighted average across its focal zone. Hence, at a given measurement position the pressure corresponding to the measured reflection coefficient is a weighted average of the point pressures across the width of the focal zone. The weighted average used was a standard Bessel function distribution as given by Silk [11]. Fig. 3 shows a parabolic pressure distribution for 4 Kg load applied to the system shown in Fig.1 and the weighted average pressure over the focal region for the same case. The effect of averaging over the focal region is to lower the peak pressure and widen the apparent contact width.
Fig. 4 shows the variation of reflection coefficient with pressure for different loads applied to the system shown in Fig. 1. At a given load, a range of pressures from zero to the peak pressure is generated and the corresponding reflection coefficients are measured. For each load there are two reflection coefficient versus pressure curves corresponding to the two sides of the reflection coefficient and pressure profiles. Most values of pressure are repeated at higher loads and in this way most of the reflection coefficient measurements at a given pressure are repeated. If the predicted pressure profiles are correct then these repeat measurements should give the same reflection coefficient. It can be seen from Fig. 4 that the agreement of these repeated measurements is reasonable (maximum error is 20% of reflection coefficient at a given pressure). Fig. 5 shows the effect of increased surface roughness on the reflection coefficient variation with pressure. Only the highest load case has been plotted for each roughness to avoid confusion. As expected, for each surface roughness, the reflection coefficient can be seen to decrease with increasing pressure towards the perfect contact value. Increased surface roughness increases the pressure needed to achieve perfect contact.

Fig. 3 Parabolic pressure distribution and the average pressure as seen by the transducer.

Fig. 4 Measured perspex-rubber reflection coefficient variation with predicted pressure for different applied loads (CLA=0.47μm)
Fig. 5 Measured perspex-rubber reflection coefficient variation with predicted pressure for three surface roughnesses.

**NUMERICAL CONTACT MODEL**

Information about the size and distribution of the gaps and contacting regions was obtained using a numerical elastic contact model. This model, developed by Webster and Sayles [12], is used widely in tribological research where it has given excellent agreement with both optical and electrical measurements of contact.

The discretised surface profile, as measured by the surface profilometer, provides the input to a numerical elastic contact model, which calculates the deformation of the surfaces as load is applied. An initial guess is made for the vertical displacement of each point on the two surfaces and iteration is then performed until the summation of contact pressures over the contact regions equals the applied load. The model outputs the deformed shape of both contacting surfaces and the pressure distribution across the contact region. From this output the sizes of all the gaps and contacts can be found and hence the percentage contact can be calculated.

From this contact model percentage contact is known for a given pressure and as reflection coefficient variations with pressure have been measured, so reflection coefficient variations with percentage contact can be plotted. Fig. 6 shows the variation of reflection coefficient with percentage contact for different loads applied to the plate. The results for these different loads can be seen to follow similar curves.

**DISCUSSION**

The previous section describes a link between reflection coefficient and percentage contact. This section will discuss the assumptions in this link and place these results in the context of other authors’ work.

In Fig. 4 reflection coefficient is plotted against pressure and it can be seen that, under different loads applied to the plate, the reflection coefficient has been measured at the same pressure a number of times. The reflection coefficient is expected to be dependent on pressure only and so, for a given roughness, a plot of reflection coefficient against pressure should be a continuous line. It can be seen from Fig. 4 that the results do not exactly follow this expected behaviour. The departure from this expected behaviour is probably due to the non-linear stress-strain behaviour of the rubber. The Young’s modulus of the rubber decreases with increased stress/strain and hence is not constant throughout
Fig. 6 Measured perspex-rubber reflection coefficient variation with predicted percentage contact for different applied loads (CLA=0.47μm).

Fig. 7 Output of the numerical contact model. Variation of percentage contact with pressure (CLA=0.47μm).

The contact region. This causes the pressure profile to be slightly non-parabolic with a lower than predicted peak pressure and higher than predicted pressures at the extremes of the profile.

The non-linear behaviour described above will also reduce the accuracy of the numerical contact model which, has assumed linear elasticity. The effects of non-linearity are not apparent in Fig. 6 due to the form of the variation of percentage contact with pressure which can be seen in Fig. 7. From Fig. 7 it can be seen that as pressure increases the percentage contact increases, but at a decreasing rate, so that as the percentage contact increases, variations in pressure have less effect on the percentage contact. This reduces the effect of the errors in Fig. 4 which increase with pressure.

The quasi-static method [3] describes how spring constants can be calculated from the static stiffness of an interface. The interfacial stiffness is defined as the average stress at the interface divided by the increase in deflection due to the interface. The numerical contact model provides the geometry of the deformed surfaces and so the increased deflection due to the interface can be found from the average closure of the interface due to a given load. The interfacial stiffness at a given load is found by increasing the load.
Fig. 8 Variation of measured perspex-rubber reflection coefficient and that predicted by spring models and using the stiffness obtained directly from the numerical contact model with predicted percentage contact (CLA=0.47mm).

slightly and noting the resulting average interfacial closure. The change in average stress across the interface is known and so the interfacial stiffness can be calculated. The interfacial stiffness can also be calculated from various approximate models. For example, at low percentage contacts the interface consists of many, approximately circular, contact regions. In two dimensions, the geometry of each of these contacts is the same as a specimen with double edge cracks and so crack growth data [13] can be used to find the increased deflection and hence stiffness of such cracks. At high percentage contacts the interface consists of many, approximately circular, gaps. This geometry is the same as a center crack and so crack data can again be used to obtain the deflection due to the crack and hence stiffness. No such data exists for more complicated geometries like those that occur at intermediate percentage contacts. At a given percentage contact the interface will consist of many, different sized, gaps and contacts each of which can be approximated as either a center crack or double edge crack. Each crack will deflect under the applied load and, assuming they all act independently, so the combined effect of all the cracks will be the average deflection of the cracks that make up the interface.

Using the interface models described above an approximate a spring constant, and hence reflection coefficient was calculated for each interfacial geometry. Fig.8 shows the predictions from center crack and double edge crack models together with the experimental results from one of the perspex-rubber interfaces and the predictions using stiffness obtained directly from the numerical contact model. The interface model curves are of similar form to the measurements and the quantitative agreement is promising. A possible explanation for the disagreement between measured and predicted reflection coefficients is in the measurement of the dynamic elastic modulus of the rubber. The modulus was calculated by measuring the shear reflection coefficient from a perfectly coupled perspex-rubber interface. Small errors in this reflection coefficient measurement become large errors in modulus. This work is continuing and it is hoped that the models will be improved.

CONCLUSIONS

The aim of this work was to allow the prediction of reflection coefficients from a solid-rubber interface under load. The numerical contact model gives the variation of percentage contact with load, so if the sensitivity of ultrasonic transmission to percentage contact is known then the load needed to achieve a given degree of coupling can be found.
However, ultrasonic transmission is not only dependent on the percentage contact but is also dependent on the size and distribution of the gaps and contacts. Reflection coefficients were calculated directly from the numerical contact model and from simple spring models and gave reasonable quantitative agreement with measurements, the agreement improving with increased percentage contact. The stiffnesses calculated using the approximate crack models agree with those calculated by obtaining average gap closure directly from the contact model.

This paper reports on work in progress and it is hoped that further investigation will lead to a fuller understanding of these results. In particular it is not known why the models agree well at 80-100% contact and less well below 80% contact.

In practice this work can be used to estimate the load required on a rubber-solid interface to achieve full coupling. Fig.6 can be used to obtain the value of percentage contact when full coupling is achieved, which is close to 100%. The numerical contact model can then be used to predict the load necessary to achieve this level of contact for any measured surface.

AKNOWLEDGEMENTS

The authors would like to acknowledge the support of the UK Engineering and Physical Sciences Research Council and thank Dr. R.B. Thompson and Dr. P.B. Nagy for useful discussions.

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