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Asymmetric Monetary Policy Rules For An Open Economy: Evidence From Canada and the UK

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Abstract

We present an analytical framework to examine the open economy monetary policy rule of a central bank under asymmetric preferences. The resulting policy rule is then empirically examined using quarterly data with regards to Canada, and the UK from 1983q1-2007q4. Our empirical investigation shows that the open economy policy rule receives support from the data and that the monetary policy makers in the UK and Canada have asymmetric preferences. Robustness checks based on model calibration provide support for the suggested policy rule.

Keywords: Monetary policy rule; asymmetric preferences; open economy; calibration.

JEL: E52; E58; F41.

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1 Introduction

It is well accepted that monetary policy plays an essential role in providing a stable macroeconomic background to facilitate the efficient allocation of resources. To demonstrate that such an economic environment can be achieved by adopting an optimal monetary policy framework, researchers have proposed several alternative models. For instance, a large number of studies advocate the adoption of inflation targeting and its implementation through variants of the Taylor rule.\(^1\) Yet researchers have mainly focused on closed economy models arguing that the impact of foreign factors on the monetary policy is small, and that their effects can be excluded.\(^2\)

However, given that exchange rates respond to foreign disturbances and, as a result affect domestic prices, it is somewhat surprising to overlook the importance of exchange rate movements on the monetary transmission mechanism. To that end, Ball (1999b) shows that although the variants of the Taylor rule are optimal in a closed economy framework these policies perform poorly in an open economy unless they are modified to account for the movements in the exchange rates. Svensson (2000) argues that the optimal reaction function in an open economy accounts for more information than that in a closed economy. He discusses the presence of various direct and indirect channels through which the exchange rate can affect monetary policy and shows that CPI inflation responds to foreign variables including the foreign inflation rate, the foreign interest rate, exchange rate and shocks from the rest of the world. Many other researchers, including Gali and Monacelli (2005), Lubik and Schorfheide (2007), and Adolfson et al. (2008), implement open economy DSGE models and show that exchange rate movements affect central bank behavior.

\(^1\)Researchers have examined different variants of the Taylor rule by introducing backward or forward looking components to linear or nonlinear objective functions. Among others see for instance Taylor (1993), Svensson (1997), Ball (1999a), Rudebusch and Svensson (1999), Ireland (1999), Clarida et al. (2000), Dolado et al. (2004), and Surico (2007).

\(^2\)For instance Taylor (2001) argues that changes in exchange rates are implicitly incorporated in prices and therefore closed economy models capture an open economy scenario. Clarida et al. (2001) document that open economy models are isomorphic to the closed economy models. Among others also see McCallum and Nelson (2000), Clarida et al. (2002), Batini et al. (2003), Dennis (2003), Leitemo and Söderström (2005), and D’Adamo (2011).
Researchers have also been challenging the common assumption that policy makers minimize a quadratic loss function subject to a linear IS equation and a linear Phillips curve. Cukierman and Gerlach (2003) suggest that a central bank responds strongly to inflation when the economy is expanding and to the output gap when the economy is contracting. Dolado et al. (2005) relax the assumption of a linear Phillips curve allowing for both inflation and the loss function to be convex functions of the output gap. Several other researchers, including Nobay and Peel (1998), Ruge-Murcia (2000, 2003), Dolado et al. (2004), Surico (2003, 2007), have also assumed that central banks accommodate a linear exponential (i.e. linex) loss function. The parameter estimates from such a model, which utilizes a linex loss function, allow the researcher to determine the asymmetric responses of the policy makers as the actual inflation or the level of output exceeds or falls short of the target. For instance, a positive value for the asymmetry parameter associated with inflation would suggest that the central bank is more worried about the inflation exceeding the set target level rather than falling below it. In this case, the policy maker is expected to take a more rigorous action when the target is exceeded in contrast to the opposite case.

In this paper, in contrast with the existing literature, we investigate the optimal open economy monetary policy rule of a central bank under the assumption that policy makers have asymmetric preferences. In doing so we first derive a closed form solution of the optimal monetary policy employing an open economy New-Keynesian model where the policy makers minimize a linear exponential loss function. Using data from two small economies, the UK and Canada, we then estimate the resulting policy rule and carry out a calibration exercise to check the validity of our modeling approach. In this framework, the certainty equivalence principle does not hold and uncertainty induces prudent behavior on the part of the central bank. Hence, the policy rule derived for our

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3 Note that since the quadratic loss function corresponds to a special case where the asymmetry parameter of the linex loss function is equal to zero, one can test the null hypothesis of quadratic preferences against the alternative of asymmetric preferences.

4 In this context, positive asymmetry implies that positive errors are more painfull than negative errors. Therefore, the central bank will implement policies that generate negative errors by overpredicting inflation.
framework differs from the earlier research as we demonstrate that the policy makers not only respond asymmetrically to deviations of inflation and the output gap from their respective targets, but they also react to changes in the real exchange rate, the base country output-gap and the base country real interest rate.

We estimate the resulting policy rule by implementing the generalized method of moments methodology for Canada, and the UK over 1983q1-2007q4. In our empirical investigation we take US as the base country. We find that monetary policy makers at the Bank of Canada (BoC) and the Bank of England (BoE) have asymmetric preferences. In particularly, we find that while policy makers at the BoC have negative output-gap asymmetry, those at the BoE have positive inflation and positive output-gap asymmetry. These observations should not be too surprising as earlier research, albeit using different analytical frameworks and empirical methodologies, have pointed out at the asymmetric preferences of policy makers. For instance, Komlan (2013), implementing threshold regression methodology, has investigated the asymmetric preferences of the Bank of Canada with regards to inflation and output gap and shown that the asymmetry parameter associated with inflation gap is statistically significant. For the UK, Taylor and Davradakis (2006), using threshold models, have provided evidence that although the stated objective of the Bank of England is to pursue a symmetric inflation target, the policy makers in practice respond rigorously once expected inflation is significantly above the 2.5% target. Likewise Boinet and Martin (2008) have shown that the Bank of England pursue an asymmetric inflation target but not for the output gap.\footnote{Also see Dolado et al. (2005) who presented evidence that several central banks behave nonlinearly since 1980’s, reacting more strongly to deviations above the targets than to deviations below the targets in inflation and output.} We differ from the above studies as our framework requires us to consider the effects of the real exchange rate and the output gap as well as the real interest rate of the base country (in our case the US) on the domestic policy rule while we examine asymmetric preferences of the policy makers. To that end our empirical investigation and the robustness checks based on model calibration provide support for the suggested policy rule.
The rest of the paper is organized as follows: Section 2 presents our analytical model. Section 3 discusses the empirical issues and the data. Section 4 lays out our empirical results while Section 5 concludes.

2 Theoretical model

In this section, we examine the asymmetric responses of policy makers to deviations of inflation and the output gap from their respective targets within the context of an open economy New-Keynesian framework similar to that in Lubik and Schorfheide (2007) and Walsh (2010). The dynamics of the open economy are given by the following three equations each of which describes the behavior of the output gap, inflation and the exchange rate, respectively.

\[ x_t = \alpha_0 + \alpha_1 E_t x_{t+1} - \alpha_2 (i_t - E_t \pi_{t+1}) - \alpha_3 (E_t q_t - q_t) + \alpha_4 (E_t y_{t+1}^f - y_t^f) + \varepsilon_{t+1}^y \]  
\[ \pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 x_t - \beta_3 (E_t q_t - q_t) + \beta_4 (q_t - q_{t-1}) + \varepsilon_{t+1}^\pi \]  
\[ q_t = E_t q_{t+1} - (i_t - E_t \pi_{t+1}) + (i_t^f - E_t \pi_{t+1}^f) \]  

It is worth noting that (1) and (2) are reduced form equations that one can obtain from an open economy new-Keynesian DSGE model. These two equations can be rewritten as:

\[ x_t = \frac{\rho}{\sigma_\gamma} + E_t x_{t+1} - \frac{1}{\sigma_\gamma} (i_t - E_t \pi_{t+1}) - \frac{\gamma}{\sigma_\gamma (1 - \gamma)} (E_t q_t - q_t) + \left[ \frac{(\sigma - \sigma_\gamma) - \gamma (1 - \nu) (\eta + \sigma_\gamma)}{\eta + \sigma_\gamma} \right] (E_t y_{t+1}^f - y_t^f) + \left( \frac{1 + \eta}{\eta + \sigma_\gamma} \right) \varepsilon_{t+1}^y \]  
\[ \pi_t = \beta E_t \pi_{t+1} + (\sigma_\gamma + \eta) \left( \frac{1 - \omega}{1 - \omega} \right) (1 - \beta \omega) x_t - \frac{\beta \gamma}{1 - \gamma} (E_t q_t - q_t) + \frac{\gamma}{1 - \gamma} (q_t - q_{t-1}) + \varepsilon_{t+1}^\pi \]
with

\[ x_t = y_t - \tilde{y}_t, \]

\[ \tilde{y}_t = \frac{\sigma - \sigma_\gamma y_f^t}{\eta + \sigma_\gamma} - \frac{1 + \eta}{\eta + \sigma_\gamma} \xi^t, \]

\[ \rho = \beta^{-1} - 1, \quad \sigma_\gamma = \frac{\sigma}{1 - \gamma^2(1 - \alpha \sigma)}, \]

\[ \nu = \alpha \sigma + (\alpha \sigma - 1)(1 - \gamma), \]

where \( \beta \) is the discount factor, \( \gamma \) is the share of foreign goods in the consumer price index, \( \alpha \) is the intertemporal elasticity of substitution between home and foreign consumption goods and \( \sigma \) is the intertemporal elasticity of substitution.\(^6\) Superscript \( f \) denotes the base country variables.

Note that \( \beta_2 = (\sigma_\gamma + \eta) \frac{(1 - \omega)(1 - \beta \omega)}{\omega} \) is the price stickyness parameter, where \( (1 - \omega) \) denotes the fraction of firms that set their prices optimally at each point of time. The intertemporal rate of substitution of labor supply is captured by \( \eta \). Equation 4 presents an open economy forward looking aggregate demand curve (IS-curve) and equation 5 is an open economy New-Keynesian Phillips curve. At any point in time \( t \), the domestic output gap is denoted by \( x_t \), the domestic nominal interest rate is \( i_t \), inflation is \( \pi_t \) and the real exchange rate, defined as the ratio of cost of foreign goods measured in domestic currency relative to domestic goods is \( q_t = SP^*/P^c \), where \( S \) is the nominal exchange rate. \( P^* \) and \( P^c \) denote the prices of goods produced in foreign and home countries, respectively. Given the information set at time \( t \) the expected value of variable \( u_{t+1} \) is denoted by \( E_t u_{t+1} \).

\(^6\)Equations (4) and (5) are similar to Walsh (2010): \( x_t = E_t x_{t+1} - \frac{1}{\gamma_\pi} (i_t - E_t \pi^*_t - \tilde{\rho}_t) \) (9.92) and \( \pi^*_t = \beta E_t \pi^*_{t+1} + k(\sigma_\gamma + \eta) x_t \) (9.93), where \( \tilde{\rho}_t = \rho + \sigma_\gamma (E_t y^t_{f+1} - y^t_f) \), \( \rho^* = \rho - \sigma_\gamma (1 - v) (E_t y^t_{f+1} - y^t_f) \) and \( \pi^*_t \) is domestic good inflation. Note \( \pi^*_t = \pi_t - \gamma \Delta \delta_t \), where \( \pi_t \) is the CPI inflation, \( \delta_t \) is the terms of trade, and \( \gamma \) is the share of foreign produced goods in the consumer price index. In this framework, the law of one price is assumed to hold such that \( P^f = S \cdot P^* \) where \( P^* \) is the foreign currency value of foreign produced goods and \( S \) is the nominal exchange rate. The terms of trade is defined as the ratio of foreign good prices to domestic good prices \( \delta_t = SP^*/P^h \) where \( P^h \) is the average price of domestically produced consumption goods, and the real exchange rate is defined as \( q_t = SP^*/P^c = \delta_t (P^h/P^c) \), where \( P^c \) denotes the price level in the domestic country. Hence, it can be shown that the real exchange rate and the terms of trade is linked as \( (1 - \gamma) \delta_t = q_t \). If we substitute \( \tilde{\rho}_t, \rho^*, \pi^*_t \) and \( \delta_t \) into (9.92) and (9.93), we obtain (4) and (5).
Equation (1), which can be rewritten as in (4), is the open economy forward-looking aggregate demand curve. This equation implies that the expected course of output ($E_t x_{t+1}$) has a positive effect while the real domestic interest rate ($i_t - E_t \pi_{t+1}$) has a negative effect on output. It also assumes that the expected changes in the real exchange rate, ($E_t q_{t+1} - q_t$), has a negative effect on the output gap. This is because for a given ($E_t \pi^F_{t+1}$) an increase in ($E_t q_{t+1} - q_t$) implies a fall of expected domestic good prices and therefore a reduction in the current domestic inflation.\(^7\) This will lead to an appreciation of domestic currency and switch demand from domestic to foreign goods. Hence, positive expected changes in the real exchange rate should exert a negative impact on output (and inflation as discussed below). Finally, the impact of foreign output growth on domestic output-gap depends on the underlying structural parameters. In particular, we expect for high values of intertemporal rate of substitution, and elasticity of substitution between domestic and foreign goods (i.e. $\alpha, \sigma$) an increase in foreign output to have positive impact on domestic output. However, for other plausible values of $\alpha, \sigma$ and $\gamma$—the share of foreign produced goods consumed in the domestic country—that researchers have noted in the literature, expected changes of foreign output may have no impact on domestic output.

Equation (2) describes an open-economy New-Keynesian Phillips curve. This equation allows the price setters to adjust the current prices accounting for future marginal costs. In that sense this equation captures a Calvo-type world in which the price adjustments take place with a constant probability in a given time. The difference of this equation from its closed economy version is the inclusion of current and expected future changes in real exchange rates.\(^8\) Here, a rise in the real exchange rate implies a relative increase in the price of foreign goods in domestic currency. Because foreign goods are consumed

\(^7\)Here, expected foreign inflation is considered as given because the foreign country is large compare to the domestic country. Note that $E_t \Delta q_{t+1} = (1 - \gamma) E_t \Delta \pi_{t+1} = (1 - \gamma) (E_t \pi^F_{t+1} - E_t \pi^D_{t+1})$. Therefore, given $E_t \pi^F_{t+1}$, an increase in $E_t \Delta q_{t+1}$ is driven by a fall of expected domestic inflation (i.e. $E_t \pi^D_{t+1}$).

\(^8\)Several other researchers, including Svensson (2000), Ball (1999b) and Leitemo et al. (2002), relate inflation to changes in real exchange rate. Ball (1999b) argues that changes in the real exchange rate affects the inflation rate by the import price pass through mechanism which constitute an indirect impact of exchange rate on domestic inflation.
by the home country, a rise in foreign goods prices will lead to an increase in consumer price inflation. However, an expected future rise in the real exchange rate should reduce current inflation. This is due to the fact that for a given $E_t \pi_{t+1}$, a rise in the expected real exchange rate implies a fall in expected future domestic inflation. Hence, the current domestic inflation should be negatively effected.\footnote{See Walsh (2010) along these lines.}

Equation (3) suggests that the real exchange rate is determined according to the UIP conditions. The foreign nominal interest rate and the foreign expected inflation rate are denoted by $i_f^t$ and $\pi_f^{t+1}$, respectively. Hence, the first and the second parenthesized terms capture the domestic and the foreign real interest rates at time $t$. Equation (3) shows that an expected future increase in the domestic real interest rate leads to an appreciation of the exchange rate as the domestic assets become more attractive. This equation also shows that an increase in the foreign (domestic) real interest rate will result in depreciation (appreciation) of the real exchange rate.

2.1 The objective function

Similar to the literature, we assume that the policy makers choose the interest rate based on the information available at the beginning of time $t$, before the economic shocks are realized. The policy authorities minimize the following intertemporal loss function:

$$\min E_{t-1} \sum_{\tau=0}^{\infty} \beta^\tau L_{t+\tau}$$

subject to the dynamics described in Equations (1-3). In Equation (6), $\beta$ is the discount factor and $L_t$ stands for the loss function of the central bank at time $t$. The objective of the central bank is to choose a path for its instrument, the short term interest rate, to minimize the expected loss.

Here, we use a linear exponential (linex) loss function that allows policy makers to weigh positive and negative deviations of output gap and inflation from their respective
targets differently.\textsuperscript{10} Setting the output gap target to zero for simplification purposes, the loss function takes the following form:

$$L(\pi_t, y_t) = \frac{e^{\mu(\pi_t - \pi^*)} - \mu(\pi_t - \pi^*) - 1}{\mu^2} + \frac{e^{\psi y_t} - \psi y_t - 1}{\psi^2}$$

(7)

where the parameters $\mu$ and $\psi$ capture asymmetries in the objective function with respect to inflation and output gap, respectively. The policy preference towards inflation stabilization is normalized to one and $\lambda$ represents the relative aversion of the policy maker towards output fluctuations around its long run level. The inflation target set by the central banker is denoted by $\pi^*$.

The significance of $\mu$ and $\psi$ identifies whether the policy maker has an asymmetric response towards inflation and output gap, respectively. For instance, a positive value for $\mu$ implies that the central bank is more worried about inflation exceeding the set target level $\pi^*$ than falling below it. This is because if $\mu > 0$ and $(\pi_t - \pi^*) > 0$ then the exponential term ($e^{\mu(\pi_t - \pi^*)}$) will rule over the linear component. Thus, positive deviations of inflation from the target level $\pi^*$ will have a dominant effect on policy makers’ loss function. The reverse is true if $\mu < 0$ and $(\pi_t - \pi^*) < 0$. In a similar vein, we can argue that should the central bank place more weight to output contractions ($y_t < 0$), then $\psi$ must take a negative value such that the exponential in the second term ($e^{\psi y_t}$) plays the dominant role. However, if the policy maker is worried about the economy overshooting its long run growth ($y_t > 0$), then we should observe a positive value for $\psi$.

Besides the idea that the policymakers can have asymmetric preferences regarding the state of inflation and output gap, the linex function allows discretion on the part of the central bank so that the higher moments of inflation and the output gap might play an important role in designing optimal policy rules (see Kim et al. (2005)). In particular, given that the certainty equivalence does not hold under asymmetric preferences, uncertainty about inflation and the output gap induces a prudent behavior on the part

\textsuperscript{10}The linex loss function was proposed by Varian (1974). Subsequently, many researchers used this function to examine optimal policy reaction function of policy makers.
of the central bank. This is because uncertainty increases the expected marginal cost of inflation and output gap when these variables deviate from their respective targets. Hence, this framework can help us understand whether the business cycle fluctuations have welfare effects beyond the first order or not. Finally, the model nests the quadratic preferences as a special case. The loss function reduces to symmetric parametrization when both \( \mu \) and \( \psi \) are equal to zero, which can be empirically tested.

2.1.1 Solution of the model

To solve the model, we first substitute Equation (3) into (1) and (2). After rearranging the terms, we obtain:

\[
y_t = \alpha_0 + \alpha_1 E_t y_{t+1} - (\alpha_2 + \alpha_3)(i_t - E_t \pi_{t+1}) + \alpha_3(i_t^f - E_t \pi_{t+1}^f) - \alpha_4(E_t y_{t+1}^f - y_{t+1}^f) + \varepsilon_{t+1}^y \quad (8)
\]

\[
\pi_t = \alpha_0 \beta_2 + \beta_1 E_t \pi_{t+1} + \alpha_1 \beta_2 y_{t+1} - [\beta_2(\alpha_2 + \alpha_3) + \beta_3](i_t - E_t \pi_{t+1}) + (\beta_2 \alpha_3 + \beta_3)(i_t^f - E_t \pi_{t+1}^f) + \beta_4(q_t - q_{t-1}) - \alpha_4 \beta_2(E_t y_{t+1}^f - y_{t+1}^f) + \beta_2 \varepsilon_{t+1}^y + \varepsilon_t^\pi \quad (9)
\]

Next, we substitute (8) and (9) into Equation (7) and obtain the following first order condition with respect to the current interest rate, \( i_t \):

\[
E_{t-1} \frac{\partial L(\pi_t, y_t)}{\partial i_t} = -(\beta_2 \alpha_2 + \beta_2 \alpha_3 + \beta_3) \mu E_{t-1} [e^{\mu (\pi_t - \pi^*)} - 1] - \frac{\lambda(\alpha_2 + \alpha_3)}{\psi} E_{t-1} [e^{\psi y_t} - 1] \quad (10)
\]

Assuming that the demand and supply shocks (\( \varepsilon_t^y \) and \( \varepsilon_t^\pi \)) are normally distributed, the expected value of the exponential terms takes the form \( e^{\mu (\pi_{t-1} - \pi^* + \frac{\mu}{2} \sigma_{\pi,t}^2)} \) and \( e^{(\frac{\psi}{2} \sigma_{y,t}^2)} \), where \( \sigma_{\pi,t}^2 \) and \( \sigma_{y,t}^2 \) denote the conditional variance of inflation and output gap, respec-
tively.\textsuperscript{11} Thus, we can rewrite Equation (10) as follows:

\begin{equation}
E_{t-1} \frac{\partial L(\pi_t, y_t)}{\partial i_t} = \frac{-(\beta_2 \alpha_2 + \beta_2 \alpha_3 + \beta_3)}{\mu} \left[ e^{(\mu(E_{t-1} \pi_t - \pi^*) + (\frac{\mu^2}{2})\sigma_{\pi t}^2) - 1} - \frac{\lambda}{\psi}(\alpha_2 + \alpha_3)[e^{(\frac{\psi^2}{2})\sigma_{y t}^2} - 1] \right]
\end{equation}

(11)

After linearizing\textsuperscript{12} Equation (11),

\begin{equation}
E_{t-1} \frac{\partial L(\pi_t, y_t)}{\partial i_t} = \frac{-(\beta_2 \alpha_2 + \beta_2 \alpha_3 + \beta_3)}{\mu} \left[ (\mu(E_{t-1} \pi_t - \pi^*) + (\frac{\mu^2}{2})\sigma_{\pi t}^2) \right] - \frac{\lambda}{\psi}(\alpha_2 + \alpha_3)(\frac{\psi^2}{2}\sigma_{y t}^2),
\end{equation}

we can solve for the expected inflation as:

\begin{equation}
E_{t-1} \pi_t = \pi^* - \frac{\mu}{2} \sigma_{\pi t}^2 - \frac{\lambda(\alpha_2 + \alpha_3)}{(\beta_2 \alpha_2 + \beta_2 \alpha_3 + \beta_3)}[(\frac{\psi}{2})\sigma_{y t}^2] \tag{12}
\end{equation}

Last, we use equations (8) and (12) to obtain the central bank’s forward looking policy rule:

\begin{equation}
i_t = \varphi_0 + \varphi_1 E_{t-1} y_{t+1} + \varphi_2 E_{t-1} \pi_{t+1} + \varphi_3 (q_t - q_{t-1}) + \varphi_4 E_{t-1}(i^f_t - \pi^f_{t+1}) + \varphi_5 (E_t y^f_{t+1} - y^f_{t+1}) + \varphi_6 \sigma_{\pi t}^2 + \varphi_7 \sigma_{y t}^2 + \text{(error)} \tag{13}
\end{equation}

The policy rule given in Equation (13) differs from the standard Taylor rule on three facets. First, it incorporates forward looking expressions of output gap and the rate of inflation. Second, the policy rule contains variables including the real exchange rate, foreign interest rate, and the foreign output gap that reflect the fact that we are examining the central bank’s policy rule in an open economy environment. Third, it allows us to examine whether the policy makers have asymmetric preferences through the significance of coefficients associated with output gap volatility and inflation rate volatility. Last, based on the parametrization of equations (1-5), we can show that the reduced form

\textsuperscript{11}Recall that output gap is assumed to be normally distributed with zero mean. Hence, \(E_{t-1} \exp(\psi y_t)\) is log normally distributed with mean equal to \(e^{\frac{\psi^2}{2}}\sigma_{y t}^2\).

\textsuperscript{12}See Uhlig (1999) for a thorough explanation of the linearization process.
parameters of the policy rule, \((\varphi_i)\), which we use to sign the coefficients and to carry out our calibration exercise, take the following form:

\[
\begin{align*}
\varphi_0 &= \frac{\alpha_0 \beta_2 - \pi^*}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}, \quad \varphi_1 = \frac{\alpha_1 \beta_2}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}, \quad \varphi_2 = \frac{\beta_1 + (\alpha_2 + \alpha_3)\beta_2 + \beta_3}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3} \\
\varphi_3 &= \frac{\beta_4}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}, \quad \varphi_4 = \frac{\alpha_3 \beta_2 + \beta_3}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}, \quad \varphi_5 = \frac{\alpha_4 \beta_2}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3} \\
\varphi_6 &= \frac{\mu/2}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}, \quad \varphi_7 = \frac{\lambda(\alpha_2 + \alpha_3)(\psi/2)}{[\{(\alpha_2 + \alpha_3)\beta_2 + \beta_3\}]^2}
\end{align*}
\]

3 Empirical issues

The policy rule in Equation (13) embodies expected future domestic and foreign output gap, and expected inflation. To compute the expected values (forecasts) of these variables we utilize autoregressive models where the model lag length is determined by the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). As an alternative, we also use bivariate vector autoregressive models (VAR) to forecast inflation and the output gap. We select optimal forecasting models on the basis of a forecast rationality test and on the root mean square forecast error (RMSFE) criterion.\(^{13}\) This exercise shows that the forecasts based on the AR approach outperforms the forecasts generated by the VAR approach.\(^{14}\) Subsequently, we use forecasts obtained from the AR models to estimate Equation (13). It should be noted that the foreign real interest rate is calculated as the deviation of nominal interest rate from the expected inflation rate of the base country (the USA).

To estimate equation (13) we must also generate a proxy for both inflation and output gap volatilities. To generate our inflation and output gap volatility measures, we implement ARCH/GARCH methodology. As Pagan (1984) and Pagan and Ullah (1988) point out, the use of generated regressors may lead to some problems in estimation and

\(^{13}\)Tests for forecast rationality indicate whether forecast errors are not significantly different from zero.

\(^{14}\)In particular, while both AR and VAR models generate unbiased forecasts there is evidence that the forecasts produced by the former approach yield lower RMSFE than the latter approach. Note that the Diebold and Mariano (1995) test accept the null hypothesis that forecasts generated by AR and VAR models are equal. These results are available from the author upon request.
statistical inference. According to Pagan (1984) although one may overcome these problems by using instrumental variables approach, the use of lagged series as instruments may not be possible when the variable under consideration is a function of the entire history of the available data. In such cases, Pagan and Ullah (1988) suggest testing the validity of the underlying assumptions of the model that is used to generate the proxy.\(^{15}\) We generate output gap and inflation volatility measures implementing GARCH\((p,q)\) or ARCH\((p)\) models. After we carefully check whether these models are well specified and there is no any neglected heteroscedasticity, we use lags of these proxies as instruments in estimating our model.

We estimate Equation (13) implementing the generalized method of moments (GMM) technique as we replace the unobserved expectations with forecasts derived from AR models and the volatility terms with proxies derived from GARCH models. In doing so we face two major issues concerning the instruments employed in estimation. First, the reliability of our econometric methodology depends crucially on the validity of the instruments which we evaluate by computing the Sargan–Hansen J test of overidentifying restrictions. A rejection of the null hypothesis that instruments are orthogonal to errors would indicate that the estimates are not consistent. We also test for the presence of the first and second order serial correlation so as to determine the correct lag structure of the instrument set. For each model we present below, the Hansen J statistic and the autocorrelation tests show that our instruments are appropriate and our models do not suffer from serial correlation.

An important problem in implementing the GMM methodology is the possibility that the instruments could be weak. Weak instruments will distort the distribution of the estimators and the test statistics will lead to misleading statistical inference.\(^{16}\) For reliability of the instrumental variables approach, the instruments should be relevant and strongly correlated with the endogenous variables. In our case, the weak identification

\(^{15}\)Many other researchers have followed a similar approach. See for instance Ruge-Murcia (2003).

\(^{16}\)For a review of weak instruments see for instance Stock et al. (2002). For the impact of weak instruments on statistical inference see for instance Hansen et al. (1996).
test proposed by Stock and Yogo (1999) might not be applicable because their approach
does not account for non i.i.d. errors from a model such as (13) which is estimated using
GMM.\textsuperscript{17} An alternative approach is to use the Kleibergen and Paap (2006) \textit{rk} statistic,
which is an F-test accounting for autocorrelation and heteroscedasticity. However, there
is no formal basis to compare the Kleibergen-Paap F-test statistic to the critical values
of Stock and Yogo (1999). Thus, when using the \textit{rk} statistic for weak identification one
should use the critical values of Stock and Yogo (1999) with a caution.\textsuperscript{18} Instead of relying
on the Kleibergen-Paap F-test statistic, we also use inference methods that are robust
to weak instruments.\textsuperscript{19} This includes the Anderson-Rubin (\textit{A} − \textit{R}) test (Anderson and
Rubin (1949)) which is robust to weak instruments because the null hypothesis is less
likely to be rejected as instruments become irrelevant.\textsuperscript{20} For all models that we report
below, we observe that the Kleibergen-Paap F-test statistic is above the critical values
given in Stock and Yogo (2005) and the \textit{A} − \textit{R} Wald test reject the null of irrelevant
instruments. Hence, we conclude that our models do not suffer from the weak instru-
mentation problem. Furthermore, test statistics for underidentification provide evidence
that the null of underidentification is rejected for all models.

3.1 Data sources and definition of variables

To estimate the policy rule given in Equation (13), we employ quarterly data for Canada
and the UK while we consider the US as the base country. To avoid the impact of
the 2008 financial crises, we set the end date of our investigation as of the last quarter
of 2007 and cover the period between 1983q1-2007q4. We implement the HP (Hodrick
\textsuperscript{17}The test statistic proposed by Stock and Yogo (1999) is an F-test, which requires the assumption
that the estimated residuals from (13) follows an i.i.d. process.
\textsuperscript{18}In particular, we apply the ‘rule of thumb’ suggested by Staiger and Stock (1997) that the F-
statistic should be at least 10 for weak identification. Note that this threshold increases with the number
instruments.
\textsuperscript{19}These procedures are robust to weak instruments in the sense that they have the correct size re-
gardless of the value of the concentration statistics. The concentration statistics can be considered as
the smallest eigenvalue of the concentration matrix which can be thought as a multivariate signal-noise
ratio obtained from the first stage regression of the endogenous variables on the instruments.
\textsuperscript{20}For a detailed discussion on robust inference with weak instruments see Stock et al. (2002) and Baum
et al. (2007).
and Prescott (1997)) filter to compute the output gap for each country. We use the overnight interbank rate for the UK and the overnight money market rate for Canada. In estimating our model, we use CPI inflation rate as suggested in Svensson (2000). The data are collected from the international financial statistics (IFS) database published by the International Monetary Fund (IMF) and the datastream database.

4 Results

To our knowledge the analytical model that we have derived is the first to explore the optimal policy rule of a central bank taking into account the asymmetric preferences of the policy makers within the context of an open economy New-Keynesian framework. In what follows, using data from Canada and the UK, we present our empirical findings and a calibration exercise that we carried out to check for the robustness of our findings.

Overall, we have two sets of key results. First, we observe that the monetary policy reacts to inflationary pressures driven by both domestic and foreign factors. That is the central bank not only reacts to changes in expected inflation and output gap but also to changes in the real exchange rate, the foreign real interest rate and the foreign output gap. Second, we provide empirical evidence that central banks have asymmetric preferences.

The empirical results for Canada and the UK are given in Tables 1 and 2, respectively. Each table presents four variants of Equation (13). In each model we use four quarter ahead forecast horizon to proxy the forward looking variables. The first column depicts results for the full open economy model which assumes that the policy makers use an asymmetric loss function with respect to both inflation and output gap. The model in column 2 relaxes the assumption of asymmetry for output gap only; the model in column 3 relaxes the assumption of asymmetry for inflation only; and the model in column 4 relaxes the assumption of asymmetry for both inflation and output gap.

\[^{21}\text{In our empirical analysis, we do not examine domestic inflation for it is more relevant in estimating the policy rule of a closed economy.}\]
4.1 Results for Canada

Table 1 provides our results for Canada. In all columns of this table, we observe that the impact of expected output gap ($\varphi_1$) on the monetary policy rule is negative. The negative sign is expected because, unlike in the case of standard policy rules, the impact of expected output on nominal interest rate in our optimal policy rule is driven by a set of underlying parameters which render the output gap coefficient to take a negative sign.

In particular, observing the underlying parameters of $\varphi_1 = \frac{\alpha_1 \beta_2}{(\alpha_2 + \alpha_3)\beta_2 + \beta_3}$ we can note that the numerator takes a positive sign $\alpha_1 \beta_2 > 0$ while the denominator takes a negative sign, $[(\alpha_2 + \alpha_3)\beta_2 + \beta_3] < 0$. Similar to the existing models, the impact of expected inflation on the policy rule should be positive. In fact, for all cases we find that the impact of expected inflation on the policy rule is positive ($\varphi_2 > 0$) and significant.

We next focus on the impact of the real exchange rate and that of the real foreign interest rate on the policy rule. The table shows for all cases that the coefficient associated with the real exchange rate is negative and significant ($\varphi_3 < 0$) and that with the real foreign interest rate is positive and significant ($\varphi_4 > 0$). These observations are consistent with our model.\textsuperscript{22} Currency depreciation will cause an increase in current and expected inflation leading to lower domestic output and nominal interest rate. In contrast, an increase in the foreign interest rate will generate an expected appreciation of the real exchange rate, putting an upward pressure on the domestic inflation and hence on the domestic interest rate.\textsuperscript{23} We also observe that the base country output gap (in our context the US output-gap) has a positive impact ($\varphi_5 > 0$) on the optimal policy rule. This might be driven by high elasticity of substitution across time and goods (i.e. domestic and foreign). Under such circumstances, foreign output will have positive effects on domestic

\textsuperscript{22} Given that $\varphi_3 = \beta_4/((\alpha_2 + \alpha_3)\beta_2 + \beta_3)$, the sign of this coefficient should be negative as the numerator is positive ($\beta_4 > 0$; see equation (1)) and the denominator is negative $((\alpha_2 + \alpha_3)\beta_2 + \beta_3) < 0$; see equations 1 and 2). Likewise, given $\varphi_4 = (\alpha_3 \beta_2 + \beta_3)/((\alpha_2 + \alpha_3)\beta_2 + \beta_3)$, the sign of this coefficient should be positive as both the numerator and the denominator take a negative sign as discussed earlier.

\textsuperscript{23} The real UIP indicates that a rise of foreign real interest rate will lead to an appreciation of the expected real exchange rate ($E_t q_{t+1} - q_t < 0$). For a given expected foreign inflation ($E_t \pi_{F,t+1}^P$), an expected appreciation will be generated by an increase in expected domestic inflation ($E_t \pi_{t+1}^P$) putting an upward pressure on current interest rate. Here, an increase of foreign interest rate will have a positive impact on domestic interest rate.
output and nominal interest rates.

When we inspect the coefficients that capture the asymmetric preferences of the policy makers we gather the following observations. We observe that the coefficient associated with inflation volatility ($\varphi_6$) is not significant, and that with output-gap asymmetry ($\varphi_7$) is negative and significant. This implies that policy makers have negative output-gap asymmetry ($\psi < 0$), supporting the view that BoC is more averse to recessions than to expansions. That is BoC acts more rigorously when output falls below the set target level than when it exceeds it. This observation can be interpreted in two ways. Firstly, one can suggest that the BoC pursues a policy that favors output growth when the output falls below its long run trend. However, this interpretation is in conflict with the mandate of the Bank which does not raise any claims about achieving output stability through the use of its instruments. Alternatively, one may argue that negative output gap implies future declines in the rate of inflation which can destabilize the economy were the interest rates to approach towards the lower bound. Therefore, a negative output gap would initiate a more rigorous response than the case when the gap exceeds the set target. This alternative interpretation agrees with the objectives of the BoC.

It should be noted that although the results presented in columns 2–4 are consistent with the full model in column 1, the Wald tests concerning the joint significance of the coefficients associated with the exchange rate and the base country real interest rate and that with the volatility terms reject the null of non-significance across all four models. In the light of this observation, we argue that the variables that relate to the open economy and those that capture the asymmetric preferences are relevant in constructing a policy rule for Canada.

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24Earlier Claus (2000) has shown for New Zealand that when the output gap is negative (positive) two times out of three inflation will decrease (increase) in the next quarter and three times out of five it will decrease (increase) the following year.
4.2 Results for the UK

Table 2 presents our findings for the UK. Inspecting the table, we see that the coefficient estimates of the expected output-gap ($\varphi_1$) is insignificant for all cases. The impact of expected inflation on the interest rate is significant and positive ($\varphi_2 > 0$). When we examine the impacts of the real exchange rate and the foreign real interest rate on the policy measure, we see that their effects are similar to that for Canada. The real exchange rate has a negative and a sizable impact on the UK interest rate ($\varphi_3 < 0$). This observation is consistent with our model and suggests that a devaluation of the real exchange rate will lead to a reduction in domestic output putting a downward pressure on the domestic interest rate. In contrast, we find that the foreign real interest rate has a positive and significant impact on the domestic interest rate ($\varphi_4 > 0$): an increase in the foreign interest rate leads to an expected appreciation of the real exchange rate putting an upward pressure on the domestic interest rate. Similar to the case of Canada we find that the impact of foreign output on domestic interest rate is positive ($\varphi_5 > 0$) and insignificant.

When we examine whether the BoE responds asymmetrically towards inflation or output gap, we find that both of the coefficients associated with inflation volatility and output-gap volatility ($\varphi_6$ and $\varphi_7$) are positive and significant. Thus, the BoE has a positive inflation and output-gap asymmetry (i.e., $\mu > 0$ and $\psi > 0$). This observation suggests that the BoE reacts more rigorously when the output-gap is positive so that future inflation can be contained. Likewise, when expected inflation is above the target, the bank acts rigorously to bring the inflation level closer to its target.

As in the case of Canada, the coefficient estimates across all variants of the model are similar. The Wald tests concerning the joint significance of the coefficients associated with the base country (the US) variables and that with the volatility terms reject the null of non-significance across all four models. Here, too, we conclude that the inclusion of the variables that capture the open economy features along with those that capture policy asymmetry is important for the UK.
4.3 Calibration

In this section we calibrate our structural models using the parameter estimates reported in Tables 1-2 to assess which of the four models represent the data better. More specifically, although the structural parameters \((\pi^*, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \alpha, \sigma, \gamma)\) are not identifiable from the estimated reduced-form coefficients \((\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7)\), we can estimate the free parameters after setting four of the structural parameters to the values reported in the literature. In particular, given the estimates for \(\alpha_i (i = 0, 1, \ldots, 4)\), we can compute the inflation asymmetry parameter \(\mu\). Also for different values of policy preference \((\lambda)\) and \(\beta_j (j = 1, 2, \ldots, 4)\) we can compute the output-gap asymmetry parameter \(\psi\). We, then, calibrate the remaining parameters to check if their values fall within a reasonable interval. In Table 3, we present our observations for models 1 and 2 for which we obtain theoretically meaningful results. We, next, discuss the implications of our findings.

The calibration of the structural parameters includes two stages. In the first stage we compute the parameters in Equations (1-3)—\((\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4)\). In the second stage, we use the estimated parameters from the first stage to compute \(\alpha, \sigma\) and \(\gamma\). In particular, we set \(\beta_1\) equal to the sample average value of \(\exp(-\frac{i}{400})\), where \(i\) is the short-term nominal interest rate \(\beta_2 \in [-0.9, 0.9]\), \(\gamma \in [0.05, 0.7]\) and \(\pi^* = \pi\) where \(\pi\) is the sample average inflation. Note that the selected range of values of \(\gamma\) is based on Lubik and Schorfheide (2007) and Chen and MacDonald (2012). We then compute \(\beta_3 = \frac{\beta_1 \gamma}{1 - \gamma}\) for different values of \(\gamma\). We assume \(\alpha_1 = 1\) and we construct \(\Phi\) for each value of \(\beta_2\) as:\(^{25}\)

\[
\Phi = \frac{\beta_2}{\varphi_1}
\]

\(^{25}\)In the New-Keynesian model \(\alpha_1 = 1\).
Thereafter, one can show that

\[ \alpha_0 = \frac{\varphi_0 \Phi + \pi^*}{\beta_2}, \quad \alpha_3 = \frac{\varphi_4 \Phi - \beta_3}{\beta_2}, \quad \alpha_2 = \frac{\Phi - \beta_3 - \alpha_3 \beta_2}{\beta_2} \]

\[ \alpha_4 = \frac{\varphi_5 \Phi}{\beta_2}, \quad \beta_4 = \varphi_3 \Phi, \quad \mu = 2\Phi \varphi_7, \quad \text{and} \quad \psi = \frac{2\Phi^2 \varphi_6}{\lambda(\alpha_2 + \alpha_3)} \]

We select the optimal parameters by minimizing the distance between the actual and fitted values of the output gap and inflation by calculating the minimum mean square error (MMSE). Having estimated the demand and supply parameters, we can compute the deep structural parameters concerning the intertemporal elasticity of substitution between home and foreign consumption goods (\( \alpha \)) and the intertemporal elasticity of substitution (\( \sigma \)). In particular, we compute\(^{26}\):

\[ \sigma = \sigma_\gamma [1 - \gamma (1 - \nu)] \]

where \( \sigma_\gamma = \frac{1}{\alpha_2} \) and \( \nu = \frac{\gamma - \alpha_4}{\gamma} \).\(^{27}\) Next, we can solve for \( \alpha \) that

\[ \alpha = \frac{1 + \nu - \gamma}{(2 - \gamma)\sigma} \]

### 4.4 Calibration results

Table 3 presents the structural parameters for Canada and the UK obtained from our calibration exercise. We find that model 1 performs better for Canada providing support for our argument that the BoC not only reacts to foreign variables but also accounts for the uncertainty concerning the deviation of inflation and output gap from their respective targets. In the case of the UK, we find that model 2 performs better than the other alternatives implying that the BoE reacts to foreign variables and to inflation uncertainty.

For both countries, the computed structural parameters are within plausible values. Although the impact of output gap on inflation is close to zero (\( \beta_2 = 0.04 \) for Canada,

\(^{26}\)For the definition of \( \sigma \) see Walsh (2010), pp. 438.

\(^{27}\)Note that \( \alpha_4 = \frac{\gamma(1 - \nu)}{\gamma} \).
and $= 0.08$ for the UK), this observation is similar to that of Rudd and Whelan (2007) and Mavroeidis et al. (2013) who find that the impact of inflation on output gap is not significantly different from zero. We also observe that in all cases both current and expected values of the real exchange rates affect output and inflation equations with the correct sign. Expected changes of the real exchange rate exerts a negative impact on output (Equation 1, $\alpha_3$) and inflation (Equation 2, $\beta_3$), while the current changes in the real exchange rate has a positive impact on inflation (Equation 2, $\beta_4$). This is consistent with our observations in section 4 regarding the reaction of central banks to changes in the real exchange rate for the UK and Canada.

It is worth noting that the impact of expected changes in real exchange rate is a function of the rate of substitution between domestic and foreign goods as well as the intertemporal rate of substitution. Estimates of the import share parameter $\gamma$ is rather low. However, this might be a reflection of the impact of deviations from the UIP on the demand and supply curves. The computed value of $\alpha$ is positive and below one. This implies that domestic and foreign goods are substitutes. Thus, an expected appreciation of the real exchange rate, driven by a decline in domestic inflation, will switch demand from foreign goods to domestic goods. The intertemporal rate of substitution coefficient ($\sigma$) found to be positive and lower than 1 for Canada and greater than 1 for the UK. This implies that agents smooth consumption more aggressively in the UK than in Canada.

Overall, the calibration exercise confirms that in formulating a monetary policy rule it is important to work within an open economy framework. This is because changes in the real exchange rate and variables such as base country real interest rate and output have significant impact on domestic output and inflation. Thus omission of open economy variables from the model would potentially lead to policy rules which may destabilize domestic growth and inflation.

\footnote{The low value of $\gamma$ may be a result of the omission of risk premium under the assumption that UIP holds. Lubik and Schorfheide (2005) provide a detailed interpretation concerning low estimates of import of share parameter.}
5 Conclusion

In this paper, in contrast with the existing literature, we construct an analytical model within the context of an open economy New-Keynesian macroeconomic framework to investigate the optimal policy rule of a central bank assuming that the policymakers entertain asymmetric preferences. Using our analytical framework, we show that monetary policymakers not only respond asymmetrically to deviations of the domestic inflation and that of the output gap from their respective targets, but they also react to the real exchange rate, foreign country output gap, foreign country real interest rate, the volatility of inflation and the volatility of the output gap. In this context, the model presents us with a policy rule with which one can empirically examine the asymmetric behavior of policymakers within an open economy framework.

We estimate the derived policy rule for Canada and the UK using the generalized method of moments (GMM) technique. The data are on a quarterly basis and span the period between 1983q1-2007q4. The empirical results can be summarized in two main categories. First, we show that the open economy variables including the real exchange rate and the foreign real interest rate affect the determination of the monetary policy rule rate in the UK and Canada. Second, we find evidence that both central banks have asymmetric preferences. In particular, we find that the BoC has negative output gap asymmetry. This result suggest that the BoC aims at responding rigorously when output gap falls below the set target to avoid low future inflation rate which can destabilize the economy. This behavior agrees with the objectives of the bank as the mandate of the BoC is to keep inflation low and stable. In the case of the UK, we find that the BoE has positive inflation and positive output-gap asymmetries. This allows one to argue that the Bank will systematically over-predict inflation and output-gap. Under such circumstances interest rate will increase and inflation and output-gap will be below the set target levels. Robustness checks based on model calibration support our modeling framework.

To broaden our understanding, we suggest that it would be fruitful to expand the
set of countries under investigation. Furthermore, it might be useful to exploit the cross-equation restrictions in estimating the model using a full-information maximum likelihood approach. These can be taken as future research.
References


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### Panel A: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_0$</td>
<td>1.285* (0.480)</td>
<td>0.301* (0.349)</td>
<td>1.434*** (0.447)</td>
<td>0.622*** (0.227)</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>-0.192** (0.087)</td>
<td>-0.108 (0.098)</td>
<td>-0.187** (0.089)</td>
<td>-0.064 (0.099)</td>
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<tr>
<td>$\varphi_2$</td>
<td>2.971** (0.359)</td>
<td>2.475*** (0.399)</td>
<td>2.832*** (0.381)</td>
<td>2.591*** (0.398)</td>
</tr>
<tr>
<td>$\varphi_3$</td>
<td>-5.534* (3.314)</td>
<td>-6.488* (3.778)</td>
<td>-5.715* (3.330)</td>
<td>-5.961* (3.651)</td>
</tr>
<tr>
<td>$\varphi_4$</td>
<td>0.718*** (0.063)</td>
<td>0.821*** (0.068)</td>
<td>0.724*** (0.068)</td>
<td>0.760*** (0.071)</td>
</tr>
<tr>
<td>$\varphi_5$</td>
<td>0.389** (0.209)</td>
<td>0.372* (0.228)</td>
<td>0.350* (0.211)</td>
<td>0.388* (0.224)</td>
</tr>
<tr>
<td>$\varphi_6$</td>
<td>0.370 (0.518)</td>
<td>0.477 (0.596)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi_7$</td>
<td>-1.552*** (0.535)</td>
<td>-1.472** (0.580)</td>
<td></td>
<td></td>
</tr>
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</table>

### Panel B: Diagnostic Tests

<p>| | | | | |</p>
<table>
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<tr>
<th></th>
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<tr>
<td>Observations</td>
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<td>104</td>
<td>104</td>
<td>104</td>
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<tr>
<td>Underidentification(p-value)†</td>
<td>0.015</td>
<td>0.004</td>
<td>0.025</td>
<td>0.001</td>
</tr>
<tr>
<td>Weakidentification(F-test)‡</td>
<td>24.406</td>
<td>33.159</td>
<td>23.784</td>
<td>38.870</td>
</tr>
<tr>
<td>Weak Inst.Test(F-test)§</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Weak Inst.Test($\chi^2$)♣</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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<tr>
<td>Stock-Wright Stats.$\chi^2$♠</td>
<td>(0.101)</td>
<td>(0.097)</td>
<td>(0.049)</td>
<td>(0.045)</td>
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<tr>
<td>J stat (p-value)</td>
<td>0.146</td>
<td>0.212</td>
<td>0.135</td>
<td>0.274</td>
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<tr>
<td>AR(1)</td>
<td>0.318</td>
<td>0.317</td>
<td>0.318</td>
<td>0.317</td>
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<tr>
<td>AR(2)</td>
<td>0.318</td>
<td>0.317</td>
<td>0.318</td>
<td>0.317</td>
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</table>

### Panel C: The Wald Test(Joint Significance)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0 : \varphi_3; \ldots; \varphi_5 = 0$</td>
<td>312.360***</td>
<td>400.480***</td>
<td>254.030***</td>
<td>116.38***</td>
</tr>
<tr>
<td>$H_0 : \varphi_6; \varphi_7 = 0$</td>
<td>8.640**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : \varphi_3; \varphi_4; \varphi_5; \varphi_6 = 0$</td>
<td>421.79***</td>
<td>341.25</td>
<td></td>
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</tr>
<tr>
<td>$H_0 : \varphi_3; \varphi_4; \varphi_5; \varphi_7 = 0$</td>
<td>472.88***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : \varphi_3; \ldots; \varphi_7 = 0$</td>
<td>532.830***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $i_t = \varphi_0 + \varphi_1 E_t y_{t+1} + \varphi_2 E_t \pi_{t+1} + \varphi_3 \Delta q_t + \varphi_4 (i_t - E_t i_{t+1}) + \varphi_5 E_t y_{t+1} + \varphi_6 \sigma^2_{\pi,t} + \varphi_7 \sigma^2_{y,t}$

In Panel A, values in parenthesis are standard errors. ***, **, and * indicate level of significance at 1%, 5%, and 10% level of significance, respectively. Panel C reports the Wald test for testing the joint significance of the underlying coefficients. † represents the the Kleibergen-Paap rk LM-statistic testing the null hypothesis that the equation is under identified. ‡ represents the Kleibergen-Paap rk Wald F-statistic testing the null hypothesis that the equation is weakly identified. § represents weak instrument robustness test based on Anderson-rubin Wald test. The critical values are based on the F-test. The values in parenthesis shows the p-values. ♣ displays the weak instrument robustness test based on Anderson-rubin Wald test. The critical values are based on the $\chi^2$ test. The values in parenthesis shows the p-values. ♠ indicates the Weak instrument robustness test based on Stock-Wright LM S statistics. The critical values are based on the $\chi^2$ test. The values in parenthesis shows the p-values.
Table 2: GMM Estimates for UK

<table>
<thead>
<tr>
<th>Panel A: Estimation Results</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_0 )</td>
<td>0.139</td>
<td>0.234</td>
<td>0.375</td>
<td>0.728**</td>
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<tr>
<td>(0.391)</td>
<td>(0.411)</td>
<td>(0.286)</td>
<td>(0.322)</td>
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<tr>
<td>( \varphi_1 )</td>
<td>0.050</td>
<td>-0.174</td>
<td>0.109</td>
<td>-0.158</td>
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<tr>
<td>(0.157)</td>
<td>(0.162)</td>
<td>(0.173)</td>
<td>(0.177)</td>
<td></td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>1.273***</td>
<td>1.574***</td>
<td>1.802***</td>
<td>2.245***</td>
</tr>
<tr>
<td>(0.294)</td>
<td>(0.330)</td>
<td>(0.261)</td>
<td>(0.278)</td>
<td></td>
</tr>
<tr>
<td>( \varphi_3 )</td>
<td>-7.393**</td>
<td>-8.481***</td>
<td>-6.521**</td>
<td>-6.063**</td>
</tr>
<tr>
<td>(2.397)</td>
<td>(2.886)</td>
<td>(2.842)</td>
<td>(2.733)</td>
<td></td>
</tr>
<tr>
<td>( \varphi_4 )</td>
<td>0.688***</td>
<td>0.699***</td>
<td>0.761***</td>
<td>0.799***</td>
</tr>
<tr>
<td>(0.063)</td>
<td>(0.069)</td>
<td>(0.051)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>( \varphi_5 )</td>
<td>-0.197</td>
<td>0.181</td>
<td>-0.590</td>
<td>-0.204</td>
</tr>
<tr>
<td>(0.151)</td>
<td>(0.167)</td>
<td>(0.167)</td>
<td>(0.168)</td>
<td></td>
</tr>
<tr>
<td>( \varphi_6 )</td>
<td>2.107***</td>
<td>2.497***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.409)</td>
<td>(0.440)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varphi_7 )</td>
<td>1.594***</td>
<td>2.403***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.508)</td>
<td>(0.494)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Diagnostic Tests

<table>
<thead>
<tr>
<th>Observations</th>
<th>Underidentification(p-value)</th>
<th>Weakidentification(F-test)</th>
<th>Weak Inst.Test(F-test)</th>
<th>Weak Inst.Test(( \chi^2 ))</th>
<th>Stock-Wright Stats.(( \chi^2 ))</th>
<th>J stat (p-value)</th>
<th>AR(1)</th>
<th>AR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>104</td>
<td>0.029</td>
<td>101.696</td>
<td>(0.000)</td>
<td>(0.027)</td>
<td>0.143</td>
<td>0.318</td>
<td>0.318</td>
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</tr>
</tbody>
</table>

Panel C: The Wald Test(Joint Significance)

| \( H_0 : \varphi_3;...; \varphi_5 = 0 \) | 120.07*** | 123.490*** | 234.990*** | 186.790*** |
| \( H_0 : \varphi_6; \varphi_7 = 0 \) | 51.24*** |                       |                        |               |
| \( H_0 : \varphi_3;\varphi_4;\varphi_5; \varphi_6 = 0 \) |                       | 243.450*** |                        |               |
| \( H_0 : \varphi_3;\varphi_4;\varphi_5; \varphi_7 = 0 \) |                       |                        | 253.370*** |               |
| \( H_0 : \varphi_3;...; \varphi_7 = 0 \) |                       |                        |                        | 355.11***     |

Notes: \( i_t = \varphi_0 + \varphi_1 E_t y_{t+1} + \varphi_2 E_t \pi_{t+1} + \varphi_3 \Delta q_t + \varphi_4 (i_t^f - E_t \pi_{t+1}^f) + \varphi_5 E_t y_{t+1}^f + \varphi_6 \sigma^2_{\pi,t} + \varphi_7 \sigma^2_{y,t} \)

In Panel A, values in parenthesis are standard errors. ***, **, and * indicate level of significance at 1%, 5%, and 10% level of significance, respectively. Panel C reports the Wald test for testing the joint significance of the underlying coefficients.

\( \dagger \) represents the Kleibergen-Paap rk LM-statistic testing the null hypothesis that the equation is under identified.

\( \ddagger \) represents the Kleibergen-Paap rk Wald F-statistic testing the null hypothesis that the equation is weakly identified.

\( \S \) displays the weak instrument robustness test based on Anderson-rubin Wald test. The critical values are based on the F-test. The values in parenthesis shows the p-values.

\( \clubsuit \) indicates the Weak instrument robustness test based on Stock-Wright LM S statistics. The critical values are based on the (\( \chi^2 \)) test. The values in parenthesis shows the p-values.
Table 3: Results from Calibration

<table>
<thead>
<tr>
<th>Parameter estimates based on (Model 1)</th>
<th>Canada</th>
<th>UK (Model 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>2.369</td>
<td>2.343</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.367</td>
<td>-0.432</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-1.619</td>
<td>-1.308</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>-0.507</td>
<td>-0.260</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.984</td>
<td>0.981</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.040</td>
<td>0.080</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.109</td>
<td>-0.097</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.153</td>
<td>3.899</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.336</td>
<td>0.728</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.780</td>
<td>3.450</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.040</td>
<td>0.090</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostic Tests</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$RMSE^e_{Infl}$</td>
<td>0.516</td>
<td>0.953</td>
</tr>
<tr>
<td>$RMSE^e_{OG}$</td>
<td>1.488</td>
<td>1.601</td>
</tr>
<tr>
<td>$Bias^a_{Infl}$</td>
<td>0.185</td>
<td>0.465</td>
</tr>
<tr>
<td>$Bias^d_{OG}$</td>
<td>0.479</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Note: Model 1 is the unrestricted model. Model 2 imposes zero restriction on output asymmetry. $a$ (b) denotes RMSE of inflation (output-gap). $c$ (d) denotes the probability that the forecast error of inflation (output-gap) is equal to zero.