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Performance analysis of cables with attached tuned-inerter-dampers

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ABSTRACT

Cables are widely used structural elements capable of bearing tensile forces and experience vibration problems due to their slenderness and low mass. In the field of civil engineering, they are mostly used in bridges where the vibrations are mainly induced by wind, rain, traffic and earthquakes. This paper proposes the use of a tuned-inerter-damper (TID) system, mounted on cables to suppress unwanted vibrations. These are to be attached transversally to the cable, in the vicinity of the support, connected between the deck and the cable. The potential advantage of using a TID system consists in the high apparent mass that can be produced by the inerter. Our analysis showed that the modal damping ratio obtained is much higher than in the case of traditional dampers or tuned mass dampers, leading to an improved overall response. An optimal tuning methodology is also discussed. Numerical results are shown with a cable subjected to both free and forced vibrations and the TID performance is improved when compared with equivalent dampers.

Keywords: cable; tuned inerter damper; vibration suppression; support excitation; viscous damper.

1 Introduction

Bridge cables are prone to large amplitude vibrations due to a series of factors. The most common factors causing these vibrations are wind and rain action, deck vibration induced by traffic or earthquakes.

Several solutions aimed at reducing cable vibrations have been proposed. One of the most widely used solutions consists of the installation of dampers. These are generally located in the vicinity of the anchorage point, connected between each cable and the deck. The efficacy of this approach has been studied both theoretically and experimentally.

An important concept in understanding the behaviour of the combined cable and damper systems refers to the maximum attainable modal damping [1]. This means that the there exists an optimum damper that can be attached at a certain location along the cable length, in order to achieve maximum damping of the vibration mode that the device is tuned for. The optimum damper size can be determined using the universal curve for estimating the modal damping of stay cables, introduced by Pacheco *et al.* [2]. Later, the universal curve formulation was extended by Cremona [3] to inclined cables. The work of Main and Jones [4] extended these studies to the case when dampers trigger large frequency shifts of the uncontrolled cable natural frequency. On the experimental side, the behaviour of cables with attached dampers was presented in several case studies [5].

One of the disadvantages of using dampers refers to the fact that their installation is restricted to the end of the cable, generally at a distance lower than 5% of the cable length from the support [6]. A solution to this problem can be the use of tuned mass dampers (TMD) [7]. These can be located anywhere along the cable length and do not need to be connected to the ground or bridge deck. Wu & Cai [8] conducted a parametric study looking into the influence of the TMD and cable parameters (mass, damping, stiffness, location, cable inclination) on the behaviour of the combined cable and TMD system performance. Anderson *et al.* [9] compared the behaviour of cables with attached viscous dampers and TMDs and concluded that TMDs potentially become more effective than dampers when located at 40% distance from the cable anchorage point. Their performance can be improved through adequate tuning of parameters or by using magneto-rheological dampers instead of viscous dampers.

However, the fact that TMDs are most efficient when located close to the cable midspan represents a disadvantage from installation point of view, as cables are usually very long. Also, the size of the TMD mass is limited to ratios of under 10% of the cable mass.

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As an alternative, this paper proposes the use of TID vibration suppression systems for cable vibration mitigation. The TID, a system comprising of a spring-inerter-damper system, was introduced by the authors in [10]. Its layout is similar to that of a passive TMD, where the mass element was replaced by an inerter.

The inerter was introduced by Smith [11] and used in modern suspension systems for Formula 1 racing cars[12]. The inerter is the mechanical equivalent of the capacitor, thus completing the force-current analogy between mechanical and electrical networks. The force produced by an inerter is

$$\mathbf{F} = b(\ddot{\mathbf{x}}_{i+1} - \ddot{\mathbf{x}}_i) \tag{1}$$

where b is the constant of proportionality of the device, named inertance and $\ddot{\mathbf{x}}_i - \ddot{\mathbf{x}}_j$ represents the relative acceleration between its nodes. The inertance is measured in kilograms.

The most important characteristic of the inerters consists of their capacity of generating high apparent masses, mainly through gearing. This leads to the generation of a high inertial force, that cannot be obtained using a traditional TMD, where the mass is restricted to low values. Gearing ratios of 200:1 have been achieved [13].

The applications of inerters span across many engineering fields from vehicle [14, 15, 16] and train suspension systems [17] to building suspension systems [18, 19]. Ikago *et al.* [13, 20] studied the performance of tuned viscous mass dampers, where the inerter is mounted in parallel to a damper, both elements being connected in series with a spring. Marian and Giaralis [21] propose the use of a tuned mass-damper-inerter systems (TMDI), where the inerter is mounted in series with a TMD, thus connecting it between two adjacent storeys. They study the performance of TMDIs when installed in support-excited structures.

The TID system provides a very good vibration suppression level of the host structure. It has been shown that it is most efficient when located at bottom storey level, this constituting an important advantage from installation point of view. Its performance was studied for a wide range of loading scenarios, including sinusoidal support and lateral excitation [22], earthquake excitation and wind loads [23]. The TID showed comparable or improved performance when compared to equivalent TMDs or damper systems.

As shown in [10], in case of multi-storey structures, the TID must be connected between the first floor and the ground. Similarly, when installed on cables, the TID is connected between the cable and the bridge deck. Therefore, its installation is restricted to locations situated in the vicinity of the cable anchorage point, as in the case of dampers. Given this, the TID is located within easy access, should maintenance or retrofitting become necessary. While in the case of dampers their location represents a disadvantage in terms of performance, in the case of TIDs the limitation can be overcome by adjusting the inerter's apparent mass. This represents an important advantage of the proposed system.

Based on the analysis of horizontal cables with attached viscous dampers [4] presented in the literature, we study the performance of cables with attached TIDs. Starting from the uncontrolled cable model, differential equations can be derived for calculating the modal damping ratio obtained using a TID. For simplicity, a finite element model has also been created. This reduces the computational effort without significant loss of accuracy.

An important aspect is represented by the device optimisation. This is more involved due to the fact that TIDs introduce a greater number of parameters (inertance, damping and stiffness), while in the case of dampers only one parameter needs to be set, namely the damping capacity. The optimisation is aimed at minimising the displacement at the cable's midspan when the cable is subjected to sinusoidal deck excitation.

Our analysis showed that better performance can be achieved by using TIDs, when connected at the same location as viscous dampers. Moreover, the optimal damping required in a TID is much lower than that of dampers. Considering the results shown, it was concluded that the TID represents a viable alternative to viscous dampers when used to limit unwanted cable vibration.

2 Structural system

The structural system considered is represented by a horizontal cable fixed at both ends, as shown in Figure 1. The TID is connected at length l_1 from the left support of the cable. In case of a bridge this would correspond to the point of anchorage to the deck. l_2 represents the distance to the right support which could be associated to the cable connection to the bridge tower.

The elements of the TID, as described in [10] are the inerter (characterised by its inertance, b_d), a spring (with stiffness k_d) and a damper (with damping c_d). The TID is also connected to the bridge deck.

The cable used in the numerical examples has a total length $L = 100$m and is subject to a tensile force $T = 5000$kN. Its mass per unit length is $m = 100$kg/m and the resulting frequency of the first mode of vibration is $\omega_c = 7.02$rad/s.

The system will be subject to sinusoidal deck excitation, which translates into a sinusoidal displacement of the left support of the cable and of the point of anchorage of the viscous damper or TID to the bridge deck.

The tuning of both devices is done such that we obtain minimum displacement at the cable's midspan, when the overall structure is subject to deck excitation.
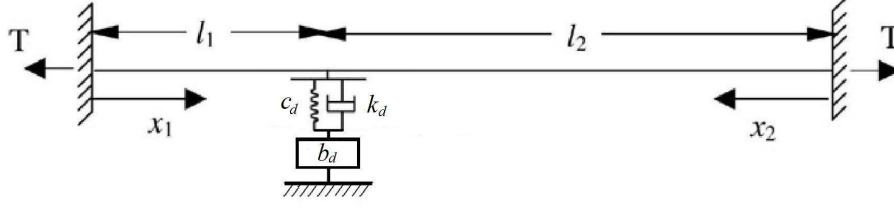
Fig. 1 Cable with an attached TID

2.1 Cable differential equations

Assuming that the tension in the cable is large compared to its weight, the bending stiffness and damping of the cable are small, and that the cable deflection is small such that the tension force caused by deflection is negligible in comparison to the static tension, the following partial differential equation is satisfied over each segment of the cable:

$$T\frac{\partial^2 v_k(x_k,t)}{\partial x_k^2} = m\frac{\partial^2 v_k(x_k,t)}{\partial t^2} \tag{2}$$

where T is the tension force in the cable, m is the cable mass per unit length, x_k is the coordinate of the k-th cable segment along the cable axis and v_k represents the transversal deflection. For simplification, a non-dimensional time parameter $\tau = \omega_c t$ is introduced. Using the new notation, the cable displacement can be expressed as

$$v_k(x_k,t) = \gamma(\tau)\frac{\sinh(\pi\lambda x_k/L)}{\sinh(\pi\lambda l_k/L)} \tag{3}$$

where λ is the dimensionless eigenvalue and γ is the displacement at the location where the device is connected [4].

The equilibrium equation at the TID connection point can be written as

$$T\left(\frac{\partial v_1}{\partial x_1}\bigg|_{x_1=l_1} - \frac{\partial v_2}{\partial x_2}\bigg|_{x_2=l_2}\right) = k_d(v_1|_{x_1=l_1} - v_d) + c_d\left(\frac{dv_1}{dt}\bigg|_{x_1=l_1} - \frac{dv_d}{dt}\right) \tag{4}$$

where k_d, c_d and v_d are the stiffness, damping and vertical displacement of the TID system. The equilibrium equation of the TID is

$$b_d\frac{d^2 v_d}{dt^2} + k_d(v_d - v_1|_{x_1=l_1}) + c_d\left(\frac{dv_d}{dt} - \frac{dv_1}{dt}\bigg|_{x_1=l_1}\right) = 0 \tag{5}$$

where b_d is the inertance of the TID system.

Following the procedure presented in [7], the TID system displacement can be expressed as $v_d = \beta\gamma(\tau)$, where β is the complex amplitude ratio between the TID and the corresponding cable point.

$$\beta = \frac{1 + 2\xi\rho\lambda}{1 + 2\xi\rho\lambda + \rho^2\lambda^2} \tag{6}$$

where $\xi = \frac{c_d}{2b_d\omega_d}$ is the TID damping ratio and $\rho = \frac{\omega_c}{\omega_d}$ is the ratio between the cable and the TID fundamental frequency $(\omega_d = \sqrt{\frac{k_d}{b_d}})$.

Substituting equations 3 and 6 into 4, we obtain

$$\coth(\pi\lambda l_1/L) + \coth(\pi\lambda l_2/L) + \frac{\pi\lambda b_d}{Lm}\frac{1 + 2\xi\rho\lambda}{1 + 2\xi\rho\lambda + \rho^2\lambda^2} = 0 \tag{7}$$

The solutions, λ, of Equation 7 represent the eigenvalues of the combined cable-TID system. Considering $\lambda = \sigma + \varphi i$ where $i = \sqrt{-1}$, and separating the real and imaginary terms, we obtain a system of equations that can be soled numerically and the modal damping ratio can be calculated as

$$\zeta_i = \left(\frac{\varphi_i^2}{\sigma_i^2} + 1\right)^{-\frac{1}{2}}. \tag{8}$$

2.2 Finite element model

Although this method provides accurate results, it is difficult to use in practice, where we need to calculate the response of the structure. To overcome this limitation, a finite element model using axial elements was created. Once the mass and stiffness matrices have been assembled, the structural response can be easily evaluated using the state space formulation. We opted for a model formed of 20 elements. The resulting frequency of the first mode of vibration is $\omega = 7.03\text{rad/s}$, very close to the frequency determined analytically, $\omega_c = 7.02\text{rad/s}$. The resulting modal damping ratios have been validated using the analytical model. This simplified model has been employed in the following calculations and examples.

3 Analysis of damper systems performance

This section is aimed at analysing the response of cables with attached viscous dampers. The approach is similar to the one presented in [4]. As explained by the authors, there is an optimal damping capacity that ensures a maximum level of modal damping in each vibration mode. This is illustrated in Figure 2(a), where we can see the variation of the modal damping ratio attained in the first, second and third mode of vibration of a cable plus damper system with damper capacity.
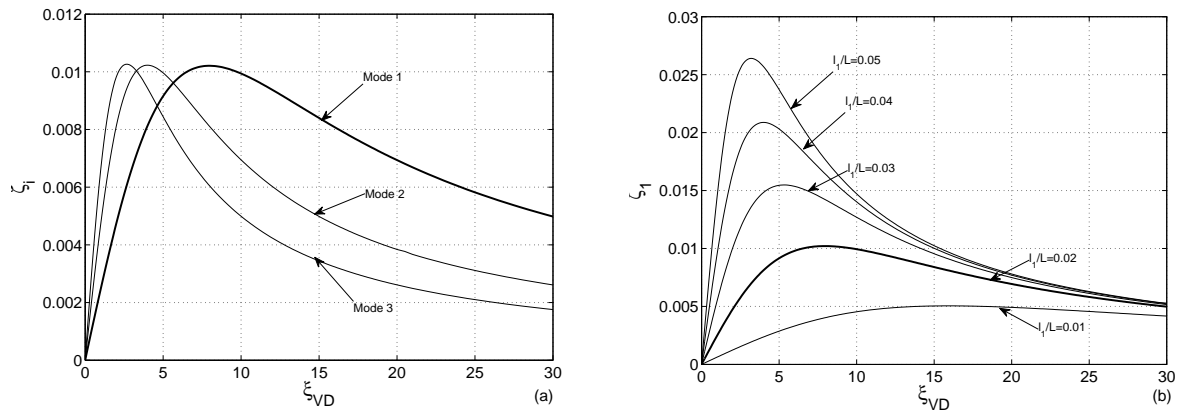

Fig. 2 (a)Modal damping ratio obtained in the first, second and third mode of vibration of a cable with an attached viscous damper, located at $0.02L$ distance from the left support; (b) Variation of the first modal damping ratio, ζ_1, with the damper location.

The damper capacity is expressed in terms of its damping ratio. This is defined as

$$\xi_{VD} = \frac{c}{2\sqrt{Tm}} \tag{9}$$

where c represents the damping coefficient of the viscous damper.

When the viscous damper is located at $l_1 = 0.02L$ distance from the left support, the maximum modal damping attached in the first three vibration modes is approximately $\zeta_i = 1\%$. However, these maxima cannot be attained concomitantly. A different damper capacity is needed in each case. For example, if the damper capacity is set to $\xi_{VD} = 7.9$, the corresponding modal damping ratios will be $\zeta_1 = 1\%$, $\zeta_2 = 0.8\%$ and $\zeta_3 = 0.6\%$. Similar results have been reported in [4] for a different scaling of the damping parameter.

In the present case, we are interested in tuning the damper such that we suppress the vibration of the first mode of vibration, therefore we will extend the system analysis in this direction. To do so, we are looking at the variation of the modal damping ratio achieved in the first mode of vibration when the damper is moved along the cable length. This is shown in Figure 2(b). The device becomes more effective when located at a larger distance with respect to the left support. The thick line representing the first modal damping ratio of the system in Figure 2(b) is shown using the same legend in Figure 2(a). When the length ratio is increased to $l_1/L = 0.05$, the first modal damping ratio increases to $\zeta_1 = 2.6\%$.

Figure 3(a) shows the variation of the midspan gain with length ratio, for optimally tuned dampers. As the length ratio increases, the gain value drops, indicating the increased efficiency of the device. A value $G_{ms} = 0.5$ is attained when the device is located at 5% distance from the left support. Also, the damping ratio necessary to obtain the minimum gain decreases as the length ratio is increased. This trend can be seen in Figure 3(b).
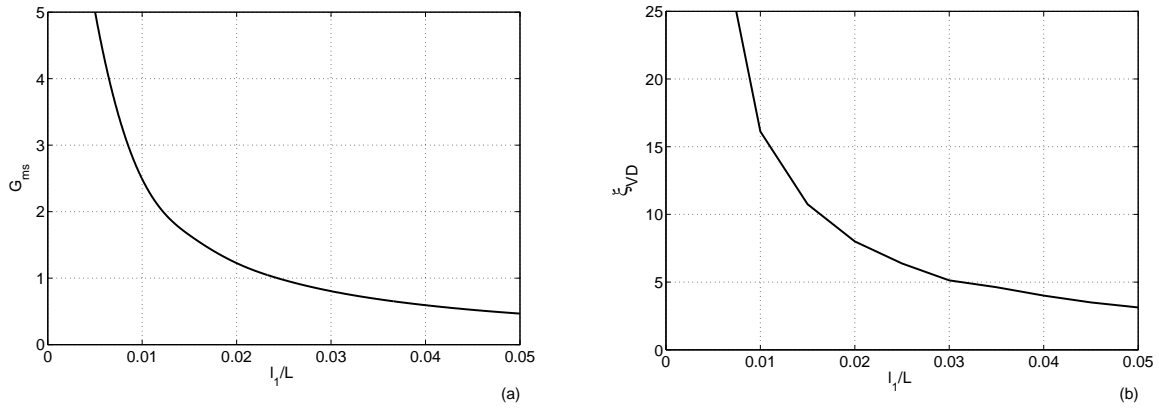
Fig. 3 (a) Variation of the midspan gain G_{ms} with length ratio; (b) Variation of the optimal damper capacity with length ratio.

4 Analysis of TID systems performance

A similar analysis is now done in case of cables with attached TIDs. The device tuning is more complex in this case due to the increased number of parameters that need to be taken into account.

As explained in [10], the TID introduces a new degree of freedom in the system. This induces a TMD-like behaviour in the vicinity of the first fundamental frequency of the uncontrolled system. The aim of our optimisation was to tune the TID such that the amplitude of response at the newly created peaks is the same. Concomitantly, the response must be minimised. Therefore, after setting the desired mass ratio, μ, we will determine the optimum frequency ratio, ρ, and the corresponding damping ratio, ξ_{TID}. The TID damping ratio is calculated as $\xi_{TID} = \xi \frac{\pi \mu}{\rho}$, where $\xi = \frac{c_d}{2\omega_d b_d}$. This scaling is done in order to comply with the damping coefficient definition for viscous dampers given in Equation 9.

Figure 4(a) shows the the variation of the modal damping ratio attained in the first, second and third mode of vibration of a cable plus TID system with the TID damping ratio. The maximum modal damping ratio in the first mode of vibration is approximately $\zeta_1 = 1.7\%$. As in the case of dampers, when tuning the device to suppress the vibration of the first mode, the corresponding second and third mode damping ratios are much smaller. If the optimisation criterion is changed, higher modal damping ratios can be obtained. However, this would not lead to an optimal response in terms of midspan gain, G_{ms}, as in the case of dampers.

Figure 4(b) gives the variation of the first modal damping ratio with the length ratio giving the TID position. As seen, the maxima obtained are considerably higher than in the case of dampers. When the TID is placed at $l_1/L = 5\%$, a modal damping ratio of $\zeta_1 = 4.5\%$ can be achieved.

The midspan gain variation with the length ratio is shown in Figure 5(a). As in the case of dampers, the gain decreases as the device is moved further from the support. The minimum value obtained using a TID with mass ratio $\mu = 0.2$, located at $l_1/L = 5\%$ from the support is $G_{ms} = 0.35$, smaller than in the case when dampers are used. This proves that the TID system is more effective.
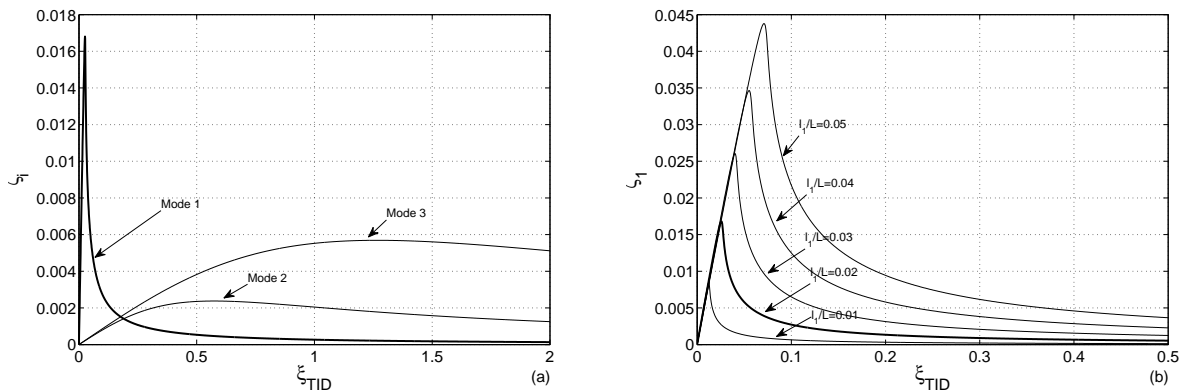

Fig. 4 (a)Modal damping ratio obtained in the first, second and third mode of vibration of a cable with an attached TID with mass ratio $\mu = 0.2$, located at $0.02L$ distance from the left support; (b)Variation of the first modal damping ratio, ζ_1, with the TID location, for a mass ratio $\mu = 0.2$.

Figure 5(b) shows the variation of the optimal damper capacity with length ratio for a TID with mass ratio
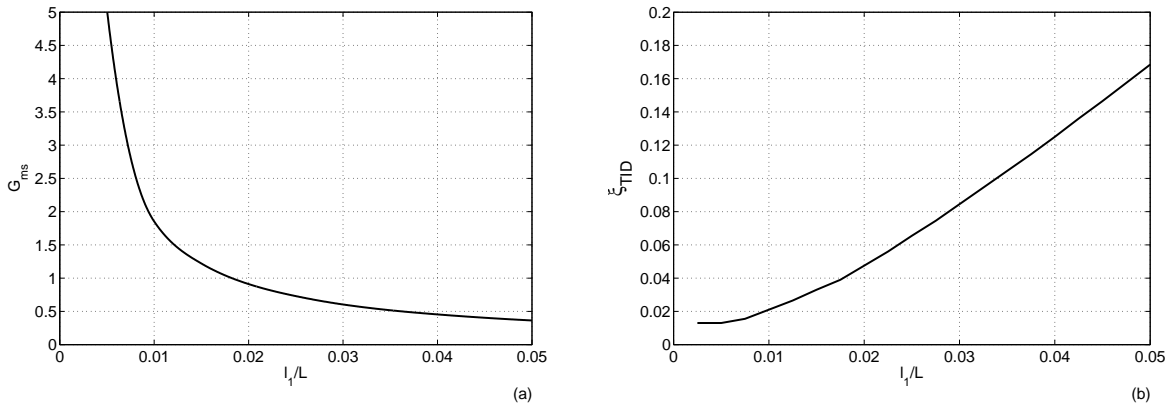
Fig. 5 (a) Variation of the midspan gain G_{ms} with length ratio for a TID with mass ratio $\mu = 0.2$; (b) Variation of the optimal damper capacity with length ratio for a TID with mass ratio $\mu = 0.2$.

$\mu = 0.2$. In this case, as the length ratio is increased, the optimal TID damping ratio increases as well. However, the necessary damper capacity is much smaller than in the case of dampers.

The results obtained indicate the fact that a TID with mass ratio $\mu = 0.2$ of the total cable mass is more efficient than viscous dampers, when connected at the same location along the cable's length.

5 Response to vibrations induced by bridge deck motion - Performance comparison

One of the most attractive aspects associated with the use of TID instead of viscous dampers is the fact that the performance of the TID can be adjusted by increasing its mass ratio. We are looking at the performance of different mass ratio TIDs, each optimised to minimise the midspan gain.
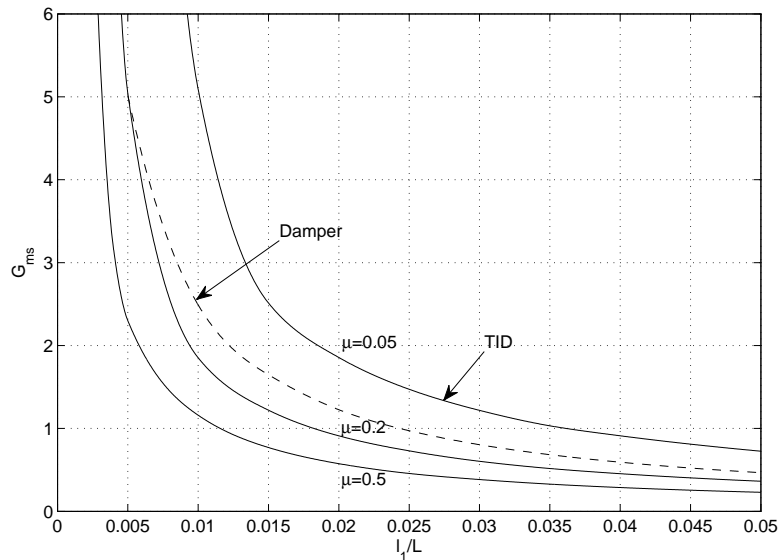

Fig. 6 Variation of midspan gain with length ratio optimal dampers and TIDs with mass ratios $\mu = 0.05$, $\mu = 0.2$ and $\mu = 0.5$.

Figure 6 shows a comparison between the performance of TIDs and dampers in terms of midspan gain, G_{ms}. The dashed line representing the optimal damper performance lies between the curves corresponding to TIDs with mass ratios of $\mu = 0.05$ and $\mu = 0.2$. In case of TIDs, an improved performance can be obtained id the mass ratio is further increased to $\mu = 0.5$. These values are feasible due to the capacity of the inerter to generate high apparent mass.

For a better understanding of the TID and damper behaviour, Figure 7 shows the frequency response to sinusoidal deck excitation for the case when the devices are located at $l_1/L = 2\%$ from the left support. It is considered that the uncontrolled cable has no inherent damping, thus the amplitude of its response near the first
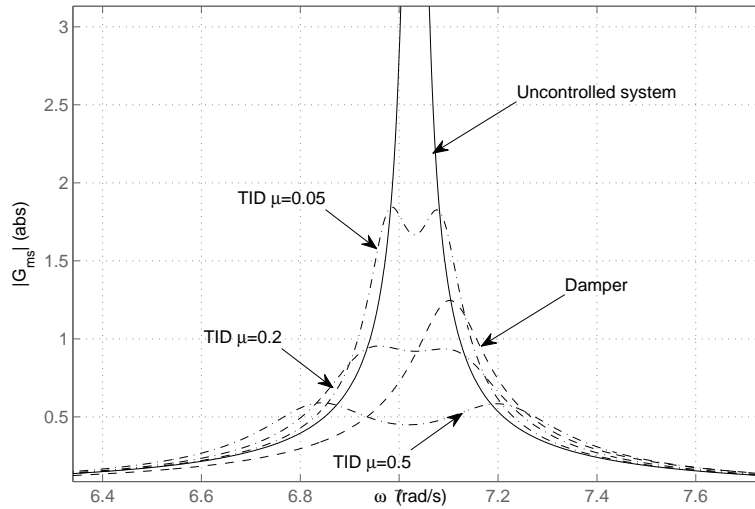
Fig. 7 Frequency response of the uncontrolled system, cable with an optimised damper and cable with an attached TID, at $l_1/L = 2\%$.

fundamental frequency is infinite. It can be seen that the high damper capacity induces a frequency shift in the overall system response. The TIDs display the equal-amplitude split-peak response characteristic for TMD-like systems. As shown in Figure 6, the dampers performance lies in between that of a TID with $\mu = 0.05$ and $\mu = 0.2$, and an improved performance is obtained for $\mu = 0.5$.

6 Conclusion

In this paper we analysed the possibility of using TIDs for cable vibration suppression, as an alternative to viscous dampers.

The inerter is an attractive device for vibration suppression application, given its ability of generating a high apparent mass compared to its physical mass. This leads to high inertial forces, proportional to the relative acceleration between the device terminals.

The performance of TIDs installed on cables is compared to that of optimised viscous dampers attached at the same location. Since the TID system introduces a higher number of design parameters and obtaining the highest modal damping ratio does not lead to an optimal response of the overall system, we introduced a new optimisation algorithm, based on the minimisation of the response gain at the cable's midspan.

As presented in previous literature, there is a maximum level of modal damping that can be achieved when a viscous damper is connected to a cable at a given location. In the case of TIDs, this is no longer valid as the device can be improved varying its mass ratio. This leads to very good vibration suppression levels.

If is shown that for the case when the system is subject to sinusoidal excitation of the left support, a better vibration suppression level can be achieved by using a TID than a viscous damper, when both devices are located at $l_1/L = 2\%$ from the support. Considering the results shown, it was concluded that the TID represents a promising alternative to viscous dampers when used to limit cable vibration.

Acknowledgement

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