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Energy-Efficient and Low Signalling Overhead Cooperative Relaying with Proactive Relay Subset Selection

Bleron Klaiqi, Xiaoli Chu, Jie Zhang
Dept. of Electronic and Electrical Engineering, University of Sheffield, Sheffield, S1 3JD, UK
Email: {b.klaiqi, x.chu, jie.zhang}@sheffield.ac.uk

Abstract—Energy efficient wireless communications have recently received much attention, due to the ever-increasing energy consumption of wireless communication systems. In this paper, we propose a new energy-efficient cooperative relaying scheme that selects a subset of relays before data transmission, through proactive participation of available relays using their local timers. We perform theoretical analysis of energy efficiency under maximum transmission power constraint, using practical data packet length, and taking account of the overhead for obtaining channel state information, relay selection, and cooperative beamforming.

We provide the expression of average energy efficiency for the proposed scheme, and identify the optimal number and location of relays that maximise energy efficiency of the system. A closed-form approximate expression for the optimal position of relays is derived. We also perform overhead analysis for the proposed scheme and study the impact of data packet lengths on energy efficiency. The analytical and simulation results reveal that the proposed scheme exhibits significantly higher energy efficiency as compared to direct transmission, best relay selection, all relay selection, and a state-of-the-art existing cooperative relaying scheme. Moreover, the proposed scheme reduces the signalling overhead and achieves higher energy savings for larger data packets.

Index Terms—Cooperative communications, decode-and-forward relays, energy efficiency, relay selection, overhead.

I. INTRODUCTION

Due to the rapid growth of energy-hungry wireless multimedia services, telecom energy consumption is increasing at an extraordinary rate. Besides negative environmental impacts and higher energy bills for operators, it also affects user experience as improvements in battery technologies have not kept up with increasing mobile energy demands. Therefore, how to increase the energy efficiency of wireless communications has gained a lot of attention [2]-[4]. Wireless cooperative communications can significantly increase system capacity and reliability [5]-[8]. It is also widely recognized as a promising technique to improve the energy efficiency of wireless networks [2]-[4].

One of the key challenges in wireless cooperative communications is relay selection. Most existing cooperative communication schemes select either the best relay [9]-[14] or all available relays [15]-[18] to cooperatively forward data to the destination. Moreover, almost all existing cooperative relaying schemes have neglected the energy consumption and signalling overhead needed for the acquisition of channel state information (CSI), relay selection, and coordination among selected relays. In [19], relay precoders and decoders were jointly optimized for amplify-and-forward (AF) cooperative systems under various CSI assumptions. In [20], energy efficient schemes were proposed for AF relays in a single carrier frequency division multiple access system. Energy efficient best relay selection schemes for DF relays were studied in [21] and [22]. In [23], energy efficiency was investigated for joint physical and network layer cooperative relaying schemes. Nevertheless, none of these works have considered related signalling overhead in the performance analysis.

It has been shown that cooperative communications lead to higher signalling overhead compared to direct transmission [24]. Therefore, the signalling overhead and associated energy consumption have to be taken into account in the design of an energy-efficient cooperative relaying system. The impact of overhead on the spectral efficiency has been investigated for different relaying schemes in [24]. A timer based scheme was proposed to reduce the overhead for best-relay selection [11], [25], where available relays each start a timer that is inversely proportional to a relay-specific performance metric and the relay with the first expiring timer is selected. However, in practical systems, such a timer based relay selection may fail in case more than one relays’ timers expire within a window of vulnerability, leading to packet collisions at the receiver [11]. In the timer-based relay selection schemes that maximize the probability of successful selection [26]-[28], each relay needs to know either the exact number of available relays [26], [27] or the range within which that number lies [28] in order to optimize their timers. That information has to be signalled to all available relays, hence increasing the signalling overhead as compared to the best-relay selection in [11], [25]. Furthermore, each relay has to maintain a lookup table that needs to be updated every time the number of available relays changes.

Selecting more than one relays may offer a higher energy efficiency than selecting only the best relay, but the overhead for CSI acquisition and feedback limits the number of relays that can be used for energy-efficient cooperative beamforming [29]. The energy-efficiency oriented cooperative
relaying scheme in [29] performs reactive relay selection, i.e., relay selection is performed after data transmission from the source. In [30] and [31], the energy efficiency of clustered cooperative beamforming (where relays can overhear each other’s transmissions) was analysed considering the related overhead. However, none of these works have performed signalling overhead analysis or explored the opportunity of further improving energy efficiency of cooperative relaying by reducing the signalling overhead.

In addition to signalling overhead, other practical limitations such as maximum transmission power, length of data packets, overhearing capabilities of relays, and relay location also affect the energy efficiency of cooperative relaying. The maximum transmission power constraint of practical communication systems was not considered in [29]. In [30] and [31], it was assumed that relays can overhear each other’s transmissions and very long data packets were used. Nevertheless, in practical systems hidden relay nodes that cannot hear and/or be heard by other relay nodes may exist [11] and the length of data packets is restricted by the channel coherence time. The assumption of extremely long data packets simplifies the analysis of energy efficiency, but may disguise the actual effect of overhead on the energy efficiency of cooperative communications. Furthermore, none of the aforementioned works has investigated how the cooperating relays’ location and data packet length would affect the optimal number of selected relays and the energy efficiency of cooperative relaying.

In this paper, we provide a comprehensive study of the number and location of relays that should be selected for cooperative communications to maximize energy efficiency, taking into account the associated signalling overhead and practical constraints such as maximum transmission power, practical data packet lengths and the case that relays cannot overhear each other’s transmissions. The main contributions of this work can be summarized as follows:

- We propose a new energy-efficient and low signalling overhead cooperative relaying scheme that proactively selects a subset of available relays before data transmission, using timers set at relays. Its performance in terms of energy efficiency and required signalling overhead is compared to the cooperative relaying scheme in [29], best relay selection, all relay selection, and direct transmission.
- We perform theoretical analysis of energy efficiency considering practical constraints such as maximum transmission power, hidden relay nodes, and practical data packet length. Furthermore, we carry out signalling overhead analysis factoring in the costs for channel estimation, relay selection and cooperative beamforming.
- We study how the optimal number of relays that maximizes the energy efficiency is affected by the number of correctly decoding relays, relay location, and data packet length. We identify the number and location of cooperating relays that maximize energy efficiency for given number of correctly decoding relays, source-to-destination distance, and data packet length. The results can be used as a guideline for developing energy-efficient transmission strategies that can dynamically switch between different communication modes: direct transmission, best-relay selection, and our proposed cooperative relaying scheme, depending on which of them offers the highest energy efficiency for a given scenario.
- We derive the expression of average energy efficiency for our proposed cooperative relaying scheme, and a closed-form approximate expression of the optimal location of cooperating relays as a function of the numbers of correctly decoding relays and selected relays that maximizes energy efficiency. The accuracy of the expressions is evaluated through simulations.

The remainder of the paper is organized as follows. The system model and the proposed cooperative relaying scheme are presented in Section II. Section III presents the energy efficiency analysis. In Sections IV and V, we derive the optimal location of cooperating relays and perform signalling overhead analysis, respectively. The simulation results are presented in Section VI. Finally, conclusions are given in Section VII.

Notations: \(|D|\) is the cardinality of the set \(D\). Floor operation is given by \(\lfloor . \rfloor\). In order statistics [32], the \(i\)th largest value among \(M\) values is denoted by \(g_{i:M}\), i.e., \(g_{1:M} \geq g_{2:M} \geq \ldots \geq g_{M:M}\). \(\mathbb{E}\{X\}\) and \(\mathbb{E}\{X|Y\}\) denote the expected value of \(X\) and the conditional expectation of \(X\) given \(Y\), respectively, where \(X\) and \(Y\) are random variables.

II. SYSTEM MODEL AND COOPERATIVE RELAYING SCHEME

We consider a wireless communication system consisting of one source-destination pair and \(N\) half-duplex decode-and-forward (DF) relays as shown in Fig. 1. Each node is equipped with a single omni-directional antenna. The channel power gains between the source and relay \(i\) \((i=1,\ldots,N)\) and from relay \(i\) to the destination are given by \(h_i\) and \(g_i\), respectively, which are independent and exponentially distributed random variables with the mean values, \(\tilde{h}_i=(\lambda_c/4\pi d_0)^2 (d_{si}/d_0)^{-\xi}\) and \(\tilde{g}_i=(\lambda_c/4\pi d_0)^2 (d_{id}/d_0)^{-\xi}\). Thereby, \(\lambda_c\) denotes the carrier wavelength, \(d_0\) is the reference distance, \(\xi\) is the path-loss exponent, and \(d_{si}\) and \(d_{id}\) are the distances between source and relay \(i\) and between relay \(i\) and destination, respectively. It is assumed that inter-relay distances are much smaller than those between the source and relays and from relays to the destination, i.e., we approximately have \(h_i=\tilde{h}\) and \(g_i=\tilde{g}\) \((i=1,\ldots,N)\), where \(\tilde{h}\) and \(\tilde{g}\) denote the mean channel power gains of all links between source and relays and all links from relays to destination, respectively. We assume that \(\tilde{h}\) and \(\tilde{g}\) are known at source, relays and destination [29]. Furthermore, channel reciprocity is assumed, i.e., the forward and reverse links between two nodes are identical and remain constant during the time period for training, relay selection, and data transmission [29]-[31]. It will be shown in Section VI that the time required for training, relay selection and data transmission by the proposed scheme is much shorter than the channel coherence time of low mobility scenarios (with typical pedestrian speed of 3km/h). For higher mobility scenarios, data packets can be split into smaller packets and more signalling overhead is necessary as channel changes...
much faster. Communications between any two nodes have a rate $R$ (bits/symbol) and bandwidth $B$ (Hz). Perfect channel estimation at each node is also assumed. We consider relatively long range transmissions. As it has been shown in [33] for this case, the circuit energy consumption can be neglected as it is dominated by the energy consumed for signal transmission.

We propose an energy-efficient and low signalling overhead cooperative relaying scheme, which can be divided into three main phases as illustrated in Fig. 1 and explained as follows.

A. Relay Channel Estimation Phase

Relays have to obtain first-hop CSI, in order to decode data from the source. To this end, source broadcasts training symbols at the minimum power required to support the target rate $R$ with outage probability $p_{out}^{tr}$ [29], i.e.,

$$P_S^T = N_0 B \frac{1 - 2R}{h \ln(1 - p_{out}^{tr})},$$  

where $N_0$ is the power spectral density of additive white Gaussian noise (AWGN). Similarly, for relays to acquire the CSI on their links to the destination, the destination broadcasts training symbols$^2$ with the following power,

$$P_D^T = N_0 B \frac{1 - 2R}{g \ln(1 - p_{out}^{tr})}.$$  

B. Relay Selection Phase

Step 1: Since we consider DF relays, only relays $j$, $1 \leq j \leq N$, that can correctly decode the received data from the source, i.e., with received signal-to-noise ratio (SNR), $\gamma_{sj}$, being able to support the rate $R$ with the maximum allowed transmission power, $P_{max}$, are suitable to forward the received data to the destination and hence become part of the decode set $D = \{1 \leq j \leq N : \gamma_{sj} \geq 2R - 1\}$. Furthermore, as the relays have estimated the channels from the source to them in the relay channel estimation phase, under the assumption of channel reciprocity the relays would also know the channels from themselves to the source. In Step 1 of the relay selection phase, each correctly decoding relay transmits one bit “1” to the source with channel inversion$^3$, i.e., compensating the channel effect before transmission so that the source can decode the transmitted bits without CSI$^4$. The source adds the received bits up to obtain the number of correctly decoding relays ($M$), and then based on $M$ determines the optimal number of relays to be selected ($K$) that maximizes energy efficiency (see Section VI). The overall relay transmission power for signalling the size of the decoding relay set, $M=|D|$, to the source is given by

$$P_M = N_0 B (2R - 1) \sum_{j=1}^{M} \frac{1}{h_j},$$  

where $\lambda = \frac{\gamma_{sj}}{\Delta_g}$, $\lambda$ is a predefined constant parameter, and $\Delta_g$ is a guard interval that depends on the processing delay, the propagation delay, and the transmitted symbol duration [11]. For the proposed scheme we set $\Delta_g = N_T T_S$, where $N_T$ and $T_S$ are the number of symbols used for training and the symbol duration, respectively. The processing delay and the propagation delay are negligible compared to the symbol duration. The correctly decoding relays are ranked in descending order of their channel strengths to the destination so that the timer of the relay with the strongest channel in the second hop expires first, followed by the timer of the relay with the second strongest second-hop channel and so on.$^5$

**Proposition 1:** The time required for selecting the $K$ best relays is obtained as

$$T_{sel,K} = \Delta_g \frac{M!}{(K - 1)!} \sum_{n=1}^{n_{max}} \sum_{i=0}^{M-K} (-1)^n \frac{(-1)^n}{n! (i + K)(M - i - K)!} \left( e^{-\frac{i+K}{n+1}} - e^{-\frac{i+K}{n}} \right),$$  

where $\theta = \frac{\Delta_g}{\lambda}, n_{max} = \left\lfloor \frac{T_{max,\theta}}{\Delta_g} \right\rfloor$, and $T_{max}$ is the maximum allowable relay selection time.

**Proof:** The proof is given in Appendix A.

Step 2: After the expiration of its timer, a relay transmits $N_T$ training symbols with transmission power of

$$P_T^R = \max\left\{ P_S^T, P_D^T \right\}.$$  

In this way, by exploiting the broadcast nature of the wireless channel, the source and the destination can use the same training symbols to perform channel estimation and obtain the corresponding CSI. The source will use the estimated first-hop CSI to adapt its data transmission power to the minimum level required for reaching the selected relays (see Section II-C).

Due to the use of discrete relay timers in (4), collisions between relay transmissions may occur if the timers of two or more relays expire at the same time.

**Proposition 2:** The collision probabilities among the $K$ best relays for $K=1$ and $K>1$ are given by

$$p_{coll,K=1,n_{max}} = 1 - M \sum_{n=0}^{n_{max}} \left( e^{\frac{-\theta}{n+1}} - e^{\frac{-\theta}{n}} \right) \left( 1 - e^{\frac{-\theta}{n+1}} \right)^{M-1},$$  

$^5$Note that as relays with the strongest second-hop channels among the correctly decoding relays (i.e., relays with the first-hop link strength that satisfy the rate $R$ with $P_{max}$) are selected, link strengths of both hops are considered in the relay selection.
informs the other
sum of the
propagation delay is negligible compared to the guard interval.

before it starts transmitting training symbols, given that the
which will be used by the selected
running and are in idle mode.

in the immediate data transmission phase
timers, not to transmit training symbols, and not to participate
K
their timers.

also able to receive and process signals. Therefore, when the source transmits
source, they switch to idle mode waiting for expiration of their timers and are
392
373
392
stopping
a single bit message would be sufficient to request the
decode the one bit message. We have used a “0” message just to indicate that
long as the source transmits with maximum allowed power
p
P

Fig. 1. Proposed cooperative relaying scheme

\[
P_{\text{coll},K>1,n_{\text{max}}} = 1 - \frac{M!}{(M-K)!(K-1)!} \sum_{n=K-1}^{n_{\text{max}}} e^{-\frac{(K-1)\theta}{n}} \left( e^{-\frac{\theta}{n+1}} - e^{-\frac{\theta}{n}} \right) \left( 1 - e^{-\frac{\theta}{n+1}} \right)^{M-K} \left( 1 - I_{\{K\geq3\}}(K)^{P_{\text{coll},K>2,n}} \right), \quad (8)
\]

where the indicator function \( I_A(x) = 1 \) if \( x \in A \), 0 otherwise.

Proof: The proof is given in Appendix B.

Step 3: Once the source has received training symbols from the first \( K \) relays, using maximum transmission power \( P_{\text{max}} \) it informs the other \( M-K \) relays via a single bit “0” to stop their timers, not to transmit training symbols, and not to participate in the immediate data transmission phase. We assume that the relay with timer \( t_{K+1:M} \) receives the single bit “0” notification before it starts transmitting training symbols, given that the propagation delay is negligible compared to the guard interval.

Prior to data transmission, the destination broadcasts the sum of the \( K \) estimated second-hop channel power gains, which will be used by the selected \( K \) relays for cooperative beamforming with the optimal transmission power, using the following transmission power

\[
P_{FB} = \frac{N_0B(2^R - 1)}{g_{K:M}}, \quad (9)
\]

where the weakest second-hop channel power gain \( g_{K:M} \) among the \( K \) selected relays is used, because this broadcast information has to reach all the \( K \) selected relays.

C. Data Transmission Phase

So far, the source and the \( K \) selected relays have obtained all necessary information to perform data transmission in an energy-efficient manner. In the first hop, the source transmits data with the minimum transmission power required for reaching the \( K \) selected relays, i.e.,

\[
P_{AD} = \frac{N_0B(2^R - 1)}{\min\{h_1, \ldots, h_K\}}, \quad (10)
\]

To some extent, this may also prevent the other \( M-K \) relays from unnecessarily decoding and buffering data packets.

In the second hop, the \( K \) selected DF relays perform cooperative beamforming to transmit the decoded source data to the destination, with the overall transmission power given by

\[
P_{CB} = \sum_{i=1}^{K} P_{CB}^i, \quad (11)
\]

where \( P_{CB}^i \) is the optimal transmission power at relay \( i \) and is calculated as follows

\[
P_{CB}^i = N_0B(2^R - 1) \left( \frac{1}{\sqrt{g_{i:M}}} \sum_{j=1}^{K} g_{j:M} \right)^{-2}. \quad (12)
\]

The optimal beamforming weights have been calculated utilizing the Lagrangian multiplier technique [35].
The minimum channel coherence time required for the proposed cooperative relaying scheme is given by

\[ T_{\text{min-coh}} = ((K + 2)N_T + (M + 1)/R + N_{FB} + 2N_D)T_S + T_{sel,K}. \]

(13)

where \( N_{FB} \) is the number of symbols used for destination feedback, and \( N_D \) is the number of symbols per data packet. The first part in the summation represents the time needed for training. The second part is the total time consumed for invalidating relay timers of not selected relays. The third and fourth parts embody the time required for destination feeding back the sum of second-hop channel power gains to the \( K \) selected relays and the time needed for cooperative data transmission, respectively. The last part is the time for selecting \( K \) relays.

III. ANALYSIS OF AVERAGE ENERGY EFFICIENCY

In this section, the average energy efficiency under maximum transmit power constraint, \( P_{\text{max}} \), is analysed for both cooperative communications and direct transmission, facilitating a quantitative comparison between them. Energy efficiency (in bits/Joule) is defined as the ratio of the number of successfully transmitted data bits to the corresponding energy consumption.

A. Cooperative Communications

Without loss of generality, we assume that \( M \geq 2 \) relays decode correctly the data transmitted from the source and that \( \{h_i\}_{i=1}^M \) and \( \{g_i\}_{i=1}^M \) are independent and identically distributed (i.i.d), i.e., \( h_i = h \) and \( g_i = g \) (\( i = 1, \ldots, M \)). The average energy efficiency of the proposed cooperative relaying scheme is given by

\[
EECC(K, M, \psi) = (1 - p_{\text{out}}^{\text{CC}})(1 - p_{\text{coll},K,n_{\text{max}}})RN_D
\]

\[
= \frac{1}{E\{E_O(K, M, \psi) + E_D(K, M, \psi)\}}
\]

\[
\approx \frac{(1 - p_{\text{out}}^{\text{CC}})(1 - p_{\text{coll},K,n_{\text{max}}})RN_D}{E\{E_O(K, M, \psi)\} + E\{E_D(K, M, \psi)\}},
\]

(14)

where the third line is obtained using the first-order Taylor approximation, \( \psi \) is the location of the \( K \) selected cooperating relays, \( p_{\text{out}}^{\text{CC}} \) is the outage probability of cooperative communications, \( E_O(.) \) denotes the energy consumption caused by signalling overhead, and \( E_D(.) \) is the energy consumed for data transmission.

**Proposition 3:** The outage probability of cooperative communications is given by

\[
p_{\text{out}}^{\text{CC}} = \frac{M!}{(K - 1)!} \sum_{j=0}^{M-K} \frac{(-1)^j}{(j + K)(M - K - j)!} \mu^j.
\]

(15)

where \( \mu = \frac{N_DB(2^R - 1)}{P_{\text{max}}} \).

**Proof:** The proof is given in Appendix C.

In (14), \( E_O(.) \) is the total energy consumed for training \( E_T(.) \), for destination feedback \( E_{FB}(.) \), for relays signalling \( M \) to source \( E_M(.) \), and for source telling non-selected relays to invalidate their timers \( E_{INV} \).

**Proposition 4:** The average energy consumption for the signalling overhead is given by

\[
E\{E_O(K, M, \psi)\} = E_T(K, M, \psi) + E\{E_M(M, \psi)\} + I_{\{2 \leq K \leq M\}}(K)E\{E_{FB}(K, M, \psi)\} + E_{INV},
\]

(16)

where

\[
E_T(K, M, \psi) = N_TN_0BT_S\left(\frac{1 - 2R}{\ln(1 - p_{\text{out}})}\right) + \left(\frac{1}{h} + \frac{1}{g} + K_{\text{max}}\left(\frac{1}{h} + \frac{1}{g}\right)\right),
\]

(17)

\[
E_{INV} = N_{INV}T_SP_{\text{max}},
\]

(18)

\[
E\{E_{FB}(K, M, \psi)\} = -N_{FB}T_SN_0B(2^R - 1)\frac{M!}{g(K - 1)!} \sum_{j=0}^{M-K} \frac{(-1)^j}{(M - K - j)!} E_i\left(\frac{j + K}{g}\right) \left(1 + M!\frac{1}{(K - 1)!} \sum_{j=0}^{M-K} \frac{(-1)^j}{(M - K - j)!} E_i\left(\frac{j + 1}{g}\right)\right)^{-1},
\]

(19)

\[
E\{E_M(M, \psi)\} = -M N_M T_S N_0 B(2^R - 1) e^\frac{\mu}{h} E_i\left(\frac{-\mu}{h}\right),
\]

(20)

in which \( N_{INV} \) and \( N_M \) are the numbers of symbols used for invalidating not-selected relays’ timers and relays signalling \( M \) to source, respectively, and \( E_i \) is the exponential integral function, defined as \( E_i(x) = \int_x^\infty e^{-t}/t \ dt \).

**Proof:** The proof is given in Appendix D.

The energy consumption for data transmission, \( E_D(.) \), comprises the energy consumed in the first hop \( E_D^1(.) \) and that in the second hop \( E_D^2(.) \).

**Proposition 5:** The average energy consumption for the data transmission is given by

\[
E\{E_D(K, M, \psi)\} = E\{E_D^1(K, M, \psi)\} + I_{\{K=1\}}(K)E\{E_D^1(K=1, M, \psi)\}
\]

\[
+ I_{\{2 \leq K \leq M\}}(K)E\{E_D^2(K > 1, M, \psi)\},
\]

(21)

where

\[
E\{E_D^1(K, M, \psi)\} = -K N_D T_S N_0 B(2^R - 1) e^\frac{\mu}{h} E_i\left(\frac{-\mu}{h}\right) K,
\]

(22)

\[
E\{E_D^2(K = 1, M, \psi)\} = -N_D T_S N_0 B(2^R - 1) \frac{M!}{g} \sum_{j=0}^{M-1} \frac{(-1)^j}{(M - j - 1)!} E_i\left(\frac{j + 1}{g}\right) \left(1 - M! \sum_{j=0}^{M-1} \frac{(-1)^j}{(M - j - 1)!} E_i\left(\frac{j + 1}{g}\right)\right)^{-1},
\]

(23)
\[ E\{E_D^{II}(K > 1, M, \psi)\} = \frac{N_D T_S N_0 B(2^R - 1)}{g} \left( \frac{M}{K} \right) \]

\[ \left(\frac{\Gamma(K - 1, \frac{g}{g})}{(K - 1)!} + \sum_{i=1}^{M-K} (-1)^i K - 1 \binom{M - K}{i} \left(\frac{M}{K} - i\right) \frac{\mu}{g} \right) \]

\[ \left(\frac{\Gamma(K - 1, \frac{g}{g})}{(K - 1)!} \right)^{K-1} \left(\frac{E_i}{-K + \frac{g}{g}} - E_i \left(\frac{K}{i} + \frac{\mu}{g}\right) \right) \]

\[ - \sum_{j=1}^{K-2} \left(\frac{-\frac{g}{g}}{j!} \Gamma\left(j, \frac{K}{i} + \frac{\mu}{g}\right) \right)^j \left(1 - \frac{M!}{(K - 1)!} \right)^{-1} \sum_{j=0}^{M-K} \frac{(-1)^j}{j!(j + K)} \left(1 - e^{-\frac{\mu}{g}}\right)^{-1}, \] (24)

with \(\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt\) being the upper incomplete gamma function [36].

**Proof:** The proof is given in Appendix E.

Lemma 1: The average energy efficiency in (14) can be upper bounded as follows

\[ \overline{E_{EC}}(K, M, \psi) \leq \frac{1 - p_{out}^{CC}(1 - p_{coll,K,n_{max}}) R N_D}{\overline{E_D}^{LB}(K, M, \psi) + \overline{E_D}^{LB}(K, M, \psi)}, \] (25)

where \(\overline{E_D}^{LB}(K, M, \psi)\) and \(\overline{E_D}^{LB}(K, M, \psi)\) denote the lower bound of average energy consumption for signalling overhead and for data transmission, respectively, and can be calculated as follows

\[ \overline{E_D}^{LB}(K, M, \psi) = E_T(K, M, \psi) + \overline{E_M}^{LB}(M, \psi) + E_{INV} + I_{(2 \leq K \leq M)}(K) \overline{E_{FB}}^{LB}(K, M, \psi), \] (26)

\[ \overline{E_D}^{LB}(K, M, \psi) = \overline{E_D}^{LB}(K, M, \psi) + I_{(K = 1)}(K) \overline{E_D}^{IL, LB}(K = 1, M, \psi) + I_{(2 \leq K \leq M)}(K) \overline{E_D}^{IL, LB}(K > 1, M, \psi), \] (27)

where

\[ \overline{E_M}^{LB}(M, \psi) = \frac{N_M M T_S N_0 B(2^R - 1)}{\mu + h}, \] (28)

\[ \overline{E_{FB}}^{LB}(K, M, \psi) = N_{FB} T_S N_0 B(2^R - 1) \left(\frac{K - 1)!}{M!} \right) \left(1 - \frac{M!}{(K - 1)!} \sum_{j=0}^{M-K} \frac{(-1)^j}{j!(j + K)} \left(1 - e^{-\frac{\mu}{g}}\right)^{-(\mu + \frac{g}{j + K})} \right), \] (29)

\[ \overline{E_D}^{IL, LB}(K, M, \psi) = N_D T_S N_0 B(2^R - 1) e^{h K} \left(\frac{K}{h + \mu K} \right), \] (30)

\[ \overline{E_D}^{IL, LB}(K = 1, M, \psi) = \frac{N_D T_S N_0 B(2^R - 1)}{M!} \left(1 - \frac{M!}{(M - j - 1)!} \sum_{j=0}^{M-1} \frac{(-1)^j}{j!(j + 1)} e^{-\frac{\mu}{g}} \left(\mu + \frac{g}{j + 1}\right)^{-1}\right), \] (31)

\[ \overline{E_D}^{IL, LB}(K > 1, M, \psi) = \frac{N_D T_S N_0 B(2^R - 1)}{M!} \left(1 - \frac{M!}{(M - j - 1)!} \sum_{j=0}^{M-1} \frac{(-1)^j}{j!(j + 1)} e^{-\frac{\mu}{g}} \left(\mu + \frac{g}{j + 1}\right)^{-1}\right), \] (32)

**Proof:** The proof is given in Appendix E.

### B. Direct Transmission

For energy efficiency analysis, we consider two transmission strategies for the direct communication between the source and the destination.

In the first strategy, source transmits training symbols at the minimum power required to satisfy the target \(R\) with outage probability \(p_{out}^{SD}\), i.e.,

\[ P_T^{SD} = N_0 B \frac{1 - 2^R}{h_0 \ln(1 - p_{out}^{SD})}, \quad h_0 = \left(\frac{\lambda_c}{4 \pi d_0}\right)^2 \left(\frac{d_{sd}}{d_0}\right)^{-\xi}, \] (33)

where \(h_0\) and \(d_{sd}\) denote the mean channel gain and the distance of the direct link from source to destination, respectively. Subsequently, data is transmitted using the maximum allowed transmission power, \(P_{max}\). The resulting average energy efficiency is given by

\[ \overline{E_{DT}}^{MAX} = \left(1 - p_{out}^{DT}\right) \frac{R N_D}{T_S} \left(\frac{N_T N_0 B}{h_0 \ln(1 - p_{out}^{DT})}\right)^{-1} + N_D P_{max} \right), \quad p_{out}^{DT} = 1 - e^{-\frac{\mu}{g}}, \] (34)

In the second strategy, during the channel estimation phase, source sends training symbols with transmission power as in (33) and destination estimates the channel gain. Thereafter, the destination feedbacks CSI to the source. This enables the source to transmit data with the minimum power required to meet the target rate \(R\). The corresponding average energy
efficiency and its upper bound are given, respectively, by
\[
E_{DT}^{ADP} \approx (1 - p_{out}^{DT}) \left( \frac{RN_D}{N_0BT_S} \right) \left( \frac{\tilde{h}_0}{1 - 2R} \right),
\]
\[
E_{DT}^{ADP,UB} = (1 - p_{out}^{DT}) \left( \frac{RN_D}{N_0BT_S} \right) \left( \frac{N_T}{\tilde{h}_0 \ln(1 - p_{out}^{DR})} - \frac{N_D + N_{FB}}{\mu + \tilde{h}_0} \right)^{-1}.
\] (35)
(36)

IV. OPTIMAL LOCATION OF RELAYS

In this section, we derive the optimal location of cooperating relays that maximizes the average energy efficiency. Without loss of generality, we assume that source is located at the origin \((0, 0)\), destination is located at \((d_{sd}, 0)\), and the selected relays are relatively close to one another so that their distances to the source are approximately the same. Furthermore, it is assumed that diversity gains offered by relays are sufficiently high to keep the outage probability very low, i.e., \(p_{out}^{DR} \approx 0\). In this case, the expressions in (28)-(32) can be simplified by replacing conditional expectations with unconditional ones. Since maximizing the average energy efficiency while maintaining the target rate \(R\), is equivalent to minimizing the lower bound of average energy consumption, the optimal location of cooperating relays is given by
\[
\psi_{opt}(K, M) \approx \arg\min_{\psi} \left( E_{O}^{LB} (K, M, \psi) + E_{D}^{LB} (K, M, \psi) \right),
\] (37)
where \(\psi\) denotes the distance from the source along the direct line connecting source and destination.

**Proposition 6:** The optimal position of cooperating relays is approximately given by
\[
\psi_{opt}(K, M) \approx \left( 1 + \left( \frac{\alpha(K, M)}{\beta(K, M)} \right)^{-\frac{1}{\xi}} \right)^{-1} d_{sd},
\] (38)
where
\[
\alpha(K, M) > 1, \quad \xi > 1,
\]
\[
\beta(K, M) = \frac{M}{R} + KN_D - \frac{N_T}{\ln(1 - p_{out}^{DR})},
\]
\[
\beta(K, M) = I_1(K=1)N_D \left( \sum_{j=1}^{M} \frac{1}{j} \right)^{-1} + I_{2 \leq K \leq M} (K)
\]
\[
= \left( \frac{N_D}{K} \left( 1 + \sum_{j=K+1}^{M} \frac{1}{j} \right)^{-1} + N_{FB} \left( \sum_{j=K}^{M} \frac{1}{j} \right)^{-1} \right)
\]
\[
- \left( 1 + K \right) \frac{N_T}{\tilde{h}_0 \ln(1 - p_{out}^{DR})}.
\]

**Proof:** The proof is given in Appendix G.

From (38) we can see that the optimal source-to-relay distance increases with \(d_{sd}\) and the path-loss exponent \(\xi\). The accuracy of (38) will be evaluated through simulation in Section VI.

V. OVERHEAD ANALYSIS

The signalling overhead for the cooperative relaying system in Fig. 1 can be calculated as
\[
\Omega_{pro} = (K + 2)N_T + \frac{M + 1}{R} + I_{2 \leq K \leq M} (K)N_{FB}.
\] (39)
It consists of three main parts. The first part is the overhead for transmitting \(N_T\) training symbols from the source to relays and from the destination to relays as well as the overhead for broadcasting \(N_T\) training symbols from the \(K\) selected relays. The second part represents the overhead for the \(M\) correctly decoding relays to inform the source of the number of correctly decoding relays (each of them uses one bit message, i.e., in total \(MR\) symbols, where \(R\) is the data rate in bits/symbol) and for the source to stop the \(M-K\) relay timers via a single-bit message, i.e., \(1/R\) symbols. The last part is the overhead needed for the destination to use \(N_{FB}\) Symbols to feedback the sum of \(K\) best second-hop channel power gains to the selected relays. Since the feedback from the destination is only required for cooperative beamforming \((K \geq 2)\), the feedback overhead is multiplied by the indicator function \(I_{2 \leq K \leq M} (K)\).

The signalling overhead for the cooperative relaying scheme in [29], which is referred to as the reference scheme hereafter, can be calculated as
\[
\Omega_{ref} = MN_T + \left( K_{ref} + I_{2 \leq K_{ref} \leq M} (K_{ref}) \right) N_{FB},
\] (40)
where the number of selected relays, \(K_{ref}\), is given by
\[
K_{ref} = \begin{cases} 1, & M \leq 2 \\ 2, & 3 \leq M \leq 6 \\ 3, & 7 \leq M \leq 15 \end{cases}.
\]

Compared to the reference scheme, the signalling overhead reduction achieved by the proposed scheme is given by
\[
\Omega_{red} = \left( \frac{\Omega_{ref} - \Omega_{pro}}{\Omega_{ref}} \right) 100\%
\]
\[
= \left( M - K - 2 \right)N_T - \frac{M + 1}{R}
\]
\[
+ \left( K_{ref} + I_{2 \leq K_{ref} \leq M} (K_{ref}) - I_{2 \leq K \leq M} (K) \right) N_{FB}
\]
\[
\left( MN_T + \left( K_{ref} + I_{2 \leq K_{ref} \leq M} (K_{ref}) \right) N_{FB} \right)^{-1} 100\%.
\] (41)

When the number of correctly decoding relays approaches infinity, the overhead reduction converges to
\[
\lim_{M \to \infty} \Omega_{red} = \left( 1 - \frac{1}{RN_T} \right) 100\%.
\] (42)
which depends only on the data rate \((R)\) and the number of training symbols \((N_T)\) used for channel estimation. Increasing \(R\) and/or \(N_T\) for both schemes would lead to more significant overhead reduction by the proposed scheme.
VI. SIMULATION RESULTS

The performance of the proposed cooperative relaying scheme and the accuracy of the analytical results are evaluated through simulation. In the simulation, source and destination are located at \((0, 0)\) and \((d_{sd}, 0)\), respectively. The \(M(>1)\) relays that can correctly decode messages from the source, are situated close to one another with approximately the same distance \(\psi\) from the source. System parameters as listed in Table I conform to 3GPP LTE-A [37]. For illustration purposes we consider a single subcarrier with 16-QAM modulation, i.e., \(R=4\). During training, one OFDM symbol \((N_{TF}=1)\) is transmitted at the target rate \(R\) with outage probability \(p_{\text{out}}^{\text{sm}}=0.12\). The destination utilizes two OFDM symbols \((N_{FB}=2)\) to feedback the sum of second-hop channel power gains to the selected relays.

<table>
<thead>
<tr>
<th>TABLE I SYSTEM PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency, (f_c)</td>
</tr>
<tr>
<td>Reference distance, (d_0)</td>
</tr>
<tr>
<td>Path-loss exponent, (\xi)</td>
</tr>
<tr>
<td>Noise power spectral density, (N_0)</td>
</tr>
<tr>
<td>Maximum transmission power, (P_{\text{max}})</td>
</tr>
<tr>
<td>Subcarrier bandwidth, (\Delta f)</td>
</tr>
<tr>
<td>Symbol length, (T_s)</td>
</tr>
<tr>
<td>Data packet length (in OFDM symbols), (N_D)</td>
</tr>
<tr>
<td>Source to destination distance, (d_{sd})</td>
</tr>
</tbody>
</table>

Fig. 2 shows both the analytically calculated and simulated collision probability \(p_{\text{coll},K,n_{\text{max}}}\) and relay selection time \(T_{\text{sel},K}\) versus \(\theta = \lambda/\Delta y\) for two different numbers of selected relays and \(M=10\). The results calculated using (5), (7), and (8) are in close agreement with those obtained from simulation. We can see that with increasing \(\theta\), the collision probability decreases, whereas the relay selection time increases. There exists a trade-off between \(p_{\text{coll},K,n_{\text{max}}}\) and \(T_{\text{sel},K}\) that is controlled by \(\lambda\) for given \(\Delta y = N_{TF}T_S\). For a given \(\theta\), selecting one more relay leads to a higher collision probability and a higher relay selection time. In the following, we set \(\theta=70\) as it provides a good trade-off between collision probability and relay selection time, both of which will be included in the evaluation of energy efficiency and spectral efficiency. With the parameter values in Table I and for \(M=10\), it can be calculated using (5) and (13) that the minimum channel coherence time required for the proposed scheme is 22ms, which is significantly shorter than the channel coherence time \(T_{\text{coh}}=76.1\)ms for low mobility scenario (with speed of 3km/h).

Fig. 3 plots the simulation results of average energy efficiency for \(\psi=50\)m over different values of \(M\) and \(K\). It can be seen that the maximum average energy efficiency is achieved by selecting the \(K=2\) best relays. We also observe that deploying all decoding relays, i.e., \(K=M\) \((M>2)\), for cooperative beamforming exhibits the lowest energy efficiency, because the energy consumption for signalling overhead outweighs the energy savings from cooperative beamforming. For a given \(K\), a larger number of correctly decoding relays \((M)\) leads to a higher energy efficiency due to increased diversity gain.

Fig. 4 plots the optimal number of selected relays that maximizes the average energy efficiency obtained through simulations versus the source-to-relay distance. We can see that for \(M=3\) and \(M=5\) (the two curves overlap with each other), selecting the best two relays is optimal for source-to-relay distances up to 150m, beyond which the best relay selection \((K=1)\) maximizes the energy efficiency. This is because for long source-to-relay distances, the overhead energy consumption required to select one additional relay plus the extra source transmission power required to reach the additional relay in the first hop outweighs the energy savings from cooperative beamforming in the second hop. In the case of \(M=10\), the threshold source-to-relay distance reduces to 130m due to increased relay transmission collision probability. The results may change with different sizes of data packets (see Fig. 9 and Fig. 10).
In Fig. 5, we evaluate the accuracy of the approximate optimal location of cooperative relays in (38) by comparing it with simulation results. There is a good match between the theoretically calculated optimal location of relay(s) and that found through simulation for both the best relay selection and the proposed scheme. Conforming to the observation in Fig. 4, the optimal location of relays is closer to the source for the proposed scheme than for the best relay selection. For both schemes, as $M$ increases (e.g., due to better first-hop channel conditions), the optimal location of relays gets only slightly closer to the source. This indicates that the optimal location of relay(s) can be predicted using (38) for both the proposed cooperative relaying scheme and the best relay selection, and the prediction does not need to be updated frequently.

![Fig. 5. Approximate optimal location of cooperative relays versus $M$.](image)

In Fig. 6, the overhead reduction offered by the proposed scheme as compared to the reference scheme [29] calculated using (41) is depicted versus $M$ for three different numbers of training symbols ($N_T$). The reduction in signalling overhead increases with increasing $M$ for all considered $N_T$, due to the stronger dependence on $M$ of the reference scheme than the proposed scheme, as shown in (40). For $M<6$, a smaller $N_T$ leads to a higher reduction in signalling overhead; while for $M>8$, a larger $N_T$ leads to a higher overhead reduction. As it can be seen from (41), for small $M$, e.g., $M=3$, $M-K-2<0$, and increasing $N_T$ decreases the overhead reduction. According to (42) for large $M$, the signalling overhead reduction increases with $N_T$ for given $R$. Significant increase of $\Omega_{red}$ occurs from $M=6$ to $M=7$ because the reference scheme increases the number of selected relays from 2 to 3 as $M$ increases from 6 to 7 (see (40)).

Table II shows the signalling overhead reduction achieved by the proposed scheme with respect to the reference scheme [29] for different modulation orders and numbers of training symbols. The results in the table conform to (42), i.e., increasing modulation order and/or number of training symbols leads to a higher reduction in signalling overhead. For instance, increasing modulation order from 4-QAM to 64-QAM for $N_T=1$, increases the overhead reduction from 36.11% to 56.48%.

The $M$ correctly decoding relays each transmit a training symbol to the destination, which performs channel estimation and selects the $K_{ref}$ relays (as shown in Section V) with the highest second-hop channel power gains. The destination feeds back first the corresponding channel power gain to each selected relay and then the sum of the $K_{ref}$ channel power gains to all of them. We can see that the performance of the reference scheme with fixed source transmit power ($P_{max}$) is nearly independent of the relay location and the value of $M$. For a more comprehensive comparison, we assume that the source knows the minimum power required to reach all $M$ correctly decoding relays, so that the reference scheme is also able to use adaptive source transmission power. We can see that the energy efficiency of the reference scheme is significantly improved due to the use of adaptive source transmission power. For $M>2$, the proposed scheme offers higher energy efficiency than the reference scheme (with adaptive source transmit power) for all three cases, and the gap between the two schemes increases with $M$ for each given

![Fig. 6. Overhead reduction of the proposed cooperative relaying scheme over the reference scheme [29] for different numbers of training symbols $N_T$.](image)

![Fig. 7. Average energy efficiency for the proposed cooperative relaying scheme and the reference scheme [29] for three different locations of cooperating relays.](image)

<table>
<thead>
<tr>
<th>Modulation order</th>
<th>4-QAM</th>
<th>16-QAM</th>
<th>64-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_T$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>36.11</td>
<td>51.39</td>
<td>56.48</td>
</tr>
<tr>
<td>2</td>
<td>44.64</td>
<td>54.46</td>
<td>57.73</td>
</tr>
</tbody>
</table>

The $M$ correctly decoding relays each transmit a training symbol to the destination, which performs channel estimation and selects the $K_{ref}$ relays (as shown in Section V) with the highest second-hop channel power gains. The destination feeds back first the corresponding channel power gain to each selected relay and then the sum of the $K_{ref}$ channel power gains to all of them. We can see that the performance of the reference scheme with fixed source transmit power ($P_{max}$) is nearly independent of the relay location and the value of $M$. For a more comprehensive comparison, we assume that the source knows the minimum power required to reach all $M$ correctly decoding relays, so that the reference scheme is also able to use adaptive source transmission power. We can see that the energy efficiency of the reference scheme is significantly improved due to the use of adaptive source transmission power. For $M>2$, the proposed scheme offers higher energy efficiency than the reference scheme (with adaptive source transmit power) for all three cases, and the gap between the two schemes increases with $M$ for each given

Table II

| SIGNALLING OVERHEAD REDUCTION $\Omega_{red}($%$)$ COMPARED TO [29] FOR DIFFERENT MODULATION ORDERS, $N_T$=1,2, AND $M$=10. |
|------------------|-------|--------|--------|
| $N_T$            |       |        |        |
| 1                | 36.11 | 51.39  | 56.48  |
| 2                | 44.64 | 54.46  | 57.73  |
relay location. This is mainly because of two reasons. First, the proposed scheme enables the source to adapt its transmission power to reach only the $K$ selected relays ($K \leq M$), while the reference scheme requires a source transmission power that can reach all the $M$ correctly decoding relays. Second, the energy consumption for signalling overhead is reduced in the proposed scheme. In contrary to the reference scheme that loses energy efficiency with increasing $M$ for large values of $M$, the proposed scheme is able to maintain a stable energy efficiency at large values of $M$, indicating a much better scalability.

Comparison of average energy efficiency between the proposed cooperative relaying scheme, best relay selection, and direct transmission using adaptive transmission power is depicted in Fig. 8, where the position of cooperating relays is set at $\psi = d_{sd}/10$ for different $d_{sd}$. Fig. 8 presents both simulation results and theoretical results calculated using (25) and (36) for cooperative and direct transmissions, respectively. We can see that the theoretical results closely match the simulation results. Direct transmission is more energy efficient than the proposed scheme and best relay selection for $d_{sd} < 300$ m, as it requires less signalling overhead. As $d_{sd}$ increases, the energy efficiency of cooperative communications decreases much slower than direct transmission, leading to a higher energy efficiency for $d_{sd} \geq 300$ m. This is because cooperative communications have lower outage probability and can use lower transmission power than direct transmission for long source-to-destination distances, due to the cooperative gains. The proposed scheme achieves higher energy efficiency than the best relay selection, because deploying one more relay offers higher cooperative gains.

Fig. 8. Energy efficiency comparison between direct transmission and cooperative communications for $\psi = d_{sd}/10$.

Fig. 9 shows the simulation results of the average energy efficiency over different data packet sizes $N_D$ for the proposed scheme and best relay selection. In all considered cases, increasing data packet size leads to higher energy efficiency, as data transmission becomes the dominant part in overall energy consumption and the impact of overhead diminishes. As shown in Fig. 3, for $N_D=140$ OFDM symbols, the optimal number of relays for cooperative beamforming is limited to $K=2$ by the related signalling overhead. For $N_D > 200$ OFDM symbols, the optimal number of relays selected for cooperative beamforming increases to 3, because the impact of overhead energy efficiency is mitigated by long data packets. The increase of $K$ leads to a higher cooperative beamforming gain, which further improves the energy efficiency.

Fig. 9. Average energy efficiency versus data packet length for different $M$ and $K$, and $\psi = 50$ m.

In the following, we include a comparison of spectral efficiency (SE) to make the performance evaluation more comprehensive. The SE of direct transmission is given by [24]

\[
SE_{DT} = (1 - p_{DT}^{out}) R \left( \frac{T_{coh}}{T_{coh} - T_{DT}^{out}} \right),
\]

where $p_{DT}^{out}$ and $T_{coh}$ are the outage probability of direct transmission and channel coherence time, respectively, and $T_{DT}^{out} = (N_T + N_{FB}) T_S$ denotes the overhead transmission time (i.e., source training and destination feedback for CSI) of direct transmission. With the half-duplex DF relays, the SE of the proposed cooperative relaying scheme can be calculated...
as

$$SE_{CC} = \frac{(1 - p_{\text{out}})(1 - p_{\text{coll},K,n_{\text{max}}})R}{2B} \left( - T_{\text{coh}} - T_{\text{sel},K} - T_{\text{CC}}^{\text{out}} \right),$$

(44)

where the factor 1/2 results from the two-hop half-duplex transmission, $p_{\text{out}}^{\text{CC}}$ is given in (15), and $T_{\text{CC}}^{\text{out}} = \left((K + 2)N_{L} + N_{FB} + \frac{M+1}{R}\right)T_{S}$ is the overhead transmission time of cooperative relaying.

Fig. 11 shows the SE of the proposed scheme, the best relay selection, and the reference scheme [29] normalized with respect to that of direct transmission (i.e., $SE_{CC}/SE_{DT}$) versus $d_{sd}$ for $M=10$ and two different modulation orders. The normalized SE of the best relay selection and the proposed scheme in the ideal case without any relay transmission collision or delay due to relay selection (i.e., $p_{\text{coll},K,n_{\text{max}}}=0$, $T_{\text{sel},K}=0$) is also plotted. In the ideal case, the proposed scheme and the best relay selection achieve the same SE, which is the highest SE that can be expected for cooperative communications in theory, because there is no loss of SE due to relay transmission collisions or relay selection time. We can see that with relay transmission collisions and relay selection time taken into account, the SE of the proposed scheme is reasonably close to that of the ideal case. This shows that the loss of SE caused by the proactive relay subset selection in the proposed scheme is reasonably low. In most cases considered in Fig. 11, the normalized SE is less than one, i.e., cooperative communications are less spectral efficient than direct transmission. This is mainly due to the factor 1/2 in (44) of half-duplex relaying. For each considered modulation, the normalized SE of the proposed scheme and the best relay selection is much higher and increases much faster with $d_{sd}$ than that of the reference scheme. This indicates that while the SE of direct transmission decreases with $d_{sd}$, the proposed scheme and the best relay selection achieve much higher SE than the reference scheme at long source-to-destination distances. The reason is that the reference scheme requires the relays to transmit on orthogonal subcarriers in order to ensure the orthogonality between relay transmissions during the training phase [29], while in the proposed scheme relays contend with each other for the same subcarrier. For 64-QAM and $d_{sd}>860\text{m}$, the proposed scheme is more spectral efficient than direct transmission. The proposed scheme exhibits slightly lower SE than the best relay selection owing to the higher collision probability and longer relay selection time for deploying more relays.

VII. CONCLUSIONS

In this paper, we have proposed an energy-efficient and low signalling overhead cooperative relaying scheme that selects a subset of DF relays for cooperative beamforming in a proactive manner by relays using their local timers. We have carried out theoretical analysis of energy efficiency under maximum transmission power constraint, with practical data packet length, and considering the overhead for obtaining CSI, relay selection, and cooperative beamforming. The accuracy of our derived expression of average energy efficiency and closed-form approximate expression for the optimal location of relays that maximizes energy efficiency has been verified by simulation results. The analytical and simulation results have shown that the proposed scheme not only reduces the signalling overhead significantly, but also exhibits higher energy efficiency compared to the existing energy-efficient cooperative relaying scheme [29], best relay selection, all relay selection, and direct transmission, especially for relays located close to the source. We have also demonstrated that energy efficiency of cooperative relaying increases with data packet size under the constraint of channel coherence time. Our results can be used as a guideline for developing dynamic energy-efficient cooperative transmission strategies that can adapt to different channel and system conditions.

APPENDIX A

PROOF OF PROPOSITION 1

Probability density function (pdf) for the $K$-th best channel power gain $g_{K:M}$ is given by [32]

$$p_{g_{K:M}}(x) = \frac{M!}{g(K-1)!} \sum_{i=0}^{M-K} \frac{(-1)^i}{(M-K-i)!} e^{-\frac{xK}{\tilde{g}}} x^i.$$  

(45)

It follows then for the average relay selection time

$$T_{\text{sel},K} = \mathbb{E}\{I_{K:M}\} = \Delta_g \mathbb{E}\left\{ \frac{\tilde{\lambda}}{g_{K:M} \Delta_g} \right\}$$

$$= \Delta_g \sum_{n=1}^{n_{\text{max}}} n \Pr\left\{ \frac{\tilde{\lambda}}{(n+1) \Delta_g} \leq g_{K:M} \leq \frac{\tilde{\lambda}}{n \Delta_g} \right\}$$

$$= \Delta_g \sum_{n=1}^{n_{\text{max}}} \int_{\Delta_g (n+1)}^{\Delta_g n} p_{g_{K:M}}(x)dx.$$  

(46)

Using (45) and evaluation of the integral in (46) leads to (5).

APPENDIX B

PROOF OF PROPOSITION 2

Let $K$ be the set containing $(K-1)$ best relays and $\mathcal{R} = \mathcal{D} \setminus (K \cup \{j\})$. For collision-free $K$ best relay selection,
the following conditions have to be satisfied: (1) for \((K-1)\) best relays \(\hat{\lambda}/g_\epsilon \leq \Delta g\) and no collisions between relays in this interval, (2) for the \(K\)th best relay \(n\Delta g \leq \hat{\lambda}/g_\epsilon < (n+1)\Delta g\), and (3) for the remaining \((M-K)\) relays \(\hat{\lambda}/g_\epsilon \geq (n+1)\Delta g\). For the best relay selection \((K=1)\) only conditions (2) and (3) are relevant.

Using multinomial distribution, the probability that all the three conditions (for \(K>1\)) are fulfilled is given by

\[
p_{\text{no-coll}, K=1, n_{\text{max}}} = \frac{M!}{(M-K)!} \left( \prod_{i \in \mathbf{R}} \Pr\{\lambda/g_i < n\Delta g\} \right) 
\]

where

\[
\Pr\{\lambda/g_{\epsilon} \leq \Delta g\} = \left( \sum_{n=0}^{n_{\text{max}}} \frac{M!}{(M-K)!} \left( \prod_{i \in \mathbf{R}} \Pr\{\lambda/g_i \geq n\Delta g\} \right) \right)^{K-1} 
\]

while the probability that only conditions (2) and (3) are satisfied for best relay selection \((K=1)\) can be calculated as follows

\[
p_{\text{no-coll}, K=1, n_{\text{max}}} = M \sum_{n=0}^{n_{\text{max}}} \left( F_g(\hat{\lambda}/n\Delta g) - F_g(\hat{\lambda}/(n+1)\Delta g) \right) 
\]

where \(F_g(x) = 1 - e^{-x/\bar{g}}\) is cumulative distribution function (cdf) of channel power gain \(g\). The collision probability can be calculated using \(p_{\text{coll}, K, n_{\text{max}}} = 1 - p_{\text{no-coll}, K, n_{\text{max}}}\).

**APPENDIX C**

**PROOF OF PROPOSITION 3**

As we have assumed that \(M \geq 2\), outage occurs only in the second hop. For the best relay selection \((K=1)\), outage is declared if channel power gain \(g_{1:M}\) cannot support the target rate \(R\) under maximum transmission power constraint, \(P_{\text{max}}\). For cooperative beamforming \((K \geq 2)\) outage occurs if the destination transmit power to feedback the sum of second-hop channel power gains does not meet the target rate \(R\) with \(P_{\text{max}}\). It follows then for the outage probability of cooperative communications

\[
p_{\text{out}}^{CC} = I_{\{K=1\}}(K) \Pr\{g_{1:M} < \mu\} + I_{\{2 \leq K \leq M\}}(K) \Pr\{g_{K:M} < \mu\} = \int_{0}^{\mu} p_{g_{K:M}}(x) \, dx, 
\]

using (45) in (49) and integration leads to (15).

**APPENDIX D**

**PROOF OF PROPOSITION 4**

Expressions (16)-(18) can be obtained easily from the Fig. 1 and discussions in Section II.

Average energy consumption for the destination feedback is calculated as follows

\[
\mathbb{E}\{E_{FB}(K, M, \psi)\} = N_{FB}T_S N_0 B(2^R - 1) \sum_{i=1}^{M} \mathbb{E}\left\{ \frac{1}{h_i} \left| g_{K:M} \geq \mu \right\} \right\} 
\]

where \(F_{g_{K,M}}(\mu) = \int_{0}^{\mu} p_{g_{K:M}}(x) \, dx\) is cdf of the \(K\)-th best channel power gain. Using (45) in (50) and performing the integration results in (19).

Average energy consumed to signal \(M\) to the source is given by

\[
\mathbb{E}\{E_M(M, \psi)\} = N_M T_S N_0 B(2^R - 1) \sum_{i=1}^{M} \mathbb{E}\left\{ \frac{1}{h_i} \left| h_i \geq \mu \right\} \right\} 
\]

where \(p_{h_i}(x) = \frac{1}{\bar{g}^i} e^{-x/\bar{g}}\) for \(i = 1, \ldots, M\). Evaluation of (51) leads to (20).

**APPENDIX E**

**PROOF OF PROPOSITION 5**

Summation of the average energy consumption for the first and second hop data transmission as well as considering both cases best relay selection \((K=1)\) and cooperative beamforming \((K \geq 2)\) leads to (21).

The average energy consumption for data transmission from the source to the \(K\) selected relays is given by

\[
\mathbb{E}\{E_D(K, M, \psi)\} = N_D T_S N_0 B(2^R - 1) \sum_{i=1}^{M} \mathbb{E}\left\{ \frac{1}{H_{\text{min}}} \left| h_{\text{min}} \geq \mu \right\} \right\} 
\]

where \(H_{\text{min}} = \min\{h_1, \ldots, h_K\}\) and [32]

\[
p_{H_{\text{min}}}(x) = \frac{M!}{\bar{g}^i} \sum_{i=0}^{M-1} \frac{(-1)^i}{(M-1)!} \int_{0}^{\mu} e^{-x/\bar{g}^i} \, dx, 
\]

Calculation of the integral in (52) yields (22).
Average energy consumed in the second hop for the data transmission for the best relay selection (K=1) can be calculated as follows

\[ E\{E_D^I(K = 1, M, \psi)\} = N_D T_S N_0 B(2R - 1) \mathbb{E}\left\{ \frac{1}{g_{1:M}} |g_{1:M} \geq \mu \right\} \]

\[ = \frac{M!}{K K! g^{K+1}} \sum_{j=K+1}^{M} \frac{(-1)^j + K - 1}{(j - K - 1)!(M - j)!} \mathbb{E}\{ (s + \frac{j}{gR})^{-K} \} \mathbb{E}\{ (s + \frac{1}{g})^{-1} \}, \quad (59) \]

Using (45) with K=1 and evaluation of (53) results in (23).

Average energy consumed in the second hop for the data transmission for cooperative beamforming (K≥2) is given by

\[ E\{E_D^I(K > 1, M, \psi)\} = N_D T_S N_0 B(2R - 1) \mathbb{E}\left\{ \frac{1}{g_{1:M}} |g_{1:M} \geq \mu \right\} \]

\[ = \frac{M!}{K K! g^{K+1}} \sum_{j=K+1}^{M} \frac{(-1)^j + K - 1}{(j - K - 1)!(M - j)!} \mathbb{E}\{ (s + \frac{j}{gR})^{-K} \} \mathbb{E}\{ (s + \frac{1}{g})^{-1} \}, \quad (59) \]

and pdf can be computed as

\[ p_{\Sigma_{i=1}^{K} g_{i:M}}(x) = \mathcal{L}^{-1}\{ \mathcal{M}_{\Sigma_{i=1}^{K} g_{i:M}}(s) \} \]

where \( \mathcal{L}^{-1} \) is inverse Laplace transformation and ‘*’ denotes convolution operator. Using Laplace transform table [36] and performing convolution leads to

\[ p_{\Sigma_{i=1}^{K} g_{i:M}}(x) = \frac{M!}{K K! g^{K+1}} \sum_{j=K+1}^{M} \frac{(-1)^j + K - 1}{(j - K - 1)!(M - j)!} \]

\[ \mathcal{L}^{-1}\{ (s + \frac{j}{gR})^{-K} \} \ast \mathcal{L}^{-1}\{ (s + \frac{1}{g})^{-1} \}, \quad (60) \]

and for the sum of K largest channel power gains

\[ \sum_{i=1}^{K} g_{i:M} = \sum_{j=1}^{K} j d_j + K \sum_{j=K+1}^{M} d_j. \quad (55) \]

Spacing pdf is given by [38]

\[ p_{d_m}(x) = \frac{m}{g} e^{-m x}, \quad x \geq 0. \quad (56) \]

It follows for moment generating function (MGF)

\[ \mathcal{M}_{\Sigma_{i=1}^{K} g_{i:M}}(s) = (1 - \bar{g} s)^{-K} \prod_{j=K+1}^{M} \left( 1 - \frac{i}{g} \frac{K}{j} \right)^{-1}. \quad (57) \]

Using partial fraction for simple roots [36] leads to

\[ \mathcal{M}_{\Sigma_{i=1}^{K} g_{i:M}}(s) = \frac{M!}{K K! g^{K+1}} \sum_{j=K+1}^{M} \frac{(-1)^j + K}{(j - K - 1)!(M - j)!} \left( s - \frac{j}{gR} \right)^{-1}, \quad (58) \]

APPENDIX F

PROOF OF LEMMA 1

Using Jensen’s inequality for conditional expectations, \( \mathbb{E}\{\frac{1}{X}\} \geq \frac{1}{\mathbb{E}\{X\}} \), where X and Y are random variables, the average energy consumption for the signalling overhead and data transmission can be lower bounded as follows

\[ \mathbb{E}\{E_O(K, M, \psi)\} \geq \mathbb{E} E_T(K, M, \psi) + \mathbb{E} E_{INV} + \mathbb{E} E_{FED}(K, M, \psi), \]

\[ \mathbb{E}\{E_D(K, M, \psi)\} \geq \mathbb{E} E_{DLB}(K, M, \psi) + \mathbb{E} I_{(2 \leq K \leq M)}(K) E_{FLB}(K, M, \psi), \]

\[ + \mathbb{E} I_{(K = 1)}(K) E_{FLB}(K = 1, M, \psi) + \mathbb{E} I_{(2 \leq K \leq M)}(K) E_{FLB}(K > 1, M, \psi), \]
where
\[ E^{LB}_M (M, \psi) = N_M T S N_0 B (2^R - 1) \sum_{i=1}^{M} \mathbb{E} \{ h_i | h_i \geq \mu \}, \]
(61) \[ E^{LB}_F (K, M, \psi) = \frac{N_F B T S N_0 B (2^R - 1)}{\mathbb{E} \{ g_{K:M} | g_{K:M} \geq \mu \}}, \]
(62) \[ E^{LB}_I (K, M, \psi) = \frac{N_D T S N_0 B (2^R - 1)}{\mathbb{E} \{ H_{\text{min}} | H_{\text{min}} \geq \mu \}}, \]
(63) \[ E^{LB}_D (K = 1, M, \psi) = \frac{N_D T S N_0 B (2^R - 1)}{\mathbb{E} \{ g_{1:M} | g_{1:M} \geq \mu \}}, \]
(64) \[ E^{LB}_D (K > 1, M, \psi) = \frac{N_D T S N_0 B (2^R - 1)}{\sum_{i=1}^{K} \mathbb{E} \{ g_{i:M} | g_{i:M} \geq \mu \}}. \]
(65)
Evaluations of \( \mathbb{E} \{ \cdot \} \) in (61)-(65) can be done similar to Appendix E and result in (28)-(32).

APPENDIX G

PROOF OF PROPOSITION 6

The optimal location of cooperating relays is given by
\[ \psi_{\text{opt}} (K, M) \approx \arg \min_{\psi} \left( E^{LB}_O (K, M, \psi) + E^{LB}_D (K, M, \psi) \right) \]
(66)
\[ = \arg \min_{\psi} \left( (C_T + C_D^I + C_M) \psi^\xi + (C_T + C_D^I) + I_{\{2 \leq K \leq M\}} (K) C_{FB} (d_{sd} - \psi)^\xi \right. \]
\[ + K C_T \max (\psi^\xi, (d_{sd} - \psi)^\xi), \]
In order to find \( \psi_{\text{opt}} (K, M) \), two different cases have to be investigated.

Case I: \( 0 \leq \psi \leq \frac{d_{sd}}{2} \)

Using the following substitutions in (66),
\[ \alpha_I = C_T + C_D^I + C_M, \]
\[ \beta_I = (1 + K) C_T + C_D^I + I_{\{2 \leq K \leq M\}} (K) C_{FB}, \]
we rewrite the optimization problem as follows
\[ \min_{\psi} \alpha_I \psi^\xi + \beta_I (d_{sd} - \psi)^\xi \]
(67)
\[ \text{s.t.} \]
\[ \psi \geq 0, \; \psi \leq \frac{d_{sd}}{2}. \]

It can be solved using Karush-Kuhn-Tucker (KKT) conditions [39]
\[ \xi \left( \alpha_I \psi^{\xi-1} - \beta_I (d_{sd} - \psi)^{\xi-1} \right) + \lambda_1 - \lambda_2 = 0, \]
\[ \lambda_1 \left( \psi - \frac{d_{sd}}{2} \right) = 0, \]
\[ \lambda_2 \psi = 0, \]
\[ \lambda_1 \geq 0, \; \lambda_2 \geq 0. \]
The above KKT conditions are only fulfilled for
\[ \psi_I = \left( 1 + \left( \frac{\alpha_I}{\beta_I} \right)^{\frac{1}{\xi}} \right)^{-1} d_{sd}. \]
(68)
Case II: \( \frac{d_{sd}}{2} < \psi \leq d_{sd} \)

Analogous to case I, it can be shown that
\[ \psi_{II} = \left( 1 + \left( \frac{\alpha_{II}}{\beta_{II}} \right)^{\frac{1}{\xi}} \right)^{-1} d_{sd}. \]
(69)
As \( \alpha_{II} > \alpha_I \) and \( \beta_{II} < \beta_I \)
\[ \psi_{II} < \left( 1 + \left( \frac{\alpha_{II}}{\beta_{II}} \right)^{\frac{1}{\xi}} \right)^{-1} d_{sd}, \]
(70)
i.e., \( \psi_{II} < \psi_I \) and this violates \( \psi_{II} > \frac{d_{sd}}{2} \). Therefore, the optimal solution is \( \psi_{\text{opt}} = \psi_I \).

REFERENCES


