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Low-complexity Distributed Beamforming for Relay Networks with Real-valued Implementation

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Abstract. The distributed beamforming problem for amplify-and-forward relay networks is studied. Maximizing output SNR (signal-to-noise ratio) for distributed beamforming can be considered as a generalized eigenvector problem (GEP) and the principal eigenvector and its eigenvalue can be derived with a standard closed-form solution. In this paper, four classes of beamforming algorithms are derived based on different design criteria and constraints, including maximizing output SNR subject to a constraint on the total transmitted signal power, minimizing the total transmitted signal power subject to certain level of output SNR, minimizing the relay node number subject to constraints on the total signal power and output SNR, and a robust algorithm to deal with channel estimation errors. All of the algorithms have a low computational complexity due to the proposed real-valued implementation.

Keywords. Distributed beamforming, relay networks, generalized eigenvector problem, robust algorithm.

I. Introduction

Distributed beamforming (also called collaborative beamforming) is a form of cooperative communication using a relay network consisting of two or more nodes forwarding the message from a transmitter to an intended receiver when there is no direct link between them or the link is so weak that it cannot support the minimum required quality of service (QoS) [1], [2], [3], [4], [5]. It can improve the QoS when channel conditions are poor, and the resultant cooperation diversity can also provide benefits of increased range and data rate or improved energy efficiency [1], [6], [7]. In general, distributed beamforming algorithms are divided into three categories: amplify-and-forward (AF) [6], [7], [8], [9], decode-and-forward [10], [9], and compress-and-forward [11], [12]. The AF scheme is of particular interest and has been studied extensively due to its simplicity in both algorithm and implementation aspects. In this paper we will focus on the AF-based one.

Depending on different design objectives, constraints and assumptions, various methods have been proposed based on the knowledge of channel state information (CSI). With the assumption that the source knows the direct link (the channel between itself and the destination), each relay knows its own channels and the destination knows all of the channels, the problem of maximizing output SNR subject to individual power transmission constraint for both with and without direct link scenarios was solved in [13]. The distributed beamforming problem of maximizing output SNR with total and individual power constraints for a three-hop relay system was investigated in [14]. Instead of maximizing the output SNR, beamforming schemes based on the minimum mean square error (MMSE) criterion were studied in [15], [16], [17]. Based on the second-order statistics of CSI, two beamforming algorithms, one for minimizing the total transmit power subject to the receiver QoS constraint and the other one for maximizing the receiver SNR subject to two types of power constraints, were proposed in [18]. This work was then extended to two-way communication systems in [19], and multiple peer-to-peer communications based on a common relay network in [20]. Given errors in the downlink (from relay nodes to destination) CSI, a robust optimization algorithm was derived with the consideration that the uplink (from source to relay nodes) coefficient could be estimated more accurately. Another robust scheme was proposed in [21] for uplink transmission with each node equipped with an antenna array.

In this paper, with the knowledge of instantaneous CSI, we propose a low-complexity real-valued implementation of the system by introducing a preprocessing stage with a set of offset phase shifts into the relay network. Four problems are then studied based on this implementation. The first one is maximizing output SNR subject to a limit on the total transmitted signal power, which can be considered as a real-valued generalized eigenvector problem (GEP) with its matrix pair consisting of a special rank-one signal correlation matrix and a diagonal noise correlation matrix. The GEP is then transformed into an eigenvector problem (EP), where the principal eigenvector and its eigenvalue can be derived with a closed-form solution. The derivation does not involve any eigen-decomposition operation and avoids the most time consuming part of the algorithm. The second problem studied is minimizing power consumption with a constraint on a certain level of output SNR. Its optimum solution has to satisfy the standard structure of the principal eigenvector, with a single unknown parameter solved by an iterative method. The third problem is based on a new consideration that the free nodes in a network are limited resources, and some of them could enter or exit due to their operation state and their own consideration such as battery status, and a new algorithm for minimizing the relay number subject to a power consumption limit and a certain level of output SNR is proposed. Two solutions are derived by setting different priorities for the two constraints. The above three problems are based on perfect instantaneous CSI for both up and down links. However, there may exist estimation errors for both links. Benefiting from the standard structure of the optimum solution based on the GEP, a low-complexity robust algorithm is derived in our fourth considered problem to combat channel estimation errors.

The rest of the paper is organized as follows. In Section

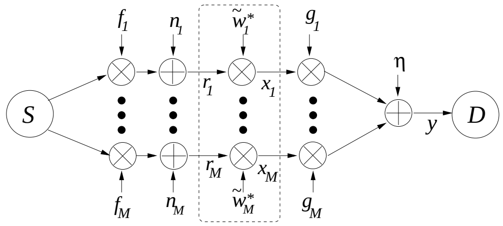

Fig. 1. A distributed beamforming structure for relay networks.

II, the distributed beamforming model and complex-valued formulation for maximizing output SNR are introduced; the latter one is then reduced to a real-valued one by our proposed method. In Section III, the four algorithms based on different design criteria are derived. Simulations are presented in Section IV and conclusions are drawn in Section V.

Notation: Vectors are denoted by lowercase bold letters and matrices by uppercase bold letters. $\{\cdot\}^H, \{\cdot\}^T, \{\cdot\}^*$ denote Hermitian transpose, transpose and complex conjugate, respectively. $\mathcal{E}\{\cdot\}, \text{Re}\{\cdot\}, \mathcal{P}\{\cdot\}, \text{tr}\{\cdot\}$ denote the operations of taking expectation, real-valued part, principal eigenvector and trace, respectively. $\mathbf{I}$ is the identity matrix and $j = \sqrt{-1}$. $\odot$ denotes the element-wise (Schur-Hadamard) multiplication of two vectors or matrices.

II. System Model and Real-valued Implementation

A. System Model

Consider a wireless network with a source $\mathcal{S}$, a relay network consisting of M nodes with each equipped with a single antenna for up and down link communications, and a destination $\mathcal{D}$, as shown in Fig. 1. The channel is based on narrowband system and assumed to be flat fading and stationary and there is no direct link between the source and the destination. First the source sends the signal to all relay nodes with an attenuation coefficient f_i for the ith node; the received signal by each relay node is weighted by a complex-valued coefficient $\tilde{w}_i^*$ and then forwarded to the destination.

At the ith relay node, the received signal is $r_i = f_i s + n_i$ with s and $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ being the transmitted signal and noise at the ith relay, respectively. In a vector form

$$\mathbf{r} = \mathbf{f}s + \mathbf{n} \; , \tag{1}$$

where $\mathbf{r} = [r_1, \cdots, r_M]^T$, $\mathbf{f} = [f_1, \cdots, f_M]^T$ and $\mathbf{n} = [n_1, \cdots, n_M]^T$. Then r_i is weighted by $\tilde{w}_i^*$ before it is forwarded to the destination, which can be expressed as

$$\mathbf{x} = \tilde{\mathbf{w}}^* \odot (\mathbf{f}s + \mathbf{n}) \; , \tag{2}$$

where $\mathbf{x} = [x_1, \cdots, x_M]^T$ and $\tilde{\mathbf{w}} = [\tilde{w}_1, \cdots, \tilde{w}_M]$. With the downlink coefficient g_i, the destination received signal $y = \sum_{i=1}^M g_i x_i + \eta$ is a linear mixture of x_i with additive noise $\eta \sim \mathcal{CN}(0, \sigma_v^2)$, i.e.,

$$y \;=\; \mathbf{g}^T(\tilde{\mathbf{w}}^* \odot \mathbf{f})s + \mathbf{g}^T(\tilde{\mathbf{w}}^* \odot \mathbf{n}) + \eta \; . \tag{3}$$

Since $\mathbf{g}$ and $\tilde{\mathbf{w}}^*$ are exchangeable, we can rewrite (3) as

$$y \;=\; \tilde{\mathbf{w}}^H(\mathbf{g} \odot \mathbf{f})s + \tilde{\mathbf{w}}^H(\mathbf{g} \odot \mathbf{n}) + \eta = s_d + n_d \; , \tag{4}$$

with $s_d = \tilde{\mathbf{w}}^H(\mathbf{g} \odot \mathbf{f})s$ and $n_d = \tilde{\mathbf{w}}^H(\mathbf{g} \odot \mathbf{n}) + \eta$ being the signal and residual noise part, respectively.

The output SNR is defined as the ratio of the output desired signal power $\mathcal{E}\{|s_d|^2\}$ and the residual noise power $\mathcal{E}\{|n_d|^2\}$

$$\text{SNR} = \frac{\mathcal{E}|s_d|^2}{\mathcal{E}|n_d|^2} = \frac{P_s \tilde{\mathbf{w}}^H(\mathbf{g} \odot \mathbf{f})(\mathbf{g} \odot \mathbf{f})^H \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathcal{E}\{(\mathbf{g} \odot \mathbf{n})(\mathbf{g} \odot \mathbf{n})^H\} \tilde{\mathbf{w}} + \sigma_v^2} \tag{5}$$

where $P_s = \mathcal{E}\{|s|^2\}$ is the power of the signal s and without loss of generality, we assume $P_s = 1$. We have also assumed that all noise components are independently distributed and they are uncorrelated with the signal. By further defining

$$\begin{aligned} \mathbf{R}_s &= (\mathbf{g} \odot \mathbf{f})(\mathbf{g} \odot \mathbf{f})^H \\ \mathbf{R}_n &= \mathcal{E}\{(\mathbf{g} \odot \mathbf{n})(\mathbf{g} \odot \mathbf{n})^H\} \\ &= \text{diag}[\sigma_1^2|g_1|^2, \sigma_2^2|g_2|^2, \cdots, \sigma_M^2|g_M|^2] \end{aligned} \tag{6}$$

we have

$$\text{SNR} = \frac{\tilde{\mathbf{w}}^H \mathbf{R}_s \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{R}_n \tilde{\mathbf{w}} + \sigma_v^2} \tag{7}$$

$\mathbf{R}_s$ and $\mathbf{R}_n$ are two correlation matrices playing key roles in distributed beamforming. Note that $\mathbf{R}_n$ is a real-valued diagonal matrix. For the rank one matrix $\mathbf{R}_s$ with principal eigenvector $\mathbf{g} \odot \mathbf{f}$, it is generally complex-valued. However, by applying a set of offset phase shifts, we can transform $\mathbf{R}_s$ into a real-valued one.

B. Real-valued implementation

The principal eigenvector of $\mathbf{R}_s$ can be written as $\mathbf{g} \odot \mathbf{f} = |\mathbf{g}| \odot |\mathbf{f}| \odot \boldsymbol{\varphi}$, where $\boldsymbol{\varphi}$ is the phase vector of $\mathbf{g} \odot \mathbf{f}$ with $\boldsymbol{\varphi} = [e^{\varphi_{f1} + \varphi_{g1}}, \cdots, e^{\varphi_{fM} + \varphi_{gM}}]^T$.

If there is an optimum solution $\tilde{\mathbf{w}}$, then we can always decompose it into the form $\tilde{\mathbf{w}} = \mathbf{w} \odot \boldsymbol{\varphi}$, where $\mathbf{w}$ is determined by the element-wise division between $\tilde{\mathbf{w}}$ and $\boldsymbol{\varphi}$. Then the optimum output signal power can be written as

$$\begin{aligned} \tilde{\mathbf{w}}^H \mathbf{R}_s \tilde{\mathbf{w}} &= (\mathbf{w} \odot \boldsymbol{\varphi})^H (|\mathbf{g}| \odot |\mathbf{f}| \odot \boldsymbol{\varphi})(|\mathbf{g}| \odot |\mathbf{f}| \odot \boldsymbol{\varphi})^H (\mathbf{w} \odot \boldsymbol{\varphi}) \\ &= \mathbf{w}^H (|\mathbf{g}| \odot |\mathbf{f}|)(|\mathbf{g}| \odot |\mathbf{f}|)^H \mathbf{w} = \mathbf{w}^H \bar{\mathbf{R}}_s \mathbf{w} \; , \end{aligned}$$

where $\bar{\mathbf{R}}_s = (|\mathbf{g}| \odot |\mathbf{f}|)(|\mathbf{g}| \odot |\mathbf{f}|)^H$ is a real-valued signal correlation matrix. Due to the diagonal and real-valued property of $\mathbf{R}_n$, using $\mathbf{w}$ instead of $\tilde{\mathbf{w}}$ will not affect the noise power in the denominator of Equation (7).

This idea can be exploited in two ways. First we can use $\bar{\mathbf{R}}_s$ instead of $\mathbf{R}_s$ in the algorithm and find the optimum solution $\mathbf{w}$ based on different design criteria, with the final optimum solution obtained by $\tilde{\mathbf{w}} = \mathbf{w} \odot \boldsymbol{\varphi}$. Or we can employ a preprocessing stage in the relay network as shown in Fig. 2, where the received data r_i is preprocessed (multiplied) by φ_i^*; then the transmitted signal is changed to $\bar{x}_i = \varphi_i^* w_i^* x_i$, where φ_i is the $i-th$ element of the vector $\boldsymbol{\varphi}$. The preprocessing is equivalent to offseting the phase shift in $\mathbf{f}$ and $\mathbf{g}$ to $|\mathbf{f}|$ and $|\mathbf{g}|$. We can see from (5) and (6) that no parameters have been changed and $\mathbf{w}$ will be the final optimum solution. Moreover,

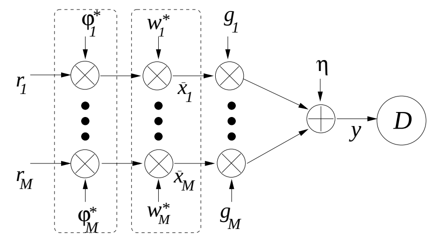

Fig. 2. Distributed beamforming with a preprocessing stage.

with the preprocessing, the total signal power transmitted by the relay network can be written as

$$P = \mathcal{E}(\bar{\mathbf{x}}^H\bar{\mathbf{x}}) = \mathcal{E}(\mathbf{x}^H\mathbf{x}) = \mathbf{w}^H\mathbf{D}\mathbf{w} \ , \quad (8)$$

where $\mathbf{D}$ is a diagonal matrix

$$\mathbf{D} = \text{diag}[|f_1|^2 + \sigma_1^2, \cdots, |f_M|^2 + \sigma_M^2] \ . \quad (9)$$

In this paper, we will adopt the preprocessing method. Therefore, $\mathbf{R}_s$ becomes a real-valued matrix, given by

$$\mathbf{R}_s = (|\mathbf{g}| \odot |\mathbf{f}|)(|\mathbf{g}| \odot |\mathbf{f}|)^H \ . \quad (10)$$

We will see later that all the optimum solutions based on the preprocessing method become real-valued, leading to low-complexity implementations. Since the error caused by estimating $\mathbf{g}$ and $\mathbf{f}$ may render the real-valued transformation ineffective, a robust algorithm will be proposed at a later stage.

III. Proposed Algorithms

A. Maximizing Output SNR

With the preprocessing, the problem of maximizing output SNR subject to a limit P_t for the total transmitted power can be formulated as

$$\max_{\mathbf{w}} \text{SNR} = \frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H\mathbf{R}_n\mathbf{w} + \sigma_v^2}, \ \ \text{subject to } \mathbf{w}^H\mathbf{D}\mathbf{w} \leq P_t \quad (11)$$

Here the the constraint is on the total transmitted power including both the desired signal component and the additive noise component, since the proposed algorithms are based on the AF scheme and the relay nodes can not eliminate the additive noise in the received signal $\mathbf{x}$. To have a true signal power constraint, we can subtract the noise power from $\mathbf{D}$. However, a problem is that the resultant total transmitted power including the noise component can be very large, which may become impractical for battery-powered devices.

Define $\hat{\mathbf{w}} = \mathbf{D}^{1/2}\mathbf{w}/\sqrt{P_t}$ and substitute it into (11). Since the maximum output SNR will occur on the boundary of the constraint equation [18], (11) can be changed to

$$\max_{\hat{\mathbf{w}}} \text{SNR} = \frac{P_t\hat{\mathbf{w}}^H\hat{\mathbf{R}}_\mathbf{s}\hat{\mathbf{w}}}{\hat{\mathbf{w}}^H(P_t\hat{\mathbf{R}}_\mathbf{n} + \sigma_v^2\mathbf{I})\hat{\mathbf{w}}}, \ \ \text{subject to } \|\hat{\mathbf{w}}\|^2 = 1 \quad (12)$$

with $\hat{\mathbf{R}}_\mathbf{s} = \mathbf{D}^{-1/2}\mathbf{R}_\mathbf{s}\mathbf{D}^{-1/2}$ and $\hat{\mathbf{R}}_\mathbf{n} = \mathbf{D}^{-1/2}\mathbf{R}_\mathbf{n}\mathbf{D}^{-1/2}$. This is a constrained real-valued GEP with its matrix pair being $\hat{\mathbf{R}}_\mathbf{s}$ and $P_t\hat{\mathbf{R}}_\mathbf{n} + \sigma_v^2\mathbf{I}$. It is well-known that the GEP can be transformed into an EP [22] (pp. 375-378). With $\hat{\mathbf{R}} = P_t\hat{\mathbf{R}}_\mathbf{n} + \sigma_v^2\mathbf{I}$, its solution is given by the principle eigenvector of the matrix $\hat{\mathbf{R}}^{-1}\hat{\mathbf{R}}_\mathbf{s}$ [18], i.e.

$$\hat{\mathbf{w}} = \mathcal{P}\{\hat{\mathbf{R}}^{-1}\hat{\mathbf{R}}_\mathbf{s}\} = \mathcal{P}\{\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}\hat{\mathbf{a}}^H\} = \alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}} \ , \quad (13)$$

where α is a scalar to be determined later and $\hat{\mathbf{a}} = \mathbf{D}^{-1/2}|\mathbf{g}| \odot |\mathbf{f}| = \mathbf{D}^{-1/2}\mathbf{a}$ with $\mathbf{a} = |\mathbf{g}| \odot |\mathbf{f}|$. The final step can be verified as follows.

$$\begin{aligned}&\hat{\mathbf{R}}^{-1}\hat{\mathbf{R}}_\mathbf{s}(\alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}) = \alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}} \\ &= (\alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}})(\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}) = (\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}})(\alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}) \ ,\end{aligned} \quad (14)$$

where $(\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}})$ is a scalar. So the principal eigenvector of $\hat{\mathbf{R}}^{-1}\hat{\mathbf{R}}_\mathbf{s}$ is given by $\alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}$.

Given the constraint $\|\hat{\mathbf{w}}\|^2 = 1$, we have

$$|\alpha\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}|^2 = \alpha^2(\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-2}\hat{\mathbf{a}}) = 1 \ , \quad (15)$$

i.e. $\alpha = 1/\sqrt{\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-2}\hat{\mathbf{a}}}$. Substituting this result into (13), the optimum solution and the maximum output SNR are then given respectively by

$$\hat{\mathbf{w}}_\text{I} = \frac{\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}}{\sqrt{\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-2}\hat{\mathbf{a}}}} \quad (16)$$

$$\hat{\text{SNR}}_\text{I} = P_t\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}} \quad (17)$$

The optimum solution (16) has a similar form to a traditional maximum output SINR beamformer [23], [24], where $\hat{\mathbf{a}}$ is the steering vector of the desired signal and $\hat{\mathbf{R}}$ is the interference-plus-noise correlation matrix. Here, for interference-free distributed beamforming, $\hat{\mathbf{R}}$ is the noise correlation matrix, while $\hat{\mathbf{a}}$ contains the relay information and could be considered as the virtual "steering vector" of the transmitted signal. Another difference comes from $\hat{\mathbf{R}}$, where for traditional beamforming, the received signal at each antenna is a coherent replica of each other and the solution is a complicated combination of the antenna signals. However, in distributed beamforming, $\hat{\mathbf{R}}$ is diagonal with each diagonal element $P_t\sigma_i^2|g_i|^2/(|f_i|^2 + \sigma_i^2) + \sigma_v^2$ contributing to the maximally allowed transmitted power separately.

Using $\hat{\mathbf{w}} = \mathbf{D}^{1/2}\mathbf{w}/\sqrt{P_t}$, the final optimal solution $\mathbf{w}_\text{I}$ for (11) and maximum output SNR are given by

$$\mathbf{w}_\text{I} = \frac{\sqrt{P_t}\mathbf{D}^{-1/2}\hat{\mathbf{R}}^{-1}\mathbf{D}^{-1/2}\mathbf{a}}{\sqrt{\mathbf{a}^H\mathbf{D}^{-1/2}\hat{\mathbf{R}}^{-2}\mathbf{D}^{-1/2}\mathbf{a}}} \quad (18)$$

$$\text{SNR}_\text{I} = P_t\mathbf{a}^H\mathbf{D}^{-1/2}\hat{\mathbf{R}}^{-1}\mathbf{D}^{-1/2}\mathbf{a} \ . \quad (19)$$

Since both $\mathbf{D}$ and $\hat{\mathbf{R}}$ (or $\mathbf{R}$) are diagonal, (18) and (19) can be further factorized into a polynomial form in terms of g_i and f_i as follows

$$w_{\text{I},i} = c_tc_i \quad (20)$$

$$\text{SNR}_\text{I} = P_t\sum_{i=1}^{M}\frac{|f_i|^2|g_i|^2}{P_t\sigma_i^2|g_i|^2 + \sigma_v^2(|f_i|^2 + \sigma_i^2)} \quad (21)$$

where

$$\begin{aligned}c_t &= \frac{1}{\sqrt{\mathbf{a}^H\mathbf{D}^{-1/2}\hat{\mathbf{R}}^{-2}\mathbf{D}^{-1/2}\mathbf{a}}} \\ &= \frac{1}{\sqrt{\sum_{i=1}^{M}\frac{|f_i|^2|g_i|^2(|f_i|^2+\sigma_i^2)}{[P_t\sigma_i^2|g_i|^2+\sigma_v^2(|f_i|^2+\sigma_i^2)]^2}}} \\ c_i &= \frac{\sqrt{P_t}|f_i||g_i|}{P_t\sigma_i^2|g_i|^2 + \sigma_v^2(|f_i|^2 + \sigma_i^2)}\end{aligned} \quad (22)$$

The optimal solution in (20) for each relay has a closed form composed of two parts: the basic part c_i determined only by each relay parameter and noise power, and the second part c_t for each relay, containing the global information due to the power constraint. (20) has quite low computational complexity since no complex-valued matrix operation is involved. The maximum output SNR in (21) is a sum of M components with each contributed by one relay only. This property will be used for the relay number minimization problem.

Based on the closed-form solution (18), $18M+1$ real-valued multiplications are needed, while the number for the complex-valued version (i.e. without preprocessing) would be $21M+2$. Note the same problem has been studied in [18] and a solution similar to (20) was derived. However, there are two major differences: firstly, our formulation is real-valued as a result of the proposed preprocessing stage and therefore has lower computational complexity; secondly, the solution in [18] is still in the form of a principle eigenvector operation, while we have given a closed-form principle eigenvector in (13) and a further simplified result in (18) and (20).

B. Minimizing Total Transmitted Power

Now consider the problem of minimizing the total transmitted power subject to the output SNR reaching a certain level, formulated as

$$\begin{aligned} &\min_{\mathbf{w}} P = \mathbf{w}^H \mathbf{D} \mathbf{w} \\ &\text{subject to SNR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w} + \sigma_v^2} \geq \gamma \end{aligned} \tag{23}$$

The original problem can be found in [18] and was solved with eigenvector decomposition based on second-order statistics of the CSI. Here we provide a less complex solution based on the GEP given in Section II.

Suppose γ is set within the following range

$$0 < \gamma < \sum_{i=1}^{M} |f_i|^2/\sigma_i^2 \ , \tag{24}$$

which can be explained as follows. According to (23), SNR$=0$ only for $\|\mathbf{w}\|^2 = 0$. For any nonzero weight vector, SNR>0; for the right side, the equality holds when $\|\mathbf{w}\|^2 \to \infty$ and it is obtained by substituting $P_t = \infty$ into (21).

Since the transmitted power by the relays has a positive correlation with the output SNR, the larger the output SNR at the destination, the more power needed at the relay. So the minimum transmitted power by relays will occur at the output SNR constraint boundary, i.e. SNR $=\gamma$, which lead to the following problem:

$$\begin{aligned} &\min_{\mathbf{w}} P = \mathbf{w}^H \mathbf{D} \mathbf{w} \\ &\text{subject to SNR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w} + \sigma_v^2} = \gamma \end{aligned} \tag{25}$$

Now suppose we have found the minimum transmitted power which is P_{II} and the corresponding solution for $\mathbf{w}$, denoted by $\mathbf{w}_{\text{II}}$. Then we consider the following problem:

$$\max_{\mathbf{w}} \text{SNR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w} + \sigma_v^2}, \quad \text{subject to } \mathbf{w}^H \mathbf{D} \mathbf{w} = P_{\text{II}} \tag{26}$$

Assume the maximum SNR obtained in the above formulation given the power constraint of P_{II} is denoted by $\tilde{\gamma}$ and the optimum solution for $\mathbf{w}$ is denoted by $\tilde{\mathbf{w}}_{\text{II}}$. Firstly, $\tilde{\gamma}$ should not be smaller than γ, since given the total transmitted power P_{II}, the system can achieve an SNR equal to γ according to (25); secondly, $\tilde{\gamma}$ should not be larger than γ, which can be proved by contradiction as follows. If $\tilde{\gamma} > \gamma$, then we can scale down $\mathbf{w}$ and reach an SNR equal to γ, which in turn leads to a reduced total transmitted power $\tilde{P}_{\text{II}} < P_{\text{II}}$. This means with an SNR equal to γ, the system can achieve a lower total transmitted power than P_{II}, which contradicts the result in (25). As a result, we have $\tilde{\gamma} = \gamma$ for the problem in (26). Then we know that $\mathbf{w}_{\text{II}} = \tilde{\mathbf{w}}_{\text{II}}$. According to the results in Section III-A, $\tilde{\mathbf{w}}_{\text{II}}$ and therefore $\mathbf{w}_{\text{II}}$ follow the same structure given in (13), i.e. $\mathbf{w}_{\text{II}} = \beta \hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}$, where β is a scalar.

However, we need to decide the value of β due to the power constraint. Suppose for the optimal solution, we have $\|\mathbf{w}_{\text{II}}\|^2 = P_w$. Then (23) is reduced to find a P_w satisfying

$$\text{SNR}(P_w) = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H (\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})\mathbf{w}} = \gamma \tag{27}$$

with $\mathbf{w} \in \mathcal{U}$, where $\mathcal{U}$ is the space of the principal eigenvector of $(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{R}_s$. P_w is the only unknown in (27). However, there is no closed-form solution for P_w and iterative methods such as the bisection or Newton methods can be used [22].

Note that in (27) P_w is a monotonous function of SNR. Moreover, $0 < \gamma < \sum_{i=1}^{M} |f_i|^2/\sigma_i^2$. For $P_w = 0$ and $P_w = \infty$, $\text{SNR}(0) = 0 < \gamma$ and $\text{SNR}(\infty) = \mathbf{w}^H\mathbf{R}_s\mathbf{w}/(\mathbf{w}^H\mathbf{R}_n\mathbf{w}) > \gamma$, which indicates that the optimum solution of P_w is unique.

After finding P_w, the optimal weight vector for (23) and the total transmitted power can be written as

$$\mathbf{w}_{\text{II}} = (\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{a} \tag{28}$$

$$P_{\text{II}} = \mathbf{a}^H(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{D}(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{a}\ . \tag{29}$$

Again, $\mathbf{w}_{\text{II}}$ and P_{II} can be calculated by factorized polynomial in terms of f_i and g_i due to diagonal matrices $\mathbf{D}$ and $\mathbf{R}_n$. The most time consuming part of the algorithm is to solve the single unknown P_w, and details of this process is given as follows.

Substituting (28) into (27), with $\mathbf{R}_s = \mathbf{a}\mathbf{a}^H$, we have

$$\begin{aligned} &\frac{\mathbf{a}^H(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{R}_s(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{a}}{\mathbf{a}^H(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{a}} \\ &= \mathbf{a}^H(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{a} = \gamma\ . \end{aligned} \tag{30}$$

Since $\mathbf{R}_n$ is diagonal, we have

$$(\mathbf{R}_n + \frac{\sigma_v^2}{P_w}\mathbf{I})^{-1} = diag[\frac{1}{\sigma_1^2|g_1|^2 + \tau}, \cdots, \frac{1}{\sigma_M^2|g_M|^2 + \tau}] \tag{31}$$

where $\tau = \sigma_v^2/P_w$. With $\mathbf{a} = |\mathbf{g}| \odot |\mathbf{f}| = [|g_1 f_1|, |g_2 f_2|, \cdots, |g_M f_M|]$, we further have

$$\frac{|g_1^2 f_1^2|}{\sigma_1^2|g_1|^2 + \tau} + \frac{|g_2^2 f_2^2|}{\sigma_2^2|g_2|^2 + \tau} + \frac{|g_M^2 f_M^2|}{\sigma_M^2|g_M|^2 + \tau} = \gamma \tag{32}$$

Depending on the accuracy of the initial value for iteration and the required accuracy for the result, the computation complexity is roughly around $O(kM)$ based on the bisection method, where k is the number of iterations. It is difficult to compare the complexity of the proposed solution with the solution in [18] directly. However, with our proposed preprocessing stage, we can use the principle eigenvector solution derived in [18] based on the real-valued $\mathbf{R}_s$, which will lead to much lower computational complexity than the case based on a complex-valued $\mathbf{R}_s$ employed in the solution in [18].

C. Minimizing Relay Number

Various methods have been proposed for minimizing power consumption or maximizing output SNR. However, the available node number in a relay network is another valuable resource especially when there are more than one relay networks working simultaneously. Moreover, the number of relay operations each node can participate within a fixed period of time can be very limited. Therefore, it is necessary to design a new scheme based on minimizing the number of relay nodes subject to some limit on output SNR and the total transmitted signal power as follows

$$\min_{\mathbf{w},1\le M_N\le M} M_N \quad \text{subject to SNR} \ge \gamma \ \text{ and } \ P=\mathbf{w}^H\mathbf{D}\mathbf{w}\le P_t \tag{33}$$

This sparsity optimization problem may be solved by transforming it into a convex problem with extremely high computational complexity. It is especially difficult for relay networks composed of handsets with limited processing power. Here we give two solutions by treating the two inequality constraints differently.

In the first case, we consider the following problem

$$\min_{\mathbf{w},1\le M_N\le M} M_N \quad \text{subject to SNR} \ge \gamma \ \text{ and } \ P=\mathbf{w}^H\mathbf{D}\mathbf{w}= P_t \tag{34}$$

where the total transmitted power is fixed to P_t. The minimum number of relays must occur under the condition of the weight vector being in the subspace of the principal eigenvector of $\hat{\mathbf{R}}^{-1}\hat{\mathbf{R}}_s$ since in that case each relay can make the most contribution to the output SNR so that the total required node number is reduced. The output SNR is a sum of M_N terms with each one being the contribution of a relay node according to (21). We can then change (34) into

$$\min_{1\le M_N\le M} M_N \quad \text{subject to } w_i=c_tc_i, (\text{for } i=1,\cdots,M_N)$$
$$\text{and SNR}=P_t\sum_{i=1}^{M_N}\frac{|f_i|^2|g_i|^2}{P_t\sigma_i^2|g_i|^2+\sigma_v^2(|f_i|^2+\sigma_i^2)}\ge\gamma \tag{35}$$

If we choose the minimum M_N relay nodes from M to satisfy the output SNR requirement with a fixed transmitted power, the optimum selection is first sorted out in a descending order $\mathbf{u}=[u_1,\cdots,u_i,\cdots,u_M]$ with

$$u_i=\frac{|f_i|^2|g_i|^2}{P_t\sigma_i^2|g_i|^2+\sigma_v^2(|f_i|^2+\sigma_i^2)}, (i=1,\cdots,M) \tag{36}$$

Then we choose from the largest one until the sum of them satisfies the minimum output SNR requirement.

There are several schemes to decide the minimum M_N. We can add one by one starting from the largest term u_1 until the output satisfies SNR$\ge\gamma$. This method may be used for the scenario that γ is small and u_1 is relatively large. If γ is large and u_1 is relatively small compared with γ, we can first add all of the SNR terms; if it is still smaller than the required output SNR level, the solution is infeasible. Suppose γ has been chosen within a reasonable range, i.e., $\gamma\le\sum_{i=1}^M u_i$; then we drop the smallest term u_M, the second smallest, and so on until $\sum_{i=1}^k u_i<\gamma$, and the minimum M_N is then $k+1$. We may use other faster searching algorithms such as the bisection method to decide the required minimum number M_N.

The second case to consider based on the optimization problem (33) is to fix the output SNR, which gives

$$\min_{\mathbf{w},1\le M_N\le M} M_N \quad \text{subject to SNR} = \gamma \ \text{ and } \ P=\mathbf{w}^H\mathbf{D}\mathbf{w}\le P_t \tag{37}$$

Similar to the minimizing total power problem formulated in Section III-B, we can see that the optimum solution must lie in the subspace of the principal eigenvector of $(\mathbf{R}_n+\frac{\sigma_v^2}{P_w}\mathbf{I})^{-1}\mathbf{R}_s$. The unknown parameter P_w is decided by the contraint equation of output SNR. Then (37) may be changed to

$$\min_{P_w\le P_t,1\le M_N\le M} M_N \quad \text{subject to } \mathbf{w}\in\mathcal{U}^{M_N\times 1}$$
$$\text{and SNR}(P_w)=\frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_n+\frac{\sigma_v^2}{P_w}\mathbf{I})\mathbf{w}}=\gamma \tag{38}$$

(38) can be further reduced in terms of f_i and g_i using (28)

$$\min_{P_w\le P_t,1\le M_N\le M} M_N$$
$$\text{subject to } w_i=\frac{P_w|f_i||g_i|}{P_w(|f_i|^2+\sigma_i^2)+\sigma_v^2} \quad (i=1,\cdots,M_N)$$
$$\text{and SNR}(P_w)=\sum_{i=1}^{M_N}\frac{P_w|f_i|^2|g_i|^2}{P_w(|f_i|^2+\sigma_i^2)+\sigma_v^2}=\gamma \tag{39}$$

Let us first define $\tilde{\mathbf{u}}=[\tilde{u}_1,\cdots,\tilde{u}_M]$ with

$$\tilde{u}_i=\frac{P_w|f_i|^2|g_i|^2}{P_w(|f_i|^2+\sigma_i^2)+\sigma_v^2} \tag{40}$$

Then we have $\text{SNR}(P_w)=\sum_{i=1}^{M_N}\tilde{u}_i$. The selection scheme would be the same as the previous case, i.e., the larger u_i is, the smaller the relay number is needed to satisfy the output SNR, and we tend to choose a large u_i as well. One problem is that u_i contains an unknown P_w for each term and it is impossible to sort out the order until it is solved by some iterative methods for each selection, which is very time consuming.

Next ,we give one possible way to order $\tilde{u}_i$ with affordable computational complexity for the relays. First we assume that for each single decision from $i=1,2\cdots,M_N$ (or in a descending order from $i=M,M-1,\cdots,M_N$), the optimum transmitted power $P_{w,o}$ for P_w is approximately the same as the maximally allowed power, i.e. $P_{w,o}=P_t$, which is a close approximation based on the following analysis: 1) If $P_{w,o}$ is much smaller than P_t, then the difference may be big enough for one chosen node to increase its power to increase the overall output SNR so that one less node can be used, leading to a smaller M_N, which is in contradiction with the assumption that $P_{w,o}$ is the optimum value; 2) given $P_{w,o} \approx P_t$, the approximation will generally not affect the order of $\tilde{u}_i$ since a little change to P_w will not cause a significant change to the value of $\tilde{u}_i$ in (40) (note that the value of σ_v^2 is very small in general); 3) even if the approximation causes a change to the order of $\tilde{u}_i$, this will only affect the final result when the affected $\tilde{u}_i$ is located at the boundary between the chosen and the unchosen nodes.

With the approximation $P_{w,o} = P_t$, no iteration will be needed in the processing and once M_N is decided, we can use the original minimizing transmitted signal power method in Section III-B to find a more accurate solution to P_w. An iterative process might be necessary, i.e., given a more accurate $P_{w,o}$, we can find a new M_N.

For the first case, the computational complexity with and without preprocessing is $18M_N + 1$ and $21M_N + 1$ real-valued multiplications, respectively; for the second case with approximation of $P_{w,o} = P_t$, the algorithm with and without preprocessing will have the complexity of $11M_N + 1$ and $13M_N + 1$ real-valued multiplications, respectively.

D. Robust Algorithm

All the algorithms we developed so far assume that the CSI is known. In practice, their performance will degrade due to errors in estimating $\mathbf{f}$ and $\mathbf{g}$. In particular, errors in the estimated phase information will cause problems for the proposed real-valued implementation, and therefore further degrade their performance. Next we develop a robust algorithm for distributed beamforming by considering both up and down link channel coefficient errors with a very low complexity.

Suppose the actual channel coefficient vector $\mathbf{f}$ and $\mathbf{g}$ and the estimated ones $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$ have the following relationships

$$\mathbf{f} = \tilde{\mathbf{f}} + \boldsymbol{\Delta}\mathbf{f} \quad\quad \mathbf{g} = \tilde{\mathbf{g}} + \boldsymbol{\Delta}\mathbf{g} \tag{41}$$

with $\boldsymbol{\Delta}\mathbf{f} = [\Delta f_1, \cdots, \Delta f_M]^T$ and $\boldsymbol{\Delta}\mathbf{g} = [\Delta g_1, \cdots, \Delta g_M]^T$ being the unknown estimation errors, ranged in

$$\boldsymbol{\Delta}\mathbf{f} \odot \boldsymbol{\Delta}\mathbf{f}^* \preceq \mathbf{e}_f, \quad \boldsymbol{\Delta}\mathbf{g} \odot \boldsymbol{\Delta}\mathbf{g}^* \preceq \mathbf{e}_g \tag{42}$$

where $\mathbf{e}_f = [e_{f,1}^2, e_{f,2}^2, \cdots, e_{f,M}^2]^T$, the element $e_{f,i}^2$ is the error boundary of $\Delta f_i \Delta f_i{}^* = \|\Delta f_i\|^2$, and $\preceq$ denotes that all of the elements $\|\Delta f_i\|^2$ are less than or equal to $e_{f,i}^2$, i.e., $\|\Delta f_i\|^2 \leq e_{f,i}^2$, for $i = 1, \cdots, M$. For $\boldsymbol{\Delta}\mathbf{g}$ we have the same definition and $\mathbf{e}_g = [e_{g,1}^2, e_{g,2}^2, \cdots, e_{g,M}^2]^T$.

Based on the preprocessing in Fig. 2 and according to the analysis in Section II, we can constrain

$$\tilde{\mathbf{f}} \in \mathbb{R}^{M\times 1} \quad \text{and} \quad \tilde{\mathbf{g}} \in \mathbb{R}^{M\times 1} \tag{43}$$

Then the robust optimization problem of maximizing output SNR can be formulated as

$$\max_{\mathbf{w}} SNR = \frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H\mathbf{R}_n\mathbf{w} + \sigma_v^2}, \quad \text{subject to } \mathbf{w}^H\mathbf{D}\mathbf{w} \leq P_t$$
$$\forall \quad \boldsymbol{\Delta}\mathbf{f} \odot \boldsymbol{\Delta}\mathbf{f}^* \preceq \mathbf{e}_f, \quad \boldsymbol{\Delta}\mathbf{g} \odot \boldsymbol{\Delta}\mathbf{g}^* \preceq \mathbf{e}_g \tag{44}$$

where the objective is to find a $\mathbf{w}$ which maximizes the worst-case SNR among all possible values of $\boldsymbol{\Delta}\mathbf{f}$ and $\boldsymbol{\Delta}\mathbf{g}$, subject to the power constraint. To solve the problem, we first find a pair of $\boldsymbol{\Delta}\mathbf{f}_o$ and $\boldsymbol{\Delta}\mathbf{g}_o$ in the errors' range in (42) which corresponds to the highest total transmitted power P_t (the worst case in terms of power consumption) given an arbitrary fixed $\mathbf{w}_o$, i.e.

$$P(\boldsymbol{\Delta}\mathbf{f}_i, \boldsymbol{\Delta}\mathbf{g}_i) = \mathbf{w}_o^H\mathbf{D}(\boldsymbol{\Delta}\mathbf{f}_i, \boldsymbol{\Delta}\mathbf{g}_i)\mathbf{w}_o \leq P(\boldsymbol{\Delta}\mathbf{f}_o, \boldsymbol{\Delta}\mathbf{g}_o)$$
$$\text{with} \quad \|\boldsymbol{\Delta}\mathbf{f}_i - \boldsymbol{\Delta}\mathbf{f}_o\|^2 + \|\boldsymbol{\Delta}\mathbf{g}_i - \boldsymbol{\Delta}\mathbf{g}_o\|^2 \neq 0 \tag{45}$$

Now for the set $\boldsymbol{\Delta}\mathbf{f}_o$, $\boldsymbol{\Delta}\mathbf{g}_o$ and $\mathbf{w}_o$, the total transmitted power can be written as

$$P(\boldsymbol{\Delta}\mathbf{f}_o, \boldsymbol{\Delta}\mathbf{g}_o) = \mathbf{w}_o^H\mathbf{D}(\boldsymbol{\Delta}\mathbf{f}_o)\mathbf{w}_o \tag{46}$$

where $\mathbf{D}(\boldsymbol{\Delta}\mathbf{f}_o)$ is a function of $\boldsymbol{\Delta}\mathbf{f}_o$ and not related to $\boldsymbol{\Delta}\mathbf{g}_o$. With $\mathbf{D}(\boldsymbol{\Delta}\mathbf{f}_o) = \text{diag}[|\tilde{f}_1 + \Delta f_{o,1}|^2 + \sigma_1^2, \cdots, |\tilde{f}_M + \Delta f_{o,M}|^2 + \sigma_M^2]$, we have

$$P(\boldsymbol{\Delta}\mathbf{f}_o, \boldsymbol{\Delta}\mathbf{g}_o) = \sum_{i=1}^{M} |w_{o,i}|^2(|\tilde{f}_i + \Delta f_{o,i}|^2 + \sigma_i^2) \tag{47}$$

Then the optimum $\Delta f_{o,i}$ $(i = 1, \cdots, M)$ can be obtained by solving the following problem

$$\max_{\Delta f_i} |\tilde{f}_i + \Delta f_i|^2, \quad \text{subject to} \quad |\Delta f_i| \leq e_{f,i} \tag{48}$$

It is not difficult to see that the optimum solution occurs at the boundary of the error range, i.e. $\Delta f_{o,i} = e_{f,i}$, $i = 1, \cdots, M$. Then (44) is reduced to

$$\max_{\mathbf{w}} SNR = \frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H\mathbf{R}_n\mathbf{w} + \sigma_v^2}, \quad \text{subject to } \mathbf{w}^H\mathbf{D}(\Delta\mathbf{f}_o)\mathbf{w} = P_t$$
$$\forall \quad \boldsymbol{\Delta}\mathbf{g} \odot \boldsymbol{\Delta}\mathbf{g}^* \preceq \mathbf{e}_g \tag{49}$$

where the objective is to find a $\mathbf{w}$ which maximizes the worst-case SNR among all possible values of Δg, subject to the new power constraint.

Based on the selection $\boldsymbol{\Delta}\mathbf{f}_o = \mathbf{e}_f$, we can take any value from the error range $\boldsymbol{\Delta}\mathbf{g}$ as $\boldsymbol{\Delta}\mathbf{g}_o$ since the transmitted power is not dependent on $\mathbf{g}$ directly. Then the inequality in (49) is removed, and an optimum solution for $\mathbf{w}$ can be found by solving the new constrained problem. However, the chosen value of $\mathbf{g}_o$ will affect the output SNR and the final transmitted signal power level. Since we cannot know its exact value, we can choose from the following three most representative values $\tilde{\mathbf{g}} - \mathbf{e}_g$, $\tilde{\mathbf{g}}$ or $\tilde{\mathbf{g}} + \mathbf{e}_g$. With a small uncertainty, $\tilde{\mathbf{g}} - \mathbf{e}_g$ may give the smallest SNR among the three cases. However, when the errors range, especially for $\mathbf{f}$, is not exactly known, it can give the most secure power consumption figure; $\tilde{\mathbf{g}} + \mathbf{e}_g$ may provide the largest SNR and the power consumption will still be less than the allowance if the error of $\mathbf{f}$ is definitely located within the range of $\mathbf{e}_f$; $\tilde{\mathbf{g}}$ is a compromise between the previous two cases. However, we will give simulations based on the three cases and show that the differences among them are not significant.

As an example, based on the selection of $\tilde{\mathbf{g}}$ and its corresponding optimum pairs $(\tilde{\mathbf{f}} + \mathbf{e}_f, \tilde{\mathbf{g}})$, the optimum solution, output SNR and the actual transmitted power of the robust optimization problem (44) are given, respectively, by

$$\mathbf{w}_{\text{IV}} = \frac{\sqrt{P_t}\mathbf{D}_R^{-1/2}\hat{\mathbf{R}}_R^{-1}\mathbf{D}_R^{-1/2}\mathbf{a}_R}{\sqrt{\mathbf{a}_R^H\mathbf{D}_R^{-1/2}\hat{\mathbf{R}}_R^{-2}\mathbf{D}_R^{-1/2}\mathbf{a}_R}} \quad (50)$$

$$\text{SNR}_{\text{IV}} = \frac{\mathbf{w}_{\text{IV}}^H\mathbf{R}_{s,A}\mathbf{w}_{\text{IV}}}{\mathbf{w}_{\text{IV}}^H\mathbf{R}_{n,A}\mathbf{w}_{\text{IV}} + \sigma_v^2} \quad (51)$$

$$\begin{aligned} P_{\text{IV}} &= \mathbf{w}_{\text{IV}}^H\mathbf{D}_A\mathbf{w}_{\text{IV}} \\ &= \frac{P_t\mathbf{a}_R^H\mathbf{D}_R^{-1/2}\hat{\mathbf{R}}_R^{-1}\mathbf{D}_R^{-1/2}\mathbf{D}_A\mathbf{D}_R^{-1/2}\hat{\mathbf{R}}_R^{-1}\mathbf{D}_R^{-1/2}\mathbf{a}_R}{\mathbf{a}_R^H\mathbf{D}_R^{-1/2}\hat{\mathbf{R}}_R^{-2}\mathbf{D}_R^{-1/2}\mathbf{a}_R} \end{aligned} \quad (52)$$

where $\mathbf{a}_R$, $\hat{\mathbf{R}}_R$ and $\mathbf{D}_R$ are the optimum values used for the robust algorithm, obtained by using $(\tilde{\mathbf{f}} + \mathbf{e}_f, \tilde{\mathbf{g}})$ instead of $(|\mathbf{f}|, |\mathbf{g}|)$ in the original definitions in $\mathbf{a}$, $\hat{\mathbf{R}}$ and $\mathbf{D}$ respectively. $\mathbf{R}_{n,A}$, $\mathbf{R}_{s,A}$ and $\mathbf{D}_A$ are the actual values for output SNR measurement, which are defined by using the true up and down link channel coefficients $(\tilde{\mathbf{f}}+\mathbf{\Delta f}, \tilde{\mathbf{g}}+\mathbf{\Delta g})$ instead of $(|\mathbf{f}|, |\mathbf{g}|)$ in the original definitions in $\mathbf{R}_n$, $\mathbf{R}_s$ and $\mathbf{D}$, respectively.

Since (50) has the same structure as in the one given in Sec. III-A, it has the same low and affordable computational complexity. If there is a noise power estimation error, with a similar idea in finding the maximum transmitted power and letting it equal to the maximally allowed one, a similar robust algorithm can be developed with the optimum solution obtained by taking the upper bound of the errors.

IV. Simulations and Results

In our MATLAB simulations, the signal power $P_s = 1$ and the input SNR varies by changing the noise power σ^2. $\mathbf{f}$ and $\mathbf{g}$ are Rayleigh fading and generated by the function $(\text{randn}(M,1)+j\text{randn}(M,1))/\sqrt{2}$, and then scaled by ε_f and ε_g, respectively.

A. Maximizing output SNR

Performance of the proposed algorithm in terms of output SNR versus the total power consumption limit P_t with different input SNRs ranging from $0, 5, 10, 15$ to 20dB is shown in Fig. 3, with two sets of channel coefficients $\varepsilon_f = \varepsilon_g = 0.1$ and $\varepsilon_f = \varepsilon_g = 1$. There are $M = 30$ relay nodes employed. Clearly, with an increase P_t, the output SNR is going up at a faster rate for smaller values of P_t, and it gets slower with a larger P_t, where the output SNR tends to reach the theoretic maximum value for a fixed input SNR. This applies to all scenarios here with different input SNRs and link coefficients. For the same P_t, the output SNR has almost a linear relationship with the input SNR. Moreover, the larger the input SNRs, the smaller the slope of the curves. This is because with the increase of input SNR, the contribution of the noise power term in (7) becomes less and less and almost zero for a very large α (e.g., $\alpha = 20$dB), and then the norm of the weight vector will not affect the output SNR any more. In this case, a large P_t tends to be not necessary.

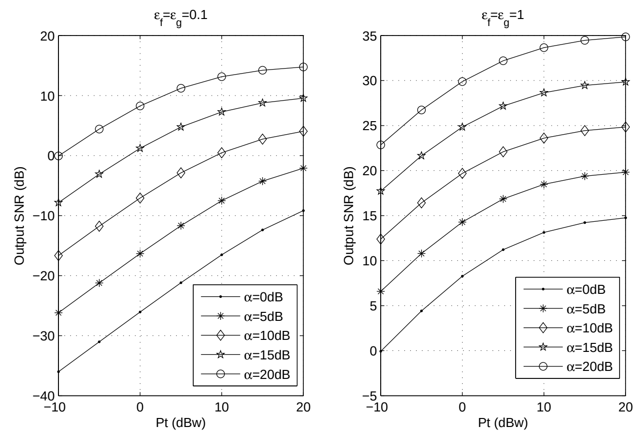

Fig. 3. Output SNR versus the power consumption limit P_t for the maximizing output SNR algorithm.

B. Minimizing total transmitted signal power P_t

By varying the input SNR from $\alpha = 0, 5, 10, 15$ to 20dB, the total transmitted power by a 30−node relay network in terms of the output SNR limit is shown in Fig. 4, with small ($\varepsilon_f = \varepsilon_g = 0.3$) and large ($\varepsilon_f = \varepsilon_g = 1$) link coefficients at left and right subplots, respectively. From the left subplot, all of the curves are up when the required output SNR increases, showing that better QoS is at the cost of larger power consumption. The value of P_t increases significantly for larger output SNRs and cut off at some specific limit value, which means that the solution for the required γ is infeasible. This infeasible value increases with the increase of input SNR limit or ε_f (ε_g). In practice, this infeasible point should be avoided. Comparing Figs. 4 and 3, we can see the equivalence of the two algorithms, i.e. if given a power constraint P_t, the maximum SNR achieved for the first algorithm is SNR_t, then given the SNR constraint SNR_t, the minimum power needed will be P_t according to the second algorithm, and vice versa, as also explained in the paragraph after (25). For example, with $\varepsilon_f = \varepsilon_g = 1$, $\alpha = 0$dB, the output SNR is 10 dB with $P_t = -10$dBw in Fig. 4, while the output $P_t = -10$dBw when $\gamma = 0$dB in Fig. 3.

C. Minimizing relay number M_N

First consider the algorithm based on the formulation in (34) with $\varepsilon_f = \varepsilon_g = 1$. The minimum required relay number M_N is shown in Fig. 5 with respect to the maximum power consumption for different input SNRs (α changing from $0, 5, 10, 15$ to 20dB), and relatively small ($\gamma = 10$dB) and large ($\gamma = 20$dB) output SNR. The maximum available relay number is $M_{\text{max}} = 200$. For all cases of input SNR, the required minimum relay number reduces with the allowed power consumption or input SNR increasing.

For the algorithm based on (37), Fig. 6 shows the minimum required relay number M_N versus the output SNR γ by varying the input SNR α with small ($P_t = 0$dBw) and large ($P_t = 10$dBw) power consumptions. The minimum required relay number increases sharply with the increase of required output SNR. We also see from Figs. 5 and 6 that with the

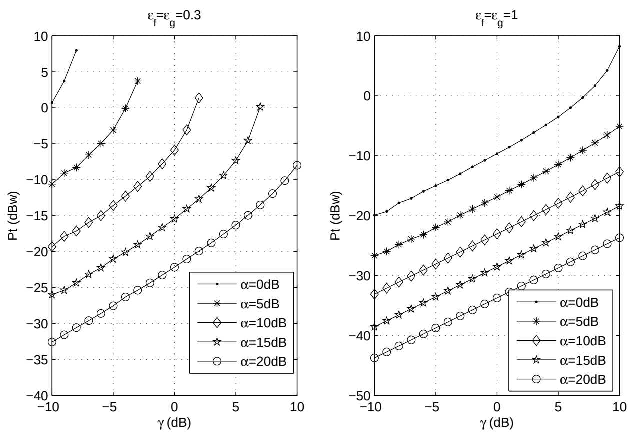

Fig. 4. The total transmitted signal power versus the output SNR limit (γ) for the minimizing power consumption algorithm.

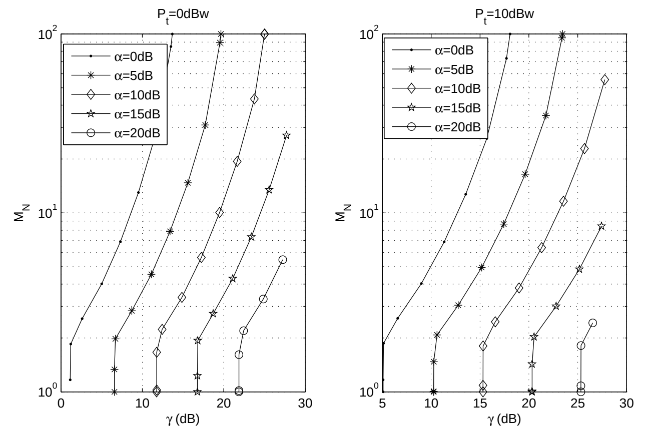

Fig. 6. The minimum required relay nodes (M_N) versus the output SNR (γ) based on formulation (37).

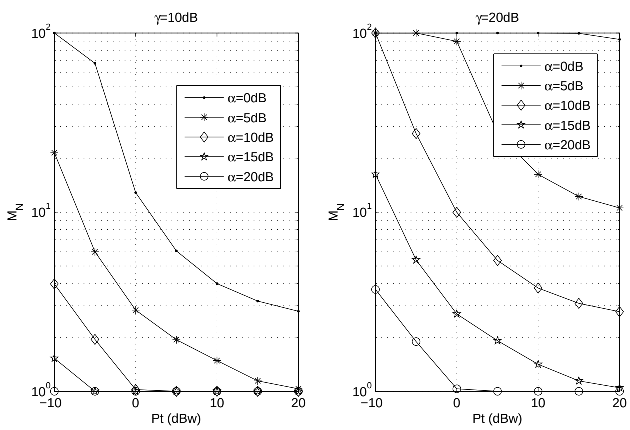

Fig. 5. The minimum required relay nodes (M_N) versus the allowed total transmitted signal power (P_t) based on formulation (34).

same γ and P_t, the required node number is nearly the same, which demonstrates the equivalence of the two algorithms. For example, with $\gamma = 20$dB and $P_t = 0$dBw, the required node number for both algorithms are 10 from Figs. 5 and 6.

D. Robust algorithm

Consider a $30-$node relay network with the input SNR $\alpha = 0$dB and $\varepsilon_f = \varepsilon_g = 1$. Suppose there is a set of randomly distributed error $\nu[\text{randn}(M,1)+j\text{randn}(M,1)]/\sqrt{2}$ added to the estimated coefficient vectors $\mathbf{f}$ and $\mathbf{g}$, where ν is a parameter for variance control. Now the actual link vectors are $\mathbf{f}_a = \mathbf{f} + \nu[\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$ and $\mathbf{g}_a = \mathbf{g}+\nu[\text{randn}(M,1)+j\text{randn}(M,1)]/\sqrt{2}$. We first consider a scenario with $\nu = -5$dB, and assume that the estimated error bound is $\mathbf{e}_f = \mathbf{e}_g = 1.2 \cdot \nu \cdot \text{ones}(M,1)$.

The robust algorithm (50), the original algorithm (18) and the optimum solution with known errors are studied in the left subplot of Fig. 7. For the robust algorithm, as we discussed in Section III-D, any selection of $\mathbf{g}$ from within the error range will keep under the allowed signal power level. For three cases by choosing $\mathbf{g}$, $\mathbf{g} + 1.2 \cdot \nu$ and $\mathbf{g} - 1.2 \cdot \nu$, denoted by robust algorithms 1, 2, and 3 respectively in Fig. 7, the one choosing $\mathbf{g} + 1.2 \cdot \nu[\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$ (robust algorithm 2) has achieved a slightly higher output SNR, while all of them have a slightly lower output SNR compared to the optimum solution. It also shows that the original algorithm (denoted by "Without robust") has a higher output SNR than the robust algorithms. However, from the right-side subplot, where it shows the actually transmitted signal power versus the constrained power level, we can see that all the three cases of the robust algorithm have almost the same total transmitted signal power. However it is smaller than the optimum one. Moreover, due to the errors, the original algorithm has a larger signal power than the allowed one for some values of P_t, which leads to a larger output SNR.

With $\alpha = 10$dB and $P_t = 10$dBw, the output SNR result with respect to ν is shown in Fig. 8. In the left, all the output SNRs decrease and move away more and more from the optimum solution as ν increases. The difference among the three robust cases is increasing as well. Moreover, it also shows that the original algorithm ("Without robust") still achieved a better output SNR than the three robust cases. However, the actual transmitted signal power of the original algorithm, as seen from the right subplot, is much higher than the allowed value (10dBw) and it goes up sharply as ν increases. For the robust algorithm, the larger the error ν, the smaller the signal power, since with the uncertainty growing, to keep the power level under the limit, the algorithm tends to choose a larger boundary, which is further away from the true value of $\mathbf{f}$ and $\mathbf{g}$, and therefore causes a lower output SNR compared with the optimum one.

E. Comparison of the proposed algorithms

At last, we examine the relationship of the output SNR of proposed algorithms versus the relay node number ($\varepsilon_f = \varepsilon_g = 1$). Since the maximizing output SNR solution is equivalent to the minimizing power consumption solution, as explained in Sec. IV-B, we only compare the algorithms of (18), (35) and (50). The algorithm for (39) is not included due to the equivalence explained in Sec. IV-C. For the algorithm (35), we have limited the maximum available node number M to

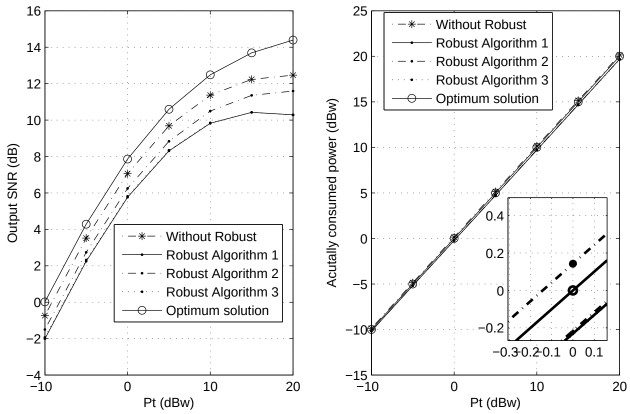

Fig. 7. Output SNR versus P_t for the robust algorithm.

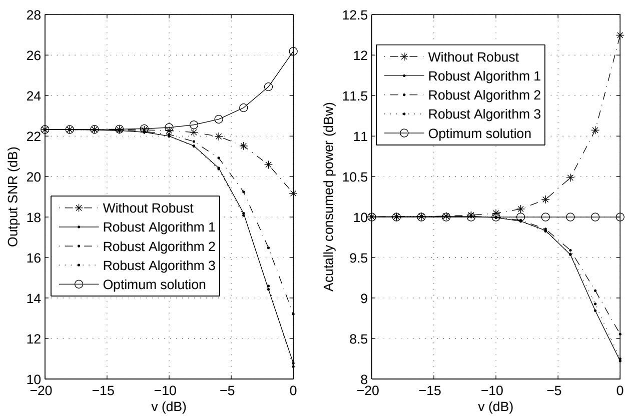

Fig. 8. Output SNR versus error scale (ν) for the robust algorithm.

20 and 40, respectively. The results are shown in Fig. 9, where the total transmitted signal power for all algorithms has been normalised to $P_t = 10$dBw for a fair comparison. For the case of error-free channel coefficients, as shown in the left side of Fig. 9, we see that with the node number increasing from 1 to 20, the advantage of algorithm (35) with $M = 20$ is vanishing compared with (18), which is because with a smaller node number, the algorithm (35) can choose the best set of channels from its maximum 20 nodes to give an output SNR as high as possible. With the node number closer to its maximum value, they are finally overlapped with (35), as (35) has to use all of the 20 nodes to reach the required output SNR, and in that case, it is equivalent to the algorithm (18). However, with the maximum available node number changed to $M = 40$, algorithm (35) has a much better performance than (18) for all real node number from 1 to 20.

The right-side subplot of Fig. 9 is based on an imperfect channel model. The actual channel coefficients are generated by $f_a = [\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$ and $g_a = [\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$. However, due to errors, we suppose the estimated channel is given by $f + 0.1 \cdot [\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$ and $g + 0.1 \cdot [\text{randn}(M,1) + j\text{randn}(M,1)]/\sqrt{2}$. For algorithm (35), the maximum available node number M is limited to 20 and again the total transmitted signal power for all algorithms has been normalised to

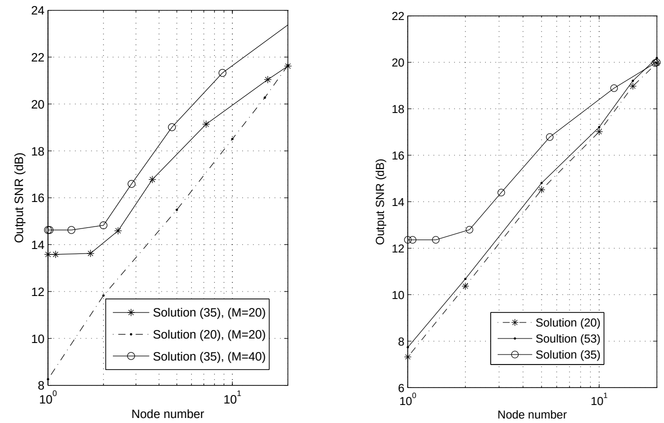

Fig. 9. Output SNR versus node number for the different algorithms for both perfect (left) and imperfect (right) estimations.

$P_t = 10$dBw. Now we can see clearly the advantage of the robust algorithm (50) compared to the non-robust one in (18). Between (18) and (35), we see a similar trend as observed from the right-side subplot.

V. Conclusions

We have studied the distributed beamforming problem for AF relay networks. Given the CSI, we can design a preprocessing stage so that the following beamforming process can be achieved with real-valued implementation, leading to a class of low-complexity algorithms. Four specific problems have been investigated, including maximizing output SNR subject to a constraint on the total transmitted signal power, minimizing the total transmitted signal power subject to certain level of output SNR, minimizing the relay node number subject to constraints on the total signal power and output SNR, and a robust beamforming algorithm to deal with channel estimation errors. The effectiveness of the proposed algorithms have been verified by extensive simulation results.

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