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Article:

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Abstract—We describe a new measurement capability which provides fully calibrated, traceable scattering parameter measurements in rectangular metallic waveguide in the frequency range 750 GHz to 1.1 THz. The instrumentation consists of a Vector Network Analyzer (VNA) with waveguide extender heads, situated at the University of Leeds, and primary measurement standards characterized by the National Physical Laboratory. The measurement standards consist of lengths of precision waveguide that are used during the calibration of the instrumentation. Traceability to the International System of units (SI) is established by performing high-precision dimensional measurements on the waveguide sections. A preliminary uncertainty budget is presented, indicating the expected sizes of the main sources of error in both reflection and transmission measurements.

Index Terms—Vector network analysis, calibration and measurement, waveguides, submillimeter-waves, terahertz, measurement traceability.

I. INTRODUCTION

ATIONAL Measurement Institutes (NMIs), from many regions around the world, have established facilities to provide high precision scattering parameter measurements at RF, microwave and millimeter-wave frequencies [1]. These facilities achieve traceability to the international system of units (SI) by relating the quantities being measured (i.e. the scattering parameters) to the relevant base units of the SI (in this case, the meter, ampere and second) [2, 3]. Through provision of national and international references for scattering parameter measurements, these systems make it possible to harmonize all measurements that can be traced to these primary standards.

Most of these NMI facilities operate at frequencies up to 110 GHz. Recently, some NMIs have established new metrology capabilities at frequencies above 110 GHz (e.g. NIST [4, 5], PTB [6], NMIJ [7, 8] and NPL [9-11]).

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Nick M. Ridler is with the Time, Quantum and Electromagnetics Division, National Physical Laboratory (NPL), Teddington, UK (e-mail: nick.ridler@npl.co.uk). R. G. Clarke is with the Institute of Microwaves and Photonics, School of Electronic and Electrical Engineering, University of Leeds, Leeds, UK (e-mail: r.g.clarke@leeds.ac.uk).

These developments have extended the availability of traceable measurements in certain waveguide bands. In response to demand from the industry (see, for example, [12-15]), instrument manufacturers have developed measurement systems, i.e. Vector Network Analyzers (VNAs), which operate at all frequencies from 110 GHz to 1.1 THz (see, for example, [16-19]). This calls for reliable measurement references and methods for quality assurance for metrology at these frequencies. Ultimately, SI traceability for the measurements is required to achieve these goals. Within the UK, this need has driven a program of research to propose suitable calibration techniques, and also establish the associated traceability to SI.

These research objectives are being delivered by means of a collaboration between the University of Leeds and the National Physical Laboratory (NPL). A millimeter- and submillimeter-wave VNA at the University of Leeds provides the measurement instrumentation. The calibration standards are lengths of precision waveguide. These standards are characterized by NPL. Researchers at both the University of Leeds and NPL have contributed to the development and operation of the traceable measurement capabilities.

Initially, this research has concentrated on waveguide bands in the upper part of the millimeter-wave region, i.e. from 110 GHz to 330 GHz [9-11]. The program has now established a facility to provide traceable S-parameter (i.e. reflection and transmission coefficient) measurements in the WM-250 [20] (or, equivalently, WR-01 [21]) waveguide band, which supports frequencies from 750 GHz to 1.1 THz. We report on the new capability in this paper. Some work has already been undertaken by other researchers (see [7, 8]) to provide metrological traceability for this range of frequencies. However, the work reported in [7, 8] utilised a new type of waveguide flange/interface that is not commonly used in this, or any other, waveguide band. The type of waveguide flange/interface used at these frequencies will have a major impact on the performance of the VNA measurement system. The work described in this paper uses the same type of flange/interface that is used for nearly all applications in waveguide bands above 110 GHz (including the 750 GHz to 1.1 THz band).

II. MEASUREMENT SYSTEM

The measurement technique employed by this facility is based on NPL’s Primary Impedance Measurement System.
(PIMMS) [22-23]. The measurement instrumentation comprises a Keysight Technologies PNA-X VNA and a pair of VDI submillimeter-wave extension modules. This system is shown in Fig. 1. The submillimeter-wave extension modules provide a complete S-parameter test set for a specific waveguide band. The measurement stimulus signal is obtained via harmonic multipliers within the extension modules. For the 750 GHz to 1.1 THz waveguide band, the instrument produces a nominal test port power of ~35 dBm.

The measurement uncertainty is established following international recommendations [24] which are adapted to account for the fact that the quantities being measured (i.e. the scattering parameters) are complex-valued quantities [25, 26]. In order to evaluate the extent of random errors in the measurements, multiple measurements are made with repeated connections of the waveguide interfaces. Separate experiments are used to ascertain the extent of systematic errors in the measurement system. These systematic errors are due to imperfections in the physical properties of the calibration standards, VNA test port mismatches, non-linearity and cross-talk. The evaluation of measurement uncertainty is described in section VI.

The NPL / University of Leeds partnership has been the UK’s national measurement reference for S-parameters. This is considered a reasonable assumption for a ‘known’ calibration standard.

III. CALIBRATION STANDARDS AND TECHNIQUES

The measurement technique has been established [31] whereby phase changes in the range from 30º to 150º are considered to provide suitable calibration schemes at frequencies above 110 GHz because the required length of the Line standards becomes very short. For example, in the 750 GHz to 1.1 THz band, a line length of approximately 100 μm is needed to provide phase changes that do not coincide with these calibration failure values. Generally, lines are chosen so that phase differences are more than 30º away from the failure frequencies, e.g. for ¼-wave TRL, phase changes that vary in the range from 30º to 150º are considered to provide suitable calibrations.

In order to evaluate the extent of systematic errors in the measurement system, these systematic errors are due to imperfections in the physical properties of the calibration standards, VNA test port mismatches, non-linearity and cross-talk. The evaluation of measurement uncertainty is described in section VI.

The NPL / University of Leeds partnership has been the UK’s national measurement reference for S-parameters. This is considered a reasonable assumption for a ‘known’ calibration standard.
needed to provide acceptable TRL calibrations. This achieves a so-called \( \frac{3}{4} \)-wave TRL calibration technique. The phase of these longer lines varies more rapidly as the frequency is increased (compared to a conventional ‘short’ \( \frac{1}{4} \)-wave line) and so one line is not able to provide stable calibrations over the full band. Therefore, two lines are used — one for the lower part of the band; one for the higher part of the band. This \( \frac{3}{4} \)-wave TRL method, for millimeter-wave frequencies, is described in [31]. The approach can also be extended to submillimeter-wave frequencies and this leads to line lengths for the 750 GHz to 1.1 THz band as shown in Table I.

<table>
<thead>
<tr>
<th>Nominal line length (( \mu \text{m} ))</th>
<th>Frequency range (GHz)</th>
<th>Phase change (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>388</td>
<td>750</td>
<td>928</td>
</tr>
<tr>
<td>298</td>
<td>838</td>
<td>1100</td>
</tr>
</tbody>
</table>

The phase changes produced by the two lines (also shown in Table I), indicate that the useable bandwidths for the lines overlap, to some extent. The 388 \( \mu \text{m} \) line has an upper frequency of 928 GHz whereas the 298 \( \mu \text{m} \) line has a lower frequency of 838 GHz. The changeover from using one line to the other line, as the Line standard, can occur at any frequency from 838 GHz to 928 GHz. In practice, the frequency is chosen as 883 GHz (i.e. approximately in the middle of the overlap frequency region).

Fig. 2 shows one of the Line standards from the \( \frac{3}{4} \)-wave TRL calibration kit. Situated in the middle of this standard is the waveguide aperture. However, because the dimensions of the aperture are very small (approximately 250 \( \mu \text{m} \times 125 \mu \text{m} \)), it is barely visible in this photograph. A close-up view (i.e. an optical scan) of the waveguide aperture is shown in Fig. 3. (This scan was obtained during the collection of the dimensional measurements of the apertures – the two vertical lines in the scan are part of the measurement frame used by the dimensional measurement system.) The scan shows some imperfections in the waveguide aperture, including some rounding of the corners of the aperture. Effects due to these dimensional imperfections, on the electrical performance of the line standards, are discussed in Section V.

A reflection standard is also employed in the TRL calibration process. This Reflect standard must produce an identical, though not necessarily quantified, value of reflection coefficient at each of the test port reference planes of the VNA. The Reflect standard is ordinarily implemented by connecting a flush short-circuit (i.e. a flat metallic sheet with no waveguide aperture) to the VNA test ports. Using the same physical Reflect standard at each test-port (in turn) permits the assumption that an identical value of reflection coefficient is presented at both of the VNA’s reference planes (neglecting electrical noise and connection repeatability errors). The complete calibration kit (i.e. two line standards and a flush short-circuit) is shown in Fig. 4. The device in the middle, in Fig. 4, is the flush short-circuit (i.e. containing no waveguide aperture).

IV. DIMENSIONAL DATA

Metrological traceability to the International System of units (SI) is achieved for the \( S \)-parameters via precision dimensional measurements of the TRL Line standards – specifically, measurements of the dimensions of the waveguide apertures and the alignment mechanisms found on the waveguide interfaces. The measurements are temperature corrected using a value for the coefficient of linear thermal expansion of \( 16.6 \times 10^{-6} \text{ K}^{-1} \) (i.e. assuming the lines are made primarily from copper). Each dimensional measurement is repeated (typically, four or five times) with the mean of the measurement data reported.

![Fig. 2. Photograph showing one of the TRL Line standards (the waveguide aperture is barely visible in the center)](image)

![Fig. 3. Close-up view of the waveguide aperture of one of the TRL Line standards](image)

Measurements of the waveguide aperture broad and narrow wall dimensions were performed using a microscope with a travelling stage and reflecting illumination. The displacement of the stage was measured by means of a helium-neon laser interferometer, the frequency of the laser having been determined using an iodine-stabilized reference laser. Measurements were made at both front and back faces of the lines. The measurements were made of the bulk wall properties of the aperture at the mid-point of the broad and narrow walls. The reported results are the average of four repeated measurements. The expanded uncertainty (using a coverage factor of \( k = 2 \)) of both broad and narrow wall dimensional measurements is expected to be approximately 2.0 \( \mu \text{m} \).
In addition, the position and size of the alignment mechanisms on the waveguide interfaces are also measured. These alignment mechanisms are the dowel holes, used in conjunction with externally fitted dowel pins, to align the standards with the VNA waveguide test ports. These measurements were made using a Zeiss F25 coordinate measuring machine (CMM) fitted with a ball tip micro-stylus of diameter 0.3 mm. Each line was measured separately after being positioned with the aperture axis aligned vertically on the CMM, as shown in Fig. 5. The expanded uncertainty (using a coverage factor of $k = 2$) in these dimensional measurements is typically 0.5 μm. Some measurements of these alignment mechanisms were also made using the laser interferometer in order to correlate the two sets of dimensional measurements.

![Fig. 5. Photograph showing the CMM measuring a waveguide Line standard using a ball tip micro-stylus](image)

There is currently in existence at least two sets of published values for the nominal aperture dimensions of waveguide used for the 750 GHz to 1.1 THz band. These waveguide sizes are known as WM-250 [20] and WR-01 [21]. The nominal mechanical dimensions of these two waveguide sizes are shown in Table II.

**TABLE II**

Nominal Values for the Aperture Dimensions of Waveguide Used for the 750 GHz to 1.1 THz Band

<table>
<thead>
<tr>
<th>Waveguide name</th>
<th>Broad wall (μm)</th>
<th>Narrow wall (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WM-250</td>
<td>250</td>
<td>125</td>
</tr>
<tr>
<td>WR-01</td>
<td>254</td>
<td>127</td>
</tr>
</tbody>
</table>

Measurements of the broad and narrow wall dimensions of the waveguide apertures, described in Section IV, showed that the apertures of the two lines used for the TRL calibration exhibit measurable departures from the nominal values for both the WM-250 and WR-01 waveguide sizes. These measured values can be summarized in terms of their observed deviation from the nominal waveguide aperture dimensions for both WM-250 and WR-01. These summary values are shown in Tables III and IV, for the broad wall and narrow wall dimensions, respectively.

**TABLE III**

Summary of the Measurements of the Broad Wall Dimensions of the Two TRL Line Standards

<table>
<thead>
<tr>
<th>Nominal line length (μm)</th>
<th>Average measured value (μm)</th>
<th>Deviation with respect to WM-250 (μm)</th>
<th>Deviation with respect to WR-01 (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>253.7</td>
<td>+3.7</td>
<td>-0.3</td>
</tr>
<tr>
<td>388</td>
<td>254.2</td>
<td>+4.2</td>
<td>+0.2</td>
</tr>
</tbody>
</table>

**TABLE IV**

Summary of the Measurements of the Narrow Wall Dimensions of the Two TRL Line Standards

<table>
<thead>
<tr>
<th>Nominal line length (μm)</th>
<th>Average measured value (μm)</th>
<th>Deviation with respect to WM-250 (μm)</th>
<th>Deviation with respect to WR-01 (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>124.7</td>
<td>+0.3</td>
<td>-2.3</td>
</tr>
<tr>
<td>388</td>
<td>128.8</td>
<td>+3.8</td>
<td>+1.8</td>
</tr>
</tbody>
</table>
These summary values show that all measured values are within ±5 μm of the nominal values (for both the broad and narrow wall dimensions) for both the WM-250 and the WR-01 waveguide aperture sizes.

Values of reflection coefficient due to tolerances in waveguide apertures have been given in [20, 32], where it is shown that, for these waveguide aperture sizes, a tolerance of ±5 μm gives rise to a maximum reflection coefficient, between two perfectly aligned waveguides, of -22 dB. This is equivalent to a linear reflection coefficient magnitude of 0.079. This value is therefore used as an input quantity for the uncertainty assessment for this system.

The alignment mechanisms used for the TRL calibration lines are similar to those specified for a design given in [33] – in particular, the design known as the IEEE 1785.2a ‘Precision Pin’ interface. This type of interface uses two tight tolerance inner alignment holes in conjunction with four looser tolerance outer alignment holes, as indicated in Fig. 6. Appropriately sized dowel pins are inserted into all six of these alignment holes during connection.

![Identification of the alignment holes used for aligning the calibration standards with the VNA test ports. The scale shown is in centimeters.](image)

Fig. 6. Identification of the alignment holes used for aligning the calibration standards with the VNA test ports. The scale shown is in centimeters.

The measurements of the diameters of the alignment holes, also described in Section IV, on the two TRL Line standards are summarized in Tables V and VI. These tables give the maximum departures from nominal diameters of both the inner and outer alignment holes, respectively.

**TABLE V**

**SUMMARY OF THE DIAMETER MEASUREMENTS OF THE INNER ALIGNMENT HOLES ON THE TWO TRL LINE STANDARDS**

<table>
<thead>
<tr>
<th>Nominal line length (μm)</th>
<th>Nominal diameter (mm)</th>
<th>Maximum, or minimum, measured diameter (mm)</th>
<th>Deviation with respect to nominal (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>1.587</td>
<td>1.582 4</td>
<td>-4.6</td>
</tr>
<tr>
<td>388</td>
<td>1.613</td>
<td>1.610 0</td>
<td>-3.0</td>
</tr>
</tbody>
</table>

According to [33], the IEEE 1785.2a interface achieves a worst case reflection coefficient of -19 dB, for this waveguide size. This is equivalent to a linear reflection coefficient magnitude of 0.112. Based on the dimensional values given in Tables V and VI, it is assumed that the interfaces on the two TRL line standards achieve this same level of performance and therefore this value is used as an input quantity for the uncertainty assessment for the system.

**VI. UNCERTAINTY ESTIMATES**

Evaluating the performance of the VNA requires the production of uncertainty budgets. These budgets indicate the expected size of individual uncertainty contributions which are attributed to systematic errors within the measurement system (e.g. imperfections in the calibration standards, residual terms in the VNA error model, isolation/crosstalk, VNA detectors’ non-linearity, etc). Random errors (e.g. connection repeatability of the device under test (DUT), noise and fluctuations in the environmental conditions) are not included in these uncertainty budgets. With the exception of electrical noise, these random errors may be considered to be ‘external’ to the VNA and consequently, they are not representative of the VNA’s performance. Connection repeatability errors are mainly influenced by the quality of the waveguide interfaces on the DUTs. The VNA system is housed in a temperature-controlled laboratory to reduce fluctuations in the ambient conditions. The impact of changes in the measurement environment is further mitigated through minimizing the time between calibration and measurement of the DUT.

The uncertainty budgets presented in this paper also do not contain a contribution to account for the frequency accuracy and spectral purity of the VNA test frequency. Although potentially an important contribution, it is considered beyond the scope of the preliminary uncertainty budgets presented in this paper. The resulting uncertainty budgets establish the Calibration and Measurement Capability (CMC) [34] for the measurement system. The uncertainty budgets can therefore be considered appropriate for establishing a Scope of Accreditation [35] for the VNA system.

**A. Reflection measurements**

The main source of uncertainty for reflection measurements (i.e. $S_11$ and $S_22$), may be attributed to reflections caused by the imperfections in the Line standards used during calibration. In particular, reflections caused by imperfections in the waveguide aperture sizes (i.e. the broad and narrow wall dimensions) and the alignment mechanisms found on the waveguide interfaces. The Line standards are used in the TRL calibration to set the characteristic impedance.
for the system and so these reflections cause there to be uncertainty in the characteristic impedance determined by the calibration. It is expected that, for such small waveguide apertures, these dimensional imperfections will be the dominant source of uncertainty in setting the characteristic impedance of the system. This uncertainty subsequently affects all reflection measurements made by the calibrated VNA.

Section V gave a maximum value for the reflection caused by the deviations in the broad and narrow wall dimensions of the apertures of the Line standards. This worst-case reflection error, \( \Delta(|\Gamma_1|) = 0.079 \), is converted to an equivalent standard uncertainty, \( u(|\Gamma_1|) \), using [24]:

\[
u(|\Gamma_1|) = \frac{\Delta(|\Gamma_1|)}{\sqrt{3}} = 0.046
\]

(1)
since it is assumed that the worst-case reflection error can be represented using a uniform probability density function (PDF).

Section V also gave a maximum value for the reflection caused by the imperfect alignment of the interfaces of the Line standards. As before, this worst-case reflection error, \( \Delta(|\Gamma_2|) = 0.112 \), is converted to an equivalent standard uncertainty, \( u(|\Gamma_2|) \), using [24]:

\[
u(|\Gamma_2|) = \frac{\Delta(|\Gamma_2|)}{\sqrt{3}} = 0.065
\]

(2)
since, as before, it is assumed that the worst-case reflection error can be represented using a uniform PDF.

The two uncertainty contributions, \( u(|\Gamma_1|) \) and \( u(|\Gamma_2|) \), are independent of each other and so the combined standard uncertainty in reflection, \( u(\Gamma) \), is given by [24]:

\[
u(\Gamma) = \sqrt{u(|\Gamma_1|)^2 + u(|\Gamma_2|)^2} = 0.079
\]

(3)
The expanded uncertainty in reflection coefficient measurements, \( U(\Gamma) \), obtained using a coverage factor of \( k = 2 \), is given by [24]:

\[
U(\Gamma) = 2 \times u(\Gamma) \approx 0.16
\]

(4)

Therefore, this value can be considered the CMC expanded uncertainty for the VNA reflection coefficient measurements (i.e. \(|S_{11}\) and \(|S_{22}\)). This uncertainty value is equivalent to a return loss of approximately 16 dB. For comparison purposes, similarly-sized dimensional errors in WR-10 waveguide (i.e. for frequencies from 75 GHz to 110 GHz) produce a return loss of approximately 40 dB.

B. Transmission measurements

For transmission measurements (i.e. \(|S_{21}\) and \(|S_{12}\)), the uncertainty is evaluated using the error model given in [36]. (The symbols and terminology presented here are consistent with that used in [36].) The three main contributions to the overall uncertainty, given in [36], are: (i) isolation/crosstalk; (ii) mismatch; (iii) non-linearity.

The system isolation/crosstalk is determined by observing \(|S_{21}|\) and \(|S_{12}|\) when both ports of the VNA are terminated with low reflecting loads. The achieved performance is shown in Fig. 7.

Fig. 7. Isolation/crosstalk assessment for the VNA with both ports terminated with low-reflecting loads

Fig. 7 suggests that the isolation/crosstalk error, \( I \), at nearly all frequencies across the waveguide band is better than \(-40\) dB. For a given DUT, the contribution to uncertainty due to isolation/cross-talk, \( dA \), will vary according to the attenuation, \( A \), following the expression in [36]:

\[
dA = 20 \log_{10} \left[ 1 + 10^{\frac{(I + A)}{20}} \right]
\]

(5)

Here, the isolation/crosstalk term is considered to be a transmission coefficient (i.e. \( I = -40 \) dB, rather than \( I = +40 \) dB) and the measured attenuation, \( A \), is expressed as a positive number, e.g. \( A = +20 \) dB.) This results in a slightly different form of the equation given in [36]. For a given attenuation value, \( dA \) is effectively a worst-case error estimate and, consequently, it may be treated as a limit value. Therefore, it is characterized using a uniform PDF. The equivalent standard uncertainty, \( u(dA) \), is therefore established in the usual way [24]:

\[
u(dA) = \frac{dA}{\sqrt{3}}
\]

(6)

The equation used to calculate the error due to mismatch, \( M_{TM} \), is also given in [36]:

\[
M_{TM} = 20 \log_{10} \left[ \frac{1 + \left| \frac{|M| S_{11}| + |F_{11} S_{12}| + |F_{12} S_{22}| + |M| F_{11}| S_{22}|}{1 - |M| F_{11}|} \right|}{1 + \left| \frac{|M| S_{11}| + |F_{11} S_{12}| + |F_{12} S_{22}| + |M| F_{11}| S_{22}|}{1 - |M| F_{11}|} \right|} \right]
\]

(7)

where \( S_{11}, S_{22}, S_{12}, \) and \( S_{21} \) are the measured \( S \)-parameters of the DUT, \( M \) is the VNA residual test port match, and \( \Gamma_{L} \) is the VNA residual load match.

The values of both \( M \) and \( \Gamma_{L} \) may be considered to be equivalent to that of the standard uncertainty for reflection.
measurements [36], which has been determined already (i.e. an uncertainty in linear reflection coefficient of 0.079). For convenience, the estimate of mismatch is limited to the case of DUTs with relatively low input and output reflection, i.e. where linear $|S_{11}| \leq 0.1$ and linear $|S_{22}| \leq 0.1$. (For devices with linear $|S_{11}| > 0.1$ and/or linear $|S_{22}| > 0.1$, the mismatch calculation is repeated using the measurements of $S_{11}$ and $S_{22}$ in equation (7).) Under these circumstances, the worst-case value of $M_{TM}$ is 0.244 dB (for all passive DUT values of $S_{21}$ and $S_{22}$).

For vector errors, where knowledge of the phase is either absent or not used (due to a lack of confidence in the reliability of the measured phase value), it is conventional to use a $U$-shaped PDF to characterize the error. Therefore, the equivalent standard uncertainty, $u(M_{TM})$, is established following [37, 38]:

$$u(M_{TM}) = \frac{M_{TM}}{\sqrt{2}}$$  \hfill (8)

The systematic non-linearity error, $L$, in transmission measurements is ordinarily assessed by means of a calibrated step attenuator, to provide different, but known, power levels to the VNA test ports. However, for this waveguide size, there are no traceable calibrated attenuation ‘steps’ available. Consequently, estimates of the likely values for $L$ are used for this contribution to the uncertainty budget. A typical value for $L$ obtained for coaxial VNA systems is 0.002 dB/dB [36]. For VNAs with waveguide extender heads operating from 110 GHz to 330 GHz, the estimated non-linearity is 0.004 dB/dB, according to [9-11]. Thus, a conservative value of 0.01 dB/dB is used here for the purpose of this preliminary uncertainty budget. The equivalent standard uncertainty, $u(L)$, is established following the procedures given in [36]:

$$u(L) = \frac{L}{2}$$  \hfill (9)

From equations (6), (8) and (9), the combined standard uncertainty for the transmission measurements, $u(T)$, is evaluated following [24]:

$$u(T) = \sqrt{(u(dA))^2 + (u(M_{TM}))^2 + (u(L))^2}$$  \hfill (10)

Therefore, this value can be considered the CMC standard uncertainty for transmission measurements (i.e. $|S_{21}|$ and $|S_{12}|$, in dB). The expanded uncertainty in transmission coefficient measurements, $U(T)$, obtained using a coverage factor $k = 2$, is given by:

$$U(T) = 2 \times u(T)$$  \hfill (11)

The measurement uncertainty is normally calculated at each frequency, at each measured value, and for each $S$-parameter. This will often lead to values of uncertainty that are somewhat different (either lower or higher) than the values shown in Figs. 8 and 9, and presented in Table VII. For example, it is evident from Fig. 7 that at many frequencies across the band, the crosstalk/isolation of the VNA is considerably better than the value of $-40\,\text{dB}$ used here in the estimate of overall measurement uncertainty.

C. Random Errors

Although the evaluation of the uncertainty presented in this paper has not included a treatment of the random errors impacting this measurement system, it is informative to provide some information about these types of error. For measurements in waveguide at submillimeter-wave frequencies, the most significant source of random errors is likely to be due to the repeatability of connection of the waveguide devices to the VNA test ports. This lack of repeatability emanates primarily from the mechanical properties of the waveguide interfaces – e.g. the mechanical tolerances on the alignment mechanisms (i.e. the dowel pins and holes), the tightness of the connection (i.e. the torques applied to the bolts used to tighten the waveguide interfaces), the roughness and flatness of surfaces (e.g. on the faces of the interfaces). These errors will vary depending on the quality (i.e. degree of precision) and condition of the waveguide interfaces, and while it is possible to control these attributes for the VNA test ports, it is not possible to have the same degree of control for the DUTs (which will often be provided by third parties). It is for this reason that it is not generally feasible to provide a complete uncertainty budget for a particular DUT before it has been measured by the system.
TABLE VII

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Estimate</th>
<th>Uncertainty</th>
<th>Distribution</th>
<th>Divisor</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>0.01 dB/dB</td>
<td>0.080 dB</td>
<td>Gaussian</td>
<td>2</td>
<td>0.040 dB</td>
</tr>
<tr>
<td>Mismatch</td>
<td>0.192 dB</td>
<td></td>
<td>U-shaped</td>
<td>√2</td>
<td>0.135 dB</td>
</tr>
<tr>
<td>Isolation/crosstalk</td>
<td>-40 dB</td>
<td>0.828 dB</td>
<td>Rectangular</td>
<td>√3</td>
<td>0.478 dB</td>
</tr>
<tr>
<td>Combined standard uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.498 dB</td>
</tr>
<tr>
<td>Expanded uncertainty (k = 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.0 dB</td>
</tr>
</tbody>
</table>

Some work on assessing DUT repeatability in this waveguide size has been undertaken recently [39-41] for some selected one-port devices. These were high-reflecting devices (a flush short-circuit and an offset short-circuit) and a low-reflecting device (a near-matched load). It was found in [39] that experimental standard deviations, calculated from a series of 12 repeat measurements made under essentially the same condition of measurement, varied from approximately 0.01 to 0.1 (in terms of linear reflection coefficient). This is equivalent to a standard uncertainty of the order of 0.029 [24].

Further work [40] investigated the situation where the aperture of the DUT was inverted (i.e. rotated through 180°) between the repeated disconnection and re-connection of the DUT to the VNA test port. In some instances, this showed a substantial increase in the observed experimental standard deviations (compared with the situation when the DUT aperture was not inverted between reconnection). This suggested that the positional alignment of the aperture of some DUTs exhibited a systematic offset from the nominal position of the aperture. The impact of this effect can be removed from the measurements by only measuring an item in the same, specified, connection orientation to the VNA test port. Alternatively, a single device could be measured twice – i.e. (i) in a non-inverted orientation; and (ii) in an inverted orientation. Each of the two orientations would be treated as a separate measurand. This, in effect, treats the DUT as providing two measurands – one, in the non-inverted orientation; and another, in the inverted orientation. Separate results, each with an associated uncertainty, could then be given for these two measurands.

Finally, some related work [41] investigated the effect of using different types and combinations of alignment dowel pins and holes during the connection of the waveguide interfaces. These investigations showed that, for the interfaces that were studied experimentally, there was no obvious ‘best choice’ combination of alignment dowels, although all connections that were investigated used at least the four outer alignment holes (shown in Fig. 6).

It was shown in [40, 41] that experimental standard deviations, calculated from series of 24 repeat measurements, can be as large as 0.4 (in terms of linear reflection coefficient). This is equivalent to a standard uncertainty of the order of 0.08 [24], which is larger than both standard uncertainty contributions given in equations (1) and (2), relating to the deviations in the aperture dimensions of the waveguides. This illustrates that the contribution to uncertainty due to connection repeatability of the waveguide interfaces can be the dominant source of uncertainty for these types of measurement.
D. Uncertainty in phase

For a given S-parameter, \( S_i \) \((i = 1, 2; j = 1, 2)\), the standard uncertainty for phase, \( u(\phi) \), can be estimated using [42]:

\[
u(\phi) = \sin^{-1}\left(\frac{|S_{ij}|}{|S_{ij}|^2}ight)
\]

(12)

where \( S_{ij} \) is the measured S-parameter and \( u(|S_{ij}|) \) is the standard uncertainty in \( |S_{ij}| \). Equation (12) assumes that the uncertainty in each of the S-parameters can be represented by a circular region of uncertainty (i.e. characterized by a circular bivariate normal PDF) in the complex plane for each S-parameter. The expanded uncertainty in phase, \( U(\phi) \), obtained using a coverage factor of \( k = 2 \), is given by [24]:

\[
U(\phi) = 2 \times u(\phi)
\]

(13)

When using equation (12) to compute the standard uncertainty in the phase of transmission measurements, a preliminary step is needed to determine the standard uncertainty in the magnitude of the linear transmission coefficient, \( \alpha \). This can be derived from the measured attenuation (in dB), \( \alpha \), and the standard uncertainty in the measured attenuation (also in dB), \( u(\alpha) \). For reciprocal devices, the magnitude of the linear transmission coefficient, \( |S_{ij}| (i \neq j) \), is related to \( \alpha \) as follows:

\[
|S_{ij}| = 10^{-\alpha/20}
\]

(14)

From the Law of Propagation of Uncertainty [24]:

\[
u^2(|S_{ij}|) = \left(\frac{d|S_{ij}|}{d\alpha}\right)^2 u^2(\alpha)
\]

(15)

and from equation (14):

\[
\frac{d|S_{ij}|}{d\alpha} \approx \frac{|S_{ij}|}{8.686}
\]

(16)

So, from equations (14), (15) and (16):

\[
u(|S_{ij}|) \approx \frac{1}{8.686} \times 10^{-\alpha/20} \times u(\alpha)
\]

(17)

Equation (12) shows that the uncertainty in phase will vary depending on the magnitude of the S-parameter being measured. For linear magnitudes close to unity (i.e. representing either complete reflection or complete transmission), the standard uncertainty approximates to:

\[
u(\phi) \approx \sin^{-1}\left(u(|S_{ij}|)\right)
\]

(18)

Equation (18) establishes a CMC for S-parameter phase measurements. However, when the magnitude of a given S-parameter is less that the uncertainty in the magnitude of the S-parameter (i.e. \( |S_{ij}| < u(|S_{ij}|) \)), the uncertainty in phase becomes indeterminate. To illustrate a calculation of uncertainty in phase, we use the uncertainty budget in Table VII, where the standard uncertainty in logarithmic transmission is given as 0.498 dB. This is equivalent to an uncertainty in linear transmission, \( S_{21} \), using equation (17), of 0.005 8 (assuming the measured value of transmission is actually 20 dB). This produces a standard uncertainty in \( S_{21} \) phase, using equation (12), of 3.3º, or equivalently, an expanded uncertainty of 6.6º (using equation (13)). More generally, Figure 10 shows a graph of expanded uncertainty in transmission phase, as a function of measured attenuation.

![Fig. 10. Overall expanded uncertainty in phase for transmission measurements](image)

VII. SUMMARY

This paper has described a new capability for providing S-parameter measurements, with traceability to the International System of units (SI), of waveguide devices in the frequency range 750 GHz to 1.1 THz. This capability is provided by a partnership between NPL and the University of Leeds. The VNA system is owned by, and operated at, the University of Leeds and the primary reference standards in the TRL calibration kit are characterized by NPL. Researchers at both NPL and the University of Leeds are involved in providing the S-parameter measurements.

It has been demonstrated that the capability achieves a Calibration and Measurement Capability (CMC) expanded uncertainty \((k = 2)\) of 0.16 for linear reflection coefficient magnitude, and 0.36 dB for low measured values of attenuation. The related CMC for S-parameter phase measurements, using equations (18) and (13), i.e. for S-parameters with linear magnitudes close to unity, is 9.1º for measurements of reflection and 21º for measurements of transmission. These CMCs are expanded uncertainties obtained using a coverage factor of \( k = 2 \).

It is fully recognized that the uncertainty values given in this paper are based only on a preliminary assessment of what is expected to be the most significant sources of error affecting measurements of this type. However, there has been some
related recent work aimed at quantifying uncertainty components for this waveguide band [7, 8, 43] and there is encouraging agreement between these independent uncertainty assessments. It will therefore be useful, at some time in the future, to verify and validate these uncertainty statements – for example, through a measurement comparison exercise involving systems operating at these frequencies belonging to other end-users – for example, as used in [7, 8, 43]. It will also be useful to undertake a rigorous review of the mechanical interactions that take place when connecting interfaces of this type at these, and similar, submillimeter-wave frequencies. Detailed models that include the imperfections in both interfaces that are involved in a waveguide connection could be used to provide a more accurate definition of the condition of measurement for the S-parameter measurands.

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The authors wish to dedicate the work described in this paper to the memory of the late Professor Roger Pollard, formerly Dean of Engineering at the University of Leeds and formerly President of the IEEE Microwave Theory and Techniques Society, who encouraged and supported the formation of a millimeter-wave and terahertz metrology partnership between NPL and the University of Leeds.

REFERENCES


Nick M. Ridler (M’03-SM’06-F’14) received the B.Sc. degree from King’s College, University of London, London, U.K., in 1981. He has since spent over 30 years working in both industrial and government scientific research laboratories. He is currently with the National Physical Laboratory (NPL), Teddington, U.K., where he is Principal Research Scientist in the Time, Quantum and Electromagnetics Division. He is also Visiting Professor at the Institute of Microwaves and Photonics, School of Electronic and Electrical Engineering, University of Leeds, Leeds, U.K., Visiting Professor in the Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, U.K., and he holds a Fellowship in Nonlinear Microwave Metrology at the Advanced Technology Institute, University of Surrey, Guildford, U.K.

His current research interests include measurements at millimeter-wave and terahertz frequencies, nonlinear microwave measurements, and high-speed digital measurements on printed circuit boards. Prof. Ridler is a past president of the Automatic RF Techniques Group (ARFTG), past chair of the IEEE MTT-11 “Microwave Measurements” Technical Committee, chair of IEEE standard Working Group P1785 “Waveguide for Millimeter and Submillimeter Wavelengths”, vice-chair of IEEE standard Working Group P287 “Precision Coaxial Connectors at RF, Microwave and Millimeter-wave Frequencies”, chair of the MTT-S Standards Coordinating Committee, and member of the IEEE MTT-4 “Terahertz Technology and Applications” Technical Committee. He is also a member of the BIPM Joint Committee for Guides in Metrology (JCGM) Working Group 1: “Uncertainty in Measurement”. He is a Chartered Engineer and a Fellow of the Institution of Engineering and Technology (formerly the Institution of Electrical Engineers, U.K.).

Roland G. Clarke (M’04) was born in Huddersfield, U.K. in 1966. He received the BSc degree from the University of Leeds, Leeds, UK in 2003.

For several years he was responsible for the technical management of high-frequency research laboratories at the University of Leeds. He is currently a Senior Teaching Fellow within the School of Electronic & Electrical Engineering at the University of Leeds and a member of the Institute of Microwaves & Photonics at the University of Leeds. His research interests are principally concerned with high-frequency metrology, particularly millimeter-wave and submillimeter-wave vector network analyzer measurements.