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Vibration absorbers for chatter suppression: a new analytical tuning methodology

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Abstract

Vibration absorbers have been widely used to suppress undesirable vibrations in machining operations, with a particular emphasis on avoiding chatter. However, it is well known that for vibration absorbers to function effectively their stiffness and damping must be accurately tuned based upon the natural frequency of the vibrating structure. For general vibration problems, suitable tuning strategies were developed by Den Hartog and Brock over 50 years ago. However, the special nature of the chatter stability problem means that this classical tuning methodology is no longer optimal. Consequently vibration absorbers for chatter mitigation have generally been tuned using ad-hoc methods, or numerical or graphical approaches. The present article introduces a new analytical solution to this problem, and demonstrates its performance using time domain milling simulations. A 40-50% improvement in the critical limiting depth of cut is observed, compared to the classically tuned vibration absorber.

Nomenclature

b	m	Depth of cut during machining
b_{lim}	m	Limiting depth of cut due to chatter stability boundary
f	-	Frequency ratio of absorber to main structure
F_0	N	Static load
f_{opt}	-	Optimal frequency ratio
g	-	Dimensionless frequency
K_s	N/m^2	Cutting coefficient
G	m/N	Frequency response function
h	m	Chip thickness during machining
k_a	N/m	Absorber stiffness
k_m	N/m	Main structure stiffness
m_a	kg	Absorber mass
m_m	kg	Main structure mass
N		Number of complete vibration cycles between successive tooth passes
u	-	Orientation coefficient
x	m	Displacement of cutting tool
δ_{st}	m	Static deflection of main structure
ε	rad	Relative phase of vibration between successive tooth passes
μ	-	Mass ratio of absorber to main structure
τ	s	Delay due to spindle rotation
ω	rad s^{-1}	Vibration frequency
ω_a	rad s^{-1}	Absorber natural frequency
ω_c	rad s^{-1}	Chatter frequency
ω_m	rad s^{-1}	Main structure natural frequency
ζ	-	Absorber damping ratio
ζ_m	-	Main structure damping ratio
ζ_{opt}	-	Optimal absorber damping ratio

Subscripts:

$_{a,b,p,n}$	Invariant or locked points

Introduction

The productivity of many machining operations is fundamentally limited by the onset of regenerative chatter, which occurs when vibration between the cutting tool and workpiece modulate the cutting force, leading to a self-excited vibration. This form of instability causes an unacceptable surface finish, along with excessive tool wear or breakage, thereby limiting the metal removal rate that can be achieved.

It is widely known that the chatter stability of machining processes can be improved by the addition of tuned vibration absorbers to the structure. For example, Tobias [1] illustrated a number of practical approaches that could be employed, with the vibration absorber fitted to various elements of the machine tool structure. More recently, practical and optimised designs for tooling with embedded absorbers have been developed [2], and non-traditional absorber designs have been proposed, such as those based upon impact dampers [3] and particle dampers [4]. Active vibration absorbers [5, 6] have also been proposed, since they can be more easily tuned than passive systems and can enable higher levels of energy dissipation. This approach is a special case of active vibration control, which has been applied to various machining chatter problems [7-9]. Despite the potential advantages of fully-active methods, passive tuned vibration absorbers remain a useful device for improving the chatter stability of machining systems, due to their lower complexity and cost.

For passive, and also active, vibration absorbers the performance is dependant upon correct tuning of the physical parameters or control gains, respectively. In the general field of vibration control it is normally desirable to suppress the response magnitude in the frequency domain, and this can be achieved using Ormondroyd and Den Hartog's classical 'equal peaks' method [10, 11]. A corresponding approach for active absorbers was proposed by Nishimura [12]. This method has been widely employed for a variety of problems in applications as diverse as civil engineering and space structures.

However, the tuning requirements for improving chatter stability differ from those for other vibration problems. To overcome this, Hahn [13, 14] proposed a basic tuning strategy for boring bars, but this was based upon Lanchester dampers rather than vibration absorbers. Tarng *et al* [15] manually tuned a vibration absorber to achieve the desired behaviour, and Liu *et al* used numerical optimisation routines based upon time domain machining simulations [16]. To the author's knowledge, the only published work that describes absorber design using analytical methods is that of Rivin and Kang [2], who went on to perform a detailed and comprehensive experimental study that demonstrated significant performance improvements using their design procedure. The present contribution will focus on an alternative analytical solution, and in a later section this will be compared to Rivin and Kang's method.

Analytical solutions for milling and turning chatter (e.g. [17-19]) have demonstrated that the critical limiting depth of cut is inversely proportional to the most negative real value of the orientated transfer function. Consequently, an optimally tuned vibration absorber will seek to replace this 'trough' in the real part of the orientated transfer function with two troughs of equal depth. The question that arises, then, is whether the optimal absorber for chatter can be tuned using analytical approaches, rather than trial and error or numerical methods. Furthermore, it is of interest to compare the analytical result with the classical method developed by Den Hartog [11] and the work of Rivin and Kang [2]. These issues will be tackled by the present article.

The new analytical method is relevant to a wide range of machining chatter problems. For turning and boring operations, passive vibration absorbers (as used in references [2, 15, 16]) could be tuned using this technique. Alternatively, the controller gains in an active absorber (such as that used by Pratt and Nayfeh [6]) could be chosen using the analytical method. For milling operations, there are a number of possible positions where an absorber could be added, such as the spindle housing [20] or machine tool column [7]. In some special cases, the workpiece itself could cause chatter, and so it may be advantageous to attach an absorber to the workpiece during the machining process [4]. The present article will consider this workpiece chatter scenario in a numerical example, but it should be pointed out that this is just one possible application of the analytical method.

The manuscript is organised as follows. First, the relevant machine-tool chatter theory is briefly summarised. A vibration absorber tuning solution for the case of chatter is then developed and the results compared to the Den Hartog solution. These results are then compared to the analytical/numerical solution of Rivin and Kang [2]. The main assumptions of the analytical result are then explored by performing a numerical optimisation, and then by simulating the performance of the absorber in a milling scenario. Following a discussion of the results, some conclusions are drawn.

It should be noted that the aim here is not to implement vibration absorbers during machining, since this has been widely reported elsewhere (e.g. [2]). Furthermore, the contribution does not claim to be the first to provide a solution to the optimisation problem. It does however provide a new analytical solution, which is considerably more elegant (and easy to apply) than other numerical or graphical approaches, and can be compared directly to the classical Den-Hartog approach.

Theory

To begin, it is worthwhile summarising the theory of regenerative chatter, which motivates the need for an alternative tuning procedure. Regenerative chatter is most commonly explained (e.g. [21, 22]) with reference to the simplified scenario of turning, depicted in Figure 1. Here, a flexible cutting tool is removing material from the workpiece, with a chip thickness h and depth of cut (normal to the plane of the diagram) b. The motion of the tool means that the chip thickness h is a function of the present displacement, x, and the displacement during its previous pass over the workpiece, $x(t-\tau)$ where τ is the time delay due to the spindle rotation. Assuming that the cutting force is proportional to the cross-sectional area of the chip, then the system can be represented by the block diagram in Figure 1b. The orientation coefficient u maps the cutting force F to the direction of the tool transfer function $G(j\omega)$ and the subsequent motion x. Instability of the feedback loop causes the self-excited vibration known as chatter. The stability can be found using the Nyquist criterion:

$$K_s b_{\lim} u G(j\omega)\left(1-e^{-j\omega\tau}\right)=-1 \tag{1}$$

Here, b_{lim} is the limiting depth of cut, i.e. the value of b beyond which the system becomes unstable. Following some algebra (see, for example, [21, 22]), the stability condition can be written as:

$$b_{\lim} = -1/(2K_s \operatorname{Re}[uG(\omega_c)]) \tag{2}$$

where ω_c is the frequency of vibration at the boundary of stability, and is referred to as the chatter frequency. The integer number N of oscillations between each tooth pass, and the phase ε of the oscillations, are given by:

$$N+\varepsilon/(2\pi)=\omega_c\tau \tag{3}$$

which can be used to determine the relation between spindle speed $1/\tau$ and chatter frequency ω_c.

Plotting the spindle speed against b_{lim} for different values of N gives the so-called stability lobe diagram. To demonstrate why special optimal absorber tuning is required for this problem, a simplified arbitrary single-degree-of-freedom (DOF) problem can be considered. If a vibration absorber is added to the structure then the resulting 2DOF system can be tuned with Den Hartog's method to give two peaks of equal magnitude in the magnitude-frequency response function (FRF). This is shown schematically in Figure 2a. However, from (2) it is the real part of the response (Figure 2b) that dictates the chatter stability (Figure 2c). To maximise the depth of cut at which the system becomes unstable requires the real part of the FRF to have two troughs of equal magnitude, as demonstrated in Figure 2.

The aim of this contribution is to provide an analytical solution to this problem in a form similar to Den Hartog's classical solution.

Chatter stability optimisation

To develop an analytical solution Den Hartog's method of derivation will be adapted for use on the real part of the FRF rather than the magnitude part. As with Den Hartog, the main structure is assumed to have single undamped mode of vibration. From (1), it is noted that for the chatter problem the relevant transfer function is scaled by a factor u which may be positive or negative. If u is positive, then the chatter stability is dictated by the negative real part of the FRF and so it is desirable to increase this value. If u is negative, then chatter stability is dictated by the most positive real part, and so it is desirable to reduce this value. In what follows, both scenarios will be investigated.

To begin, the absorber and host structure are defined by the following non-dimensional terms:

$$\begin{aligned} \text{mass ratio} \quad & \mu = m_a / m_m \\ \text{static deflection} \quad & \delta_{st} = F_0 / k_m \\ \text{absorber natural frequency} \quad & \omega_a = \sqrt{k_a / m_a} \\ \text{main structure natural frequency} \quad & \omega_m = \sqrt{k_m / m_m} \\ \text{frequency ratio} \quad & f = \omega_a / \omega_m \\ \text{non - dimensional excitation freqeuency} \quad & g = \omega / \omega_m \end{aligned} \tag{4}$$

The main structure's mass, stiffness, and natural frequency are denoted m_m, k_m, and ω_m, respectively, whilst the equivalent terms for the absorber are assigned the subscript $_a$. The excitation frequency is ω, and F_0 is the static load on the main structure. The non-dimensional response as a function of non-dimensional frequency, $R(g)$, can then be presented as [23]:

$$R(g) = \frac{X_1}{\delta_{st}} = \frac{\left(f^2 - g^2 + i2\zeta g\right)}{\left(1-g^2\right)\left(f^2 - g^2\right) - \mu f^2 g^2 + i2\zeta g\left(1 - g^2 - \mu g^2\right)} \tag{5}$$

where ζ is the absorber damping ratio. The real part of (5) is given by:

$$\mathrm{Re}(R(g)) = \frac{(f^2 - g^2)((1-g^2)(f^2-g^2) - \mu f^2 g^2) + 4\zeta^2 g^2 (1 - g^2 - \mu g^2)}{((1-g^2)(f^2-g^2) - \mu f^2 g^2)^2 + 4\zeta^2 g^2 (1 - g^2 - \mu g^2)^2} \tag{6}$$

In Figure 3 this is plotted for three levels of absorber damping: ζ=0, ζ=∞, and ζ=0.03, for f=1 and μ=0.01. It can be seen that there are three invariant, or 'locked points' [24] on the response. Closer inspection reveals that at these locked points the ζ=0 or ζ=∞ curves have an infinite gradient and pass through zero. Consequently, the non-dimensional frequencies of the locked points can be determined by evaluating the roots of (6) when ζ=0 or ζ=∞. These will be considered in turn.

The roots of (6) when ζ=0 are given by the solution of:

$$(f^2 - g^2)((1-g^2)(f^2-g^2) - \mu f^2 g^2) = 0 \tag{7}$$

Defining the three roots as g_i, g_p, and g_n, and solving (7) gives:

$$\begin{aligned} g_i &= f \\ {g_p}^2 &= \frac{(1 + f^2 + \mu f^2) - \sqrt{(1 + f^2 + \mu f^2)^2 - 4f^2}}{2} \\ {g_n}^2 &= \frac{(1 + f^2 + \mu f^2) + \sqrt{(1 + f^2 + \mu f^2)^2 - 4f^2}}{2} \end{aligned} \tag{8}$$

Of the three roots, g_i is inadmissible as a locked frequency because it does not correspond to a location on the response with an infinite gradient. Meanwhile, g_p is the locked frequency where the real part of the response is positive, and g_n is a locked frequency where the real part of the response is negative.

Next, this analysis is repeated for the case when ζ=∞. Dividing the numerator and denominator of (6) by ζ^2 indicates that the zero is given by the solution of:

$$4g^2(1 - g^2 - \mu g^2) = 0 \tag{9}$$

In this case the solution g=0 is inadmissible as it requires an infinite natural frequency for the main structure. This leaves the solution defined as g_a:

$$g_a = \sqrt{\frac{1}{1+\mu}} \tag{10}$$

which is shown on Figure 3 along with g_p and g_n.

To recap, the three locked frequencies on Figure 3 have been determined analytically. To obtain the optimal response for chatter stability, it is desirable to ensure that the response at g_a matches that at either g_p or g_n, depending on whether the direction factor u is negative or positive, respectively. These two alternatives are shown in Figure 4.

The next step, then, is to equate (6) at g=g_a to (6) at g=g_p or g=g_n, and solve to find f. Unfortunately, the mathematics become immensely protracted, and so symbolic algebra computer software is required. It transpires that despite the lengthy intermediate equations, the optimal value for f can be expressed relatively concisely:

$$
\begin{aligned}
f_{opt,n}{}^2 &= \frac{\mu + 2 + \sqrt{2\mu + \mu^2}}{2(1+\mu)^2} \\
f_{opt,p}{}^2 &= \frac{\mu + 2 - \sqrt{2\mu + \mu^2}}{2(1+\mu)^2}
\end{aligned}
\qquad \textbf{(11)}
$$

Here, the optimum frequency so that the response at g_a matches that at g_p is denoted $f_{opt,p}$, and the optimum frequency so that the response at g_a matches that at g_n is denoted $f_{opt,n}$. The symbolic computations were performed using Maple [25]. The solutions are compared to Den Hartog's classical solution in Figure 5a.

Having determined the optimum frequency ratio, it is now desirable to adjust the damping ratio ζ until the real response at the locked points is flat. As Den Hartog pointed out [11] this can be achieved by evaluating d(Re(R))/dg at the locked points, equating to zero, and solving to find the damping ratio. For the locked frequency g_a, this problem can be solved analytically (again with the help of symbolic computations), giving:

$$\zeta_{opt,a,n}{}^{2}=\frac{\mu\left(\mu+3+\sqrt{2\mu+\mu^{2}}\right)}{4(1+\mu)\left(\mu+2+\sqrt{2\mu+\mu^{2}}\right)}$$
$$\zeta_{opt,a,p}{}^{2}=\frac{\mu\left(\mu+3-\sqrt{2\mu+\mu^{2}}\right)}{4(1+\mu)\left(\mu+2-\sqrt{2\mu+\mu^{2}}\right)} \qquad \textbf{(12)}$$

where the subscript $_n$ or $_p$ denotes which frequency ratio is used from equation (11). Unfortunately, obtaining a flat response at g_n or g_p means substituting (8) into d(Re(R))/dg, and the resulting closed form solution could not be simplified to a useful form by the symbolic computation software. However, the solutions are shown graphically in Figure 5b, along with those from equation (12). It transpires that within the limits of machine precision the solutions are numerically equivalent to:

$$\zeta_{opt,n,n}\approx\zeta_{opt,a,p}$$
$$\zeta_{opt,p,p}\approx\zeta_{opt,a,n} \qquad \textbf{(13)}$$

The mean of the optimum tuning at g_a and that at g_p or g_n is therefore the mean of (12):

$$\zeta_{opt}=\sqrt{\frac{3\mu}{8(1+\mu)}} \qquad \textbf{(14)}$$

which is also shown in Figure 5b. Remarkably the result is the same as the optimum damping ratio for a classically tuned vibration absorber.

To summarise, the analytical formulations for optimal stiffness and damping have been derived for the case of chatter mitigation. However, this analytical result is based upon two key assumptions: the main structure has zero damping, and the main structure is a single-degree-of freedom system. Furthermore, it is useful to compare the result to the work of Rivin and Kang. These issues will now be addressed.

Comparison with Rivin and Kang's method [2]

Rivin and Kang considered the behaviour of the system during metal cutting and represented the self-excitation (chatter) mechanism as an effective cutting stiffness and effective cutting damping rate. They derived the equations of motion in non-dimensional form by dividing the absorber natural frequency ω_a by the 'chatter frequency' ω_c (denoted ω in reference [2]) which was a function of the effective cutting stiffness (an empirical measurement). They showed that frequency ratio ω_a/ω_c and absorber damping ratio ζ influenced the value of a performance index ζ_0 which was determined using the Routh-Hurwitz stability criterion. An example of their results is given in Figure 6, for two different ratios of the absorber mass to main structure mass. A more negative value of ζ_0 indicates that a greater depth of cut could be achieved without chatter. It can be seen that higher mass ratios increase the effectiveness of the absorber (as expected), and that for each mass ratio there exists an optimum combination of frequency ratio and absorber damping ratio. Finding this optimum absorber design requires a graphical (as in Figure 6) or numerical optimisation approach. Furthermore, transforming this non-dimensional result into an absorber design (i.e. stiffness and damping) requires knowledge of the chatter frequency ω_c. This can only be determined by first identifying the effective cutting stiffness from a series of experiments.

Figure 7 shows the optimum values for frequency ratio ($f_{opt}=\omega_a/\omega_c$) and damping ratio ζ, using Rivin and Kang's method [2]. The new analytical solution is also shown and it should be noted that the frequency ratio for this case is $f_{opt}=\omega_a/\omega_m$. Despite the different definition of the frequency ratio, it is clear that the analytical result proposed in the present contribution is fundamentally different to that of Rivin and Kang. Consequently, the solution of Rivin and Kang does not minimise the peaks or troughs in the frequency response function, even though it has been shown [2] to offer superior performance than Den Hartog's method for machining problems.

Effects of main structure damping

One advantage of Rivin and Kang's solution, compared to the new analytical approach, is that it did not assume that the main structure was undamped. To investigate the role of main structure damping on the analytical approach, equation (5) can be readily modified to include damping, ζ_m of the main structure. However, optimum values for the design parameters can no longer be found in closed form, and so numerical optimisation must be used instead. Standard optimisation routines such as the Matlab *fminsearch* function [26] were found to be appropriate for this task. Three different optimisations were performed: minimise the magnitude of the FRF; minimise the positive real part of the FRF; and maximise the negative real part of the FRF. In each case optimal values of the variables ζ (absorber damping) and f (absorber frequency ratio) were sought.

The optimisation can be repeated for a range of values of mass ratio μ and main structure damping ratio ζ_m. The results are shown in Figure 7. It can be seen that with no main structure damping the analytical solutions are correct (as expected), and that as damping is added to the main structure there is only a small change in the optimum absorber damping. However, for chatter optimisation the optimum frequency ratio is slightly more sensitive to the damping ratio of the main structure. This suggests that in practice it might be more difficult to accurately tune the damper, especially when the main structure is heavily damped or if its damping changes significantly under different conditions.

Milling simulations

The final issue that must be tackled is how the analytical solution performs for multiple degree of freedom structures. To investigate this issue, and to provide more insight as to how the absorber can increase chatter stability, a time-domain simulation of a milling scenario was performed.

Time domain models of milling have been widely reported as providing an accurate reflection of the stability of the process [27]. Furthermore, the experimental and commercial application of vibration absorbers to milling and machining problems has been previously reported, and the aim of this contribution is not to repeat these efforts. Consequently a time domain numerical study will serve the purpose of investigating the assumptions in the new analytical approach.

The numerical simulation used a time-domain formulation based upon the method of Campomanes and Altintas [27], along with a recently described signal conditioning method [28] to analyse the chatter stability. For comparison purposes, an analytical solution was developed using the approach described by Tlusty [17] and implemented in commercially available software [29].

The milling scenario that was considered was based on the very flexible cantilever workpiece that is shown schematically in Figure 8. It is worth reiterating that this is just one possible application where optimally tuned vibration absorbers could be used – others include boring bars, milling machine columns, and milling spindle housings. For the example problem of the workpiece shown in Figure 8, chatter is likely to be caused by the bending vibrations of the workpiece. This vibration could be suppressed by an appropriately tuned vibration absorber mounted on the uncut side of the workpiece as shown in Figure 8. To specify the vibration absorber parameters and predict the chatter stability, it is therefore desirable to measure and model the frequency response function at this location. The workpiece exhibited three main modes of vibration: a bending mode at 410Hz, a torsional mode at 1556Hz, and then a second bending mode at 3255Hz. The first two modes were modelled using modal analysis techniques [30], and a corresponding state-space representation extracted:

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6.654e6 & 0 & -22.919 & 0 \\ 0 & -9.5554e7 & 0 & -70.47 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -2.1162 \\ -1.6015 \end{bmatrix} F \\ y &= \begin{bmatrix} -2.1162 & -1.6015 & 0 & 0 \\ 0 & 0 & -2.1162 & -1.6015 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F \end{aligned} \qquad \textbf{(15)}$$

The state-space system has two outputs (displacement and velocity at the absorber location) and one input (applied force F). To control the first bending mode at 410Hz, a vibration absorber with mass ratio μ=0.05 can be specified. The effective mass of the main structure at this location is 0.223 kg, giving a required absorber effective mass of 0.011kg. Three different tuning strategies can be used for the absorber: classical Den Hartog tuning, equal real peaks, or equal real troughs (equations (11) and (14)). These lead to absorber stiffnesses of 67.3N/mm, 58.2N/mm and 79.9N/mm respectively.

The absorbers could be readily designed as small cantilever beams with appropriate damping treatments [24]. In practice the natural frequency of the workpiece will change if large amounts of material are removed during machining, but for problematic finish-machining operations the volume of material removed will be small enough that the de-tuning of the absorber can be neglected.

The predicted frequency responses for the damped workpiece are compared to the un-damped workpiece model in Figure 9. Here it can be observed that the frequency response curves are not perfectly tuned, in that one peak is slightly greater than the other. For classical tuning the difference in peaks is very small. Since this tuning is not sensitive to the main structure damping (Figure 7a), the de-tuning effect can be attributed to the influence of the second mode of vibration. For the equal real peaks/troughs tuning, the main structure damping has a stronger influence (as shown by Figure 7a), and the second mode of vibration will also have an effect. Nevertheless, the performance of the tuning methodology is good, in that the difference between the real peaks or troughs is small.

To investigate the chatter stability of the workpiece, the same state space models (for damped and un-damped conditions and with different tuning methods) were implemented in the analytical and time domain milling simulations. The tool geometry and cutting conditions used in the simulations are given in Table 1. Both up-milling and down-milling scenarios were considered, since for the chosen cutting configurations up-milling will lead to a negative orientation coefficient whereas down-milling will lead to a positive orientation coefficient.

The numerical and analytical results for the up-milling simulation are shown in Figure 10 for all three tuning conditions. The discrepancy between the analytical result and the time domain simulation can be attributed to the approximations used in determining the orientation coefficient for the analytical model. However, the trends observed in the time domain model are very similar to those in the analytical model. Whilst the classically tuned absorber is effective in increasing the chatter stability, it can be seen that a properly tuned absorber can provide a 40% improvement in the critical limiting depth of cut. This serves to validate the tuning methodology presented in this article. However, Figure 10b illustrates how if the absorber is tuned to give equal real troughs, rather than equal real peaks, then the response is actually worse than the classically tuned case. Consequently care must be taken to ensure that the sign of the orientation coefficient is taken into consideration when tuning the absorber.

For the down-milling simulation, the orientation coefficient is expected to be positive and so it is desirable to tune the absorber so as to achieve equal real troughs. The chatter predictions for this case are shown in Figure 11 along with the un-damped and classically tuned scenarios. The properly tuned vibration absorber provides a 50% performance improvement in the critical limiting depth of cut compared to the classically tuned absorber. The analytical prediction again suffers from an inaccurate estimation of the orientation factor which leads to a different stability prediction compared to the time domain model.

Discussion

The results presented have clearly demonstrated that the new analytical tuning approach is effective in optimising chatter stability. However, a number of issues are worthy of further comment:

The new tuning procedure is applicable to vibration absorbers in a wide variety of machining applications, such as boring bars, milling tool spindles, machine tool columns, or the flexible workpiece scenario that was considered in the present study. At this stage, it is useful to briefly mention some of the issues associated with this flexible workpiece scenario. Since material is removed from the workpiece during machining, its natural frequency will constantly change. This may make it difficult to use a vibration absorber, unless it can be adaptively tuned e.g. by using an active vibration absorber. However, during the more problematical finishing cuts, very little material is removed and so the absorber parameters could be constant. As the tool moves around the workpiece, different modes of vibration could cause chatter which would again raise the need for an adaptively tuned absorber. Different absorber designs would also be needed for different workpiece configurations, and the absorber location would have to be chosen such that it did not interfere with the cutting process. However, these shortfalls are all specific to the problem of absorbers mounted on the workpiece, and the contribution of this work is equally relevant to other machining applications which do not encounter such problems.

It is worth reiterating that the novelty of this contribution lies in the straightforward analytical solution to optimally tuning the vibration absorber for chatter stability. Similar results could obviously be obtained using graphical or numerical optimisation of the design parameters [16], or by manually adjusting the absorber to achieve the desired behaviour. The work presented here provides an attractive alternative that would substantially reduce the design, prototype, and testing effort required. Furthermore, the approach may be applicable to other vibration problems and the comparison that can be drawn with the classical Den Hartog method may be of interest to the wider vibration community.

One of the consequences of optimally tuning the absorber is that the shape of the stability lobe changes substantially. Whilst the critical limiting depth of cut is raised, the stable 'pockets' within the lobe are lost. For example, in Figure 11 it can be seen that for the classically tuned absorber it would be possible to machine at up to 3mm depth under certain spindle speeds, whereas the optimally tuned absorber provides a maximum stable depth of only 1.5mm, throughout the spindle speed range. This is a direct consequence of the 'flattening' of the negative real part of the orientated transfer function. For turning operations this effect is inconsequential as the chatter stability is dominated by the critical limiting depth of cut [17]. In contrast, for milling operations it can be desirable to machine in the stable lobe, in which case optimally tuning the absorber will not be helpful. In practice, however, vibration absorbers are likely to be used in scenarios where one wishes to raise the critical depth of cut above the desired cutting depth, so that any spindle speed can be selected. For this problem, the new analytical tuning strategy is well suited as it offers a 40-50% improvement compared to classical tuning.

The main complication that arises when optimally tuning the absorber is that the sign of the orientation factor for the damped mode dictates the absorber natural frequency. Consequently the absorber stiffness must be changed if the cutting conditions are such that the sign of the orientation factor changes. However, this same problem would arise if the damper tuning was optimised numerically or experimentally, rather that analytically, and the analytical solution that is now available means that retuning the damper would be more straightforward.

Before drawing conclusions, it is worth emphasising again that although the present study has focussed purely on the analytical aspects of vibration absorber tuning, the practical implementation of the absorber is no different to that for devices tuned by other means. For example, application in boring could follow Pratt and Nayfeh [6], whilst application in turning could follow Tarng *et al* [15]. The only difference would be that the tuning algorithm of the absorber would be optimised from a chatter perspective, thus leading to improved chatter stability.

Conclusions

This article has described a new analytical solution to tuning vibration absorbers from a regenerative chatter perspective. The theoretical approach is identical to that originally proposed by Ormondroyd [10], Den Hartog [11], and Brock [31], except that the real part of the response function is considered rather than its magnitude. The specific conclusions are as follows:

1. Closed form analytical expressions are derived for optimally tuning the absorber frequency and damping to achieve desirable behaviour in either the positive real part or the negative real part of the frequency response function. As with Den Hartog's original approach, two optimal damping values emerge for each case and the average of these provides a useful damping value.

2. A non-dimensional numerical study has served to demonstrate that the optimum absorber natural frequency is slightly sensitive to the damping of the main structure, unlike Den Hartog's classical method.

3. The performance of the analytical approach has been demonstrated by analytical and time domain simulations of the milling of a flexible workpiece. A 40-50% performance improvement was observed compared to Den Hartog's classical optimisation approach. It is noted that the approach is not specifically aimed at milling workpiece problems but is equally applicable to turning, boring, or milling machine structures.

4. Compared to previous work on optimal absorber design, this contribution describes an analytical solution that does not require a numerical, iterative or graphical approach.

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Table 1

Tool diameter	16mm
Number of teeth	4
Flute helix	0° (axial flutes)
Radial immersion	4mm
Feed per tooth	0.05mm
Tangential cutting stiffness	796.1 N/mm^2 (Al7075-T6 [20])
Radial cutting stiffness	168.8 N/mm^2 (Al7075-T6 [20])

Table 1: Milling simulation parameters

Figure 1

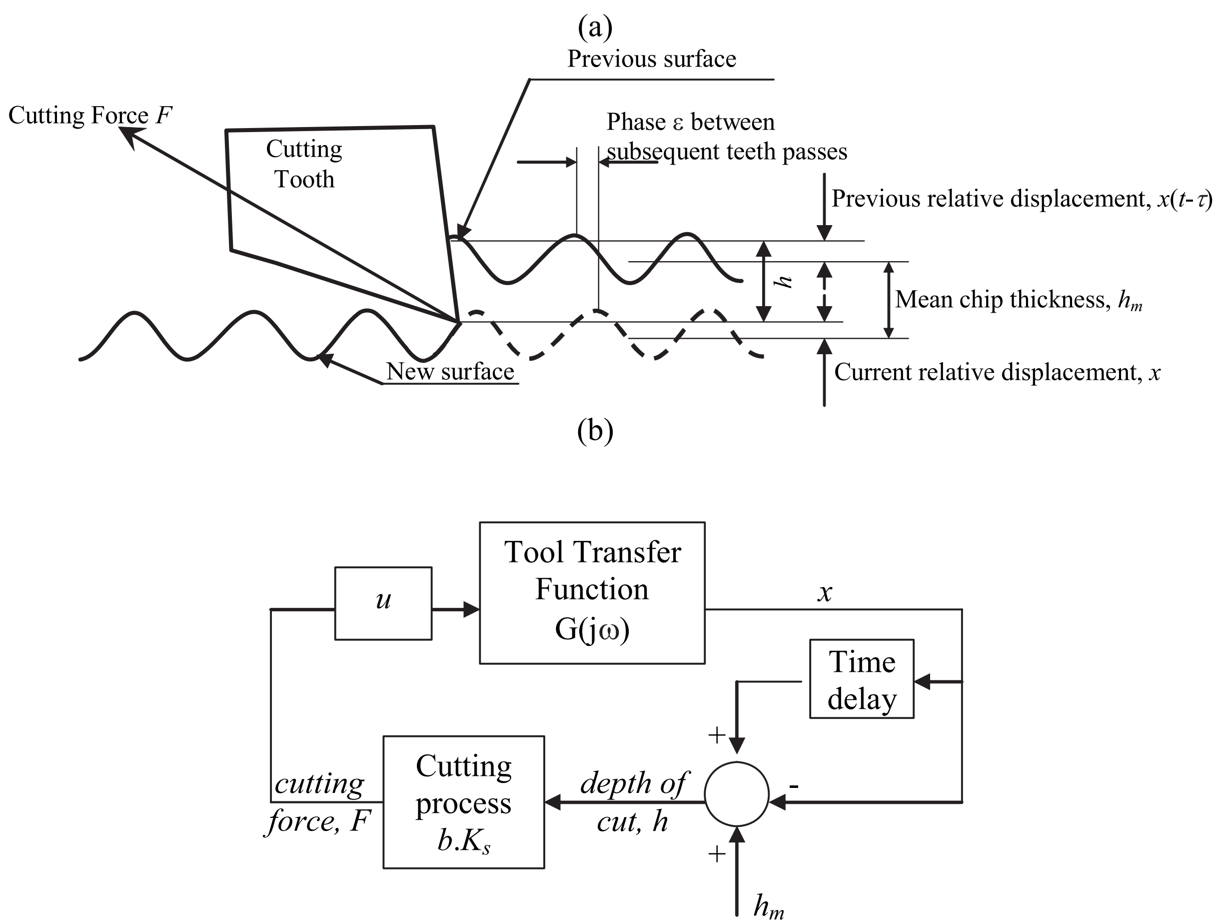

Figure 1: Schematic representation of turning. (a) surface generation and chip thickness, (b) block diagram

Figure 2

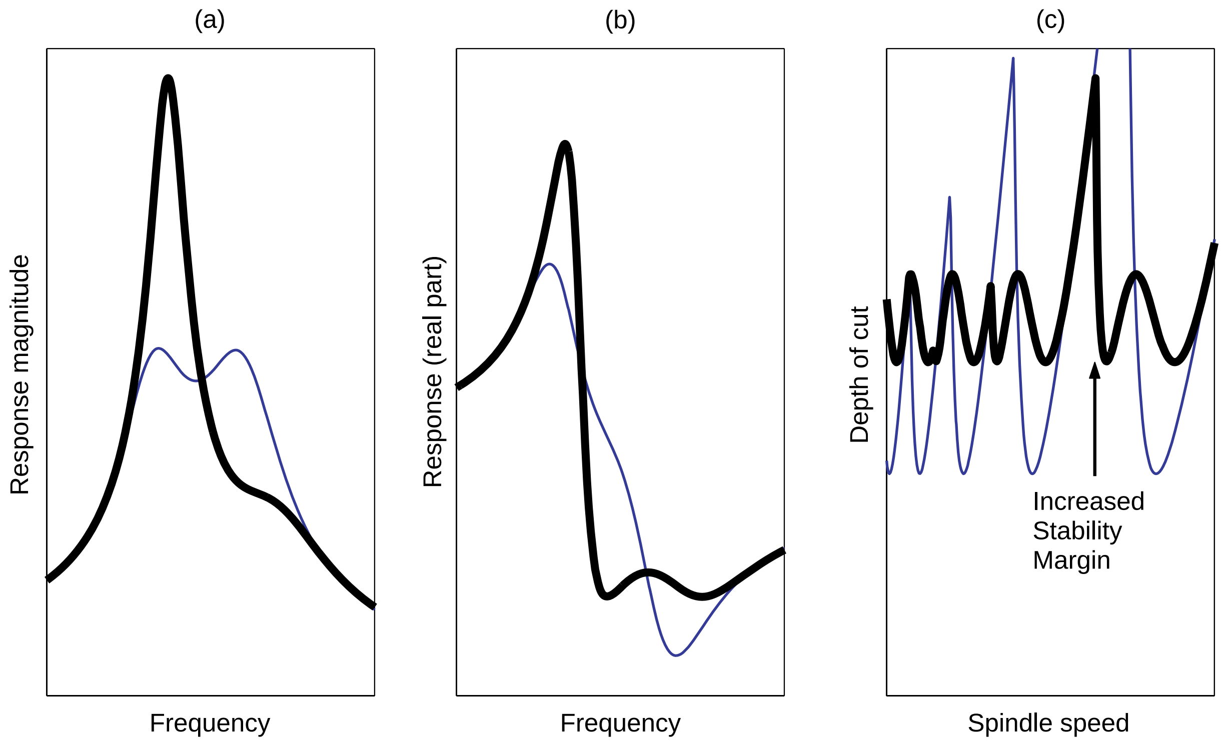

Figure 2: Optimal absorber tuning
(a) response magnitude (b) real part of the response (c) stability lobes
Chatter tuning (equal troughs in real part)
Classical tuning (equal peaks in magnitude)

Figure 3

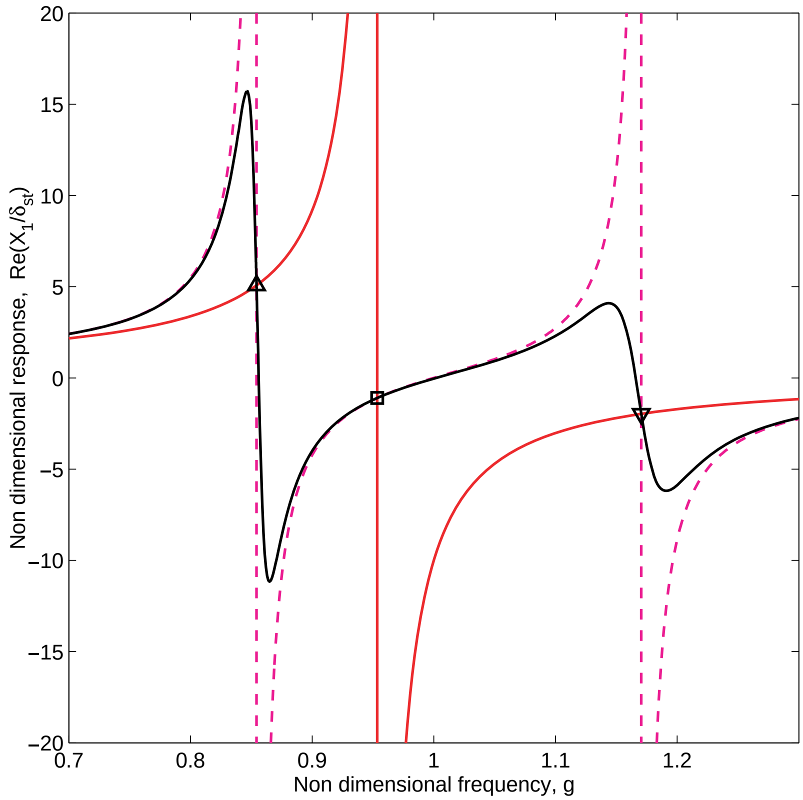

Figure 3: Real part of the frequency response function, showing the three locked frequencies.
$\zeta=\infty$; $\zeta=0$; $\zeta=0.03$; △ g_p; ▽ g_n; □ g_a;

Figure 4

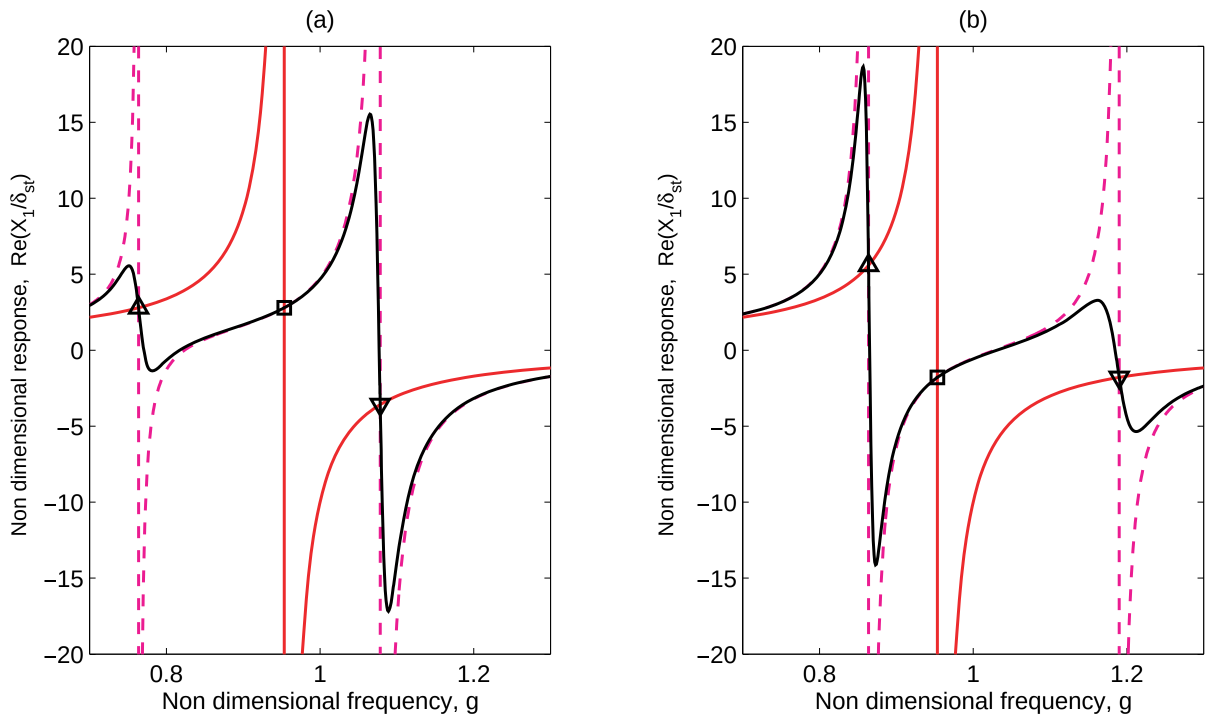

Figure 4: Optimal frequency tuning
(a) equal positive locked frequencies (b) equal negative locked frequencies
$\zeta=\infty$; $\zeta=0$; $\zeta=0.03$; △ g_p; ▽ g_n; □ g_a;

Figure 5

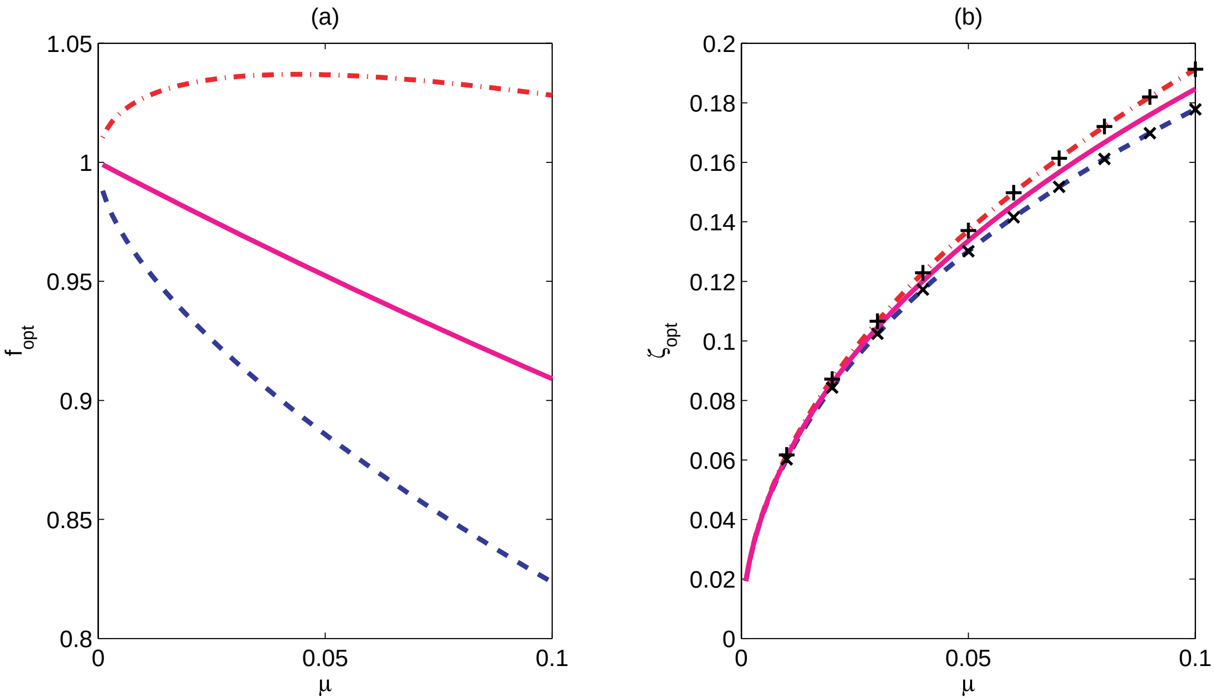

Figure 5: Analytical results. (a) optimum frequency ratio. —— Den Hartog; - · - $f_{opt,n}$; - - - $f_{opt,p}$
(b) optimum damping ratio. —— ζ_{opt}; - · - $\zeta_{opt,a,n}$; - - - $\zeta_{opt,a,p}$; × $\zeta_{opt,n,n}$; + $\zeta_{opt,p,p}$

Figure 6

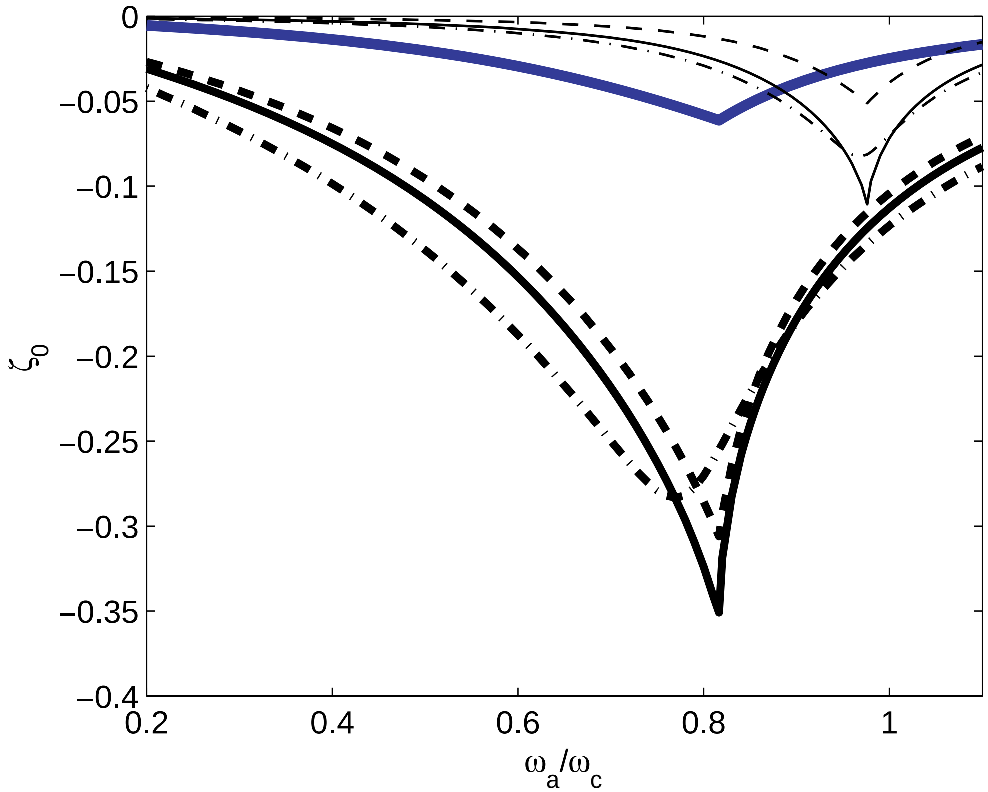

Figure 6: Optimisation using Rivin and Kang's method [2]. Values of ζ_0 above the line are stable; for a given mass ratio μ, optimum values of ω_a/ω_c and ζ give the lowest critical value of ζ_0.
μ=0.5, ζ=0.05 [2, Fig. 5]; – – μ=0.5, ζ=0.25; —— μ=0.5, ζ=0.29 (optimum); — • — μ=0.5, ζ=0.4; - - - μ=0.05, ζ=0.05; —— μ=0.05, ζ=0.11 (optimum); – · – μ=0.05, ζ=0.15.

Figure 7

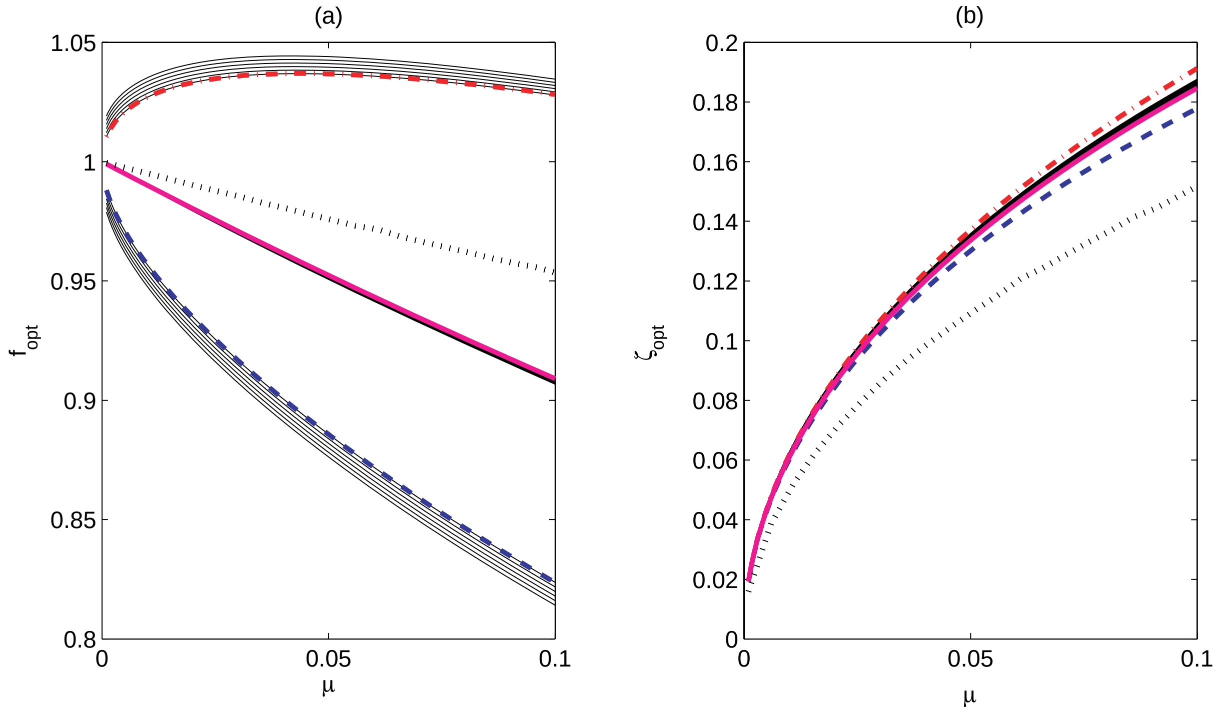

Figure 7: Numerical and analytical optimisation. Numerical optimisation is shown for different damping ratios of the main structure: ζ_m=0%,0.2%...1%. Rivin and Kang's solution is also shown, which requires a similar numerical optimisation approach and has a different definition of f_{opt}.

—— Classical tuning; - • - optimum tuning for negative real troughs; - - - optimum tuning for positive real peaks; — numerical solutions; · · · · · Rivin and Kang [2].

Figure 8

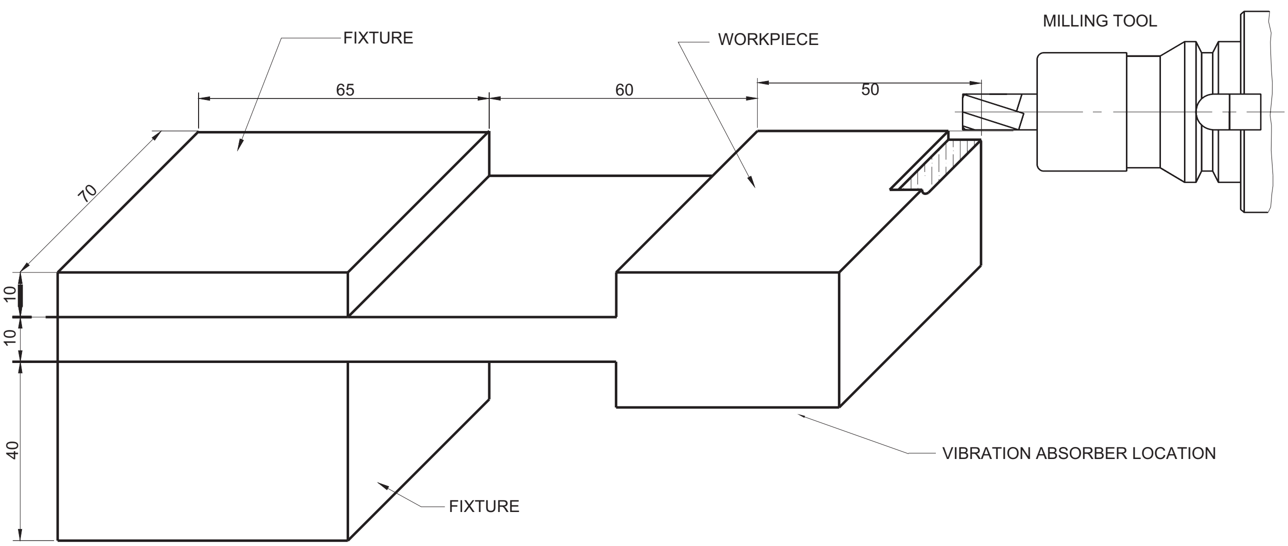

Figure 8 Milling scenario

Figure 9

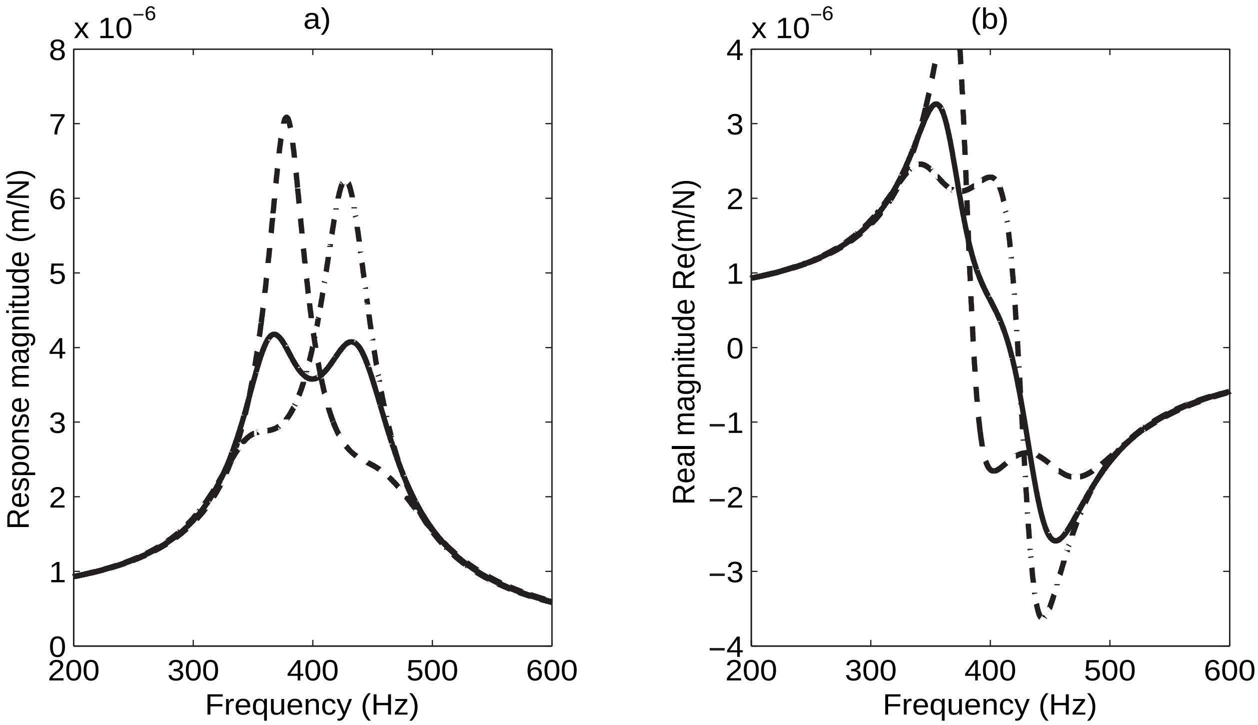

Figure 9: Simulated workpiece response. (a) magnitude, (b) real part.

Equal real troughs **Equal real peaks** **Den Hartog**

Figure 10

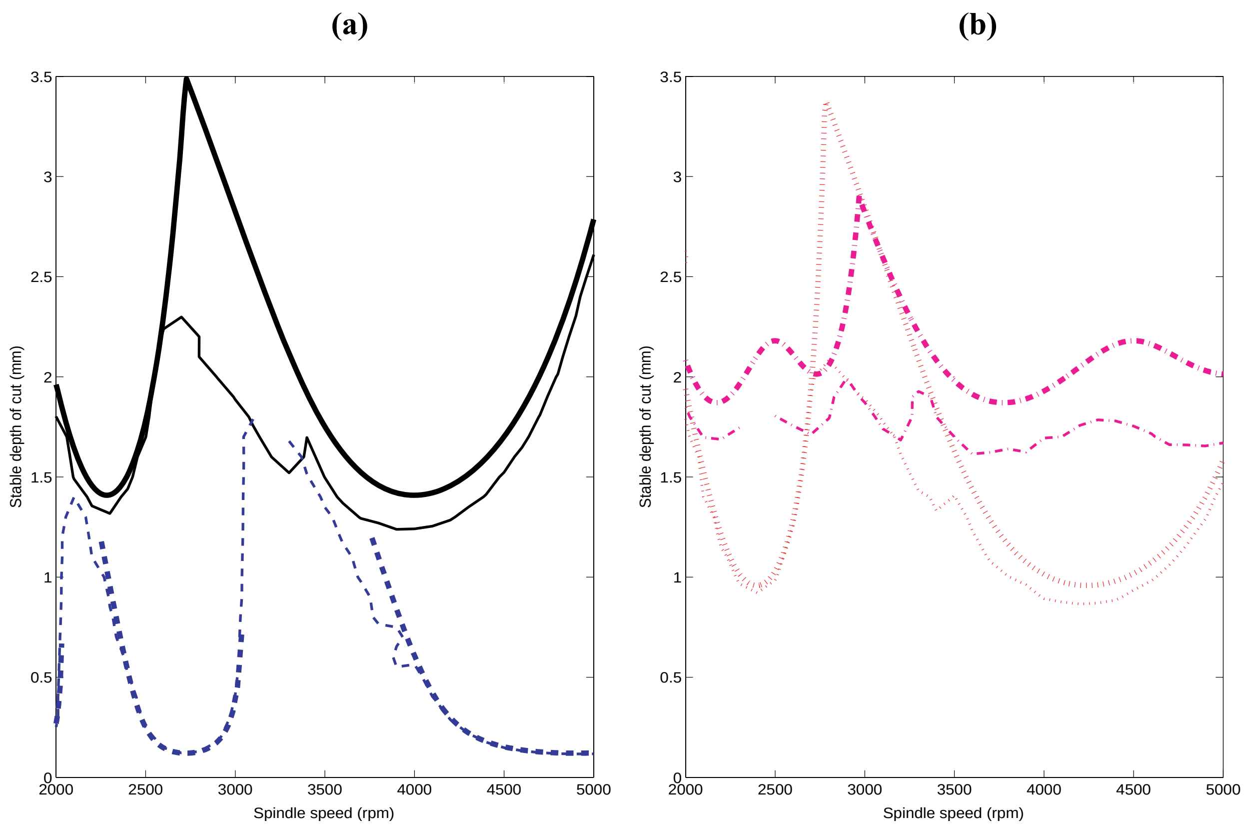

Figure 10: Stability predictions for up-milling – analytical (thick lines) and time domain simulation (thin lines). (a) open loop and classically tuned (b) chatter tuned no absorber; Den Hartog tuned absorber; equal real troughs; equal real peaks.

Figure 11

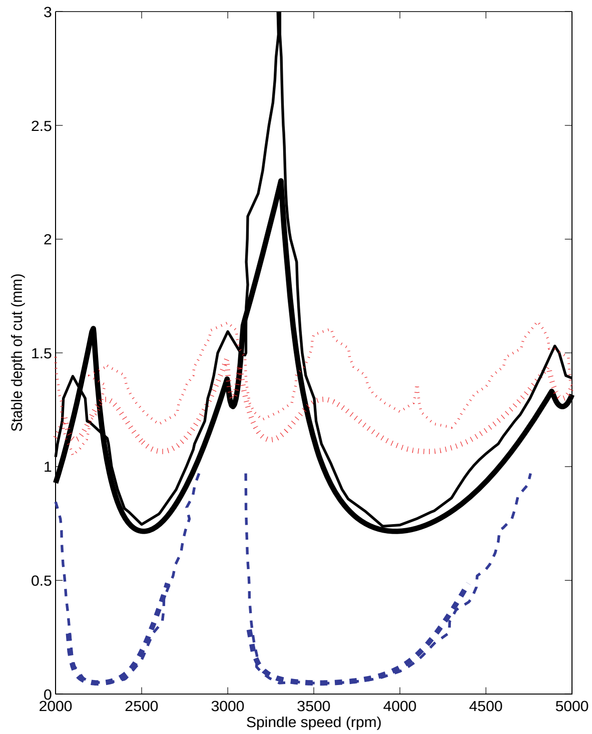

Figure 11: Stability predictions for down-milling: analytical (thick lines) and time domain simulation (thin lines)

no absorber; Den Hartog tuned absorber; equal real troughs.