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Cogging Torque Mitigation of Modular Permanent Magnet Machines

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This paper proposes a novel cogging torque mitigation method for modular permanent magnet (PM) machines with flux gaps in alternate stator teeth. The slot openings of the modular PM machines are divided into two groups in a special way. By shifting the slot openings of two groups in opposite directions with the same angle, the machine cogging torque can be significantly reduced. Analytical formula of the desired shift angle is derived, and can be applicable to other modular machines with different slot/pole number combinations. Meanwhile, the influence of the proposed method on phase back-EMF is investigated and the three phase back-EMF waveforms remain balanced for all slot/pole number combinations after slot shifting. Experiments based on existing prototypes are carried out to validate the finite element modelling.

Index Terms — Cogging torque, modular machine, permanent magnet (PM), slot opening shift, torque ripple.

I. Introduction

COGGING torque is the consequence of interaction between the permanent magnet MMF harmonics and the air gap permeance harmonics resulted from slot openings [1], [2]. Due to the fact that the cogging torque causes torque ripple and hence acoustic noises and vibration [2], the reduction of cogging torque is of significant importance for designing permanent magnet (PM) machines. Various methods to reduce the cogging torque have been proposed in recent years. For instance, employing the auxiliary slots [1], [3], [4]-[9], shaping the rotor magnets or stator teeth [10]-[13]. Another effective method for reducing cogging torque is skewing [1], [3], [14]-[16]. Moreover, by appropriately selecting slot/pole number combination [1], [17] or the optimized ratio of pole arc to pole-pitch [3], [18], the cogging torque can be effectively mitigated as well. In addition, the distribution of PMs on the rotor [9] of the PM machines can also affect the cogging torque significantly.

However, the existing methods of mitigating cogging torque still have some drawbacks. Taking the method of skewing as an example, skewing the magnets helps to reduce the cogging torque, along with the reduction in the fundamental of phase back-EMF and hence average torque. Furthermore, it is more difficult to wind the machines after skewing the stator since the effective slot opening width is decreased slightly [19]. In terms of manufacturing, skewed stator or rotor will be harder to build, which also increases the cost of production.

Another effective method of minimizing cogging torque by shifting slot openings or PMs is proposed for classic surface mounted permanent magnet (SPM) machines [3], [20]-[22]. The slot openings are divided into several groups and each group is shifted by a proper angle so the cogging torque produced by different slot openings can be cancelled each other. As a result, the resultant cogging torque can be mitigated. The most significant advantage of using this method is that the symmetry of the three-phase back-EMF remains the same and the no extra harmonics are introduced to the back-EMF.

However, there is no cogging torque mitigation method has been proposed for modular PM machines. Due to additional air-gaps introduced by modular topologies, the cogging torque could be significantly increased. In [23], the influence of additional air gaps between the stator teeth and stator back iron on cogging torque has been analysed. When the additional stator air-gaps are uniform, only the magnitude of cogging torque is increased but the non-uniform additional stator air-gaps sharply increase both the amplitude and the periodicity of cogging torque. However, although the modular stator affects the performance of the PM machines, the superiorities of using modular topologies cannot be neglected, i.e. using modular stator can significantly ease the manufacture process, especially the stator winding, particularly for large machines such as wind power generators.

In order to simplify the manufacturing while still maintaining or even improving machine performance such as average torque, novel modular topologies have been proposed [24]-[31]. The typical modular structures are shown in Fig. 1. The advantages and disadvantages of applying the modular structures and the influence of flux gaps (FGs) on the performance of modular PM machines with various slot/pole number combinations have been comprehensively investigated. It is found that the flux gaps between the stator segments have negative effects on the electromagnetic performance for the modular PM machines with slot number higher than pole number. In contrast, if the slot number is lower than the pole number, the electromagnetic performance can be improved by choosing proper flux gap width.

Nevertheless, the previous works were focused on studying the influence of various modular topologies on the electromagnetic performances such as cogging torque, phase back-EMF, on load torques, iron losses, etc., no method of reducing the cogging torque has been proposed. As discussed above, cogging torque is a main design parameter which cannot be overlooked and it is also true for the modular PM machines. Therefore, to fill this gap and to further improve the performance of modular PM machine, a novel method of mitigating the cogging torque is proposed in this paper. It is worth mentioning that different from other mitigation methods, the proposed method uses the cogging torque produced by slot openings to compensate that produced by flux gaps so to reduce the resultant cogging torque.

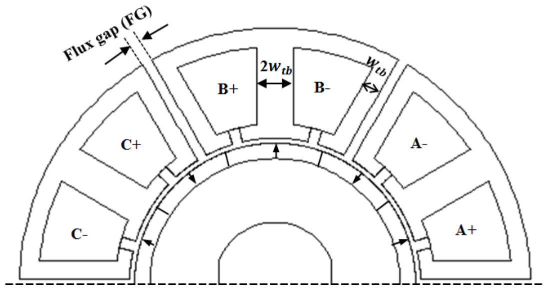


Fig. 1. The cross-section of 12-slot/10-pole modular PM machines [24], [28]-[31].

TABLE I
MAIN PARAMETERS OF MODULAR PM MACHINES

Slot number	12	Stack length (mm)	50
Pole number	10/14	Air-gap length (mm)	1
Stator outer radius (mm)	50	Rotor outer radius (mm)	27.5
Stator inner radius (mm)	28.5	Magnet thickness (mm)	3
Slot opening (mm)	2	Magnet remanence (T)	1.2

II. COGGING TORQUE PREDICTION OF MODULAR PM MACHINES

A. Effects of Flux Gaps on Cogging Torque

The previous cogging torque mitigation methods are feasible because of the assumption that resultant cogging torque can by synthesized from the cogging torque produced by each individual slot opening [19]. So by shifting slots in specific ways, the cogging torque produced by different slot openings can be cancelled out. According to [1], without accounting for magnetic saturations, the cogging torque for classic non-modular PM machines can be expressed by (1), which is the sum of the cogging torques produced by all slot openings.

$$T_{cogg} = \sum_{n=1,2,3\cdots}^{\infty} T_{N_c n} \sin(N_c n \theta) \quad (1)$$

where N_c is the least common multiple of the pole number $(2p)$ and the slot openings number (N_s), and equals to the number of periods of cogging torque over one mechanical revolution. $T_{N_c n}$ is the amplitude of its $N_c n^{th}$ harmonic. θ is the mechanical angle between the stator and rotor.

However, with regard to the modular PM machines, the stator symmetry has been changed since the stator is segmented into several identical sections by the flux gaps. Taking 12-slot/10-pole PM machine as an example, without stator segmentation, the stator periodicity repeats 12 times over the entire circumference (360 Mech. Deg.). Whereas using stator segmentation, the stator is divided into 6 identical segments (see Fig. 1.), hence, the stator periodicity repeats 6 times instead of 12 times. Therefore, the value of N_c becomes 30 instead of 60. For this reason, the cogging torque equation of modular PM machine needs to be rewritten by:

$$T_{cogg-modular} = \sum_{n=1,2,3\cdots}^{\infty} T_{N_{cm} n} \sin(N_{cm} n \theta) \quad (2)$$

where N_{cm} is the least common multiple of the flux gap number (N_{FG}) and the rotor pole number $(2p)$. Similar to non-modular PM machine, the frequency of cogging torque depends on the value of N_{cm}. However, the amplitude is determined by the widths of both the slot openings and flux gaps and also their interactions. According to [1], the larger the N_{cm}, the higher the frequency of cogging torque and the lower the amplitudes of resultant cogging torque. Therefore, the method of choosing an appropriate combination of pole and flux gap numbers is still applicable to mitigate the resultant cogging torque for modular PM machines regardless of flux gap width. By way of example, for 18-slot/16-pole modular PM machine, it has 9 flux gaps, and the value of N_{cm} is 144. As a result, its peak resultant cogging torque is nearly zero, so this kind of modular machines will not be investigated further in this paper.

B. Synthesis of Cogging Torque for Modular PM Machines

For the modular PM machines, since the flux gaps cut off the flux path (main and leakage fluxes) in the stator iron core, the entire flux distribution inside the stator has been modified. This means that, due to flux gaps, the cogging torque produced by one single slot opening will be influenced by the presence of adjacent slot openings and flux gaps. Therefore, different from classic non-modular SPM machines, the resultant cogging torque of modular machine cannot be simply written as a sum of cogging torques due to slot openings and flux gaps. It should be the resultant cogging torque generated by slot openings (C_{SO}), flux gaps (C_{FG}) and the cogging torque due to slot openings accounting for flux gaps influence (C_e), as shown in (3).

$$C_{SO} + C_{FG} + C_e = C_{total} \quad (3)$$

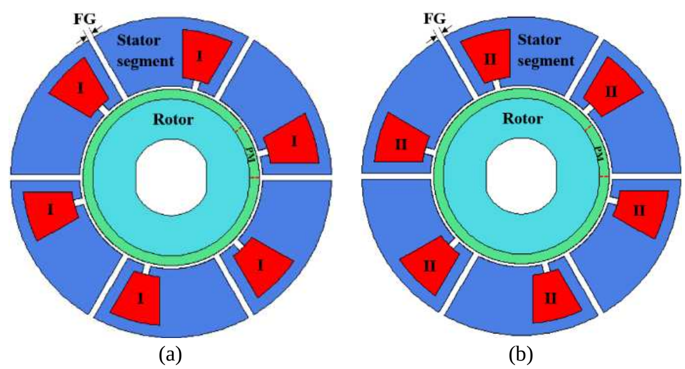


Fig. 2. The two-substructures of stator core for cogging torque synthesis. The rotor is exactly the same for 2 groups. (a) Group I. (b) Group II.

This paper introduces a special way to synthesis the resultant cogging torque waveform of modular PM machines from the cogging torque waveforms generated by both slot openings and flux gaps. To do so, the stator core has been split into two sub-structures, as shown in Fig. 2. Each sub-structure has 6 slot opening and 6 flux gaps (the 6 flux gaps are the same in both sub-structures).

To reassemble groups I and II into one entire stator core so that the resultant cogging torque can be calculated, the equation (3) should be rewritten by

$$(C_{groupI} - C_{FG}) + (C_{groupII} - C_{FG}) + C_{FG} = C_{total} \quad (4)$$

where $C_{groupI} - C_{FG}$ represents the cogging torque of slot openings under the influence of flux gaps in group I, and is named as C_{groupI}' for simplicity. Similarly, for group II, $C_{groupII} - C_{FG} = C_{groupII}'$. Therefore, (4) can be rewritten by:

$$C_{groupI}' + C_{groupII}' + C_{FG} = C_{total} \quad (5)$$

Fig. 3 proves the feasibility and accuracy of using (4) to predict resultant cogging torque waveform of modular PM machines. This has been validated for both 12-slot/10-pole and 14-pole machines.

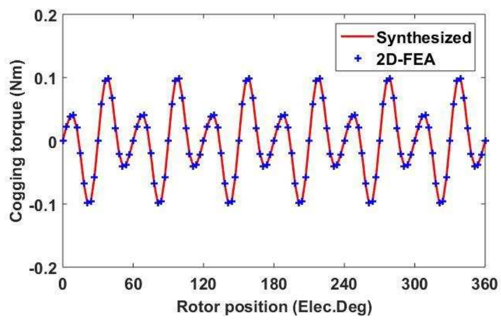


Fig. 3. Comparison of synthesized and direct FE calculated cogging torque waveforms. The flux gap width is FG = 2 mm for both machines. The machine is 12-slot/10-pole.

There is no doubt that the quantification of the influence of flux gaps on cogging torque using exact analytical expression is very difficult. However, applying the equation (4) to predict resultant cogging torque of modular PM machine can take into account the effect of C_e without the need to calculate its exact expression. This arrangement makes the proposed cogging torque mitigation method possible, as will be detailed later.

III. Cogging Torque Mitigation

As mentioned previously, cogging torque of modular machines is produced by both slot openings and flux gaps. Therefore, only compensating the cogging torque due to slot openings will not be sufficient to mitigate the resultant cogging torque. The cogging torque due to flux gaps is often more dominant and needs to be mitigated. To do so, the slot openings of Group I and Group II are shifted in opposite directions with the same shift angle (γ). Whereas, the positions of flux gaps are fixed so to maintain the two tooth body widths on two sides of flux gaps unchanged, as shown in Fig. 4.

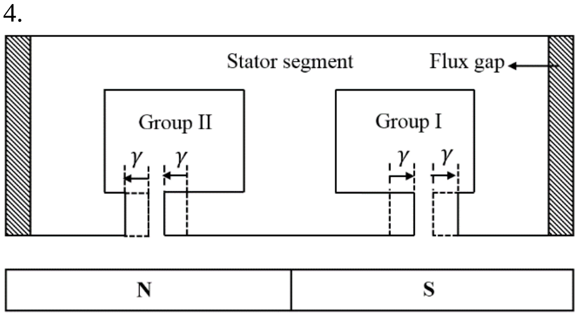


Fig. 4. Stator core with slot openings shifted. Slot opening of Group II shifted left by γ Mech. Deg., while slot opening of Group I shifted right by γ Mech. Deg.

It is worth mentioning that during the shifting process, the slot opening widths are unchanged. By employing this compensation strategy, effective mitigation of the resultant cogging torque can be achieved if an appropriate shift angle is chosen.

A. Theoretical Analysis by FEM

For conventional PM machines, when shifting the slot openings, the cogging torque components can be shifted in phase accordingly without changing the amplitude. This is still applicable for modular PM machines although the flux gaps have influence on cogging torque produced by slot openings, as shown in Fig. 5.

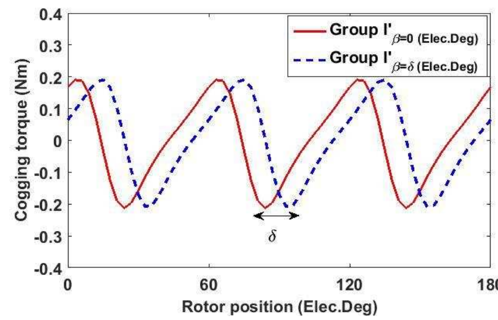


Fig. 5. Relevant cogging torque waveforms before and after shifting slot openings of Group I. β is slot opening shift angle in Elec. Deg. and δ is an electrical shift angle which can be any value.

Based on (1), the cogging torque expressions of C_{groupI}' and $C_{groupII}'$ are given as:

$$C_{groupI}' = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{I\frac{N_{cm}n}{p}}\right) \quad (6)$$

and

$$C_{groupII}' = \sum_{n=1,2,3\cdots}^{\infty} T^{II}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{II\frac{N_{cm}n}{p}}\right) \quad (7)$$

where $T^{I}_{\frac{N_{cm}n}{p}}$ and $T^{II}_{\frac{N_{cm}n}{p}}$ are the respective magnitudes of $\frac{N_{cm}n}{p}^{th}$ order harmonics of C_{groupI}' and $C_{groupII}'$ before the slot opening shift. $T^{I}_{\frac{N_{cm}n}{p}}$ and $T^{II}_{\frac{N_{cm}n}{p}}$ can be obtained through Fourier Series analysis of the FE results of cogging torque. α is the electrical angle between the stator and rotor. $\varphi_{I\frac{N_{cm}n}{p}}$ and $\varphi_{II\frac{N_{cm}n}{p}}$ are the electrical phase angles of C_{groupI}' and $C_{groupII}'$, respectively and can be acquired by Fourier analysis as well.

It is worth mentioning that the slot openings of the two groups have the same dimension. Moreover, the effect of flux gaps on cogging torque produced by slot openings in each group is nearly the same as well due to the symmetrical distribution of flux gaps in the stator. As a result, the values of $T^{I}_{\frac{N_{cm}n}{p}}$ and $T^{II}_{\frac{N_{cm}n}{p}}$ can be regarded as identical. Hence, (7) can be rewritten by:

$$C_{groupII}' = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{II\frac{N_{cm}n}{p}}\right) \quad (8)$$

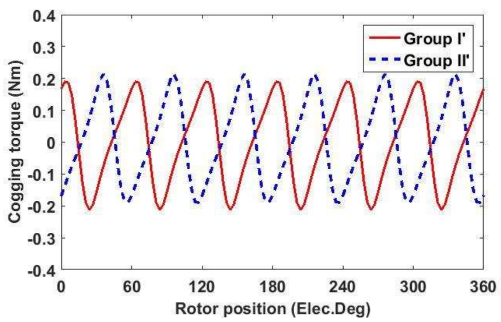


Fig. 6. Cogging torque waveforms of GroupI′ and GroupII′. The flux gap width is FG = 2 mm for both machines. The machine is 12-slot/10-pole.

Fig. 6 depicts the cogging torque waveforms of C_{groupI}' and $C_{groupII}'$ before shifting slot openings for the 12-slot/10-pole while 12-slot/14-pole modular PM machines has exactly the same phenomena. It can be observed that C_{groupI}' and $C_{groupII}'$ has the following relationship:

$$C_{groupI}'\Big|_{\alpha=\alpha_1} = -C_{groupII}'\Big|_{\alpha=\frac{2\pi}{\frac{Ncm}{p}}-\alpha_1} \tag{9}$$

where α_1 is an electrical angle which can be any value depending on the relative position between the stator and rotor of Group I'.

Therefore, based on (9), the relation between C_{groupI}' and $C_{groupII}'$ can be described by (10), which can be further simplified into (11), as detailed in appendix.

$$\sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha_1 + \varphi_{I\frac{N_{cm}n}{p}}\right) = -\sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\left(\frac{2\pi}{\frac{N_{cm}}{p}} - \alpha_1\right) + \varphi_{II\frac{N_{cm}n}{p}}\right) \tag{10}$$

$$\sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha_1 + \varphi_{I\frac{N_{cm}n}{p}}\right) = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha_1 - \varphi_{II\frac{N_{cm}n}{p}}\right) \tag{11}$$

Then, from (11), the relationship between the phases angles of C_{groupI}' and $C_{groupII}'$ ($\varphi_{I\frac{Ncmn}{p}}$ and $\varphi_{II\frac{Ncmn}{p}}$) can be obtained:

$$\varphi_{I\frac{N_{cm}n}{p}} = -\varphi_{II\frac{N_{cm}n}{p}} + 2k\pi, k \in Z \tag{12}$$

as will be verified in the following sections.

Since the focus of this paper is not on analytically predicting the cogging torque of modular PM machine but on minimizing the resultant cogging torque, the values of $\varphi_{I\frac{Ncmn}{p}}$ and $\varphi_{II\frac{Ncmn}{p}}$ are calculated by Fourier analysis directly without giving their exact analytical formula. Then, they will be used for the following analysis related to cogging torque mitigation.

By replacing $\varphi_{II\frac{Ncmn}{p}}$ using $\varphi_{I\frac{Ncmn}{p}}$ such as described by (12), (8) becomes:

$$C_{groupII}' = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin(\frac{N_{cm}n}{p}\alpha - \varphi_{I\frac{N_{cm}n}{p}}) \tag{13}$$

By employing similar expression as for C_{groupI}' and $C_{groupII}'$, the cogging torque due to flux gaps can be written as:

$$C_{FG} = \sum_{n=1,2,3\cdots}^{\infty} T^{III}_{\frac{N_{cm}n}{p}} \times \sin(\frac{N_{cm}n}{p}\alpha + \varphi_{III\frac{N_{cm}n}{p}}) \tag{14}$$

where $T^{III}_{\frac{N_{cm}n}{p}}$ is the amplitude of $\frac{N_{cm}n}{p}^{th}$ order harmonic, $\varphi_{III\frac{N_{cm}n}{p}}$ is the electrical phase angle of C_{FG} which can be acquired by Fourier analysis as well. However, by defining the position of one single flux gap as the reference position, the value of $\varphi_{III\frac{N_{cm}n}{p}}$ can only be equal to 0° or 180° Elec. Deg., and hence the analysis can be simplified.

B. Calculation of Desired Shifting Angle

When the desired shift angle is defined as $\beta_{\frac{N_{cm}n}{p}}$ Elec. Deg., (6) and (13) are modified as:

$$C_{groupI}' = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{I\frac{N_{cm}n}{p}} + \frac{N_{cm}n}{p}\beta_{\frac{N_{cm}n}{p}}\right) \tag{15}$$

and

$$C_{groupII}' = \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha - \varphi_{I\frac{N_{cm}n}{p}} - \frac{N_{cm}n}{p}\beta_{\frac{N_{cm}n}{p}}\right) \tag{16}$$

By replacing C_{groupI}', $C_{groupII}'$ and C_{FG} in (5) using (14)-(16) respectively (see appendix for more details), the resultant cogging torque expression of modular PM machine can be obtained as:

$$C_{total} = \sum_{n=1,2,3\cdots}^{\infty} \sin(\frac{N_{cm}n}{p}\alpha)\left(2T^{I}_{\frac{N_{cm}n}{p}}\cos(\varphi_{I\frac{N_{cm}n}{p}} + \frac{N_{cm}n}{p}\beta_{\frac{N_{cm}n}{p}}) \pm T^{III}_{\frac{N_{cm}n}{p}}\right) \tag{17}$$

When $\varphi_{III\frac{N_{cm}n}{p}}$ equals to 0° Elec. Deg., the value of $T^{III}_{\frac{N_{cm}n}{p}}$ is positive. However, when the value of $\varphi_{III\frac{N_{cm}n}{p}}$ is 180° Elec. Deg., $T^{III}_{\frac{N_{cm}n}{p}}$ will be negative.

Let

$$2T^{I}_{\frac{N_{cm}n}{p}}\cos(\varphi_{I\frac{N_{cm}n}{p}} + \frac{N_{cm}n}{p}\beta_{\frac{N_{cm}n}{p}}) \pm T^{III}_{\frac{N_{cm}n}{p}} = 0, n = 1,2,3,\cdots \tag{18}$$

the resultant cogging torque can be mitigated.

Hence, the desired slot opening shift angle (Elec. Deg.) can be calculated by

$$\beta_i = \frac{acos(\frac{\mp T^{III}_{\frac{N_{cm}n}{p}}}{2T^{I}_{\frac{N_{cm}n}{p}}}) - \varphi_{I\frac{N_{cm}n}{p}}}{\frac{N_{cm}n}{p}} = p \times \gamma_{\frac{N_{cm}n}{p}}.\ n = 1,2,3,\cdots.\ i = \frac{N_{cm}n}{p}. \tag{19}$$

where $\frac{\mp T^{III}_{\frac{N_{cm}n}{p}}}{2T^{I}_{\frac{N_{cm}n}{p}}}$ can be replaced by μ to simplify the analysis.

It has been found out that in some special cases, the value of μ could exceed the range of arccosine function [-1, 1]. In these cases, the maximum or minimum value of the cosine function, i.e. 1 or -1, will be employed to calculate the desired shift angle. It is found when applying the desired shift angle calculated based on μ=1 or -1, the method can still be efficient to mitigate resultant cogging torque but the mitigation effect could be unsatisfactory. Another similar phenomenon may appear during the application of this method. For some cases, the value of desired shift angle is beyond the maximum available shift range of slot openings. In such cases, the solution will be similar, i.e. the boundary value (maximum achievable slot shift angle) will be applied. It is found that the targeted harmonic of cogging torque can also be reduced.

IV. Case Study

To validate the proposed method, two typical modular PM machines with different slot/pole combinations, i.e. 12-slot/10-pole and 12-slot/14-pole, have been adopted as examples. The main parameters of the modular PM machines are listed in Table I.

A. 12-Slot/10-Pole Modular PM machine

1) First slot opening shift

With regard to the 12-slot/10-pole modular PM machine, the least common multiple of pole number $(2p)$ and the flux gap number (N_{FG}) is 30. The main coefficients for calculating the desired shift angle are shown in Table II. Two slot shift angles are also given to remove the relevant harmonics in resultant cogging torque.

TABLE II
DESIRED SHIFT ANGLE AND MAIN COEFFICIENTS (12-SLOT/10-POLE AND FG=2MM)

	n=1	n=2
$T^{I}_{\frac{N_{cm}n}{p}}$ (Nm)	0.17	0.06
$T^{III}_{\frac{N_{cm}n}{p}}$ (Nm)	0.02	0.04
$\varphi_{I\frac{N_{cm}n}{p}}$ (Elec.Deg)	98.29	5.35
$\varphi_{II\frac{N_{cm}n}{p}}$ (Elec.Deg)	-98.30	-5.27
$\varphi_{III\frac{N_{cm}n}{p}}$ (Elec.Deg)	0	180
Desired shift angle (Mech.Deg)	-0.17	1.03
Original peak cogging torque (Nm)	0.1	0.1
Shifted Peak cogging torque (Nm)	0.08(-20%)	0.22(120%)

When applying the desired shift angle (-0.17 Mech. Deg.), the targeted $\frac{N_{cm}}{p}^{th}$ (6th) order harmonic is almost eliminated, as shown in Fig. 7 (b). This can prove the efficiency of the proposed method for eliminating specific harmonics in cogging torque. However, since the $\frac{N_{cm}\times 2}{p}^{th}$ (12th) order harmonic is dominant in the resultant cogging torque, the mitigation of resultant cogging torque by only removing the 6th harmonic is not satisfactory, as shown in Fig. 7 (a). Where γ_6 is the desired mechanical shift angle for reducing the 6th order harmonic cogging torque.

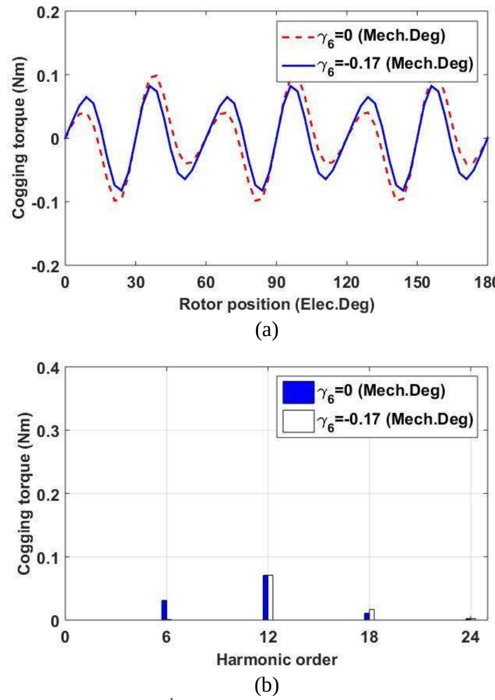


Fig. 7. The reduction of 6th order harmonic of the resultant cogging torque. (a) Comparison of cogging torque waveforms with or without slot shifting. (b) Spectra. The machine has 12-slot/10-pole & FG = 2mm.

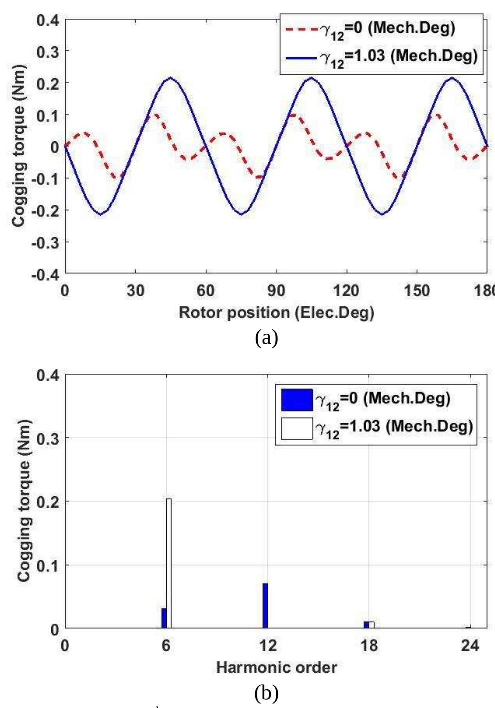


Fig. 8. Reduction of 12th order harmonic of the resultant cogging torque. (a) Comparison of cogging torque waveforms with or without slot shifting. (b) Spectra. The machine has 12-slot/10-pole & FG = 2mm.

Therefore, the question here is why not directly applying the desired shift angle to eliminate the 12th order harmonic? This is mainly due to the fact that when focusing on the elimination of the dominant harmonic, the magnitudes of other harmonics are largely out of control due to the interaction between slot opening and flux gaps. Thus, it is difficult to predict the variation trend of other harmonics after the first slot shifting even if the 12th order harmonic can be completely removed. By way of example, if the desired shift angle of 1.03 Mech. Deg. has been chosen, the 12th order harmonic is completely eliminated, as shown in Fig. 8 (b). Where γ_{12} is the shift angle for reducing 12^{th} order harmonic. However, the amplitude of the 6th order harmonic is increased dramatically. As a result, the peak cogging torque after shifting the slot openings becomes even bigger than that before shifting. In conclusion, the slot shifting method cannot be directly applied to eliminate the 12th order harmonic without eliminating the 6th order harmonic first.

2) Second slot opening shift

In order to reduce the dominant order harmonic (12th), the second shift of slot openings based on the first shift is necessary. However, after the first shift, the symmetry of the modular PM machines has been changed. This means that for the second shift of slot openings, the groups need to be re-arranged, as shown in Fig. 9. The six flux gaps are numbered from FG1 to FG6. Similar grouping method has been proposed in [21]. The re-arranged 2 groups have the same shift angle but shift in the opposite directions as well. Similar to the first shift, the following equation can be derived.

$$(C''_{groupI} - C_{FG135}) + (C''_{groupII} - C_{FG246}) + C_{FG123456} = C''_{total} \tag{20}$$

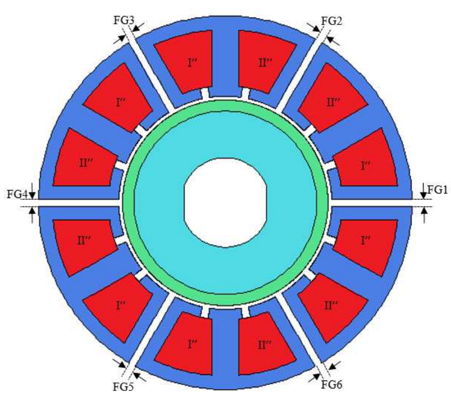


Fig. 9. Machine cross section after the first shift to eliminate the 6th order harmonic in cogging torque.

The waveforms of cogging torque obtained by direct FEM and by synthesis method based on (20) are compared in Fig. 10. An acceptable agreement can be observed with minor discrepancy between the amplitudes (the phases of two cogging torques are exactly the same), which will not compromise the mitigation effectiveness.

The second shift angle is calculated by (19) as well. After shifting the slot openings of two groups in opposite directions with the calculated shift angle (1.03 Mech. Deg.), the synthesized cogging torque waveform of slot openings is almost the mirror image of the cogging torque waveform due to flux gaps, as shown in Fig. 11 (a), where γ_6 and γ_{12} represent the shift angles for reducing the first two harmonics by employing the second shift based on the first shift. As a result, the resultant peak to peak cogging torque is reduced by~82% after the second shift of slot openings. However, it is worth mentioning that unless the higher order harmonics are dominant, the second shift is not necessary due to its complexity, such as the case for 12-slot/14-pole in the following section.

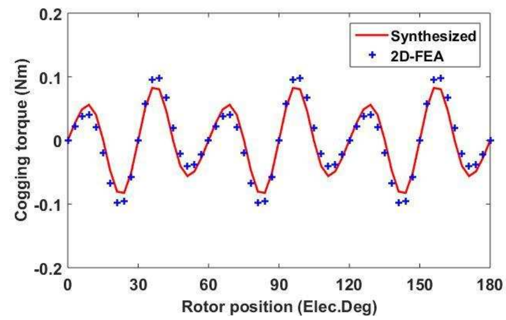
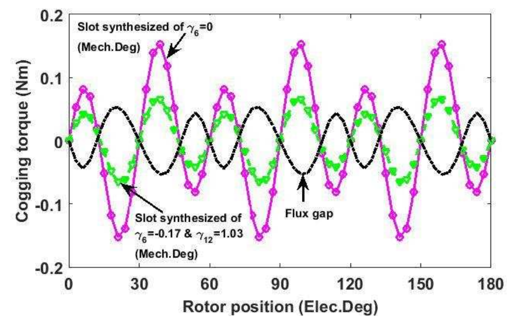
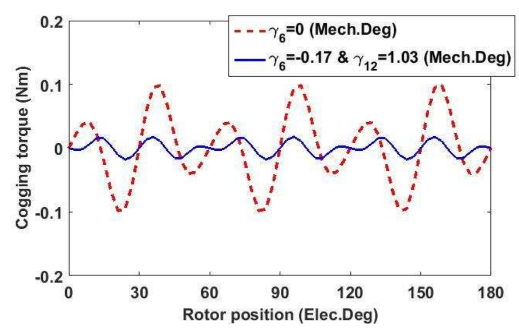


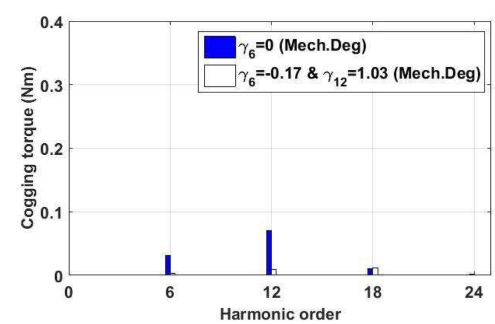
Fig. 10. Comparison of synthesized and direct FE calculated cogging torque waveforms before second shift based on (20). (12-slot/10-pole & FG=2mm).



(a)



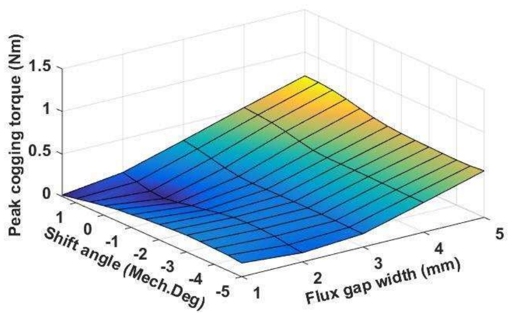
(b)



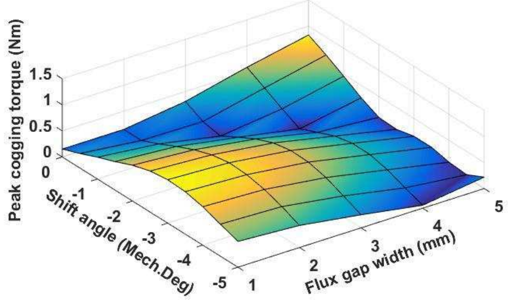
(c)

Fig. 11. Reduction of the first two order harmonics of the resultant cogging torque. (a) Cogging torque waveforms of flux gaps and slot openings before and after shifting. (b) Comparison of resultant cogging torque waveforms. (b) Spectra. (12-slot/10-pole & FG=2mm).

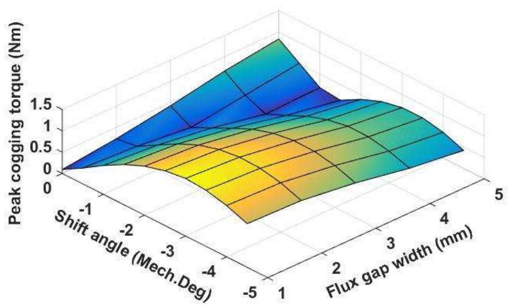
Fig. 12 shows the peak cogging torque versus shift angles and flux gap widths of the 12-slot/10-pole modular PM machine with various slot opening (SO) widths. It can be found that for different flux gap and slot opening widths, although the slot opening shift angles are different, the proposed method can always be used to reduce effectively the resultant cogging torque. Nevertheless, for the case such as SO = 1mm and FG = 5mm, the cogging torque reduction is limited. This is mainly due to the fact the resultant cogging torque of slot opening is much smaller than that due to flux gaps. As a result, they cannot be cancelled each other.



(a)



(b)



(c)

Fig. 12. Peak cogging torque vs flux gap widths and shift angles of 12-slot/10-pole modular PM machine. (a) SO = 1mm, (b) SO = 3mm, (c) SO = 5mm.

The back-EMF waveforms before and after shift are shown in Fig. 13. Here, only the results for phase A are given, for phases B and C, the EMFs of which have the same amplitude but with a phase shift angle of 120 Elec. Deg. It is found that the value of the fundamental decreases slightly, so do the other harmonics. However, this influence on phase back-EMF remains largely negligible, which is different from the stator or rotor skewing method. The tiny decrease on phase back-EMF leads to a slight decrease of average torque from 4.93Nm to 4.89Nm.

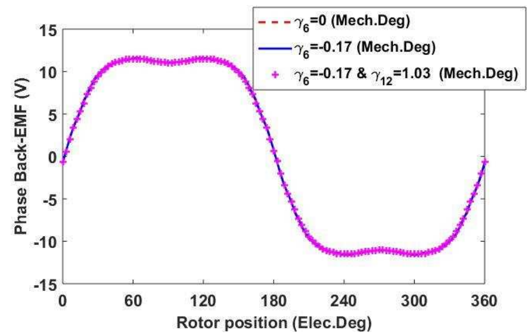


Fig. 13. Phase back-EMFs before and after slot opening shift (12-slot/10-pole & FG=2mm).

B. 12-Slot/14-Pole Modular PM machine

Another typical slot/pole number combination that has been widely investigated in literature is 12-slot/14-pole. For this machine, the least common multiple (N_{cm}) of pole number ($2p$) and the number of flux gaps (N_{FG}) is 42. Using previously mentioned methods, the main parameters of calculating the desired shift angle have been achieved and listed in Table III.

TABLE III
DESIRED SHIFT ANGLE AND MAIN COEFFICIENTS (12-SLOT/14-POLE AND FG=2MM)

	n=1	n=2
$T^{I}_{\frac{N_{cm}n}{p}}$ (Nm)	0.18	0.02
$T^{III}_{\frac{N_{cm}n}{p}}$ (Nm)	0.09	0.02
$\varphi_{I\frac{N_{cm}n}{p}}$ (Elec.Deg)	-94.5	-2.32
$\varphi_{II\frac{N_{cm}n}{p}}$ (Elec.Deg)	94.48	2.41
$\varphi_{III\frac{N_{cm}n}{p}}$ (Elec.Deg)	180	180
Desired shift angle (Mech.Deg)	-4.55	0.98
Original peak cogging torque (Nm)	0.13	0.13
Peak cogging torque with shifting (Nm)	0.02(-85%)	-

For the 12-slot/14-pole modular PM machine with 2 mm flux gaps, the 6th order harmonic is the most dominant one. Therefore, only the first shift is necessary to mitigate its resultant cogging torque. When the desired shift angle for cancelling 6th order harmonic is applied, the amplitude of 6th order harmonic can be significantly decreased, as shown in Fig. 14. Furthermore, the 12th order harmonic is reduced slightly at the same time. Consequently, 85% of the resultant cogging torque reduction has been achieved.

Peak cogging torque versus shift angles and flux gap widths of the 12-slot/14-pole modular PM machine with various slot opening widths have been calculated as well. Again, as for the 12-slot/10-pole modular machines, the slot-opening shift method can be applied to effectively reduce the resultant cogging torque. Other slot/pole number combinations have also been investigated. However, they are not presented to avoid repetition. It can be concluded that the proposed method can be applicable for different modular machines with different slot/pole number combinations, flux gap width, slot opening widths, etc. Although for some specific topologies, the reduction by employing the proposed method may not be as significant as expected. This is mainly due to the difference between the cogging torque produced by flux gaps and that could be potentially achieved by synthesising the cogging of individual slot openings.

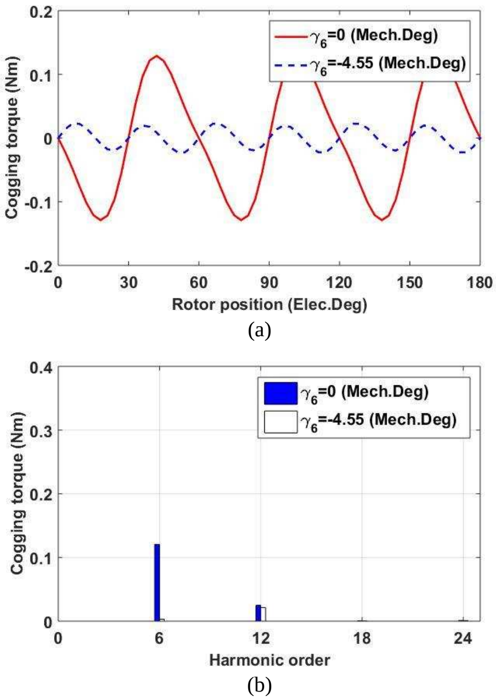


Fig. 14. Reduction of the first two order harmonic of the resultant cogging torque. (a) Comparison of resultant cogging torque waveforms. (b) Spectra. (12-slot/14-pole & FG=2mm).

Slightly different from the 12-slot/10-pole machine, the shift of slot openings weakens the magnitude of the fundamental back-EMF of 12-slot/14-pole machine leading to decrease in the average torque from 5.98 Nm to 5.69 Nm. This decrease in phase back-EMF is mainly due to the reduced winding factor caused by slot opening shift. Thus, in practice, the compromise between reducing cogging torque and phase back-EMF needs to be considered. Moreover, similar to the previous case studied, there are no extra orders of harmonic have been introduced and the balance of 3-phase back-EMF waveforms is also maintained.

Non-linear analysis has been carried out as well for the proposed cogging torque mitigation method, which has been found to be effective although the reduction in targeted cogging torque harmonics is not as good as that in linear cases.

V. Experimental Validation

A. Prototype of Modular PM Machine

In order to verify the slot opening shift method discussed in this paper, the 12-slot/10-pole and 12-slot/14-pole modular prototype PM machines with the flux gaps of 3 mm have been used. The design parameters are shown in Table I and the structures of stator, rotor and the stator segment which are built based on the limited cases analysed in this paper, can be referred to [31]. It is worth mentioning that due to cost issues, the prototype machines with desired slot shift angle have not been built. The used prototype machines were initially built to achieve the highest phase back-EMF by shifting the slot openings, which is similar to the proposed method of mitigating cogging torque in this paper. Therefore, this shift angle in prototype machines will not be optimal for reducing the resultant cogging torque as will be seen in Fig. 15. However, the FE results after shifting slot opening by a specific angle can still be validated by experiments.

B. Cogging Torque and Back-EMF

The cogging torque is measured according to the method described in [32]. The cogging torques of 12-slot/10-pole and 12-slot/14-pole modular PM machines are measured and compared against their predicted counterparts in Fig. 15. A good agreement has been observed with only minor discrepancy due to manufacturing tolerance, measuring errors, etc.

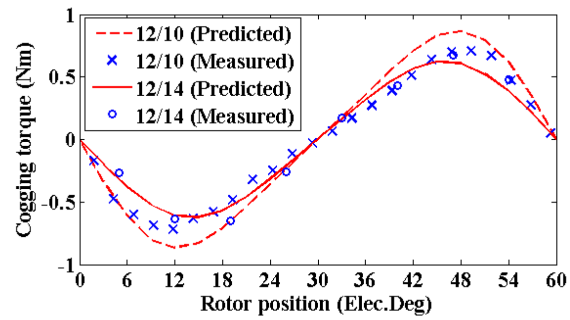


Fig. 15 Predicted and measured cogging torques for 12-slot/10-pole and 12-slot/14-pole modular machines.

In terms of EMF, by way of example, only the predicted and measured results of 12-slot/10-pole modular PM machine are shown in Fig. 16. It is found that after the slot opening shift, the balance of 3-phase back-EMFs is maintained, as expected.

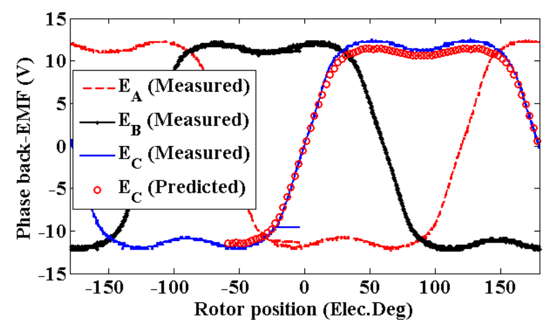


Fig. 16. Predicted and measured phase back-EMF waveforms at 400 rpm.

VI. Conclusion

In this paper, an effective method of mitigating cogging torque for modular PM machines is presented. The slot openings have been grouped into two groups. By shifting the two groups in opposite directions with a desired shift angle, the synthesised cogging torque due to slot openings could be almost opposite to that produced by flux gaps (fixed during slot opening shift), and hence cancel each other. The general expression of the desired shift angle has also been derived.

Several slot/pole number combinations have been investigated and the 12-slot/10-pole and 12-slot/14-pole have been presented to validate the proposed method. Although slightly different mitigation approaches are needed for the investigated machines due to different dominant cogging torque harmonics, a reduction by up to 85% in the resultant cogging torque can be achieved. The performances of back-EMF are studied as well. After employing slot opening shift, the balance of the three phase back-EMF waveforms is maintained and no extra back-EMF harmonics are introduced. The FE results have been validated by experiments.

APPENDIX

Equation (10) can be derived as:

$$\sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha_1 + \varphi_{I\frac{N_{cm}n}{p}}\right) = -\sum_{n=1,2,3\cdots}^{\infty}\left(T^{I}_{\frac{N_{cm}n}{p}}\sin\left(\frac{N_{cm}n}{p}\times\frac{2\pi}{\frac{N_{cm}}{p}}\right)\cos\left(\varphi_{II\frac{N_{cm}n}{p}} - \frac{N_{cm}n}{p}\alpha_1\right) + \cos\left(\frac{N_{cm}n}{p}\times\frac{2\pi}{\frac{N_{cm}}{p}}\right)\sin\left(\varphi_{II\frac{N_{cm}n}{p}} - \frac{N_{cm}n}{p}\alpha_1\right)\right) \quad (21)$$

Equation (21) can be simplified to

$$\sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha_1 + \varphi_{I\frac{N_{cm}n}{p}}\right) = \sum_{n=1,2,3\cdots}^{\infty}\left(T^{I}_{\frac{N_{cm}n}{p}}\cos\left(\frac{N_{cm}n}{p}\times\frac{2\pi}{\frac{N_{cm}}{p}}\right)\sin\left(\frac{N_{cm}n}{p}\alpha_1 - \varphi_{II\frac{N_{cm}n}{p}}\right)\right) \quad (22)$$

(22) can be further developed, and hence (11) is obtained, as shown in section III.

(5) can be rewritten as:

$$C_{total=} \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{I\frac{N_{cm}n}{p}} + \frac{N_{cm}n}{p}\beta\right) + \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} \sin\left(\frac{N_{cm}n}{p}\alpha - \varphi_{I\frac{N_{cm}n}{p}} - \frac{N_{cm}n}{p}\beta\right) + \sum_{n=1,2,3\cdots}^{\infty} T^{III}_{\frac{N_{cm}n}{p}} \times \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{III\frac{N_{cm}n}{p}}\right) \quad (23)$$

Equation (23) can be derived further as:

$$C_{total=} \sum_{n=1,2,3\cdots}^{\infty} T^{I}_{\frac{N_{cm}n}{p}} 2\sin\left(\frac{N_{cm}n}{p}\alpha\right)\cos\left(\varphi_{I\frac{N_{cm}n}{p}} + \frac{N_{cm}n}{p}\beta\right) + \sum_{n=1,2,3\cdots}^{\infty} T^{III}_{\frac{N_{cm}n}{p}} \times \sin\left(\frac{N_{cm}n}{p}\alpha + \varphi_{III\frac{N_{cm}n}{p}}\right) \quad (24)$$

After simplifying (24), (17) is obtained as detailed in section III.

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