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The Merits and Limitations of Local Impact Ionization Theory

Stephen A. Plimmer, John P. R. David, and D. S. Ong

Abstract—Multiplication measurements on GaAs p^+i-n^+ s with i -region thicknesses, w , between $1\ \mu\text{m}$ and $0.025\ \mu\text{m}$ and Monte Carlo (MC) calculations of the avalanche process are used to investigate the applicability of the local ionization theory. The local expressions for multiplication are able to predict the measured values surprisingly well in p^+i-n^+ s with i -region thicknesses, w , as thin as $0.2\ \mu\text{m}$ before the effect of dead-space, where carriers have insufficient energy to ionize, causes significant errors. Moreover, only a very simple correction to the local expressions is needed to predict the multiplication accurately where the field varies rapidly in abrupt one-sided p^+n junctions doped up to $10^{18}\ \text{cm}^{-3}$. However, MC modeling also shows that complex dead-space effects cause the local ionization coefficients to be increasingly unrepresentative of the position dependent values in the device as w is reduced below $1\ \mu\text{m}$. The success of the local model in predicting multiplication is therefore attributed to the dead-space information already being contained within the experimentally determined values of local coefficients. It is suggested that these should therefore be thought of as *effective* coefficients which, despite the presence of dead-space effects, can be still be used with the existing local theory for efficiently quantifying multiplication and breakdown voltages.

Index Terms—Avalanche diodes, avalanche photodiodes, hot carriers, impact ionization, Monte Carlo methods, power semiconductor devices.

I. INTRODUCTION

ACCURATE determination of the electron and hole ionization coefficients, α and β respectively, is important, since these are used to determine avalanche multiplication characteristics and breakdown. Conventionally α and β are assumed to depend only on the local electric field, F , and the mean multiplication due to an electron-hole pair generated at position x_o is given by

$$M(x_o) = \frac{\exp\left[-\int_{x_o}^W (\alpha - \beta) dx\right]}{1 - \int_0^W \alpha \exp\left[-\int_0^x (\alpha - \beta) dx'\right] dx} \quad (1)$$

as described by Stillman and Wolffe [1] where W is the total depletion width. The electric-field exists between $x = 0$ and $x = W$ causing electrons to move from left to right. This expression is also traditionally used to derive the values of α and β from photomultiplication measurements performed with carrier injection from the depletion region edges [2]–[4]. For electron

multiplication, $M_e, x_o = 0$ and for pure hole multiplication, $M_h, x_o = W$.

However, carriers entering the high field region with energy much less than the ionization threshold must traverse a dead space distance, d_e for electrons or d_h for holes, before they acquire sufficient energy to impact ionize. The simplest estimation for this dead space distance is given by equating it to the ballistic distance a carrier requires to reach the ionization threshold energy, E_{th} , i.e., $d = E_{th}/qF$. Several authors have attempted to account for dead-space effects in determining α and β from measured multiplication curves by modifying the local analysis. For a device where electrons are injected at $x = 0$, these corrections usually disallow electron ionization in a dead-space region from $0 < x < d_e$ and hole ionization in the region from $W - d_h < x < W$. Okuto and Crowell [5] presented an approximate expression relating multiplication to the ionization coefficients while accounting for the reduced multiplication by these regions. Bulman *et al.* [6] simplified their expression to interpret the measured multiplication results from p^+n junctions by assuming no electron initiated ionization occurs within a distance d_e from their injection point. Hole dead-space effects in the region from $W - d_h$ to W were ignored since the electric field there was small so its contribution to the multiplication was assumed negligible.

In recent years, several groups have suggested that the effect of dead-space is to reduce the mean value of multiplication below the prediction of a local model [7]–[10]. To account for dead-space regions within the local framework, Di Carlo and Lugli [9] and Wilson [10] indicated that they should be included in the electron and hole current multiplication equations which are then solved numerically. They concluded that the simple modifications to (1) of the type implemented by Bulman *et al.* [6] do not fully correct for dead-space and lead to an overestimation of the high multiplication values and thus to an underestimation of the breakdown voltage. However, the validity of their comparisons is unclear since coefficients which enter the theories of [7]–[10] describe carriers which have already travelled the dead-space and so are generally different to those in the conventional local theory described in [1] or used by [6]. More generally, several investigators have implemented more computer intensive models which account for the dead-space of all carriers such as Monte Carlo [9], [11], lucky-drift [10], [12] or the technique developed by the Wisconsin group [7], [8] in which probability distribution functions (PDF's) of ionization path lengths are used to formulate integral equations. While these predict that the mean gain of thin structures is overestimated by local models, Flitcroft *et al.* [13] have recently showed that the multiplication characteristics and breakdown voltages from the submicron p^+n

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base collector regions of heterojunction bipolar transistors could be reproduced using only the simple correction to the local expression as described in [6].

As the size of devices continues to shrink leading to higher electric fields, impact ionization will become increasingly important in device design. In submicron devices it would be expected that nonlocal aspects of carrier transport will have to be considered. However, practically all published ionization data to date have been in the form of local coefficients. Moreover, the complexity of the alternative methods of analysis make it difficult to interpret multiplication measurements especially where structures are investigated where the field varies rapidly, as argued in [6]. It is therefore important to understand both when nonlocal effects become important and their effect on the multiplication characteristics and breakdown voltage. From this understanding, the limitations on the applicability of the local model can be identified.

In Section II we show that the simplest local expression for multiplication works surprisingly well by comparing it to measurements on a range of GaAs p^+i-n^+ s in which w varies from $1\ \mu\text{m}$ to $0.025\ \mu\text{m}$. In Section III, we use a simple Monte Carlo model to clarify the limitations to the local model by generating the multiplication characteristics of ideal p^+i-n^+ s. Deviations from the local model can thus be attributed to the dead-space effect alone rather than nonuniform fields associated with depletion into the p^+ or n^+ contacts. In Section IV, the model is used to illustrate how the effect of dead-space gives rise to position dependent ionization coefficients in thin structures. Finally, in Section V, we discuss why the simple local model works so well, despite significant dead-space effects, and its consequent merits and limitations.

II. COMPARISON BETWEEN MEASURED MULTIPLICATION AND PREDICTIONS BY THE LOCAL MODEL

We have previously reported the measured photomultiplication characteristics in a range of GaAs p^+i-n^+ and n^+i-p^+ diodes with w as thin as $0.025\ \mu\text{m}$ in [14]. In that work, increasingly significant tunnelling currents were observed when w was reduced $\leq 0.1\ \mu\text{m}$ which restricted the maximum multiplication values which could be measured in the $0.05\ \mu\text{m}$ and $0.025\ \mu\text{m}$ p^+i-n^+ s to ≈ 6 . The laser light was thus chopped, and the resulting signal detected using a lock-in amplifier to ensure that only the multiplied primary photocurrents were measured. Measurements were also taken at varying laser excitation powers, to vary the current density, on several devices from all layers to ensure that any space-charge effects were insignificant. To obtain M_e and M_h , all measured photocurrent characteristics had to be corrected for a linear increase in their magnitude of $\approx 1\%$ with increasing bias from zero applied volts to that corresponding to the onset of multiplication. This was because the collection efficiency increased as the depletion region expanded into the cladding regions. Because this effect was very small however (because of the high cladding doping values $> 2 \times 10^{18}\ \text{cm}^{-3}$ in our p^+i-n^+ structures) we believe that this assignment of the unity gain point will not significantly affect the accuracy of the multiplication. The normalized multiplication was generally found to be indistinguishable from adjacent devices on the same

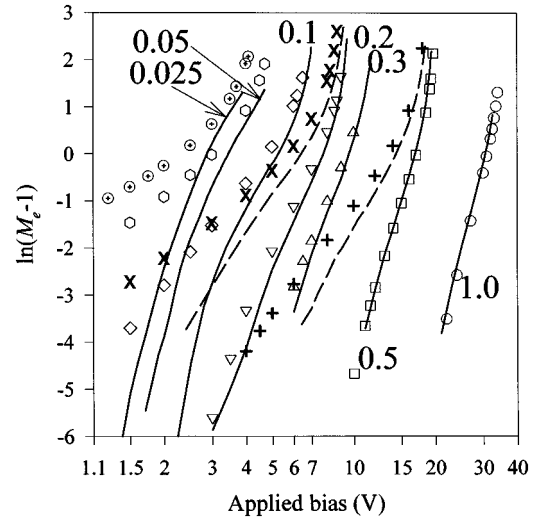


Fig. 1. Electron multiplication plotted as $\ln(M_e - 1)$ calculated using a local model for GaAs p^+i-n^+ s with nominal w of $1\ \mu\text{m}$ (\circ), $0.5\ \mu\text{m}$ (\square), $0.3\ \mu\text{m}$ (\triangle), $0.2\ \mu\text{m}$ (∇), $0.1\ \mu\text{m}$ (\diamond), $0.05\ \mu\text{m}$ (hexagonal), $0.025\ \mu\text{m}$ (dotted hexagon) and from p^+n junctions doped at $5.2 \times 10^{16}\ \text{cm}^{-3}$ ($+$) and $2.2 \times 10^{17}\ \text{cm}^{-3}$ (\times). The measured values are also plotted for p^+i-n^+ s (—) and the p^+n junctions (---). Numbers on the graph indicate the nominal w for the local p^+i-n^+ calculations.

sample for values of $\ln(M - 1)$ down to between -3.5 and -5 (corresponding to M down to 1.03 and 1.007) before experimental noise caused these characteristics to diverge from one another.

In [14], it was shown that the local model predicts measurements of M_e and M_h in GaAs p^+i-n^+ diodes with $w \geq 0.5\ \mu\text{m}$ but increasingly overestimates the multiplication as w is reduced $< 0.1\ \mu\text{m}$, especially at low fields. Since that work, we have grown and fabricated p^+i-n^+ s with $w = 0.3\ \mu\text{m}$ and $0.2\ \mu\text{m}$ and measured M_e from these to determine more precisely when dead-space effects become significant. In Fig. 1, the local model is tested directly against measurements by calculating M_e from (1) with $x_o = 0$. Also plotted on this graph are measured results from Flitcroft *et al.* [13] for the two most heavily doped p^+n junctions investigated in their work at $5.2 \times 10^{16}\ \text{cm}^{-3}$ and $2.2 \times 10^{17}\ \text{cm}^{-3}$ with the local prediction calculated in the same way for these structures. The electric field profiles for all these diodes were accurately determined by modeling the measured capacitance–voltage (C – V) characteristics via solution of Poisson's equation which was validated by secondary ion mass spectroscopy (SIMS) measurements on the p^+i-n^+ s as described in [14]. The values of w , the cladding doping values, p^+ and n^+ , and the unintentional doping in the i -region, p^- for these diodes are listed in Table I. For the local calculation the values of α and β were taken from Bulman *et al.* [6] for electric fields, F , up to $500\ \text{kV/cm}$ since this investigation involved measuring both the multiplication and excess noise characteristics of several samples with overlapping field regions and represents the most extensive and rigorous to date. For $F > 500\ \text{kV/cm}$, the α of Milledge *et al.* [15] was used and it was assumed that $\beta = \alpha$ since our experimental results in [14] showed M_e and M_h become indistinguishable in thin structures. In this work, we continue to use these two sets of parameters for the coefficients since they enable the data to be more accurately quantified over

TABLE I
TRANSPORT PARAMETERS USED IN THE MONTE CARLO MODEL FROM FITTING
MEASURED MULTIPLICATION

Nominal device	w from CV (μm)	$p^+(n^+)$ from SIMS ($\times 10^{18}\text{cm}^{-3}$)	w in MC (μm)	$p^+(n^+)$ in MC ($\times 10^{18}\text{cm}^{-3}$)	\bar{p} of SIMS & MC (cm^{-3})
1.0 μm PIN	1.13	1(3)	1.14	1(3)	10^{15}
0.5 μm PIN	0.57	1(3)	0.56	1(3)	10^{15}
0.3 μm PIN	0.285	4(4) ^(*)	0.285	4(4)	$10^{16(4)}$
0.2 μm PIN	0.205	2(2) ^(*)	0.205	2(2)	$10^{15(2)}$
0.1 μm PIN	0.106	1(3)	0.106	0.85(2.7)	10^{15}
0.1 μm NIP	0.104	4(2)	0.104	3.5(1.5)	5×10^{15}
0.05 μm PIN	0.053	3(4)	0.053	3(4)	10^{17}
0.05 μm NIP	0.055	4(2)	0.055	3.6(1.6)	5×10^{15}
0.025 μm PIN	0.026	3(3)	0.026	3(3.6)	10^{17}
5.2 $\times 10^{22}$ P ⁺ N	-	$10(5.2 \times 10^{16})^{(*)}$	-	$10(5.2 \times 10^{16})^{(*)}$	-
2.2 $\times 10^{23}$ P ⁺ N	-	$10(2.2 \times 10^{17})^{(*)}$	-	$10(2.2 \times 10^{17})^{(*)}$	-

^(*) p (and n) were extracted from the solution of Poisson's equation in these layers.

the wide field range. The coefficients are parameterised in (/meters) as

$$\alpha = 1.89 \times 10^7 \exp \left[- \left(\frac{5.75 \times 10^7}{F} \right)^{1.82} \right] \quad F \leq 5 \times 10^7 \text{ V/m} \quad (2a)$$

$$\beta = 2.21 \times 10^7 \exp \left[- \left(\frac{6.57 \times 10^7}{F} \right)^{1.75} \right] \quad F \leq 5 \times 10^7 \text{ V/m} \quad (2b)$$

$$\alpha = \beta = 2.3 \times 10^8 \exp \left[- \left(\frac{4.08 \times 10^8}{F} \right)^{0.634} \right] \quad F > 5 \times 10^7 \text{ V/m}. \quad (2c)$$

The electric field regions over which multiplication was measured in the diodes shown in Fig. 1 overlap. Therefore, in the thin devices, where dead-space occupies significant but different fractions of the overall depletion region, the use of local ionization coefficients is expected to become invalid to an extent which depends on w . (Equivalently, coefficients deduced from a measured multiplication curve using the local analysis will apply to only the electric field profile in that device.) However, in contrast to the recent publications which suggest that dead-space effects cause local models to overestimate the multiplication, it is surprising to find that the local model works very well in the p^+ - i - n^+ s with w down to and including the 0.2 μm structure. In the devices even thinner than 0.2 μm , the local model overestimates the multiplication at low bias values but the difference decreases with increasing bias so that the local model gives good agreement with the measured breakdown voltage, V_{bd} , for the entire range of device widths. Agreement between the local model and measured V_{bd} for p^+ - i - n^+ s down to $w = 0.025 \mu\text{m}$ thick was reported previously by Milledge *et al.* [15]. M_e calculated using the local model for the less heavily doped p^+ - n junction also shows reasonable agreement with experiment although the differences become more appreciable with increasing doping. The effect of the

field gradient in p^+ - n junctions therefore appears to enhance the effect of the dead-space. The behavior of M_h was found to very similar to that of M_e .

III. VALIDATION OF THE LOCAL MODEL LIMITATION BY MONTE-CARLO MODELLING

To investigate the surprising agreement between the local model and experiment even in submicron structures, we initially need to eliminate the effects of field variations in the p^+ and n^+ depletion layers by studying ideal p^+ - i - n^+ s in which the field is uniform. Since growing ideal p^+ - i - n^+ s is clearly impossible, we chose to model their behavior with a simple Monte-Carlo model which accurately reproduces the measured multiplication characteristics of all the real diodes listed in Table I. Full-band Monte Carlo models could potentially give these calculations more accurately but long computational times restrict their use by for our purposes which requires several tens of thousands of ionization events to be simulated. Our simple Monte Carlo model uses a simple band-structure with two, effective nonparabolic valleys, V1 and V2, of the usual form $\hbar^2 k^2 / 2m^* = E(1 + \gamma E)$ for each carrier type where γ is the nonparabolicity factor. V1 is an "initial The mass and nonparabolicity of V2, which contain the effects of the whole band structure, are found from fitting to the measured multiplication data. Phonon scattering is assumed to be due to events which randomise momentum and has a rate $R_{\text{ph}} = C_{\text{ph}}(N)(E_f(1 + \gamma E_f))^{1/2}(1 + 2\gamma E_f)$ for absorption processes where N is the phonon number, C_{ph} represents the coupling strength, and E_f is the final kinetic energy after a scattering event while for emission, $R_{\text{ph}} = C_{\text{ph}}(N + 1)(E_f(1 + \gamma E_f))^{1/2}(1 + 2\gamma E_f)$. Carriers are injected into V1, accelerate ballistically, undergo nonequivalent intervalley transfer to V2 and remain there undergoing further scattering within V2 only. For scattering from V1 to V2, $E_f = E - \Delta E \pm \hbar\omega$ where E is the initial kinetic energy, $\hbar\omega$ is the phonon energy taken as 29 meV from Ridley [17] and ΔE is the Γ - L separation energy for electrons and zero for holes. For carriers in V2, $E_f = E \pm \hbar\omega$. Impact ionization is included via a rate of the form $R_{\text{ii}} = C_{\text{ii}}((E - E_{\text{th}})/E_{\text{th}})^\gamma$ in V2 where E_{th} is the ionization threshold energy measured from the V2 minima and γ is taken as 4 after Stobbe [18]. E_{th} is set to the average band-gap of 1.75 eV for electron and holes after Allam [19] while C_{ii} and C_{ph} are adjustable parameters obtained from fitting the measured multiplication results. After ionization, the excess energy, given by the difference in the energy of the initiating carriers and the average band-gap, E_{th} , is divided equally between the recoil and two created carriers. All three of these particles are placed in V2, so the properties of V2 are the more important for determining the multiplication. This pragmatic, "fitting" model was found to be the simplest which was able to account for all the necessary features to replicate the experimental results in the range of structures.

M_e is calculated as the ratio of electrons leaving the device at one end to the number injected at the other end and M_h is calculated in the same way for holes. To simulate the multiplication process in ideal p^+ - i - n^+ s, the model's material parameters were obtained by fitting the measured results of real p^+ - i - n^+ s

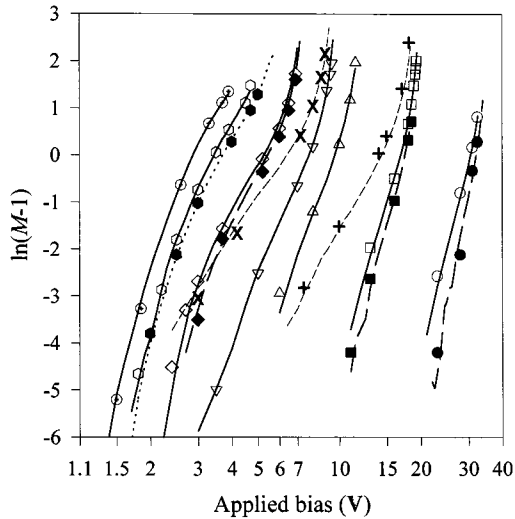


Fig. 2. Monte Carlo calculations of $\ln(M_e - 1)$ (open symbols) and $\ln(M_h - 1)$ (closed symbols) from $p^+ - i - n^+$ with w : $1 \mu\text{m}$ (\circ, \bullet), $0.5 \mu\text{m}$ (\square, \blacksquare), $0.3 \mu\text{m}$ (\triangle), $0.2 \mu\text{m}$ (∇), $0.1 \mu\text{m}$ (\diamond), $0.05 \mu\text{m}$ (open hexagonal), $0.025 \mu\text{m}$ (dotted hexagon); $n^+ - i - p^+$ with $w = 0.1 \mu\text{m}$ (\blacklozenge) and $0.05 \mu\text{m}$ (closed hexagon); $P^+ - N$ junctions doped at $5.2 \times 10^{16} \text{ cm}^{-3}$ ($+$) and $2.2 \times 10^{17} \text{ cm}^{-3}$ (\times). Lines denote experimental results from $p^+ - i - n^+$ and $n^+ - i - p^+$ for electrons (—) and holes (---) and electrons from $p^+ - n$ junctions (---).

by comparing measured and calculated plots of $\ln(M_e - 1)$ and $\ln(M_h - 1)$. Since the low multiplication values are highly sensitive to the dead-space distance and the subsequent dependence of ionization probability on distance, comparing the data in this way best indicates that these nonlocal effects are modeled realistically. Therefore the excellent fits which are achieved to the measured multiplication values, as illustrated in Fig. 2 for the range of devices listed in Table I, evidence that the Monte Carlo accurately accounts for the dead-space effects. The electric fields for the Monte Carlo calculations used the depletion approximation with the values for w and the doping given in Table I along with the values extracted from CV and SIMS. The values used in the MC model differ only very slightly and are well within the experimental errors of their extraction. The comparisons of the calculated and measured M_e for the two most heavily doped P^+N junctions investigated by Flitcroft *et al.* [13] provides further confirmation that the model accurately reproduces the avalanche process in GaAs. The model parameters are given in Table II.

This model is now used to calculate M_e and M_h for a range of ideal $p^+ - i - n^+$ with w from $1 \mu\text{m}$ down to $0.05 \mu\text{m}$. The results are shown in Fig. 3 together with the local model calculations, using (1) with 2(a)–(c), as $\ln(M_e - 1)$ and $\ln(M_h - 1)$. The results are plotted against electric field to emphasise that the electric field regions from different devices overlap significantly. Nevertheless, it appears that the use of local data can reproduce the multiplication surprisingly well for $w \geq 0.2 \mu\text{m}$. Plotting these data against electric field, rather than against voltage, emphasises the underestimation of the local model due to dead-space effects for diodes with $w < 0.5 \mu\text{m}$. Practically, however, the difference between the local result and Monte Carlo results is a very small voltage offset for $w \geq 0.2 \mu\text{m}$. The local model underestimates the multiplication in structures thinner than $0.2 \mu\text{m}$ but it should be emphasised that even the difference in the

TABLE II
VALUES OF i -REGION THICKNESS, w , CLADDING DOPING, $p^+(n^+)$ AND UNINTENTIONAL DOPING IN THE i -REGION, p^- OBTAINED FROM MODELING THE CV AND SIMS MEASUREMENTS. ALSO SHOWN ARE THE VALUES USED IN THE MONTE CARLO (MC) MODEL TO REPLICATE THE MEASURED MULTIPLICATION

	electrons		holes	
	V1	V2	V1	V2
Phonon rate constant, $C_{ph} (\text{s}^{-1} \text{eV}^{-1/2})$	1.07×10^{14}		1.16×10^{14}	
Effective mass ratio, m^*/m_0	0.067	0.4	0.4	0.4
Non-parabolicity, $\gamma (\text{eV}^{-1})$	1.16	0.35	0.7	0.35
V1-V2 separation energy, $\Delta E (\text{eV})$	0.3		0	
Ionisation threshold, $E_{th} (\text{eV})$	1.75		1.75	
Ionisation softness factor, $C_i (\text{s}^{-1})$	4.4×10^{13}		4.0×10^{13}	

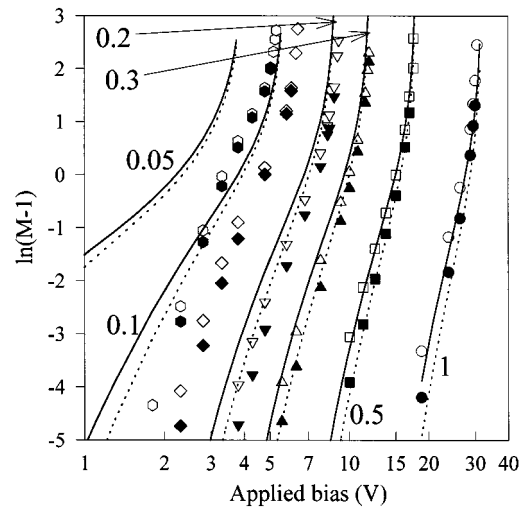


Fig. 3. Monte Carlo calculations of $\ln(M_e - 1)$ (open symbols) and $\ln(M_h - 1)$ (closed symbols) against electric field for ideal $p^+ - i - n^+$ with w : $1 \mu\text{m}$ (\circ, \bullet), $0.5 \mu\text{m}$ (\square, \blacksquare), $0.3 \mu\text{m}$ ($\triangle, \blacktriangle$), $0.2 \mu\text{m}$ ($\nabla, \blacktriangledown$), $0.1 \mu\text{m}$ (\diamond, \blacklozenge), $0.05 \mu\text{m}$ (open hexagonal, closed hexagon). Lines denote the local model prediction for electrons (full line) and holes (dashed line). Numbers indicate w for these calculations in microns.

breakdown field for the $0.1 \mu\text{m}$ structure is $\approx 50 \text{ kV/cm}$ which represents a difference in V_{bd} of only 0.5 V . For the $0.05 \mu\text{m}$ structure, the local and MC prediction of the breakdown field differs by 200 kV/cm which represents a larger relative difference in V_{bd} at 1 V .

Differences therefore occur between the local and Monte Carlo calculations of V_{bd} for ultrathin ideal $p^+ - i - n^+$ but not for the real devices in Fig. 1. This apparent anomaly can be understood by noting that the local ionization coefficients of [15] which are used in these calculations were deduced from the measured V_{bd} of $w = 0.1 \mu\text{m}$ and $0.05 \mu\text{m}$ $p^+ - i - n^+$ with very similar electric field profiles to those we measured and showed in Fig. 1. Consequently, the agreement between the data of Milledge *et al.* [15] and our measurements in Fig. 1 only indicates good agreement between measured V_{bd} values. The differences between the values of V_{bd} calculated by the Monte Carlo and local models in Fig. 3 illustrate that at very high fields α and β are only valid for the electric field profiles from which they are extracted because of dead-space effects. The results in Fig. 3 confirms that the local model does not necessarily give V_{bd} in a thin structure, as implied by Milledge *et al.* [15], and

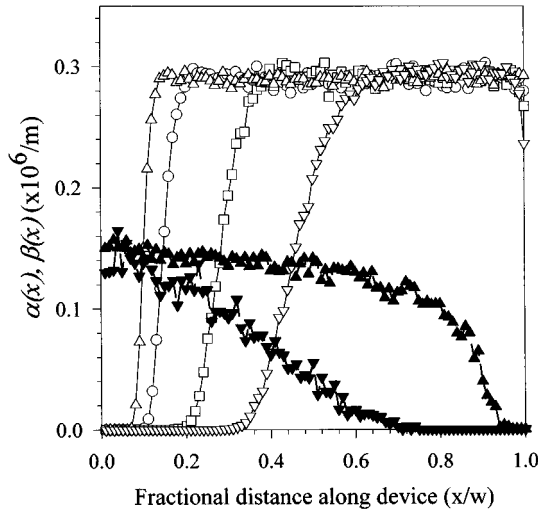


Fig. 4. $\alpha(x)$ (open symbols) and $\beta(x)$ (closed symbols) at 250 kV/cm for $w = 1.5 \mu\text{m}$ (Δ , \blacktriangle), $1 \mu\text{m}$ (\circ), $0.5 \mu\text{m}$ (\square), $0.3 \mu\text{m}$ (∇ , \blacktriangledown).

the limit to its use for calculating both multiplication and V_{bd} is actually when $w = 0.2 \mu\text{m}$ for GaAs p^+i-n^+ s.

IV. NONLOCAL IMPACT IONIZATION COEFFICIENTS

To investigate the effect of dead-space at a microscopic level, α and β were calculated as a function of distance in the ideal p^+i-n^+ s by logging the position of ionization events and calculating the electron and hole currents at each point. The electron coefficient is given by

$$\alpha(x) = \frac{1}{J_e(x)} \frac{dJ_e(x)}{dx} \quad (3)$$

where $J_e(x)$ is the position dependent electron current that results from injected electrons and those created at distances less than x by electron and hole initiated ionization events. $dJ_e(x)$ is the created electron current in a distance dx at x by electron initiated ionization events. An expression that is analogous to (3) gives $\beta(x)$ in terms of $J_h(x)$. The significance of $\alpha(x)$ and $\beta(x)$ is that their values solve the current multiplication equations and return the multiplication when used in (1). The local model assumes that $\alpha(x)$ and $\beta(x)$ are constant in an ideal p^+i-n^+ while simple modifications for dead-space assume that they are constant in all regions except for an electron dead-space region from $0 < x < d_e$ after the p^+i junction, where $\alpha(x) = 0$, and a hole dead-space region before the $i-n^+$ junction from $w - d_h < x < w$ where $\beta(x) = 0$. In this local description, $\alpha(x)$ and $\beta(x)$ depend only on F at all other points and so are assumed to be independent of w . In Fig. 4, the Monte Carlo calculated $\alpha(x)$ values for electron injection at 250 kV/cm are shown for a range of ideal p^+i-n^+ s with $w = 1.5 \mu\text{m}$, $1 \mu\text{m}$, $0.5 \mu\text{m}$ and $0.3 \mu\text{m}$ along with $\beta(x)$ for the $1.5 \mu\text{m}$ and $0.3 \mu\text{m}$ structures. The electron dead-space region, shown to the left of the plot where $\alpha(x) = 0$, is a significant fraction of device width at $\approx 10\%$ at this field even for the $w = 1.5 \mu\text{m}$ structure while it obviously becomes even more significant as w is reduced such that it occupies nearly 50% of the structure at $w = 0.3 \mu\text{m}$. The results for $\beta(x)$ are also shown for the thinnest and thickest structures for electron injection and the hole dead-space region

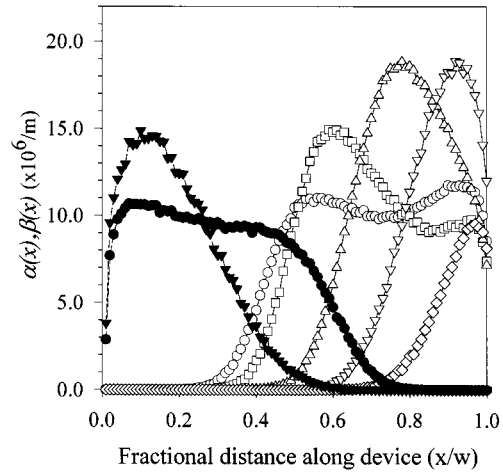


Fig. 5. $\alpha(x)$ (open symbols) and $\beta(x)$ (closed symbols) at 700 kV/cm for $w = 0.115 \mu\text{m}$ (\circ , \bullet), $0.1 \mu\text{m}$ (\square), $0.075 \mu\text{m}$ (Δ), $0.06 \mu\text{m}$ (∇ , \blacktriangledown), $0.05 \mu\text{m}$ (\diamond).

is seen to the right of the plot. However, $\beta(x)$ is not constant but increases gradually as the distance x goes from $w - d_h$ to 0 (right to left) especially for the thinnest p^+i-n^+ . This gradual increase from right to left is attributed to the dead-space of both carrier types; dead-space increases the probability that an electron leaves the structure before creating a hole while also increasing the chance that any created holes will leave the structure before initiating further ionization events. Therefore, both the hole current $J_h(x)$ and the amount by which this is changed at each point $dJ_h(x)/dx$ are decreased below the local prediction causing an overall decrease in $\beta(x)$ compared to its saturated value. The lowering of $\beta(x)$ becomes more significant as the ratio d_h/w increases in thin structures. It is therefore surprising that the local model, which assumes position independent ionization coefficients, predicts the multiplication as accurately as it does for the p^+i-n^+ s shown in Fig. 3.

When the field is increased to 700 kV/cm, the behavior of $\alpha(x)$ and $\beta(x)$ becomes much more complex as shown in Fig. 5 for a range of p^+i-n^+ s with w from $0.115 \mu\text{m}$ to $0.05 \mu\text{m}$. Since M_e is already 23 at $w = 0.115 \mu\text{m}$, thicker structures cannot be investigated at this field since the device would breakdown. In the thinnest $0.05 \mu\text{m}$ structure where the multiplication is lowest at $M_e = 1.1$, the coefficients have not reached their saturation value. For $w = 0.06 \mu\text{m}$ and $0.075 \mu\text{m}$, $\alpha(x)$ peaks but this peak is reduced when $w = 0.1 \mu\text{m}$ while $\alpha(x)$ is almost constant at $w = 0.115 \mu\text{m}$. The $\alpha(x)$ peak occurs in the $0.06 \mu\text{m}$ and $0.075 \mu\text{m}$ structures at low M_e because the ionization events are initiated by mainly primary carriers injected from the left: Primary carriers starting with negligible kinetic energy are accelerated rapidly in the high electric field while undergoing fewer phonon collisions than at lower fields. They are consequently less spread in energy when they ionize at high fields and thus ionize within a narrower region. Therefore, when ionization occurs by mainly these primary carriers, $\alpha(x)$ reflects the primary carrier behavior by showing a dead-space region, a high peak followed a drop as these carriers are returned to low energies after initiating ionization events. For the $w = 0.06 \mu\text{m}$ and $0.075 \mu\text{m}$ devices, the majority of events are initiated by these primary electrons to the right of the

device. Since the dead-space is a significant fraction of w , these carriers most probably exit the high field region without initiating further events while the hole dead-space distance, which is comparable to the electron dead-space, causes most of these created holes to also leave the device. As w is increased, secondary hole ionization increases at the left of the structure, since the dead-space is a smaller fraction of w , while the created holes go on to create more electrons until breakdown occurs at some value of w . Due to the stochastic nature of impact ionization, the positions where the carriers are created become less deterministic and $\alpha(x)$ and $\beta(x)$ are smoothed. The form of the nonlocal behavior shown in this figure becomes increasingly significant as the field increases and the dead-space distance, d , becomes an increasing fraction of the inverse ionization coefficient, $1/\alpha$. The ratio $r = d/(1/\alpha)$ was described by Okuto and Crowell [5] to determine how well the local model described the ionization behavior. When r is large the local coefficients no longer represent the ionization probability of a carrier at any point. While Figs. 4 and 5, for which $r = 0.02$ at 250 kV/cm and 0.45 at 700 kV/cm respectively, suggest that this increasing ratio is indeed causing the local model to break down on a microscopic level, Fig. 1 shows that it does not prevent the local model predicting the multiplication characteristics of a $p^+ - i - n^+$ for w down to 0.2 μm .

$\beta(x)$ from the 0.115 μm and the 0.06 μm ideal $p^+ - i - n^+$ are also plotted on Fig. 5 where the behavior reflects that of the electrons. It can be seen from the 0.115 μm structure that electron and hole ionization behavior appear very similar in thin structures. It is also noted that each carrier type ionizes in only about half of the structure even in the 0.115 μm $p^+ - i - n^+$ close to breakdown and so the conventionally assumed breakdown condition from a local model of $\alpha w = 1$, when $\alpha = \beta$ no longer holds.

Simulations were also carried out for hole injection where, at 250 kV/cm, $\beta(x)$ is zero but quickly assumes a saturated, constant value for all other points in the structure, similar to $\alpha(x)$ in Fig. 3. The $\alpha(x)$ profile is zero for a region and gradually increases with increasing distance so the electron and hole behavior is reversed from Fig. 4. At 700 kV/cm, $\alpha(x)$ and $\beta(x)$ for hole injection shows similar behavior as for electron injection except that the electron and hole behavior is again reversed. These results confirm that the behavior of $\alpha(x)$ and $\beta(x)$ is due to nonlocal aspects rather than the transport properties of either carrier type.

V. DISCUSSION

The consequence of dead-space effects at high fields is that ionization coefficients deduced from the conventional current multiplication equations, by assuming α and β depend solely on F , also become dependent on w . No matter how the dead-space regions are accounted for, the extracted local ionization coefficient will only represent some average value for the region $d_e < x < W$ for electrons and $0 < x < w - d_h$ for holes. The peaked behavior in Fig. 5 means that this average will vary with the device thickness, w , of these ideal $p^+ - i - n^+$ s and more generally with the device geometry of any diode. Consequently, no simple expression exists which can relate M_e and M_h to α

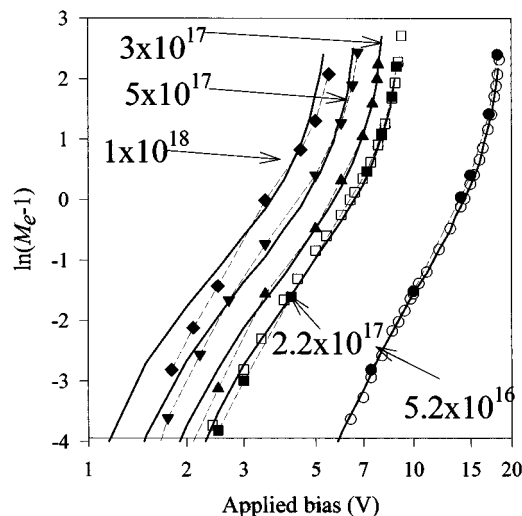


Fig. 6. Measured (open symbols) and Monte Carlo calculations (closed symbols) of $\ln(M_e - 1)$ in $p - n^+$ junctions doped at $5.2 \times 10^{16} \text{ cm}^{-3}$ (\circ, \bullet), $2.2 \times 10^{17} \text{ cm}^{-3}$ (\square, \blacksquare), $3 \times 10^{17} \text{ cm}^{-3}$ ($\blacktriangle, \blacktriangledown$), $1 \times 10^{18} \text{ cm}^{-3}$ (\blacklozenge). Local predictions using the correction of [6] are given by (—) with the doping value indicated in cm^{-3} on the plot for each diode.

and β at high F where the ratio of dead-space distance to the inverse ionization coefficient, r , becomes large.

Given that the nonlocal behavior is significant in even $w = 1.5 \mu\text{m}$ $p^+ - i - n^+$ s, it is surprising that the local model can reproduce M_e and M_h reasonably well in devices with w down to 0.2 μm as shown in Figs. 1 and 3. This is due to the way in which α and β (2(a)–(c)) were deduced from experiment and the electric field ranges that these devices use. The work of Bulman *et al.* [6] involved measuring the multiplication in several $p^+ - n$ and $p^+ - n - n^+$ junctions covering different overlapping electric field regions which were then used to obtain the ionization coefficients via a local model. A simple correction for the primary dead-space affected only the low electron multiplication values while it had very little effect at high values and allowed coefficients to be extracted which subsume dead-space effects and are practically device independent. Obviously, using these parameters in a local model and reversing the dead-space correction of Bulman *et al.* [6] will always give good agreement with multiplication for $p^+ - n$ junctions as shown by Flitcroft *et al.* [13]. In fact, this analysis can be used to give the multiplication reasonably well in even heavier doped structures than used by Flitcroft *et al.* as shown in Fig. 6 where M_e is obtained from the Monte Carlo and plotted against the corrected local model of [13]. The calculation of the corrected local model for $M_e > 1.2$ is always in good agreement with the Monte Carlo result for structures which are doped up to 10^{18} cm^{-3} .

The reason that the same coefficients work for various thicknesses of $p^+ - i - n^+$ with $w \geq 0.2 \mu\text{m}$ is because the fields used by for $p^+ - i - n^+$ s are sufficiently low so that the ratio of $r = d/(1/\alpha)$ remains relatively small, reaching a maximum of 0.33 at the breakdown field of the 0.2 μm $p^+ - i - n^+$. Consequently, the effective values α and β which already contain dead-space information, do not become significantly device dependent so as to greatly affect calculations of M_e and M_h . To illustrate this point, plotted in Fig. 7 are the α 's obtained using M_e and M_h from the ideal $p^+ - i - n^+$ s shown in Fig. 3. The α 's from the $w = 1 \mu\text{m}$

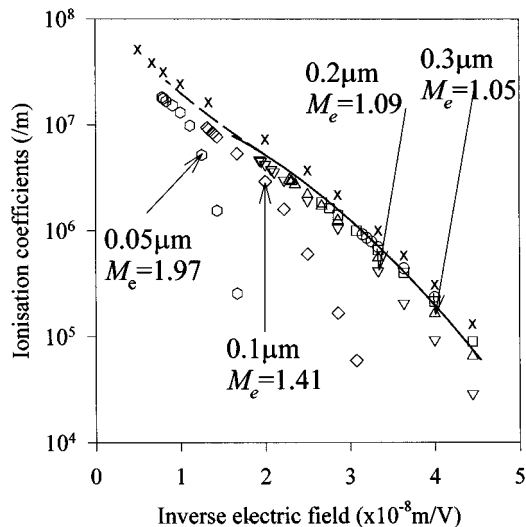


Fig. 7. Electron ionization coefficients deduced using the purely local model from the Monte Carlo calculations for ideal p^+i-n^+ s with $w = 1 \mu\text{m}$ (\circ), $0.5 \mu\text{m}$ (\square), $0.3 \mu\text{m}$ (\triangle), $0.2 \mu\text{m}$ (∇), $0.1 \mu\text{m}$ (\diamond) and $0.05 \mu\text{m}$ (open hexagons). The value of M_e below which these data show dependence on w (denoted in microns on the plot) is labeled. Also plotted are the mean distance between ionization events in a uniform field, $\alpha_{MC}(x)$ and the local electron data of [6] (—) and [14] (---) for comparison.

and $0.5 \mu\text{m}$ p^+i-n^+ s are independent of w and agree closely with the data of (2). Even for the $w = 0.3 \mu\text{m}$ and $w = 0.2 \mu\text{m}$ structures, the device dependence becomes apparent only at very low values of M_e as marked on Fig. 7. This device independence allows continuous, parameterised expressions to quantify both α and β which returns the multiplication via (1). These experimentally determined ionization coefficients are not the same however as the “true” ionization coefficients, which are the reciprocal of the mean distance between ionization events which a carrier initiates in a uniform electric field, and are the values experimentalists attempt to extract; a point made previously by Spinelli [20]. Our Monte Carlo model allows the true values to be obtained by simulating a single carrier in an infinite region of electric field until it has initiated events many times to give a converged value for the reciprocal of the mean distance between these ionization events, α_{MC} . On Fig. 7, α_{MC} is denoted as (x) to show that the α from experiment is always significantly lower because of dead-space effects. The effect of dead-space can be seen in Fig. 4 at 250 kV/cm, where the local model assumes a constant position independent α of $2 \times 10^5 \text{ m}^{-1}$ and a β of $9.8 \times 10^4 \text{ m}^{-1}$ (from (2)) which are clearly different to the values of $\alpha(x)$ and $\beta(x)$ shown. At low fields where $\alpha(x)$ is reasonably constant beyond the electron dead-space region $0 < x < d_e$, α has to effectively quantify the average of the $\alpha(x)$ curve to give the multiplication. Since $d_e \ll 1/\alpha_{MC}$, $\alpha(x) \approx \alpha_{MC}$. This inclusion of dead-space information in α 's and β 's evidently allows the local model to be used to quantify multiplication characteristics accurately for a wide range of diodes, even when dead-space effects are significant. It is also noted that their effectiveness in calculating multiplication is aided by the strong dependence of the multiplication on the highest fields in a structure; the device geometry dependence of $\alpha(x)$ and $\beta(x)$ at other points does not significantly affect the calculation of M_e and M_h . It is finally noted

regarding Fig. 7 that it is at fields above the $0.2 \mu\text{m}$ p^+i-n^+ breakdown field of 500 kV/cm, where $r = 0.33$, that the slope of both true and local ionization coefficients apparently increases. This gradient change is somewhat misleading however because the electric field range is condensed on the x -axis by plotting its inverse; when the same data is plotted against α versus the field, the coefficients appear to saturate at very high fields. This behavior of the ionization coefficient corresponds closely to that calculated by McKenzie and Burt [21] who predicted different gradients in the ionization coefficient at low fields, where the ionising carriers are those which lucky-drift from the average energy, and at high fields, where the ionising carriers do not undergo significant energy relaxation before ionising. These two regimes are reflected by the requirement for separate parameterised expressions to quantify the local coefficients in (2).

The suggestion by several authors that the dead-space would significantly suppress the multiplication and increase V_{bd} beyond the local prediction is therefore misleading. Their techniques are all valid but require different ionization coefficients to those given by [6], [14] which should be determined using consistent analysis. On the other hand, the coefficients α and β deduced by experimentalists already contain a significant amount of dead-space information which allow them to predict M_e and M_h even when the avalanche region is thin.

The correction of Bulman *et al.* [6] or Flitcroft *et al.* [13] is required in the p^+n junction because the dead-space region occupies the region where the electric field is highest in the structure. Since ionization coefficients have an exponential dependence on the electric field, the contribution to the multiplication that a purely local model assumes in the electron dead-space region of a p^+n junction represents a greater fraction of the overall multiplication compared to that in a p^+i-n^+ . For p^+i-n^+ s, the dead-space information which is included in the coefficients themselves is sufficient to allow the multiplication curves to be reproduced for $w \geq 0.2 \mu\text{m}$. In contrast, the electron multiplication of P^+N s is more sensitive and leads to device dependent ionization coefficients at the low values of measurable multiplication. It is noted from Figs. 3 and 4 that the dead-space is equivalent to that required to accelerate ballistically to about 3.2 eV which is almost double the dead-space distance assumed by both Bulman *et al.* and Flitcroft *et al.* Therefore, for p^+n 's the dead-space information is partially contained within the coefficients and partially in the assumed threshold energy of 1.7 eV. The longer dead-space distance in Figs. 3 and 4 is not surprising given that theoretical work in recent years has showed that carriers ionize at energies significantly greater than E_{th} [22], [23].

While the main advantage of the local model is its simplicity and the surprising accuracy with which it allows multiplication to be quantified for a wide range of diodes, its main drawback is that it does not provide an accurate description of the spatial distribution of ionization events as shown in Figs. 4 and 5. To model $J_e(x)$ and $J_h(x)$ in this regime, the problem can be solved numerically [7], [8] or using a random number generator [24] by the class models that account for the ionization PDF's of all carrier's. Even these models should be used cautiously because they use spatial ionization probabilities that are assumed to depend only on the local field and therefore neglect correlation effects; it has been shown by Scrobhaci and Tang [25]

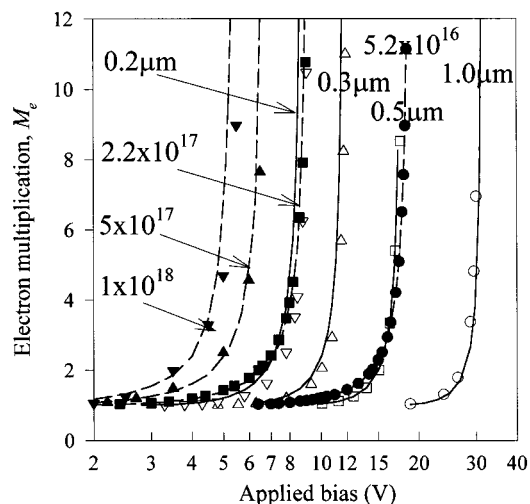


Fig. 8. Range of M_e data for which the local model works well for p^+i-n^+ s and the modified expression works well for p^+n junctions. The Monte Carlo model results for ideal p^+i-n^+ s (open symbols) are $w = 1 \mu\text{m}$ (\circ), $0.5 \mu\text{m}$ (\square), $0.3 \mu\text{m}$ (\triangle) and $0.2 \mu\text{m}$ (∇) and for P^+N junctions (closed symbols), $n = 5.2 \times 10^{16} \text{ cm}^{-3}$ (\bullet), $2.2 \times 10^{17} \text{ cm}^{-3}$ (\blacksquare), $5 \times 10^{17} \text{ cm}^{-3}$ (\blacktriangle) and $1 \times 10^{18} \text{ cm}^{-3}$ (\blacktriangledown). Lines denote the local prediction for p^+i-n^+ s (—) and the local prediction with the correction from [6] for p^+n junctions (---) with the w and n values indicated in microns and cm^{-3} , respectively, for these calculations.

that the ionization probability at two points in the same structure can be different even though the fields are the same. We also stress that the argument for maintaining the local model as a pragmatic calculation tool does not hold for excess noise calculations where the local model is unable to predict the measured behavior. Hayat *et al.* [7] and more recently Ong *et al.* [26] showed that the excess noise figures in thin structures were reduced below the local values when high fields cause determinism in the avalanche process, rather than low effective α/β ratios. In our view, attempts to correlate the excess noise with effective α/β ratios, such as those in [10], [27], are consequently misleading. The local ionization model will therefore be adequate for most calculations of the current gain by avalanche multiplication, such as those required to calculate the breakdown voltage in transistors and even in single photon avalanche detectors, where the time at which a given current is reached is required, as described by Spinelli [28]. It is not, however, adequate when the spatial distribution of ionization events is important in calculations of the excess noise in APD's with depletion thicknesses $\leq 1 \mu\text{m}$.

The use of experimentally determined local ionization coefficients in a local model for multiplication calculations is far more likely to be limited by uncertainties in the electric field profile than dead-space effects. In Fig. 8, we emphasise the ability of the local theory to quantify multiplication by plotting its prediction of M_e with the characteristics from the Monte Carlo previously shown on Figs. 3 and 6 for the diodes where the local theory works well. The biggest error shown by the local theory on this plot is for the $0.2 \mu\text{m}$ p^+i-n^+ but this is only revealed by the exact knowledge of the electric field profile in this numerical experiment. In a real device of similar dimensions, it would be doubtful that this small error due to dead-space could be separated from uncertainties in the electric field. Moreover,

the ability of the local model to reproduce the results of a more complex model in Fig. 8 is at least as good as that of an approximate model which explicitly accounts for the dead-space of all carriers [20]. We therefore suggest that local coefficients continue to be deduced from measured multiplication but on data from a range of device geometries with overlapping field regions and that the local analysis in p^+i-n^+ s or the simple correction for p^+n junctions can be used to quantify those data. The subsequent coefficients should, however, be considered *effective* coefficients rather than *local*. This analysis would enable the measured multiplication curves to be accurately, quickly and simply reproduced for device design purposes or for validating microscopic models. We note that in GaInP, the ratio $r = d/(1/\alpha)$ is lower than in GaAs enabling these local or *effective* coefficients to quantify the avalanche process accurately, including the V_{bd} of p^+i-n^+ s with w down to $0.1 \mu\text{m}$ (Ghin *et al.* [29]). Both the success of the local model, as demonstrated in Fig. 8, and these GaInP results suggest that it will now be desirable to obtain the limits of local ionization theory model in terms of general α, β and d values. Invoking complex dead-space models to quantify either the GaAs results in Fig. 8 or the GaInP results would ultimately only make the task of interpreting and using them more difficult.

CONCLUSION

The local model is shown to reproduce the multiplication surprisingly accurately in p^+i-n^+ structures as thin as $0.2 \mu\text{m}$ and in p^+n junction doped to 10^{18} cm^{-3} when a simple correction is implemented. This is in spite of significant dead space regions which can comprise more than 50% of the total device width. The success of the local model is attributed to the fact that dead-space information is already contained within the *local* or experimentally derived ionization coefficients which should consequently be thought of as *effective* coefficients. A comparison of these *local* or *effective* ionization coefficients extracted from multiplication characteristics generated by a Monte Carlo model and the reciprocal of the mean distance between ionization events obtained by the same model show that the local values are always lower. At high fields in thin structures, the dead-space narrows the regions over which ionization can occur and causes highly nonlocal ionization behavior across the device. No simple analytical expression can accurately replicate the multiplication characteristics in such thin structures. However, inaccuracies in determining the electric field profile rather than dead-space effects are more likely to limit the accuracy in determining M_e, M_h and breakdown voltage for most practical purposes.

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