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# Unspanned macroeconomic factors in the yield curve
## Online Appendix

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## D    Unrestricted vs block-diagonal macro-yields model

In this Appendix we report results for two alternative macro-yields models that use different assumptions about the matrix of factor loadings $\Gamma$ in Equation 4 in the main text. The first model that we consider is an unrestricted macro-yields model, which does not impose the zero restrictions on the factor loadings of yields on macro factors, i.e. $\Gamma_{yx}$ is allowed to be different from zero. This model allows the macro factors $F_t^x$ to directly affect the cross-section of yields. The second model imposes block-diagonality of the matrix of factor loadings $\Gamma$, i.e. $\Gamma_{yx} = 0$ and $\Gamma_{xy} = 0$. This implies that the yield curve factors and the macro factors are treated separately, as in Mönch (2012). Notice that when estimating the block-diagonal macro-yields model, we omit the federal funds rate from the macro variables used to estimate the model, as in Mönch (2012). The reason is that we cannot reliably impose a zero restriction on the factor loadings of the federal funds rate on the yield curve factors.

Results in Tables 1–4, show that the in-sample and out-of-sample performances of the unrestricted macro-yields model are equal to the ones of the macro-yields model with unspanned macro

factors in Tables 1, 2, and 5 in the main text. This provides evidence that the zero restrictions on the factor loadings are satisfied by the data, implying that the macro factors $F_t^x$ are unspanned by the cross-section of yields.

On the contrary, imposing a block-diagonal structure on the matrix of factor loadings has substantial implications for the the identification of the factors. Table 1 shows that the information criterion selects the model with six factors, i.e. three yield curve factors and three macro factors. This indicates that this model is less parsimonious than our macro-yields model. In addition, as shown in Table 2, the macro factors in the block-diagonal macro-yield model are highly correlated with the yield curve factors and are, thus, spanned by the cross-section of yields. This happens because the macro variables have a factor in common with the yields which the block-diagonal model treats as an additional macro factor. Table 2 also shows that the correlation of the first macro factor of the block-diagonal macro-yields model with the industrial production index is weaker than in our model. More importantly, the second and third macro factors do not have a clear-cut interpretation.

The fit of the block-diagonal macro-yields model is comparable to the fit of our macro-yields, as shown in Table 3. However, in spite of the fact that our model has less factors than the block-diagonal model, the forecasting performances of the macro-yields model are not worse than the ones obtained from the block-diagonal macro-yields model, on the contrary there is some evidence of improved accuracy. This suggests that in our model the eventual loss of information is more than compensated by the gains in the reduction of estimation uncertainty implied by the more parsimonious structure. In summary, this confirms that the extra macro-factor in the block diagonal model is capturing mainly information that is already contained in the yield curve.

Table 1: Model selection

| | Unrestricted | | Block-diagonal | |
|---|---|---|---|---|
| Number of factors | IC* | V | IC* | V |
| 3 | 0.02 | 0.44 | 1.17 | 1.36 |
| 4 | -0.02 | 0.31 | 1.30 | 1.16 |
| 5 | -0.11 | 0.22 | 1.50 | 1.05 |
| 6 | 0.01 | 0.18 | 0.89 | 0.43 |
| 7 | 0.25 | 0.17 | 1.06 | 0.38 |
| 8 | 0.43 | 0.16 | 1.42 | 0.41 |

This table reports the information criterion IC*, as shown in equation (7) and in equation (8), and the sum of the variance of the idiosyncratic components (divided by NT), V , when different numbers of factors are estimated. The first two columns refer to the unrestricted macro-yields model. The last two columns refer to the block-diagonal macro-yields model.

Table 2: Factor identification

| | | | Unrestricted | | |
|---|---|---|---|---|---|
| | L | S | C | M1 | M2 |
| NS L | 0.98 | 0.00 | 0.32 | -0.08 | 0.01 |
| NS S | 0.03 | 0.97 | 0.28 | 0.11 | 0.08 |
| NS C | 0.25 | 0.37 | 0.86 | 0.12 | 0.06 |
| IP | -0.08 | 0.03 | 0.16 | 0.91 | -0.09 |
| R | 0.47 | 0.05 | 0.10 | 0.20 | -0.76 |

| | | | Block-diagonal | | | |
|---|---|---|---|---|---|---|
| | L | S | C | M1 | M2 | M3 |
| NS L | 0.99 | -0.03 | 0.21 | -0.07 | 0.46 | 0.57 |
| NS S | -0.03 | 0.99 | 0.38 | 0.51 | 0.44 | -0.38 |
| NS C | 0.18 | 0.36 | 0.98 | 0.36 | 0.22 | 0.13 |
| IP | -0.10 | 0.04 | 0.20 | 0.69 | -0.49 | 0.20 |
| R | 0.44 | 0.08 | 0.15 | -0.05 | -0.35 | 0.20 |

This table reports the correlation of the estimated factors with their corresponding empirical proxies–the Nelson and Siegel level (NS L), slope (NS S) and curvature (NS C), the industrial production index (IP) and the real interest rate (R). The top panel refers to the unrestricted macro-yields model. The bottom panel refers to the block-diagonal macro-yields model.

Table 3: In sample performance

Unrestricted Model

|  | L | S | C | M1 | M2 |
|---|---|---|---|---|---|
| Average Hourly Earnings:Total Private | 0.07 | 0.29 | 0.33 | 0.33 | 0.67 |
| Consumer Price Index: All Items | 0.19 | 0.48 | 0.48 | 0.50 | 0.86 |
| Real Disposable Personal Income | 0.00 | 0.02 | 0.03 | 0.34 | 0.36 |
| Effective Federal Funds Rate | 0.54 | 0.93 | 0.96 | 0.96 | 0.97 |
| Hose Sales - New One Family Houses | 0.00 | 0.19 | 0.19 | 0.22 | 0.22 |
| Industrial Production Index | 0.02 | 0.02 | 0.03 | 0.70 | 0.70 |
| M1 Money Stock | 0.17 | 0.25 | 0.25 | 0.25 | 0.31 |
| ISM Manufacturing: PMI Composite Index (NAPM) | 0.03 | 0.05 | 0.05 | 0.61 | 0.65 |
| Payments All Employees: Total nonfarm | 0.00 | 0.02 | 0.10 | 0.71 | 0.71 |
| Personal Consumption Expenditures Price Index | 0.16 | 0.23 | 0.33 | 0.47 | 0.79 |
| Producer Price Index: Crude Materials | 0.03 | 0.13 | 0.13 | 0.20 | 0.43 |
| Producer Price Index: Finished Goods | 0.03 | 0.31 | 0.31 | 0.32 | 0.81 |
| Capacity Utilization: Total Industry | 0.02 | 0.16 | 0.20 | 0.62 | 0.63 |
| Civilian Unemployment Rate | 0.44 | 0.53 | 0.55 | 0.64 | 0.67 |

Block-diagonal Model

|  | L | S | C | M1 | M2 | M3 |
|---|---|---|---|---|---|---|
| Average Hourly Earnings: Total Private | 0.00 | 0.00 | 0.00 | 0.09 | 0.57 | 0.68 |
| Consumer Price Index: All Items | 0.00 | 0.00 | 0.00 | 0.07 | 0.76 | 0.76 |
| Real Disposable Personal Income | 0.00 | 0.00 | 0.00 | 0.13 | 0.37 | 0.42 |
| House Sales - New One Family Houses | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.60 |
| Industrial Production Index | 0.00 | 0.00 | 0.00 | 0.63 | 0.80 | 0.80 |
| M1 Money Stock | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.48 |
| ISM Manufacturing: PMI Composite Index (NAPM) | 0.00 | 0.00 | 0.00 | 0.59 | 0.61 | 0.68 |
| Payments All Employees: Total nonfarm | 0.00 | 0.00 | 0.00 | 0.68 | 0.74 | 0.74 |
| Personal Consumption Expenditures Price Index | 0.00 | 0.00 | 0.00 | 0.32 | 0.53 | 0.79 |
| Producer Price Index: Crude Materials | 0.00 | 0.00 | 0.00 | 0.42 | 0.65 | 0.75 |
| Producer Price Index: Finished Goods | 0.00 | 0.00 | 0.00 | 0.15 | 1.00 | 1.00 |
| Capacity Utilization: Total Industry | 0.00 | 0.00 | 0.00 | 0.64 | 0.64 | 0.73 |
| Civilian Unemployment Rate | 0.00 | 0.00 | 0.00 | 0.08 | 0.18 | 0.42 |

This table reports the cumulative share of variance of macro variables explained by the yield curve factors (level, slope and curvature) and the macroeconomic factors. The top panel refers to the unrestricted macro-yields model. The bottom panel refers to the block-diagonal macro-yields model.

Table 4: Out-of-sample performance

Unrestricted

| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
|----------|-----|-----|-----|-----|-----|-----|
| h=1 | 1.21 | 1.04 | 1.05 | 1.01 | 1.05 | 1.12 |
| h=3 | 0.81* | 0.93 | 0.99 | 0.97 | 0.99 | 1.02 |
| h=6 | 0.79** | 0.89 | 0.95 | 0.93 | 0.93 | 0.94 |
| h=12 | 0.67** | 0.73** | 0.78** | 0.79*** | 0.79*** | 0.79*** |
| h=24 | 0.59*** | 0.63*** | 0.71*** | 0.78*** | 0.84** | 0.92 |

Block-diagonal

| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
|----------|-----|-----|-----|-----|-----|-----|
| h=1 | 0.84 | 1.10 | 1.09 | 1.04 | 1.04 | 1.06 |
| h=3 | 0.80 | 1.01 | 1.07 | 1.03 | 1.03 | 1.04 |
| h=6 | 0.76 | 0.97 | 1.04 | 1.03 | 1.02 | 1.02 |
| h=12 | 0.68*** | 0.82 | 0.92 | 0.94 | 0.95 | 0.95 |
| h=24 | 0.77 | 0.83 | 0.94 | 1.00 | 1.05 | 1.10 |

This table reports the relative MSFE of the unrestricted macro-yields model (top panel) and of the block-diagonal macro-yields model (bottom panel) over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon $h$. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

## D.1 The block-diagonal model and principal components

This Appendix compares the principal components extracted exclusively from the macroeconomic variables used in this paper with our unspanned macro factors (top panel), and with the factors extracted from a macro-yield model with block-diagonal structure (bottom panel).

Table 5 reports the pairwise correlations between the macro-yields factors and the first eight principal components extracted from our dataset of macro variables. The factors extracted from the block-diagonal model are very similar to the first three principal components. As stressed in the paper, this is the consequence of the fact that the block-diagonal model treats the macroeconomic factors separately from the bond yield factors.

Instead, the unspanned macroeconomic factors are significantly correlated with the first four principal components. This is not surprising since principal components contains also information that is already spanned by the yield curve.

Table 5: Correlation of principal components with other factors

| Unspanned macro-yields factors | | | | | | | |
|---|---|---|---|---|---|---|---|
| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 |
| UM1 | 0.32 | -0.87 | 0.19 | 0.04 | 0.14 | 0.09 | 0.04 | 0.02 |
| UM2 | 0.63 | 0.22 | 0.19 | 0.44 | -0.07 | 0.17 | -0.40 | 0.16 |

| Block-diagonal macro-yields factors | | | | | | | |
|---|---|---|---|---|---|---|---|
| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 |
| M1 | 0.96 | -0.08 | -0.06 | -0.03 | -0.08 | 0.02 | 0.00 | 0.01 |
| M2 | 0.04 | -0.97 | 0.07 | -0.14 | 0.04 | -0.01 | 0.05 | -0.06 |
| M3 | 0.07 | -0.01 | 0.93 | -0.07 | 0.07 | 0.03 | -0.19 | 0.13 |

This table reports the correlation of principal components extracted from macro variables with the unspanned macro-yields factors (top panel) and the block-diagonal macro-yields factors (bottom panel).

## D.2 LN factor and principal components

This Appendix compares our macro-yields factors with the Ludvigson and Ng (2009) factor and the principal components used to construct it.

Table 6 reports the pairwise correlations between the our macro-yields factors and the first eight principal components extracted from a large dataset of 131 variables.[1] Results in Table 6 show that the principal components are highly correlated with the yield curve factors extracted from our macro-yields model. This implies that also the LN factor is highly correlated with the yield curve factors as it just aggregates information from the principal components without separating the information already spanned by the cross-section of yields.

Table 6: Correlation PC and LN factors with our macro-yield factors

|      | PC1   | PC2  | PC3   | PC4   | PC5   | PC6   | PC7   | PC8   | LN    |
|------|-------|------|-------|-------|-------|-------|-------|-------|-------|
| L    | -0.18 | 0.04 | -0.06 | -0.26 | 0.30  | -0.12 | 0.08  | 0.10  | 0.05  |
| S    | -0.20 | 0.70 | -0.23 | 0.01  | 0.33  | 0.08  | 0.13  | 0.06  | -0.39 |
| C    | 0.00  | 0.18 | -0.08 | 0.01  | 0.08  | -0.10 | -0.08 | 0.15  | -0.12 |
| UM1  | 0.71  | 0.35 | -0.06 | 0.39  | 0.05  | -0.01 | -0.09 | 0.10  | -0.58 |
| UM2  | 0.04  | 0.18 | -0.04 | 0.04  | -0.03 | 0.07  | 0.19  | -0.10 | -0.34 |

This table reports the correlation of the macro-yields factors with the principal components extracted from a large dataset of macro variables and used to construct the Ludvigson and Ng (2009) factor.

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[1]The 131 macroeconomic data series used to construct the LN factor have been downloaded from Sydney C. Ludvigson's website at http://www.econ.nyu.edu/user/ludvigsons/Data&ReplicationFiles.zip.

# E   Comparison between initial and final estimates

In this Appendix we compare the performance of the initial and the final estimates of the macro-yields model. The initial estimates of the yield curve factors are computed using the two-steps OLS procedure introduced by Diebold and Li (2006). We then project the macroeconomic variables on the NS factors and use the principal components of the residuals of this regression as the initial estimates of the unspanned macroeconomic factors. These estimated factor are then treated as if they were the true observed factors. The initial parameters are hence estimated by OLS. The final estimates are obtained using the EM algorithm where the initial estimates of the factors are used in order to initialize the algorithm, as described in Section 3 and Appendix A in the main text.

Table 7 reports the cumulative variance of yields and macro variables explained by the initial estimates of the macro-yields factors. Comparing these results with the ones of Table 2, it is clear that the fit of the initial estimates is at least as good as the fit of the final estimates of the model. Figure 1 shows the initial estimates of the macro-yields factors are much more volatile than the final estimates. However, the correlation between the initial estimates and the final estimates of the macro-yields factors is very high, as shown in Table 8.

The out-of-sample performance of the macro-yields model improves when using the QML estimator compared to the initial estimates obtained by OLS and principal components, see Table 9. This is due to the fact that the QML estimator take appropriately into account the dynamics of the common factors and the cross-sectional heteroscedasticity of the idiosyncratic component.

Table 7: Cumulative variance explained by the initial estimates of the macro-yields factors

|  | Level | Slope | Curv | UM1 | UM2 |
|---|---|---|---|---|---|
| Government bond yield with maturity 3 months | 0.63 | 0.95 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 1 year | 0.64 | 0.83 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 2 years | 0.69 | 0.79 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 3 years | 0.74 | 0.80 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 4 years | 0.78 | 0.82 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 5 years | 0.82 | 0.84 | 1.00 | 1.00 | 1.00 |
| Average Hourly Earnings: Total Private | 0.07 | 0.30 | 0.35 | 0.35 | 0.71 |
| Consumer Price Index: All Items | 0.20 | 0.50 | 0.50 | 0.52 | 0.92 |
| Real Disposable Personal Income | 0.00 | 0.02 | 0.05 | 0.42 | 0.47 |
| Effective Federal Funds Rate | 0.57 | 0.94 | 0.97 | 0.97 | 0.97 |
| House Sales - New One Family Houses | 0.00 | 0.21 | 0.21 | 0.28 | 0.28 |
| Industrial Production Index | 0.02 | 0.02 | 0.05 | 0.84 | 0.86 |
| M1 Money Stock | 0.18 | 0.29 | 0.31 | 0.33 | 0.44 |
| ISM Manufacturing: PMI Composite Index (NAPM) | 0.03 | 0.06 | 0.06 | 0.74 | 0.77 |
| Payments All Employees: Total nonfarm | 0.00 | 0.02 | 0.15 | 0.83 | 0.83 |
| Personal Consumption Expenditures Price Index | 0.17 | 0.23 | 0.34 | 0.50 | 0.83 |
| Producer Price Index: Crude Materials | 0.03 | 0.16 | 0.16 | 0.21 | 0.45 |
| Producer Price Index: Finished Goods | 0.03 | 0.33 | 0.33 | 0.36 | 0.91 |
| Capacity Utilization: Total Industry | 0.03 | 0.19 | 0.26 | 0.69 | 0.69 |
| Civilian Unemployment Rate | 0.47 | 0.61 | 0.61 | 0.68 | 0.76 |

This table reports the cumulative share of variance of yields and macro variables explained by the initial estimates of the macro-yields factors. The first three columns refer to the Nelson-Siegel yield curve factors (level, slope and curvature) estimated by OLS, the last two column refer to the unspanned macroeconomic factors ($UM1$ and $UM2$) estimated by principal components.

Table 8: Correlation of the final factor estimates with the initial factor estimates

|  | L | S | C | UM1 | UM2 |
|---|---|---|---|---|---|
| Initial | 0.97 | 0.96 | 0.85 | 0.94 | 0.96 |

This table reports the correlation between the final factor estimates with the corresponding initial factor estimates.

Figure 1: Comparison between initial and final estimates of the macro-yields factors
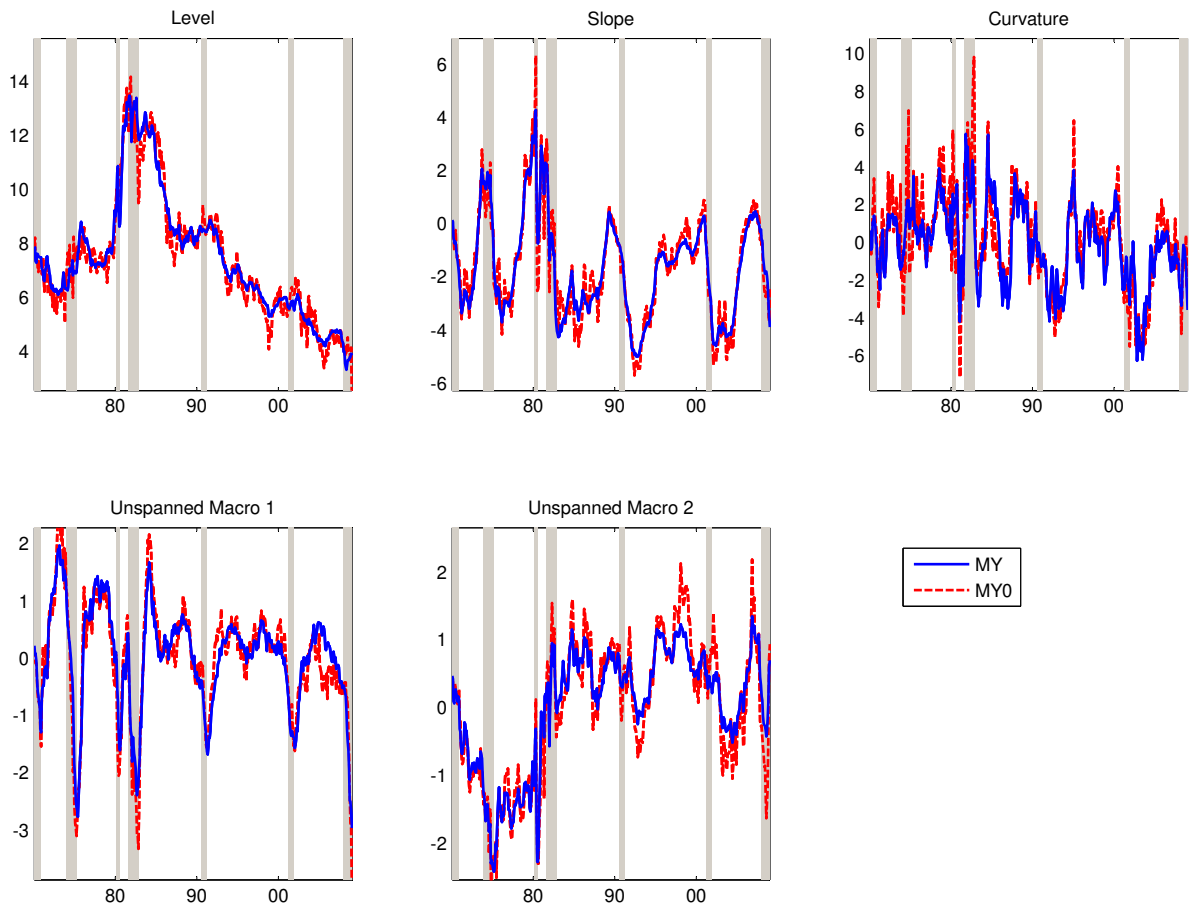


This figure displays initial (MY0) and the final (MY) estimates of the macro-yields factors. The top panel refers to the yield curve factors and the bottom panel to the unspanned macro factors.

Table 9: Out-of-sample performance of the initial and the final estimates

| | | | Initial Estimates | | | |
|---|---|---|---|---|---|---|
| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
| h=1 | 1.00 | 1.10 | 1.10 | 1.04 | 1.03 | 1.01 |
| h=3 | 1.09 | 1.18 | 1.14 | 1.07 | 1.06 | 1.04 |
| h=6 | 1.13 | 1.22 | 1.19 | 1.14 | 1.12 | 1.07 |
| h=12 | 0.86 | 0.92 | 0.95 | 0.95 | 0.96 | 0.94 |
| h=24 | 0.62** | 0.65** | 0.72** | 0.77* | 0.81 | 0.87 |

| | | | Final Estimates | | | |
|---|---|---|---|---|---|---|
| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
| h=1 | 1.17 | 1.05 | 1.06 | 1.00 | 1.05 | 1.14 |
| h=3 | 0.79* | 0.93 | 0.99 | 0.96 | 0.99 | 1.02 |
| h=6 | 0.78** | 0.89 | 0.94 | 0.93 | 0.93 | 0.94 |
| h=12 | 0.69** | 0.74** | 0.79** | 0.80*** | 0.80*** | 0.80*** |
| h=24 | 0.62*** | 0.66*** | 0.74** | 0.82** | 0.88* | 0.97 |

This table reports the relative MSFE of the macro-yields model initial and final estimates over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon $h$. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according to the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

# F    Macro factor identification

In the macro-yields model in equations (4)-(6), the two macroeconomic factors are not identified since any transformation $HF_t^x$, with $H$ non-singular, gives an observationally equivalent model. In order to achieve identification, additional restrictions are required. In the main text, we do not impose such restrictions and the EM algorithm converges to the Maximum Likelihood solution that is close to the initialisation, i.e. the principal components of the residuals of the macro variables after regressing them on the NS factors.

In this appendix, we show results for the macro-yields model when identification restrictions are imposed on the matrix of factor loadings $\Gamma_{xx}$. We identify the macro factors such that the first macro factor has a loading of one for industrial production, and the second macro factor has a loading of one for CPI and is not loaded by industrial production. We impose these restrictions in the EM algorithm and we initialize the unspanned macro factors using the same rotation for the principal components extracted from the residuals of the regression of the macro variables on the NS factors.

Table 10 reports the cumulative variance of yields and macro variables explained by the macro-yields factors when identification restrictions are imposed on the macro factors. Comparing Table 10 with Table 2, we can notice that the imposition of the identification restrictions slightly changes the proportion of variance explained by the macro factors. Table 11 shows that the correlation of the first unspanned macro factor with the industrial production marginally increases from 91% to 92% when the identification restrictions are imposed and the correlation of the second unspanned macro factor with the real rate slightly decreases from 75% to 72% when we impose such restrictions.

Table 10: Cumulative variance explained by the macro-yields factors with identification

|  | Level | Slope | Curv | UM1 | UM2 |
|---|---|---|---|---|---|
| Government bond yield with maturity 3 months | 0.59 | 0.94 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 1 year | 0.61 | 0.83 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 2 years | 0.65 | 0.78 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 3 years | 0.70 | 0.79 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 4 years | 0.74 | 0.80 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 5 years | 0.78 | 0.82 | 0.99 | 0.99 | 0.99 |
| Average Hourly Earnings: Total Private | 0.07 | 0.29 | 0.33 | 0.33 | 0.67 |
| Consumer Price Index: All Items | 0.19 | 0.48 | 0.48 | 0.53 | 0.85 |
| Real Disposable Personal Income | 0.00 | 0.02 | 0.03 | 0.36 | 0.36 |
| Effective Federal Funds Rate | 0.53 | 0.93 | 0.96 | 0.96 | 0.97 |
| House Sales - New One Family Houses | 0.00 | 0.19 | 0.19 | 0.23 | 0.23 |
| Industrial Production Index | 0.02 | 0.02 | 0.03 | 0.70 | 0.70 |
| M1 Money Stock | 0.17 | 0.25 | 0.25 | 0.25 | 0.31 |
| ISM Manufacturing: PMI Composite Index (NAPM) | 0.03 | 0.05 | 0.05 | 0.57 | 0.65 |
| Payments All Employees: Total nonfarm | 0.00 | 0.02 | 0.10 | 0.68 | 0.70 |
| Personal Consumption Expenditures | 0.16 | 0.23 | 0.33 | 0.42 | 0.78 |
| Producer Price Index: Crude Materials | 0.03 | 0.14 | 0.14 | 0.18 | 0.43 |
| Producer Price Index: Finished Goods | 0.03 | 0.33 | 0.33 | 0.35 | 0.80 |
| Capacity Utilization: Total Industry | 0.02 | 0.16 | 0.21 | 0.60 | 0.64 |
| Civilian Unemployment Rate | 0.44 | 0.54 | 0.55 | 0.67 | 0.68 |

This table reports the cumulative share of variance of yields and macro variables explained by the macro-yields factors when identification restrictions are imposed on the macro factors.

Table 11: Correlation of the factor estimates with the proxies

|  | L | S | C | UM1 | UM2 |
|---|---|---|---|---|---|
| MY | 0.97 | 0.96 | 0.84 | 0.91 | 0.75 |
| MY* | 0.97 | 0.96 | 0.84 | 0.92 | 0.72 |

This table reports the correlation between the factor estimates with the corresponding proxies. The first row refers to the MY model without the identification restrictions, the second row MY* refers to the factor estimates when the identification restrictions are imposed. The first three columns report the correlations with the NS yield curve factors estimated by ordinary least squares as in Diebold and Li (2006). The last two column report the correlation with the industrial production index (IP) and the real interest rate (FFR-CPI).

# G    Estimated parameters

In Tables 12–13 we report the estimated parameters of the macro-yields model described in Equations 4-6. Standard errors are computed using a Monte Carlo procedure, as follows: we estimate the macro-yields model in equations 4-6 and save the idiosyncratic innovations and the state innovations. We then bootstrap the state innovations and simulate the state variables. In the same way, we bootstrap the idiosyncratic innovations and simulate the idiosyncratic components. We then obtain a sample of artificial yields and macro variables by adding the simulated idiosyncratic components to the simulated state variables multiplied by the estimated factor loadings. We generate a 1000 simulated samples of yields and macro variables and for each simulated sample we estimate the macro-yields model. The standard deviations of parameters reported in Tables 12–13 are the standard deviations of the empirical distribution of each model parameter.

Results in Table 12 show that most macro variables have statistically significant factor loadings on the level and slope of the yield curve. The autocorrelation coefficient in the idiosyncratic components is significant for all variables, except for one yield. Results in Table 13 show that the level is mainly Granger caused by the second unspanned macro factor, i.e. the real interest rate. The slope and the curvature are mainly Granger caused by the first factor, i.e. industrial production.

Table 12: Estimated parameters: measurement equation

| | $\Gamma$ | | | | | | $B$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| | $L_t$ | $S_t$ | $C_t$ | $UM1_t$ | $UM2_t$ | $a_x$ | | |
| Govt bond yield maturity 3 m | 1.000 | 0.914 | 0.081 | 0.000 | 0.000 | 0.000 | 0.425 | 0.050 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.166) | (0.018) |
| Govt bond yield maturity 1 year | 1.000 | 0.709 | 0.228 | 0.000 | 0.000 | 0.000 | 0.591 | 0.021 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.057) | (0.003) |
| Govt bond yield maturity 2 years | 1.000 | 0.526 | 0.294 | 0.000 | 0.000 | 0.000 | 0.549 | 0.007 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.061) | (0.001) |
| Govt bond yield maturity 3 years | 1.000 | 0.405 | 0.294 | 0.000 | 0.000 | 0.000 | -0.194 | 0.002 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.198) | (0.001) |
| Govt bond yield maturity 4 years | 1.000 | 0.324 | 0.270 | 0.000 | 0.000 | 0.000 | 0.548 | 0.008 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.051) | (0.001) |
| Govt bond yield maturity 5 years | 1.000 | 0.267 | 0.241 | 0.000 | 0.000 | 0.000 | 0.739 | 0.009 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.050) | (0.001) |
| Av Hourly Earnings: Tot Private | 0.113 | 0.250 | 0.035 | -0.048 | 0.592 | -0.432 | 0.963 | 0.022 |
| | (0.095) | (0.064) | (0.043) | (0.215) | (0.223) | (0.665) | (0.019) | (0.004) |
| CPI: All Items | 0.189 | 0.292 | -0.010 | -0.217 | 0.622 | -0.921 | 0.943 | 0.009 |
| | (0.076) | (0.061) | (0.041) | (0.293) | (0.246) | (0.530) | (0.032) | (0.003) |
| Real Disposable Personal Inc | -0.016 | -0.085 | 0.087 | 0.599 | -0.287 | -0.029 | 0.778 | 0.262 |
| | (0.082) | (0.066) | (0.063) | (0.375) | (0.314) | (0.526) | (0.034) | (0.041) |
| Effective Federal Funds Rate | 0.320 | 0.328 | 0.017 | 0.001 | 0.049 | -1.843 | 0.654 | 0.014 |
| | (0.013) | (0.012) | (0.010) | (0.028) | (0.027) | (0.094) | (0.043) | (0.003) |
| House Sales | -0.249 | -0.065 | -0.036 | 0.267 | -0.158 | 1.750 | 0.952 | 0.054 |
| | (0.100) | (0.069) | (0.049) | (0.220) | (0.237) | (0.663) | (0.021) | (0.006) |
| Industrial Production Index | -0.055 | -0.031 | 0.087 | 0.898 | -0.054 | 0.350 | 0.941 | 0.021 |
| | (0.096) | (0.066) | (0.073) | (0.369) | (0.242) | (0.625) | (0.032) | (0.003) |
| M1 Money Stock | 0.182 | -0.157 | -0.041 | 0.101 | 0.343 | -1.603 | 0.976 | 0.044 |
| | (0.134) | (0.079) | (0.056) | (0.216) | (0.339) | (0.914) | (0.019) | (0.009) |
| ISM Man. PMI Composite Index | -0.080 | -0.089 | 0.073 | 0.819 | 0.211 | 0.437 | 0.846 | 0.103 |
| | (0.089) | (0.064) | (0.067) | (0.302) | (0.208) | (0.591) | (0.032) | (0.010) |
| Paym All Emp: Total nonfarm | -0.017 | 0.073 | 0.122 | 0.833 | 0.035 | 0.228 | 0.963 | 0.011 |
| | (0.104) | (0.058) | (0.066) | (0.308) | (0.191) | (0.728) | (0.021) | (0.002) |
| PCE | 0.174 | 0.127 | 0.085 | 0.363 | 0.578 | -1.092 | 0.797 | 0.086 |
| | (0.091) | (0.060) | (0.047) | (0.174) | (0.171) | (0.687) | (0.033) | (0.008) |
| PPI: Crude Materials | -0.077 | 0.178 | -0.043 | 0.238 | 0.505 | 0.871 | 0.897 | 0.144 |
| | (0.098) | (0.083) | (0.058) | (0.306) | (0.310) | (0.675) | (0.030) | (0.021) |
| PPI: Finished Goods | 0.069 | 0.296 | -0.023 | -0.191 | 0.732 | -0.028 | 0.907 | 0.024 |
| | (0.084) | (0.068) | (0.047) | (0.340) | (0.284) | (0.563) | (0.041) | (0.006) |
| CU: Total Industry | -0.068 | 0.199 | 0.069 | 0.642 | 0.117 | 0.823 | 0.982 | 0.011 |
| | (0.104) | (0.055) | (0.059) | (0.245) | (0.191) | (0.714) | (0.022) | (0.002) |
| Civilian Unemp Rate | 0.290 | -0.174 | 0.029 | -0.294 | 0.256 | -2.441 | 0.947 | 0.017 |
| | (0.057) | (0.040) | (0.035) | (0.177) | (0.140) | (0.382) | (0.021) | (0.002) |

This table reports the estimated parameters and corresponding standard errors for the measurement equation of the macro-yields model in equations (4) and (6). Standard errors are computed by 1000 Monte Carlo simulations.

Table 13: Estimated parameters: state equation

|  | A | | | | | $\mu$ | Q | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  | $L_{t-1}$ | $S_{t-1}$ | $C_{t-1}$ | $UM1_{t-1}$ | $UM2_{t-1}$ |  |  |  |  |  |  |
| $L_t$ | 0.998 | 0.013 | -0.004 | -0.020 | 0.051 | 0.029 | 0.055 | 0.050 | 0.034 | 0.004 | -0.032 |
|  | (0.010) | (0.010) | (0.014) | (0.024) | (0.023) | (0.064) | (0.019) | (0.024) | (0.050) | (0.016) | (0.021) |
| $S_t$ | -0.016 | 0.966 | 0.025 | 0.144 | 0.032 | 0.049 | 0.050 | 0.102 | -0.054 | 0.017 | -0.040 |
|  | (0.022) | (0.017) | (0.022) | (0.050) | (0.039) | (0.159) | (0.024) | (0.049) | (0.076) | (0.032) | (0.041) |
| $C_t$ | 0.033 | 0.030 | 0.889 | 0.155 | 0.078 | -0.221 | 0.034 | -0.054 | 0.750 | -0.079 | -0.007 |
|  | (0.037) | (0.038) | (0.048) | (0.079) | (0.071) | (0.248) | (0.050) | (0.076) | (0.241) | (0.077) | (0.070) |
| $UM1_t$ | 0.005 | -0.038 | 0.006 | 0.996 | 0.007 | -0.106 | 0.004 | 0.017 | -0.079 | 0.032 | 0.000 |
|  | (0.009) | (0.023) | (0.019) | (0.014) | (0.016) | (0.074) | (0.016) | (0.032) | (0.077) | (0.020) | (0.015) |
| $UM2_t$ | 0.002 | 0.009 | -0.012 | 0.009 | 0.979 | -0.004 | -0.032 | -0.040 | -0.007 | 0.000 | 0.035 |
|  | (0.011) | (0.024) | (0.020) | (0.016) | (0.021) | (0.079) | (0.021) | (0.041) | (0.070) | (0.015) | (0.034) |

This table reports the estimated parameters and corresponding standard errors for the state equation of the macro-yields model in equation (5). Standard errors are computed by 1000 Monte Carlo simulations.

# H   Alternative macro dataset

This Appendix contains results using an alternative macroeconomic dataset. Following Banbura, Giannone, Modugno and Reichlin (2012), we aim at selecting the headline macroeconomic variables. Accordingly, the series we collect are marked on Bloomberg website as Market Moving Indicators. We only include the headlines of each macroeconomic report since these are the data followed by financial markets and extensively commented by the newspapers. Table 14 contains a complete list of the macroeconomic variables along with the transformation applied to ensure stationarity. We transform all the variables in annual growth rates, except for the federal funds rate, the unemployment rate, housing starts, houses sold, Philadelphia Fed. survey, conference board consumer confidence and the PMI manufacturing index which we keep in levels. The monthly U.S. Treasury zero-coupon yield curve data are the same as in the main text.

Table 14: Macroeconomic variables in the alternative dataset

| Series N. | Mnemonic | Description | Transformation |
|:---:|:---|:---|:---:|
| 1 | IP | Industrial Production Index | 1 |
| 2 | PMI | ISM Manufacturing: PMI Composite Index (NAPM) | 0 |
| 3 | PI | Real Disposable Personal Income | 1 |
| 4 | Unem | Civilian Unemployment Rate | 0 |
| 5 | Paym | Payments All Employees: Total nonfarm | 1 |
| 6 | PCE | Personal Consumption Expenditures Price Index | 1 |
| 7 | Hstat | Housing Starts | 0 |
| 8 | Hsold | House Sales - New One Family Houses | 0 |
| 9 | PPI | Producer Price Index: Finished Goods | 1 |
| 10 | CPI | Consumer Price Index for All Urban Consumers | 1 |
| 11 | FFR | Effective Federal Funds Rate | 0 |
| 12 | Imp | Imports | 1 |
| 13 | Exp | Exports | 1 |
| 14 | Phil | Philadelphia Fed. Survey, General business activity | 0 |
| 15 | CB | Conference Board, Consumer confidence | 0 |
| 16 | RS | Sales of retail stores | 1 |
| 17 | NORD | Mfrs' new orders durable goods industries | 1 |

This table lists the alternative 17 macro variables used to estimate the macro-yields. Most series have been transformed prior to the estimation, as reported in the last column of the table. The transformation codes are: 0 = no transformation and 1 = annual growth rate.

Results in Tables 15– 17 and Figure 2 are in line with the results obtained with the original

Table 15: Model selection with the alternative dataset

| Number of factors | $IC^*$ | $V$ |
|---|---|---|
| 3 | 0.10 | 0.49 |
| 4 | -0.07 | 0.31 |
| 5 | -0.11 | 0.23 |
| 6 | -0.02 | 0.19 |
| 7 | 0.13 | 0.17 |
| 8 | 0.36 | 0.16 |

This table reports the information criterion $IC^*$, and the sum of the variance of the idiosyncratic components (divided by $NT$), $V$, when different numbers of factors are estimated.

dataset. Table 15 indicates that two unspanned factors are necessary in order to capture the comovement of yields and the alternative macro variables. Table 16 shows that the yield curve factors explain most of the variance of the yields and the federal funds rate. They also explain part of the variance of price indices and unemployment. The first unspanned macro factor captures the dynamics of industrial production and other real variables, while the second unspanned factor mainly explains prices, as with our original dataset. Figure 2 shows that the first unspanned macroeconomic factor closely tracks the industrial production index, with a correlation of 92%, and the second unspanned macroeconomic factor proxies the real interest, with a correlation of 74%. Also the out-of-sample results, reported in Table 17, are in line with the results obtained with the original dataset.

Table 16: Cumulative variance of yields and macro variables explained by the macro-yields factors

|  | Level | Slope | Curv | UM1 | UM2 |
|---|---|---|---|---|---|
| Government bond yield with maturity 3 months | 0.59 | 0.96 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 1 year | 0.61 | 0.84 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 2 years | 0.65 | 0.79 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 3 years | 0.70 | 0.79 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 4 years | 0.74 | 0.80 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 5 years | 0.78 | 0.82 | 1.00 | 1.00 | 1.00 |
| Industrial Production Index | 0.04 | 0.04 | 0.04 | 0.68 | 0.68 |
| ISM Manufacturing: PMI Composite Index (NAPM) | 0.06 | 0.08 | 0.08 | 0.63 | 0.66 |
| Real Disposable Personal Income | 0.00 | 0.04 | 0.04 | 0.36 | 0.46 |
| Civilian Unemployment Rate | 0.47 | 0.56 | 0.58 | 0.61 | 0.62 |
| Payments All Employees: Total nonfarm | 0.01 | 0.02 | 0.08 | 0.64 | 0.65 |
| Personal Consumption Expenditures Price Index | 0.13 | 0.20 | 0.29 | 0.47 | 0.57 |
| Housing Starts | 0.14 | 0.14 | 0.14 | 0.47 | 0.48 |
| House Sales - New One Family Houses | 0.44 | 0.47 | 0.52 | 0.55 | 0.54 |
| Producer Price Index: Finished Goods | 0.04 | 0.36 | 0.36 | 0.37 | 0.63 |
| Consumer Price Index for All Urban Consumers | 0.20 | 0.52 | 0.52 | 0.53 | 0.70 |
| Effective Federal Funds Rate | 0.55 | 0.98 | 1.00 | 1.00 | 1.00 |
| Imports | 0.01 | 0.04 | 0.05 | 0.22 | 0.48 |
| Exports | 0.02 | 0.19 | 0.20 | 0.26 | 0.46 |
| Philadelphia Fed. Survey, General business activity | 0.02 | 0.18 | 0.18 | 0.53 | 0.53 |
| Conference Board, Consumer confidence | 0.15 | 0.15 | 0.15 | 0.37 | 0.51 |
| Sales of retail stores | 0.05 | 0.18 | 0.18 | 0.53 | 0.56 |
| Mfrs' new orders durable goods industries | 0.03 | 0.04 | 0.04 | 0.65 | 0.65 |

This table reports the cumulative share of variance of yields and alternative macro variables explained by the macro-yields factors. The first three columns refer to the yield curve factors (level, slope and curvature) and the last two to the unspanned macroeconomic factors ($UM1$ and $UM2$).

Table 17: Out-of-sample performance with the alternative dataset

| | Macro-Yields | | | | | |
|---|---|---|---|---|---|---|
| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
| h=1 | 0.85 | 0.99 | 1.10 | 1.08 | 1.12 | 1.18 |
| h=3 | 0.79 | 0.96 | 1.07 | 1.08 | 1.12 | 1.14 |
| h=6 | 0.77 | 0.91 | 1.02 | 1.05 | 1.07 | 1.09 |
| h=12 | 0.73 | 0.78 | 0.85 | 0.89 | 0.91 | 0.93 |
| h=24 | 0.76 | 0.70 | 0.73 | 0.77 | 0.81 | 0.89 |

This table reports the relative MSFE of the macro-yields model over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon $h$. The sample starts on Jan 1970 and the evaluation period is Jan 1990 to Dec 2008.

Figure 2: Macro-yields factors with the alternative dataset
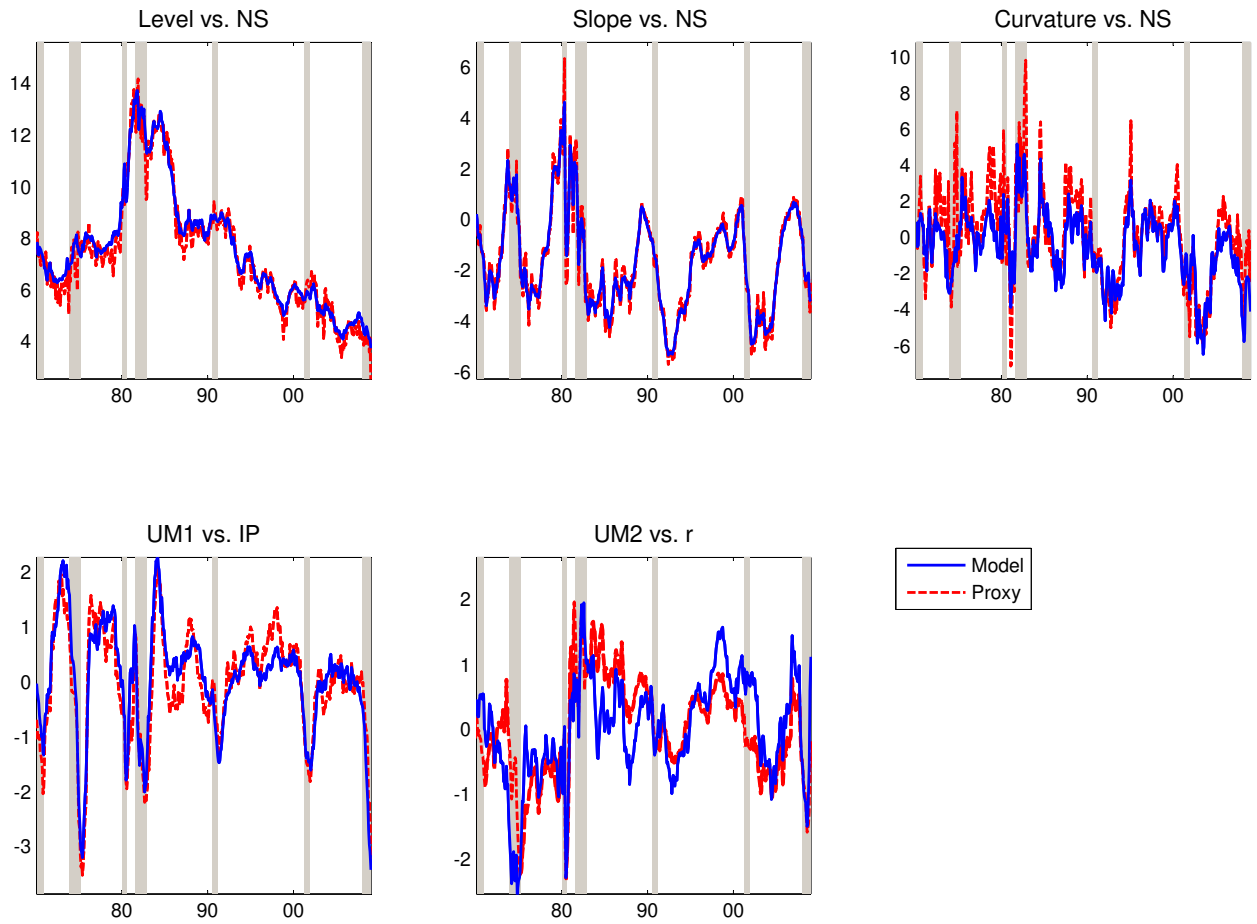


This figure displays the estimated factors of the macro-yields model. The dashed red lines in the three top graphs refer to the NS yield curve factors estimated by ordinary least squares. The red dashed line in the bottom left plot refers to the industrial production index (IP), while the red dashed line in the bottom plot refers to the real interest rate (FFR-CPI). The grey-shaded areas indicate the recessions as defined by the NBER.

# I  Alternative dataset with more yields

This appendix reports results with an alternative dataset that contains more yields than macro variables. We use a dataset composed by 17 yields (3 month and six month yields from CRSP, and 1 to 15 year yields from Gürkaynak, Sack and Wright (2007)) and 5 macro variables(Consumer Price Index, Industrial Production Index, Payments All Employees, Personal Consumption Expenditures and Producer Price Index: Finished Goods). Table 18 shows that also with this dataset the information criterion selects the model with two unspanned macro factors. Table 19 shows that the first three factors capture most of the variation in the 17 yields included in this dataset and that the two unspanned macro factors are mainly related to industrial production (the first) and the component in prices that is not explained by the yield curve factors (the second). The top plots Figure 3 compare the two unspanned macro factors obtained from the macro-yields model with the unrestricted macro factors obtained from the macro-yields model without imposing the unspanning restriction. The two set of factors are very close. This provides evidence that the zero restrictions on the factor loadings are satisfied also by this dataset and that the macro factors are unspanned by the cross-section of yields. This happens because yields have a strong factor structure and they can be fully characterized by a very small number of common factors, regardless of the number of yields considered in the analysis. As a final robustness check, the bottom plot of Figure 3 compare the unspanned macro factors obtained with this alternative dataset with the unspanned macro factors obtained in the paper. The extracted factors are very similar to the ones extracted in the paper. The only difference is that the factors extracted with the dataset proposed by the referee are more volatile, reflecting the less precise extraction of the factors due to the smaller number of macroeconomic variables.

Table 18: Model selection with more yields

| # of factors | IC* | V |
|---:|---:|---:|
| 3 | -0.84 | 0.19 |
| 4 | -0.65 | 0.17 |
| 5 | -0.87 | 0.11 |
| 6 | -0.66 | 0.10 |

This table reports the information criterion $IC^*$ and the sum of the variance of the idiosyncratic components (divided by $NT$), $V$, when different numbers of factors are estimated.

Table 19: Cumulative variance of yields and macro variables explained by the macro-yields factors

| | Level | Slope | Curv | UM1 | UM2 |
|---|---:|---:|---:|---:|---:|
| Government bond yield with maturity 3 months | 0.59 | 1.00 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 6 months | 0.53 | 0.86 | 0.97 | 0.97 | 0.97 |
| Government bond yield with maturity 1 year | 0.54 | 0.78 | 0.98 | 0.98 | 0.98 |
| Government bond yield with maturity 2 year | 0.59 | 0.73 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 3 year | 0.63 | 0.72 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 4 year | 0.67 | 0.73 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 5 year | 0.70 | 0.75 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 6 year | 0.73 | 0.77 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 7 year | 0.76 | 0.79 | 0.98 | 0.98 | 0.98 |
| Government bond yield with maturity 8 year | 0.79 | 0.82 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 9 year | 0.82 | 0.84 | 0.99 | 0.99 | 0.99 |
| Government bond yield with maturity 10 year | 0.84 | 0.86 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 11 year | 0.86 | 0.88 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 12 year | 0.88 | 0.89 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 13 year | 0.89 | 0.90 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 14 year | 0.90 | 0.91 | 1.00 | 1.00 | 1.00 |
| Government bond yield with maturity 15 year | 0.91 | 0.92 | 1.00 | 1.00 | 1.00 |
| Consumer Price Index: All Items | 0.19 | 0.48 | 0.50 | 0.61 | 0.94 |
| Industrial Production Index | 0.05 | 0.05 | 0.07 | 0.86 | 0.92 |
| Payments All Employees: Total nonfarm | 0.03 | 0.05 | 0.16 | 0.79 | 0.94 |
| Personal Consumption Expenditures | 0.06 | 0.14 | 0.38 | 0.41 | 0.87 |
| Producer Price Index: Finished Goods | 0.02 | 0.31 | 0.32 | 0.47 | 0.91 |

This table reports the cumulative share of variance of yields and alternative macro variables explained by the macro-yields factors. The first three columns refer to the yield curve factors (level, slope and curvature) and the last two to the unspanned macroeconomic factors (UM1 and UM2).

Figure 3: Estimated macro factors
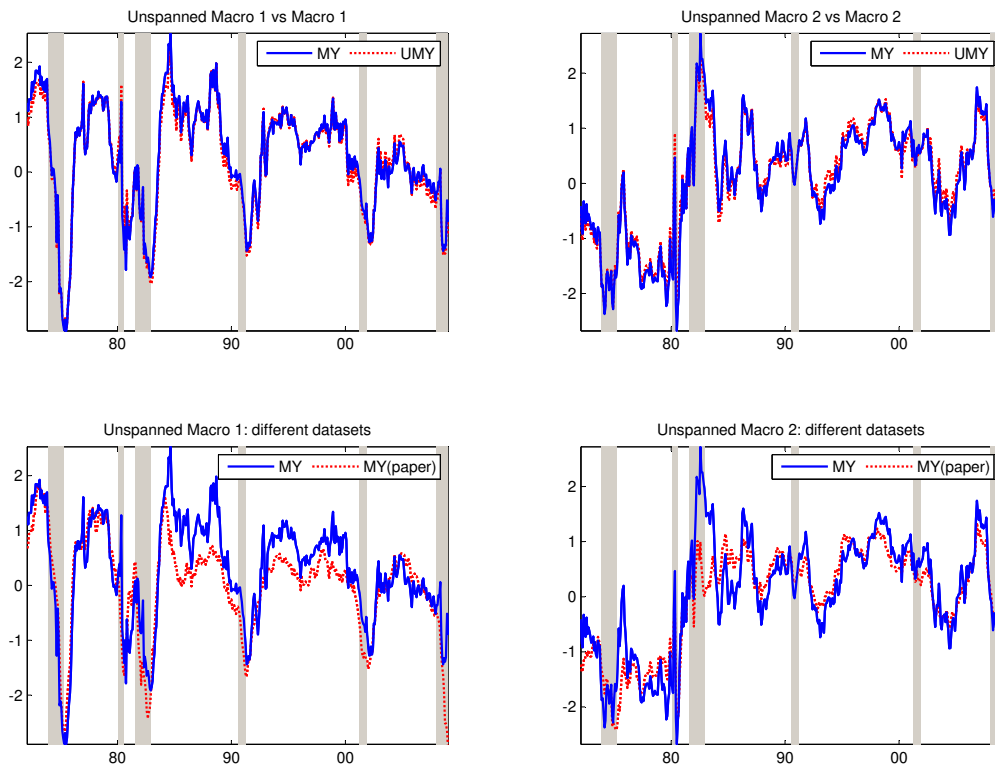
This figure displays the estimated unspanned factors of the macro-yields model using a dataset of 17 yields and 5 macro variables. The dotted red lines in the top plots refer to the unrestricted macro factors. The dotted red lines in the bottom plots refer to estimated unspanned macro factors with the dataset in the paper. The grey-shaded areas indicate the recessions as defined by the NBER.

# J  Only-yields model with a unit root in the level

Duffee (2011) finds that imposing a unit root restriction on the level factor results in superior yield predictions. To compare our results with this type of models, we estimate a only-yields version of our model with a unit root in the level factor. This is a restricted version of the macro-yields model in equations 4-6 with $Q_{yx} = A_{yx} = \Gamma_{xy} = 0$ and the parameters in the state equation are restricted as follows

$$\mu_y = \begin{bmatrix} 0 & \mu_S & \mu_C \end{bmatrix} \text{ and } A_{yy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{SS} & A_{CS} \\ 0 & A_{SC} & A_{CC} \end{bmatrix}$$

The out-of-sample forecasting performance of this model is reported in Table 20. Comparing these results with the ones in Table 5, we can notice that imposing random walk dynamics to the level of interest rates improves the predictive ability of the only-yield model in the long horizon. However this model is not able to outperform neither the macro-yields model neither the random walk.

Table 20: Out-of-sample performance

| Only-yields model with unit root in the level | | | | | |
|---|---|---|---|---|---|
| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
| h=1 | 0.99 | 1.29 | 1.23 | 1.14 | 1.11 | 1.15 |
| h=3 | 1.03 | 1.27 | 1.26 | 1.19 | 1.15 | 1.18 |
| h=6 | 1.04 | 1.28 | 1.30 | 1.27 | 1.22 | 1.24 |
| h=12 | 0.97 | 1.11 | 1.19 | 1.22 | 1.21 | 1.23 |
| h=24 | 0.82 | 0.85 | 0.92 | 0.99 | 1.03 | 1.10 |

This table reports the relative MSFE of the only-yields model with a unit root in the level factor over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon $h$. The sample starts on Jan 1970 and the evaluation period is Jan 1990 to Dec 2008.

# K  Recursive model selection

In this Appendix we report the out-of-sample performances of the macro-yields model and the block-diagonal macro-yields model when the number of macro factors is selected every time the model is re-estimated.

As explained before, we estimate the model at the beginning of each year of our evaluation sample. Given that we report the performances up to two-year horizon, the first time the model is estimated is in January 1988 (with this parameters we produce the two-year ahead forecast for January 1990). Table 21 shows that while for the block-diagonal macro-yields model there is some instability in the selection of the number of macro factors, prior to 2000, the macro-yield model is stable: the information criterium selects always 5 factors. This result leads to a slight deterioration of the out-of-sample performances for the block diagonal model, see Table 22, probably due to the fact that for 1998 and 1999 the number of factors selected is equal to eight. For the macro-yields model the out-of-sample performances are equal to the ones reported in the main text, given that the number of factors selected is always equal to five.

Table 21: Model selection for the block-diagonal and the macro-yields model

| | Number of factors | |
| year | Block-diagonal | Macro-yields |
| --- | --- | --- |
| 1988-1992 | 5 | 5 |
| 1993 | 6 | 5 |
| 1994-1997 | 4 | 5 |
| 1998-1999 | 8 | 5 |
| 2000-2008 | 6 | 5 |

This table reports the number of macro factors selected for the macro-yield and the block-diagonal model every time we re-estimate the model, i.e., once per year, every year, using the information criterion IC*, as shown in equation 7 and in equation 8.

Table 22: Out-of-sample performance with recursive model selection

Macro-Yields Model

| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
|----------|------|--------|--------|---------|---------|---------|
| h=1 | 1.17 | 1.05 | 1.06 | 1.00 | 1.05 | 1.14 |
| h=3 | 0.79* | 0.93 | 0.99 | 0.96 | 0.99 | 1.02 |
| h=6 | 0.78** | 0.89 | 0.94 | 0.93 | 0.93 | 0.94 |
| h=12 | 0.69** | 0.74** | 0.79** | 0.80*** | 0.80*** | 0.80*** |
| h=24 | 0.62*** | 0.66*** | 0.74** | 0.82** | 0.88* | 0.97 |

Block-diagonal Macro-yields Model

| Maturity | 3m | 1y | 2y | 3y | 4y | 5y |
|----------|---------|--------|------|------|------|------|
| 1 | 0.90 | 1.14 | 1.10 | 1.06 | 1.03 | 1.08 |
| 3 | 0.85* | 1.04 | 1.08 | 1.04 | 1.03 | 1.04 |
| 6 | 0.76** | 0.95 | 1.03 | 1.02 | 1.00 | 1.00 |
| 12 | 0.70*** | 0.84* | 0.95 | 0.99 | 0.99 | 1.01 |
| 24 | 0.87 | 0.93 | 1.07 | 1.17 | 1.25 | 1.36 |

This table reports the relative MSFE of the macro-yields model (top panel) and the block-diagonal model (bottom panel) over the MSFE of the random walk for multi-step predictions of the yields, when the number of macro factors are inferred every time the parameters are estimated. The first column reports the forecast horizon $h$. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according to the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

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