Resource Allocation for Cognitive Small Cell Networks: A Cooperative Bargaining Game Theoretic Approach

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Abstract—Cognitive small cell networks have been envisioned as a promising technique to meet the exponentially increasing mobile traffic demand. Recently, many technological issues pertaining to cognitive small cell networks have been studied, including resource allocation and interference mitigation, but most studies assume non-cooperative schemes or perfect channel-state information (CSI). Different from the existing works, we investigate the joint uplink subchannel and power allocation problem in cognitive small cells using cooperative Nash bargaining game theory, where the cross-tier interference mitigation, minimum outage probability requirement, imperfect CSI and fairness in terms of minimum rate requirement are considered. A unified analytical framework is proposed for the optimization problem, where the near optimal cooperative bargaining resource allocation strategy is derived based on Lagrangian dual decomposition by introducing time-sharing variables and recalling the Lambert-W function. The existence, uniqueness, and fairness of the solution to this game model are proved. A cooperative Nash bargaining resource allocation algorithm is developed, and it is shown to converge to a Pareto-optimal equilibrium for the cooperative game. Simulation results are provided to verify the effectiveness of the proposed cooperative game algorithm for efficient and fair resource allocation in cognitive small cell networks.

Index Terms—Cognitive small cell, cooperative game, fairness, Nash bargaining, OFDMA, power control, resource allocation.

I. INTRODUCTION

Driven by the rapid development of wireless terminal equipments and wide usage of bandwidth-hungry applications of mobile Internet, wireless data traffic is increasing in an exponential manner. Traditional deployment of macrocell base stations (MBS) suffers from poor quality of service (QoS) and coverage for indoor and cell edge users, especially for potential use of high carrier frequency in 5G [1]. Therefore, off-loading the traffic from primary macrocells, improving the capacity and enhancing the coverage of indoor and cell edge scenarios are critically needed. In this context, deployment of low-power, low-cost small access points (e.g., microcell, picocell and femtocell) becomes a promising technique [2]. Small cells can significantly improve the efficiency of frequency reuse and spectrum sharing. Heterogeneous networks, comprised of small base stations (SBS) and MBSs, is also an important candidate technique for 5G mobile communications. Compared with the orthogonal deployment, spectrum sharing between macrocells and small cells is more attractive due to easy implementation and more efficient utilization of spectrum. In spectrum sharing, the macrocells can be considered as the primary network and small cells can be regarded as the secondary cognitive network [3]. However, cross-tier interference could be severe in spectrum sharing cognitive heterogeneous small cell networks. Therefore, the benefits of cognitive small cell deployments come with a number of fundamental challenges, which include resource management and cross-tier interference mitigation.

Game theory based resource allocation and interference mitigation in small cells have been widely investigated in existing works [4]–[11]. In [4], non-cooperative power allocation with signal-to-interference-plus-noise ratio (SINR) adaptation is used to alleviate the interference from femtocells to macrocells, while in [5] Stackelberg game based power control is formulated to maximize femtocells’ capacity under a cross-tier interference constraint. In [6], a non-cooperative power and subchannel allocation scheme for co-channel deployed femtocells is proposed, together with macrocell user transmission protection. In [7], the authors consider a capacity maximizing power allocation based on a Stackelberg game, where the MBS is the leader and the FBSs are assumed as followers. Subchannel allocation in femtocells is formulated into a correlated equilibrium game-theoretic approach to minimize their interference to the primary MBS in [8].
A unique Nash equilibrium (NE) is achieved and a hybrid access protocol is designed for the Stackelberg game in [10]. A non-cooperative game based power control algorithm is proposed in [11] together with a base station association scheme for heterogeneous networks. In our previous work [12] [13], resource scheduling (based on uniform pricing and differential pricing game) and power control were proposed for small cell networks.

Moreover, game theory based energy efficient resource allocation has also been investigated for small cells. In [9], the energy efficiency aspect of spectrum sharing and power allocation was studied using a Stackelberg game in heterogeneous cognitive radio networks with femtocells. While in [14], NE of a power adaptation game was derived to reduce power consumption and an admission control algorithm was proposed. However, most of the aforementioned resource allocation algorithms are based on non-cooperative game, where the NEs are not always efficient, while cooperative bargaining game modeling [15]–[18] is more suitable for resource allocation in small cell networks. Moreover, most of the existing works do not consider the fairness for users in small cells.

Although some works have been done for fair resource allocation in cognitive radio [20] and femtocell networks [19], these papers mainly focus on the resource allocation with the assumption of perfect channel state information (CSI). However, in practice, it is difficult for cognitive small cell users to have perfect knowledge of a dynamic radio environment due to hardware limitations, short sensing durations and network connectivity issues in cognitive small cell networks. To the best of our knowledge, interference-aware resource allocation for small cell networks considering fairness, imperfect CSI and outage limitations has not been studied in previous works.

In this paper, we investigate the joint subchannel scheduling and power allocation problem for orthogonal frequency division multiple access (OFDMA) cognitive small cell networks based on a cooperative bargaining game model with consideration of fairness for users in each small cell, cross-tier interference limitation, QoS in terms of outage constraint, imperfect CSI and maximum power constraints. The main contributions of this paper are summarized as follows.

- We formulate the uplink subchannel and power allocation problem in cognitive small cells as a cooperative Nash bargaining game, where a cross-tier interference temperature limit is imposed to protect the primary macrocell, a minimum outage probability requirement is employed to provide reliable transmission for cognitive small cell users, a minimum rate requirement is considered to guarantee fairness for users in each small cell, and imperfect CSI is considered.
- We present a unified analytical framework for the optimization problem in cognitive small cell networks, where the near optimal cooperative bargaining resource allocation strategy is derived by introducing time-sharing variables and the Lambert-W function. The existence, uniqueness, and fairness of the solution to this game are proved analytically. Accordingly, a cooperative Nash bargaining resource allocation algorithm is developed, and is shown to converge to a Pareto-optimal equilibrium for the cooperative game.
- Small cells are enabled with cognitive capabilities, thus the spectrum sharing primary macrocell can be protected by cross-tier interference temperature. Moreover, imperfect CSI results in outage in the small cells. In our proposed joint power and subchannel allocation scheme, the achievable sum rate is maximized subject to not only the minimum data rate but also an acceptable outage probability in the cognitive small cells.
- The proposed algorithm is evaluated by extensive simulations, which show that the proposed cooperative bargaining resource allocation algorithm outperforms the existing centralized maximal rate (MR) approach, and round-robin (RR) fairness, giving a good trade-off between throughput and fairness.

The rest of the paper is organized as follows. Section II presents the system model and the problem formulation. Section III provides basics for the Nash bargaining solution (NBS) of cooperative game theory and the utility design of the cooperative game. Section IV provides the solutions and algorithm implementation of the cooperative bargaining game in small cell networks, while in Section V, performance of the proposed algorithms is evaluated by simulations. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As shown in Fig. 1, we consider an OFDMA cognitive small cell network where $K$ co-channel cognitive small base stations (CSBSs) are overlaid on a primary macrocell. We focus on resource allocation in the uplink of cognitive small cells. Let $M$ denotes the numbers of active primary macro users in the macrocell. Each small cell contains the same $F$ number of users. The OFDMA system has a bandwidth of $B$, which is divided into $N$ subchannels. The channel fading of each subcarrier is assumed the same within a subchannel, but may vary across different subchannels.

We denote $g_{k,i,n}$ and $g_{k',i,n}$ as the channel power gains on subchannel $n$ from cognitive small cell user $i$ in cognitive small cell $k$ to the primary MBS and CSBS $k'$, respectively, where $k, k' \in \{1, 2, ..., K\}$, $i \in \{1, 2, ..., F\}$, $n \in \{1, 2, ..., N\}$; denote $g_{k,j,n}^M$ as the channel power gain on subchannel $n$ from user $j \in \{1, 2, ..., M\}$ in the macrocell to CSBS $k$; denote $p_{k,i,n}^S$ and $p_{j,n}^M$ as cognitive user $i$'s transmit power on subchannel $n$ in cognitive small cell $k$ and primary macro user $j$'s power on subchannel $n$, and $\mathbf{P} = [p_{k,i,n}^S]_{K \times F \times N}$ is the power allocation matrix of the $K$ cognitive small cells. Denote $\mathbf{A} = [a_{k,i,n}]_{K \times F \times N}$ as the subchannel allocation matrix, where $a_{k,i,n} = 1$ means that subchannel $n$ is assigned to cognitive user $i$ in cognitive cell $k$, and $a_{k,i,n} = 0$ otherwise.

Then, the received SINR at the $k^{th}$ CSBS for cognitive small cell user $i$ occupying the $n^{th}$ subchannel is given by

$$\gamma_{k,i,n}^S = \frac{p_{k,i,n}^S g_{k,i,n}^S}{p_{j,n}^M g_{k,j,n}^M + \sigma^2}$$

(1)
where $I_{k,i,n}^{MMS} = P_{j,n}^{M} g_{k,j,n}^{M}$ is the co-channel interference caused by the primary macrocell, and $\sigma^2$ is the additive white Gaussian noise (AWGN) power. Note that in (1), co-channel interference between small cells is assumed as part of the thermal noise because of the severe wall penetration loss and low power of CSBSs [21]. This is particularly the case for sparse deployment of small cells in suburban environments [5], where co-tier inter-small cell interference is negligible as compared with cross-tier interference [22], [23].

Based on Shannon’s capacity formula, the achievable capacity of small cell user $i$ on subchannel $n$ in small cell $k$ is given by:

$$ C_{k,i,n}^S = \log_2 \left( 1 + \gamma_{k,i,n}^S \right) \text{ (bps/Hz)}. $$

(2)

The channel-to-interference-plus-noise ratio (CINR) is

$$ h_{k,i,n}^S = \frac{g_{k,i,n}^S}{I_{k,i,n} + \sigma^2} $$

(3)

where $I_{k,i,n} = I_{k,j,n}^{MMS}$. Therefore, eq. (2) can be rewritten as

$$ C_{k,i,n}^S = \log_2 \left( 1 + p_{k,i,n}^S h_{k,i,n}^S \right) \text{ (bps/Hz)}. $$

(4)

The $k^{th}$ small cell allocates the radio resources based on an imperfect estimation of the CINR $h_{k,i,n}^S$, where

$$ h_{k,i,n}^S = \hat{h}_{k,i,n}^S + \Delta_{k,i,n} $$

(5)

and where $\Delta_{k,i,n}$ is the channel estimation error, which is modeled as a zero-mean complex Gaussian random variable with variance $\delta_{k,i,n}$. $\Delta_{k,i,n}$ are independent and identically distributed (i.i.d.) for different subchannels, different users and different cognitive small cells. Assuming a minimum mean square error (MMSE) estimator, the CSI estimation error and the actual CSI are mutually uncorrelated [25], [24].

The achievable rate of user $i$ on subchannel $n$ in small cell $k$ can be defined as [32]

$$ r_{k,i,n}^S = \log_2 \left( 1 + p_{k,i,n}^S \hat{h}_{k,i,n}^S \right) \text{ (bps/Hz)}. $$

(6)

The outage probability [26] imposed by imperfect CSI for user $i$ on subchannel $n$ is defined as

$$ P_{outage_{k,i,n}} = \Pr \{ r_{k,i,n}^S \geq C_{k,i,n}^S \}. $$

(7)

Due to the fundamental role of primary macrocells in providing blanket cellular coverage, a macrocell users QoS should not be affected by small cell deployments. Therefore, to implement cross-tier interference protection, we impose an interference temperature limit to constrain the cross-tier interference suffered by primary MBS. Let $I_{n}^{th}$ denote the maximum tolerable interference level on subchannel $n$ for the primary macrocell, we have,

$$ \sum_{k=1}^{K} \sum_{i=1}^{F} a_{k,i,n} p_{k,i,n}^S g_{k,i,n}^{MMS} \leq I_{n}^{th}, \forall n. $$

(8)

**B. Problem Formulation**

In this paper, our target is to maximize the cognitive small cells’ utilities while protecting primary macrocells’ QoS. We assume that the cross-tier interference temperature limit is sent by a primary MBS periodically, which requires little overhead in the primary macrocell. In this case, the subchannel assignment and power control in primary macrocells are not part of the optimization. Thus, the corresponding joint subchannel scheduling and power allocation problem for uplink CSBS can be mathematically formulated as,

$$ \max_{\Delta \mathbf{S}} \sum_{k=1}^{K} U_k $$

(9)

s.t. $C_1: \sum_{n=1}^{N} a_{k,i,n} p_{k,i,n}^S \leq P_{\max}, \forall k, i$

$C_2: p_{k,i,n} \geq 0, \forall k, i, n$

$C_3: \sum_{k=1}^{K} \sum_{i=1}^{F} a_{k,i,n} p_{k,i,n}^S g_{k,i,n}^{MMS} \leq I_{n}^{th}, \forall n$

(10)

$C_4: \sum_{i=1}^{F} a_{k,i,n} \leq 1, \forall k, n$

$C_5: a_{k,i,n} \in \{ 0, 1 \}, \forall k, i, n$

$C_6: P_{outage_{k,i,n}} \leq \varepsilon, \forall k, i, n$

where $U_k$ is the objective function, which will be designed in Section III.B. Constraint $C_1$ limits the transmit power of each cognitive small cell user to be below $P_{\max}$; $C_2$ represents the non-negative power constraint of the transmit power on each subchannel; $C_3$ sets the tolerable interference temperature level on each subchannel of a primary macrocell; $C_4$ and $C_5$ are imposed to guarantee that each subchannel can only be assigned to at most one user in each cognitive small cell. $C_6$ expresses the outage probability constraint of each cognitive user in cognitive small cells, where $\varepsilon$ is the outage probability limit for user $i$ on subchannel $n$ in small cell $k$.

**III. GAME THEORETIC RESOURCE ALLOCATION IN SMALL CELL NETWORKS**

In this section, we briefly review the basic definition and concepts of cooperative bargaining games and their application in resource allocation problems. Then, the utility function is designed based on the bargaining games.
A. Basics of Bargaining Games

Let $\mathcal{K} = \{1, \ldots, k, \ldots, K\}$ be the set of players, which are the small cells in this paper. Let $S$ be the resource allocation strategy of the players, with $\mathbf{A}_k$ and $\mathbf{P}_k$ being the subchannel assignment space and the power allocation strategy space, respectively; Let $S_k$ be the resource allocation strategy of the player $k$; Let $U_k$ be the utility/payoff function of player $k$, and $U_k^{\min}$ is the minimum payoff that player $k$ expects, where $U_k^{\min}$ is defined as a minimum QoS requirement in terms of data rate. In a cooperative game, if the minimal payoff $U_k^{\min}$ is not achieved, player $k$ would not cooperate.

In non-cooperative games, players do not collaborate with one another. The stable solution for a non-cooperative game is the NE, if the NE exists and it is unique. A NE in a non-cooperative game is defined as,

$$U_k(S_k^{NE}, S_{-k}^{NE}) \geq U_k(S_k, S_{-k}^{NE}), \forall S_k$$

(11)

where $S_k^{NE}$ is the resource allocation strategy of player $k$ in NE, and $S_{-k}^{NE}$ is the strategy of the other $K - 1$ players under Nash Equilibrium except for player $k$. Nash Equilibrium is defined as the fixed points where no player can improve its utility by changing its strategy unilaterally [27].

1) Nash Bargaining Solutions: It is known that the NE in a non-cooperative game is not always efficient, that is, the strategy under NE may not be efficient. Therefore, we resort to cooperative bargaining games [29]. Let $\mathcal{U}$ be a closed and convex subset of $\mathcal{R}^N$ that represents the set of feasible payoff allocations that the players can get if they all cooperate. Suppose $\{U_k \in \mathcal{U} | U_k \geq U_k^{\min}, \forall k \in \mathcal{K}\}$ is a nonempty bounded set. Define $\mathcal{U} = (U_1^{\min}, \ldots, U_K^{\min})$, then the pair of $(\mathcal{U}, U^{\min})$ constructs a $K$-player bargaining game. Here, we define the Pareto efficient point [27], where a player can not find another point that improves the utility of all the players at the same time.

Definition 1: (Pareto Optimality) A point is said to be Pareto optimal if and only if there is no other allocation $\tilde{U}_k$ such that $U_k \geq U_k, \forall k \in \mathcal{K}$, and $U_k > U_k, \exists k \in \mathcal{K}$, i.e., there exists no other allocation that leads to superior performance for some players without causing inferior performance for some other players [27].

There may be an infinite number of Pareto optimal points in a game of multi-players. Thus, we must address how to select a Pareto point for a cooperative bargaining game. We need a criterion to select the best Pareto point of the system. A possible criterion is the fairness of resource allocation. Specially, the fairness of bargaining games is NBS, which can provide a unique and fair Pareto optimal point under the following axioms.

Definition 2: $\mathbf{u}$ is an NBS in $\mathcal{U}$ for $U^{\min}$, that is, $\mathbf{u} = f(\mathcal{U}, U^{\min})$, if the following axioms are satisfied [27].

1) Individual Rationality: $\tilde{U}_k \geq U_k^{\min}$, where $\tilde{U}_k \in \mathbf{u}, \forall k \in \mathcal{K}$.
2) Feasibility: $\mathbf{u} \in \mathcal{U}$.
3) Pareto Optimality: $\mathbf{u}$ is Pareto optimal.
4) Independence of Irrelevant Alternatives: If $\mathbf{u} \in \mathcal{U} \subset \mathcal{U}$, $\mathbf{u} = f(\mathcal{U}, U^{\min})$, then $\mathbf{u} = f(\mathcal{U}', U^{\min})$.

5) Independence of Linear Transformations: For any linear scale transformation $\psi$,

$$\psi(f(\mathcal{U}, U^{\min})) = f(\psi(\mathcal{U}), \psi(U^{\min})).$$

6) Symmetry: If $\mathcal{U}$ is invariant under all exchanges of players (small cells), $f_i(\mathcal{U}, U^{\min}) = f_j(\mathcal{U}, U^{\min}), \forall i, j$.

Axioms 1), 2) and 3) define the bargaining set $\mathcal{B}$. Hence, the NBS locates in the bargaining set. Axioms 4), 5), and 6) are called axioms of fairness. Axiom 5) ensures that the bargaining solution is scale invariant. The symmetry axiom 6) ensures that if the feasible ranges for all players are completely symmetric, then all users have the same solution. Axiom 6) implies that if players have the same QoS requirements and utility functions, they will have the same utility regardless of their indices. This represents an important fairness criterion for our cooperative game that gives incentives to players to collaborate, as they can rely on the network to treat them fairly when their utility-resource trade-offs vary over time.

B. Utility Design and Resource Allocation Game Formulation

The following theorem shows the existence and uniqueness of the NBS that satisfies the axioms 1)-6).

Theorem 1: There is a unique and fair solution function $f(\mathcal{U}, U^{\min})$ that satisfies all the axioms in Definition 2, and the solution can be obtained by

$$f(\mathcal{U}, U^{\min}) \in \arg \max_{U \in \mathcal{U}, U \geq U^{\min}} \prod_{k=1}^{K} (U_k - U_k^{\min}).$$

(12)

Proof: The proof of the theorem is omitted due to space limitations. A similar detailed proof can be found in [28].

Here, we relax $a_{k,i,n}$ to be a continuous real variable in the range $[0,1]$. In this case, $a_{k,i,n}$ can be interpreted as the fraction of time that subchannel $n$ is assigned to user $i$ in small cell $k$ during one transmission frame. We first introduce the following Lemma.

Lemma 1: Define $U_k = \sum_{i=1}^{F} \ln(V_{k,i}(S_{k,i})) = \sum_{i=1}^{F} \ln \left( \frac{R_{k,i} - R_{k,i}^{min}}{R_{k,i}^{min}} \right), \forall k \in \mathcal{K},$ where $V_{k,i}(S_{k,i}) = R_{k,i} - R_{k,i}^{min}$. The expression of $R_{k,i}$ is concave over $S_{k,i}, U_k$ will satisfy the Nash axioms required in Theorem 1.

Proof: As can be seen as a given condition, $R_{k,i}^{min}$ is concave over $S_{k,i}, \ln(V_{k,i}(S_{k,i}))$ is concave, and thus $U_k$ is also concave in $S_k$. Therefore, $U_k$ defined here can satisfy all the axioms in Definition 2 and Theorem 1.

Fig. 2 shows a simple example of a two-small-cell case, where $U_1$ and $U_2$ are the with different utilities of the two small cells [29]. Area $S$ is the feasible region for $U_1$, and $U_2$. When $U^{\min} = 0$, the objective function in (12) is reduced to $\prod_{k=1}^{K} (U_k - U_k^{\min})|_{U_k^{\min}=0,K=2} = U_1U_2 = \tilde{C}$, where $\tilde{C}$ is a constant. The optimal point of the NBS is $B$ at $(\tilde{U}_1, \tilde{U}_2)$. The physical meaning of this is that “after the small cells are
assigned with the minimal rate, the remaining resources are divided between the two small cells in a ratio equal to the rate at which the utility can be transferred.” [29]. The optimal point for the Max-Rate approach is at \((U_1^*, U_2^*)\), which is the tangent of line \(U_1 + U_2 = C^*\) and the feasible region \(S\). As it can be seen from Fig. 2, the sum rate of NBS solution, \(\bar{U}_1 + \bar{U}_2 = C_{NBS}\), is smaller than the Max-Rate approach, because of the tradeoff of sum rate and fairness in NBS. Moreover, \(C_{NBS}\) is much larger at \((U_1^*, U_2^*)\), which induces the most fair solution of \(U_1 = U_2\). That is, the NBS solution can well balance throughput and fairness.

According to Lemma 1 and Theorem 1, the unique Nash bargaining equilibrium with fairness can be found over the strategy space. By adopting the objective utility function in Lemma 1, the optimization problem in (9) under the constraints (10) can be rewritten as,

\[
\max_{A, Q} \sum_{k=1}^{K} \sum_{i=1}^{F} \ln \left( \left( \sum_{n=1}^{N} a_{k,i,n} r_{k,i,n}(q_{k,i,n}) \right) - R_{k,i}^{min} \right)
\]

\[\text{s.t. } C1, C2, C3, C4, C5, C6 \] (13)

It can be seen that the problem defined in (13) under the constraints of (14) is a non-convex mixed integer programming problem because of the discrete characteristics of the subchannel constraints in \(C4\) and \(C5\). The optimal solution can be obtained by exhaustive search, which has a high complexity. To reduce the complexity and meet the requirements in Definition 2, the optimization problem above should be transformed into a convex problem.

Before Lemma 1, we relax \(a_{k,i,n}\) to be a continuous real variable in the range \([0,1]\). This time-sharing relaxation was first proposed in [30]. After introducing the time-sharing method, the transformed optimization problem is regarded as a low bound of the original problem [30]. Time-sharing method has been widely used to transform non-convex combinatorial optimization problems into convex optimization problems for multiuser subchannel allocation in multichannel OFDMA systems [32]. According to [31], it is shown that the duality gap for a nonconvex optimization problem approaches zero in multichannel systems when the number of subchannels is large enough. In the real systems, it’s a typical configuration of 50 resource blocks (RBs) for LTE/LTE-Advanced. Similarly, in this paper, we assume there are 50 subchannels (\(N = 50\)) in our considered system. It’s large enough for the dual problem to have a near-zero-gap. For notational brevity, denote the actual power allocated to user \(i\) in cognitive small cell \(k\) on subchannel \(n\) as \(q_{k,i,n} = a_{k,i,n} p_{k,i,n}^{S}\). Similarly, denote \(I_{k,i,n} = p_{j,n}^{S} g_{k,j,n}^{S} + \sigma^2\) and \(r_{k,i,n}^{S} = \log_2 \left( 1 + \frac{q_{k,i,n} g_{k,i,n}^{S}}{a_{k,i,n} r_{k,i,n}^{S}} \right)\) the received interference power and capacity of user \(i\) on subchannel \(n\) in small cell \(k\), respectively. Now, the problem (13) subject to the constraints in (14) can be converted into

\[
\max_{A, Q} \sum_{k=1}^{K} \sum_{i=1}^{F} \ln \left( \left( \sum_{n=1}^{N} a_{k,i,n} r_{k,i,n}(q_{k,i,n}) \right) - R_{k,i}^{min} \right)
\]

\[\text{s.t. } C1 : \sum_{n=1}^{N} q_{k,i,n} \leq P_{max}, \forall k, i
\]

\[C2 : p_{k,i,n}^{S} \geq 0, \forall k, i, n
\]

\[C3 : \sum_{k=1}^{K} \sum_{i=1}^{F} q_{k,i,n} g_{k,i,n}^{S} \leq I_{th}, \forall n
\]

\[C4 : \sum_{i=1}^{F} a_{k,i,n} \leq 1, \forall k, n
\]

\[C5 : 0 \leq a_{k,i,n} \leq 1, \forall k, i, n
\]

\[C6 : P_{outage_{k,i,n}} \leq \varepsilon, \forall k, i, n
\]

where \(Q = [q_{k,i,n}]_{K \times F \times N}\).

Theorem 2: The problem in (15) under the constraints (16) is a convex optimization problem.

Proof: It can easily be proved that the Hessian matrix of (15) over \(a_{k,i,n}\) and \(q_{k,i,n}\) is negative semidefinite, thus, the objective function of (15) is concave. Moreover, the feasible set of the objective function in (15) is convex, and the corresponding optimization problem is a convex optimization problem.

Therefore, there is a unique optimal solution of problem (15) under the constraints (16), because the problem and its feasible set are convex.

Theorem 3: The utility function in (15) meets the Nash bargaining axioms defined in Definition 2, and the NBS is reduced into proportional fairness, when \(R_{k,i}^{min} = 0\) in the utility function of (15).

Proof: Since the objective function in (15) is concave and injective, it meets all the Nash Bargaining axioms defined in Definition 2. When \(R_{k,i}^{min} = 0\), the utility function in (15) can be written as \(\prod_{k=1}^{K} \ln(R_{k,i} - R_{k,i}^{min})|_{R_{k,i}^{min}=0} = \prod_{k=1}^{K} \ln(R_{k,i})\).

When \(R_{k,i}^{min} = 0\), the NBS is the same as proportional fairness, which requires that \(\prod_{k=1}^{K} \frac{(R_{k,i} - R_{k,i}^{min})}{R_{k,i}} \geq 0\) for the interested
utility \( R_{k,i}, \forall i \).

IV. BARGAINING RESOURCE ALLOCATION SOLUTIONS FOR SMALL CELL NETWORKS

We first substitute (4) and (7) into C6 and have,

\[
\Pr \{ r_{k,i,n}^s \geq \log_2 (1 + p_{k,i,n}^s h_{k,i,n}^s) \} \leq \varepsilon
\]  

(17) can be rewritten as

\[
\Pr \left\{ h_{k,i,n}^s \leq \frac{2r_{k,i,n}^s - 1}{p_{k,i,n}^s} \right\} \leq \varepsilon.
\]  

(18)

and (17) can be rewritten as

\[
\Pr \left\{ h_{k,i,n}^s \leq \frac{2r_{k,i,n}^s - 1}{p_{k,i,n}^s} \right\} \leq \varepsilon.
\]  

(19)

Here, we assume that \( h_{k,i,n}^s \) is a non-central chi-squared distributed random variable [33]; Following the simplification used in many previous works on the effects of imperfect CSI [25], we only consider the case of \( P_{\text{outage}} = \varepsilon \), and have

\[
F_{h_{k,i,n}^s} \left( \frac{2r_{k,i,n}^s - 1}{p_{k,i,n}^s} \right) = \varepsilon.
\]  

(20)

where \( F_{h_{k,i,n}^s} \) is the CDF of \( h_{k,i,n}^s \). Therefore, the data rate that satisfies the outage probability requirement can be given as

\[
r_{k,i,n}^s = \log_2 \left( 1 + p_{k,i,n}^s F_{h_{k,i,n}^s}^{-1} (\varepsilon) \right).
\]  

(21)

Substituting (20) into (15), we transform the optimization problem in (15)-(16) into

\[
\max_{\lambda, \mu, \eta} \sum_{k=1}^{K} \left( \sum_{i=1}^{F} \sum_{n=1}^{N} a_{k,i,n} \log_2 \left( 1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1} (\varepsilon) - R_{k,i}^{\min} \right) \right),
\]  

(22)

s.t. \( C1, C2, C3, C4, C5 \)

Since the optimization problem in (15)-(16) is convex, the transformed problem in (21) and (22) is also convex.

A. Solution of the Cooperative Resource Allocation Game

The solution gap between the primal problem and its dual problem can be considered zero for most engineering problems. Since the duality gap is zero, we can solve the problem in the dual domain. Moreover, the system considered here is a multi-subchannel network; therefore, dual decomposition can be an effective method. The Lagrangian function of the primal problem in (21)-(22) is given by (23) at the top of next page. Where \( \lambda = [\lambda_{k,i}]_{K \times F} \) is the Lagrange multipliers corresponding to the joint power constraint, and \( \mu = [\mu_n]_{N \times 1} \) and \( \eta = [\eta_k]_{K \times N} \) are Lagrange multipliers vectors associated with the cross-tier interference limit and subchannel usage constraints, respectively.

Thus, the dual problem is given by

\[
\min_{\lambda, \mu, \eta \geq 0} \Xi(\lambda, \mu, \eta)
\]  

(24)

where the dual function \( \Xi(\lambda, \mu, \eta) \) can be given as

\[
\Xi(\lambda, \mu, \eta) = \max_{A, Q} \mathcal{L}(A, Q, \lambda, \mu, \eta)
\]  

(25)

where \( \Psi(A, Q, \lambda, \mu, \eta) \) for a fixed set of Lagrange multipliers \( \lambda, \mu, \eta \) is given by (26) at the top of next page. Based on standard optimization techniques and the Karush-Kuhn-Tucker (KKT) conditions [34], the power allocation for user \( i \) in small cell \( k \) on subchannel \( n \) is obtained by taking the first derivative of (26) with respect to \( q_{k,i,n} \), which can be given as

\[
\frac{\partial \Psi(Q, \lambda, \mu)}{\partial q_{k,i,n}} = \frac{F_{h_{k,i,n}^s}^{-1}(\varepsilon) \ln 2 (1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1}(\varepsilon))}{\log_2 (1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1}(\varepsilon)) - R_{k,i}^{\min}} - \lambda_{k,i} - \mu_n g_{k,i,n}.
\]  

(26)

According to the KKT conditions,

\[
\frac{F_{h_{k,i,n}^s}^{-1}(\varepsilon) \ln 2 (1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1}(\varepsilon))}{\log_2 (1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1}(\varepsilon)) - R_{k,i}^{\min}} - \lambda_{k,i} - \mu_n g_{k,i,n}.
\]  

(27)

Note that (28) is a transcendental algebraic equation over \( q_{k,i,n} \), which can be solved by recursive numerical methods. The solution of (28) can be obtained as follows. Let

\[
\Lambda_{k,i,n} = 1 + \frac{q_{k,i,n}}{a_{k,i,n}} F_{h_{k,i,n}^s}^{-1}(\varepsilon),
\]  

(29)

and

\[
\Gamma_{k,i,n} = \frac{F_{h_{k,i,n}^s}^{-1}(\varepsilon)}{\ln 2 (\lambda_{k,i} + \mu_n g_{k,i,n})}.
\]  

(30)

Substituting (29) and (30) into (28), we get

\[
\Lambda_{k,i,n} \log_2 (\Lambda_{k,i,n}) - R_{k,i}^{\min} = \frac{\Gamma_{k,i,n}}{a_{k,i,n}}.
\]  

(31)

Letting \( \Upsilon = 2^{R_{k,i}^{\min}} \), we have

\[
\Lambda_{k,i,n} \log_2 \left( \frac{\Lambda_{k,i,n}}{\Upsilon} \right) = \frac{\Gamma_{k,i,n}}{a_{k,i,n}}.
\]  

(32)

Multiplying both sides of (32) with \( \frac{a_{k,i,n}}{\Upsilon} \), we have

\[
\log_2 \left( \frac{\Lambda_{k,i,n}}{\Upsilon} \right) = \frac{\Gamma_{k,i,n}}{a_{k,i,n}} \Upsilon.
\]  

(33)

Letting \( \varphi = \frac{\Lambda_{k,i,n}}{\Upsilon} \), we can get

\[
\varphi \varphi = 2^{\frac{\Gamma_{k,i,n}}{a_{k,i,n}}}.
\]  

(34)

Using the Lambert-W function properties, \( \varphi \) can be given as

\[
\varphi = \exp \left( W \left( \frac{\Gamma_{k,i,n}}{a_{k,i,n}} \right) \right)
\]  

(35)
where \( W(\cdot) \) is the Lambert’s \( W \) function given by \( W(\cdot) = \sum_{i=1}^{+\infty} \left( (-i)^{i-1} e^{1/i} \right)^i \). Substituting \( \varphi = \frac{\lambda_k,i,n}{\mu_n} \) and (29) into (35), we can obtain  

\[
1 + \frac{q_{k,i,n}}{a_{k,i,n}} F^{-1}_k h_m (\varepsilon) = Y \exp \left( W \left( \ln \left( \frac{r_{k,i,n} \eta S}{a_{k,i,n}} \right) \right) \right).
\]

Therefore, given the optimal subchannel allocation \( \hat{a}_{k,i,n} \), the optimal power allocation \( \hat{p}_{k,i,n}^S \) can be obtained as  

\[
\hat{p}_{k,i,n}^S = \frac{\hat{q}_{k,i,n}}{\hat{a}_{k,i,n}} = \frac{1}{F^{-1}_k h_m (\varepsilon)} \left( Y \exp \left( W \left( \ln \left( \frac{r_{k,i,n} \eta S}{\hat{a}_{k,i,n}} \right) \right) \right) \right)^{-1}
\]

where \( (x)^+ = \max(0, x) \).

Given the optimal power allocation, the first derivative of (26) with respect to \( a_{k,i,n} \) is given as  

\[
\frac{\partial \Psi(A, \lambda, \mu)}{\partial a_{k,i,n}} = H_{k,i,n} - \eta_{k,i,n}
\]

where

\[
H_{k,i,n} = \frac{\log_2 \left( 1 + \frac{\hat{p}_{k,i,n}^S}{\hat{a}_{k,i,n}} F^{-1}_k h_m (\varepsilon) \right)}{\log_2 \left( 1 + \frac{\hat{p}_{k,i,n}^S}{\hat{a}_{k,i,n}} F^{-1}_k h_m (\varepsilon) \right) - R_{k,i,n}^m} - \lambda_{k,i,n} \hat{p}_{k,i,n}^S - \mu_n \hat{p}_{k,i,n}^S g_{k,i,n}^M.
\]

\[
\hat{a}_{k,i,n} = \underset{a_{k,i,n}}{\text{max}} \left\{ \frac{\lambda_{k,i,n}}{\mu_n} \right\} \quad \text{subject to} \quad \sum_{a_{k,i,n}} = 1
\]

Subchannel \( n \) is assigned to the user with the largest \( H_{k,i,n} \) in small cell \( k \) [30] [32]; that is,  

\[
\hat{a}_{k,i,n} = 1 \mid \hat{a}_{k,i,n} = \text{arg max} H_{k,i,n}, \forall k, n.
\]

B. Update of the Dual Variables

Both the ellipsoid and subgradient method can be adopted in the update of dual variables [34]. Here, we choose the subgradient method to update the dual variables, as formulated in Lemma 2.

**Lemma 2:** The subgradient of \( \lambda_{k,i} \) and \( \mu_n \) are respectively given by

\[
\lambda_{k,i}^{(l+1)} = \left[ \lambda_{k,i}^{(l)} - \beta_1^{(l)} \left( P_{max} - \sum_{n=1}^{N} \hat{p}_{k,i,n}^S \right) \right]^+, \forall k, i
\]

and

\[
\mu_n^{(l+1)} = \left[ \mu_n^{(l)} - \beta_2^{(l)} \left( I_{k,i,n}^h - \sum_{k=1}^{K} \sum_{i=1}^{F} \hat{p}_{k,i,n}^S g_{k,i,n}^M \right) \right]^+, \forall n
\]

**Proof:** The proof is provided in Appendix A. \(\blacksquare\)
where $\beta_1^{(l)}$ and $\beta_2^{(l)}$ are the step sizes of iteration $l (l \in \{1, 2, \ldots, L_{\text{max}}\})$, $L_{\text{max}}$ is the maximum number of iterations, and the step sizes should satisfy the condition,

$$\sum_{l=1}^{\infty} \beta_1^{(l)} = \infty, \lim_{l \to \infty} \beta_1^{(l)} = 0, \forall l \in \{1, 2\}. \quad (45)$$

C. Cooperative Bargaining Resource Scheduling Algorithm

Although (37), (40), (43)-(44) give a solution to the joint subchannel scheduling and power allocation problem of (13)-(14), it still remains to design an algorithm to provide the execution structure and the executing entity for the equations. Therefore, we propose Algorithm 1 as an implementation of our cooperative bargaining resource scheduling solution. The proposed iterative Algorithm 1 will guarantee convergence by using the subgradient method.

Algorithm 1 Cooperative Bargaining Resource Scheduling Algorithm

1: Initialize $I_{\text{max}}$ and Lagrangian variables vectors $\lambda, \mu$, set $i = 0$
2: Initialize $p_{k,i,n}$ with an uniform power distribution among all subchannels
3: Initialize $a_{k,i,n}$ with subchannel allocation method in [38], $\forall k, i, n$
4: repeat
5: for $k = 1$ to $K$ do
6: for $n = 1$ to $N$ do
7: for $i = 1$ to $F$ do
8: a) Cognitive users update $\hat{S}_{k,i,n}$ according to (37);
9: b) Calculate $H_{k,i,n}$ according to (39);
10: c) CSBS updates $\hat{a}_{k,i,n}$ according to (40);
11: d) CSBS updates $\lambda$ according to (43);
12: end for
13: end for
14: MBS updates $\mu$ according to (44), and broadcasts those values to all CSBSs via backhaul link, $l = l + 1$.
15: until Convergence or $l = L_{\text{max}}$

Note that $g_{k,i,n}^{S}$ required in Algorithm 1 can be estimated at cognitive user $i$ in small cell $k$ by measuring the downlink channel power gain of subchannel $n$ from the macrocell and utilizing the symmetry between uplink and downlink channels, or by using site specific knowledge [35]. Furthermore, it can be assumed that there is a direct wire connection between a CSBS and the MBS for the CSBS to coordinate with the central MBS [5], [36], according to a candidate scheme proposed for 3GPP small cell mobility enhancement [37].

Algorithm 1 can be implemented by each CSBS utilizing only local information and limited interaction with the MBS; therefore, Algorithm 1 is distributed and the practicality is ensured.

V. SIMULATION RESULTS AND DISCUSSION

In this section, simulation results are given to evaluate the performance of the proposed algorithms. In the simulations, spectrum-sharing CSBSs and primary users are randomly distributed in the range of MBS, and cognitive small cell users are uniformly distributed in the coverage area of their serving small cell; the carrier frequency is 2 GHz, $B = 10$ MHz, $N = 50$, $M = 50$, and $\sigma^2 = B N_0$, where $N_0 = -174$ dBm/Hz is the AWGN power spectral density. The CINR $h_{k,i,n}^S$ is assumed as a non-central chi-squared distributed random variable. The primary macro users’ maximum transmit powers are set at 30 dBm. The coverage radius of the MBS is 500 m, and that of a small cell is 10 m. We assume that all small cell users have the same QoS requirement.

In Fig. 3, the convergence of Algorithm 1 is evaluated with the outage probability constraint $\varepsilon = 0.01$, the cross-tier interference limit $I_{th} = 7.5 \times 10^{-14}$ W, minimum data rate requirement $P_{th}^{\text{min}} = 0.5 bps/Hz$, the variance of channel estimate error $\delta_{k,i,n} = 0.05$, number of users per small cell $F = 4$ and maximal transmit power of small cell user $P_{\text{max}} = 20$ dBm. As can be seen from Fig. 3, the total capacity of the small cells converges after 30 iterations. This result, together with the previous analysis, indicates that the proposed Algorithm 1 converges in heterogeneous networks.

Fig. 4 shows the total uplink capacity of $K$ small cells

![Fig. 3. Convergence of the proposed Algorithm 1.](image)

![Fig. 4. Total capacity of small cells vs variance of estimation error.](image)
when the variance of estimation error $\delta_{k,i,n}$ increases from 0.01 to 0.2 for all $k, i, n$ with the user number per small cell $F = 2, 4, 6$. The simulation parameters are set as $K = 10$, $R_{k,i}^{\min} = 0.5$ bps/Hz for all $k, i$, $\varepsilon = 0.01$, $P_{\max} = 20$ dBm and $I_{th} = 7.5 \times 10^{-14}$ W (-101.2 dBm) for all $n$. The capacity of all small cells decreases with the increase of the variance of estimation error, because of the imperfect estimation of CSI. We also observe that a higher capacity is obtained with a larger number of $F$ because of the multi-user diversity.

for the outage probability limit $\varepsilon = 0.05, 0.1, 0.2$. The other simulation parameters are set as $K = 10$, $R_{k,i}^{\min} = 0.5$ bps/Hz for all $k, i$, $\delta_{k,i,n} = 0.05$ for all $k, i, n$, $P_{\max} = 20$ dBm and $I_{th} = 7.5 \times 10^{-14}$ W (-101.2 dBm) for all $n$. The capacity of all small cells increases with the increase of $F$ because of the multi-user diversity. Similar to the results in Fig. 5, a higher outage probability limit $\varepsilon$ induces a higher total capacity of small cells.

Fig. 5 shows the total uplink capacity of $K$ small cells when the number of small cells $K$ increases from 10 to 40, for the outage probability constraint $\varepsilon = 0.01, 0.05, 0.1$. The simulation parameters are set as $F = 4$, $R_{k,i}^{\min} = 0.5$ bps/Hz for all $k, i$, $\delta_{k,i,n} = 0.05$ for all $k, i, n$, $P_{\max} = 20$ dBm and $I_{th} = 7.5 \times 10^{-14}$ W (-101.2 dBm) for all $n$. The total capacity of all small cells increases with the increase of the number of small cells. It also can be seen from Fig. 5 that a higher outage probability limit $\varepsilon$ induces a higher total capacity of small cells, because a larger value of $\varepsilon$ enlarges the feasible region of the variables in the original problem defined in (13)-(14), etc.

Fig. 6 shows the total uplink capacity of $K$ small cells when the number of users per small cell $F$ increases from 2 to 6, for the outage probability limit $\varepsilon = 0.05, 0.1, 0.2$. The other simulation parameters are set as $K = 10$, $R_{k,i}^{\min} = 0.5$ bps/Hz for all $k, i$, $\delta_{k,i,n} = 0.05$ for all $k, i, n$, $P_{\max} = 20$ dBm and $I_{th} = 7.5 \times 10^{-14}$ W (-101.2 dBm) for all $n$. The capacity of all small cells increases with the increase of $F$ because of the multi-user diversity. Similar to the results in Fig. 5, a higher outage probability limit $\varepsilon$ induces a higher total capacity of small cells.

Fig. 7 shows the total uplink capacity of $K$ small cells when the minimum rate requirement of each cognitive user $R_{k,i}^{\min}$ increases from 0.2 bps/Hz to 1 bps/Hz, for the outage probability limit $\varepsilon = 0.01, 0.05, 0.2$. The other simulation parameters are set as $K = 10$, $\delta_{k,i,n} = 0.05$ for all $k, i, n$, $P_{\max} = 20$ dBm and $I_{th} = 7.5 \times 10^{-14}$ W (-101.2 dBm) for all $n$. The capacity of all small cells increases with increasing $R_{k,i}^{\min}$ because larger $R_{k,i}^{\min}$ enlarges the feasible region of the optimizing variable. Similar to the results in Fig. 5 and Fig. 6, a higher outage probability limit $\varepsilon$ induces a higher total capacity of small cells. In this figure, we also compare the proposed Algorithm 1 with existing cooperative resource allocation methods when the CSI is perfectly known. The existing scheme is composed of cooperative power allocation in [39] and subchannel allocation in [38]. As can be seen from Fig. 7, proposed Algorithm 1 has better performance in terms of capacity than the existing schemes with the assumption of perfect CSI. With the assumption of perfect CSI, Fig. 7 also showed about 1% 2% performance loss from the original solution of (13) to the convex optimization solution of (15), where the original solution is solved by exhaustive method with high complexity.

Fig. 8 shows the total uplink capacity of $K$ small cells when the maximum transmit power per cognitive small cell user $P_{\max}$ increases from 25 dBm to 40 dBm, for the interference temperature limit $I_{th} = 7.5 \times 10^{-13}$ W, $7.5 \times 10^{-14}$ W, $7.5 \times 10^{-15}$ W. The other simulation parameters are set as $K = 10$, $R_{k,i}^{\min} = 0.5$ bps/Hz for all $k, i$, $\delta_{k,i,n} = 0.05$ for all $k, i, n$, and $P_{\max} = 20$ dBm. The total capacity of the small cells increases with the increase of $P_{\max}$, because higher $P_{\max}$ enlarges the feasible region $S$. It also can be seen
algorithms. 

evaluate the level of fairness achieved by resource allocation

This fairness index is widely applied in the literature to

ness index among the three schemes. Here, the centralized MR approach, while round-robin(RR) method has the highest fair-

1 achieves a higher fairness index than the centralized MR approach, while round-robin(RR) method has the highest fair-

from the figure that higher interference temperature limit $I_{n}^{th}$ induces higher total capacity of the small cells.

In order to evaluate the fairness of users in small cells, we use the fairness index (FI) [40], which is defined as

$$
(\sum_{k=1}^{K} \sum_{i=1}^{F} \left( \frac{R_{k,i}}{R_{k,i}^{\text{min}}} \right)^{2})^{2} / \left(KF \left(\sum_{k=1}^{K} \sum_{i=1}^{F} \left( \frac{R_{k,i}}{R_{k,i}^{\text{min}}} \right)^{2}\right)\right).
$$

This fairness index is widely applied in the literature to evaluate the level of fairness achieved by resource allocation algorithms.

Fig. 9 shows the fairness among all the users in $K$ small cells when the number of small cells increases from 10 to 50, with the outage probability limit $\varepsilon = 0.01$, $I_{n}^{th} = 7.5 \times 10^{-13}$ W, $R_{k,i}^{\text{min}} = 1\text{bps/Hz}$, $F = 4$, $\delta_{k,i,n} = 0.05$ for all $k,i,n$, and $P_{\text{max}} = 30\text{dBm}$. As can be seen from the figure, Algorithm 1 achieves a higher fairness index than the centralized MR approach, while round-robin(RR) method has the highest fairness index among the three schemes. Here, the centralized MR approach is composed by exhaustive method of subchannel allocation and waterfilling power allocation. RR method is composed of round-robin subchannel and the proposed power allocation proposed in this paper. Therefore, it’s verified that the cooperative bargaining resource scheduling algorithm can achieve higher fairness index with the cost of small reduction in throughput, compared with the centralized MR approach.

![Fig. 8. Total capacity of small cells vs maximum transmit power of each user.](image)

![Fig. 9. Fairness index vs number of small cells.](image)

![Fig. 10. Capacity vs number of small cells.](image)
VI. CONCLUSION

In this paper, we have investigated the joint subchannel and power allocation problem in cognitive small cell networks. The resource allocation problem was formulated as a cooperative Nash bargaining game, where a cross-tier interference temperature limit is imposed to protect the primary macrocell, a minimum outage probability requirement is employed to provide reliable transmission for cognitive small cell users, a minimum rate requirement is considered to guarantee intra-small cell fairness, and imperfect CSI is considered in the analysis and algorithm design. The near optimal cooperative bargaining resource allocation solutions are derived by relaxing subchannel allocation variables and using the Lambert-W function. The existence, uniqueness, and fairness of the solution to this game model were proved analytically. Accordingly, a cooperative Nash bargaining resource allocation algorithm was developed, and was shown to converge to a Pareto-optimal equilibrium for the cooperative game. Simulation results showed that the proposed algorithms not only converge within a few iterations, but also achieve a better trade-off between capacity and fairness than the existing algorithms.

APPENDIX A

PROOF OF LEMMA 2

Based on (25), we can get (47) on the top of this page. From (29), we get (48) on the top of this page. Therefore, we have

\[
\Xi(\lambda', \mu') \geq \sum_{k=1}^{K} \sum_{i=1}^{F} \left( \lambda_{k,i} - \lambda_{k,i} \right) P_{max} - \sum_{n=1}^{N} \tilde{q}_{k,i,n} + \sum_{n=1}^{N} \left( \mu_{n} - \mu_{n} \right) \left( I_{n}^{th} - \sum_{k=1}^{K} \sum_{i=1}^{F} g_{k,i,n}^{MS} \right) + \Xi(\lambda, \mu).
\]

Eq. (49) verifies the definition of subgradient and completes the proof.

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