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Article:

Outage Probability for Multi-hop D2D Communications with Shortest Path Routing

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Abstract—In this letter, we provide a tractable theoretical framework for analysing the performance of device-to-device (D2D) communications in the presence of co-channel interference from other D2D and conventional cellular (CC) transmissions. In particular, we consider multi-hop D2D using the shortest-path-routing (SPR) algorithm in both the uplink (UL) and the downlink (DL) channels. Closed-form expressions for the number of hops and the outage probability are presented. Our analytical and numerical results both show that while the D2D links are reasonably reliable (outage probability <5%), they can severely degrade the performance of CC transmissions (outage probability >25%). Accordingly, we exploit and provide insights into the trade-off between D2D and CC transmission reliability.

I. INTRODUCTION

In view of the proliferation of smart user equipments (UEs), D2D communications underlaying cellular networks have emerged as a promising capacity enhancement technology. Due to the scarcity of the cellular spectrum, D2D links typically share the same frequency resources with CC communications, which operate between the base stations (BSs) and UEs. Despite recent progress in D2D research, there is a lack of tractable theoretical frameworks for analysing multi-hop D2D communications underlaying cellular networks. The majority of existing analysis is based either on simulation [1]–[3] or simple geometric abstractions [4]. Thus far, stochastic geometry based analysis has only considered a single-hop D2D scenario [5]. The more general scenario of multi-hop D2D with partner selection for routing in a particular direction has been neglected. In this letter, our main contributions are two fold: (i) we present a tractable theoretical framework that enables comprehensive modelling of interference and performance analysis for multi-hop D2D communications with SPR [6]; and (ii) we exploit and provide insights into the trade-off between D2D and CC transmission reliability, which can result from the sharing of either the UL or DL cellular radio resources by D2D UEs.

II. SYSTEM SETUP

We consider Long Term Evolution-Advanced (LTE-A) networks with an orthogonal frequency division multiple access (OFDMA) DL and a single-carrier frequency division multiple access (SC-FDMA) UL, operating with frequency division duplexing (FDD). As shown in Fig. 1, some UEs can act

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as a non-cooperative decode-and-forward (DF) relay for D2D communications, which can choose to use either DL or UL bands (but not dynamic). Hence, there are two different transmission tiers in *co-existence*: (i) CC which uses both the UL and the DL bands, and (ii) D2D which can use either the UL or the DL bands. The signal-to-interference-plus-noise ratio (SINR) of a link from UE n to UE m is given by

$$\gamma_{n,m} = \frac{h_{n,m}P_n\lambda r_{n,m}^{-\alpha}}{\sigma^2 + \sum_{\substack{t\in\Phi_m \\ t\neq n}} h_{t,m}P_t\lambda r_{t,m}^{-\alpha}}, \tag{1}$$

where $h_{n,m}$ and $r_{n,m}$ are the fading power gain and distance of the link from UE n to UE m, respectively, P_n is the transmit power of UE n, λ is the frequency dependent pathloss, α is the pathloss distance exponent, Φ_m denotes the set of interferers to UE m, and σ^2 is the additive white Gaussian noise (AWGN) power. Hereafter, we assume that the AWGN power is negligible as compared to the interference power and that the traffic follows a full buffer model.

III. CC OUTAGE PROBABILITY

We first present a brief review of the stochastic geometry framework for CC communications, where all the BSs are distributed randomly and uniformly. This framework will be enhanced to encompass multi-hop D2D in next section. Assuming that each UE is served by the closest BS, the probability density function (PDF) of the distance r between a UE and its serving BS can be derived as a 2-D Poisson process, i.e., $f_R(r) = 2\Lambda\pi r\exp(-\Lambda\pi r^2)$, where Λ is the BS density. We consider two arbitrarily located UEs m and m' who wish to communicate with each other. The distances of UE m and UE m' to their serving BS are $r_{m,n}$ and $r_{n,m'}$, respectively. The density of co-channel BSs is Λ_{BS}, and the density of co-channel UEs is Λ_{CC}, which we assume to be of the same density and distribution as co-channel BSs, i.e., no intra-cell UL or DL interference, as shown in (1). The outage probability of a link is defined as the probability that the average received SINR ($\overline{\gamma}$) falls below a threshold (ζ), i.e., $\mathbb{P}(\overline{\gamma} < \zeta)$. For DL transmissions, the interference comes from adjacent BSs. For UL transmissions, the interference is caused by UEs served by neighboring BSs. Assuming a Rayleigh fading channel, the probability of successful transmission in the UL and the DL is respectively given by:

$$\mathbb{P}(\overline{\gamma}_{m,n}^{\mathrm{UL}} > \zeta) = e^{-\Lambda_{\mathrm{CC}}\pi r_{m,n}^2\mathcal{A}(\zeta,\alpha)}$$
$$\mathbb{P}(\overline{\gamma}_{n,m'}^{\mathrm{DL}} > \zeta) = e^{-\Lambda_{\mathrm{BS}}\pi r_{n,m'}^2\mathcal{A}(\zeta,\alpha)}, \tag{2}$$

a) Uplink Band: Additional Interference to and from CC Channels

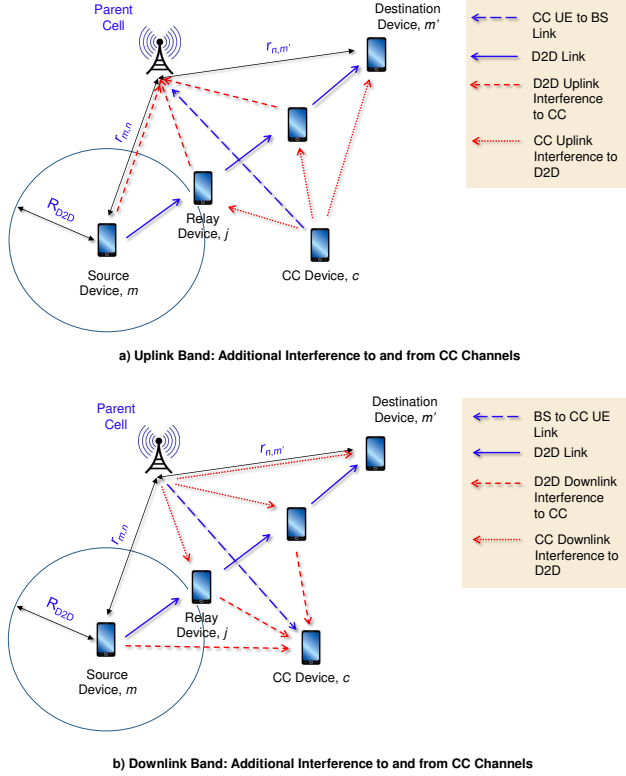b) Downlink Band: Additional Interference to and from CC Channels

Fig. 1. Illustration of SPR between UE m and m'. The additional interference caused to CC UEs is shown for UL and DL radio resources.

where $\mathcal{A}(\zeta, \alpha) = \int_{\zeta^{-2/\alpha}}^{+\infty} \frac{\zeta^{2/\alpha}}{1+u^{\alpha/2}} \, du$. For $\alpha = 4$, $\mathcal{A}(\zeta, 4) = \sqrt{\zeta} \arctan(\sqrt{\zeta})$. Therefore, the outage probability of the CC communication between UE m and UE m' is given by (conditioned on $\Lambda_{\rm CC} = \Lambda_{\rm BS}$):

$$\begin{aligned}
{\rm P}_{\rm CC,out} &= 1 - \mathbb{P}(\overline{\gamma}_{m,n} > \zeta)\mathbb{P}(\overline{\gamma}_{n,m'} > \zeta) \\
&= 1 - e^{-\Lambda_{\rm BS}\pi(r_{m,n}^2 + r_{n,m'}^2)\mathcal{A}(\zeta,\alpha)}.
\end{aligned} \quad (3)$$

The outage probability averaged over all possible UE locations is obtained as:

$$\mathbb{E}_R[{\rm P}_{\rm CC,out}] = 1 - \left[\frac{1}{1+\mathcal{A}(\zeta,\alpha)}\right]^2. \quad (4)$$

IV. D2D OUTAGE PROBABILITY

A. Single Hop

D2D communications underlaying cellular networks can be considered as a temporary 2-tier heterogeneous network. Generally, in a K-tier network with spatial Poisson point process (SPPP) intensities Λ_k ($k = 1, ..., K$), the coverage probability at a distance r from a transmitting node of the $i^{\rm th}$-tier is given by [7]:

$$\mathbb{P}(\overline{\gamma}_i > \zeta, r) = \exp\left[-\sum_{k=1}^{K} \Lambda_k \left(\frac{P_k}{P_i}\right)^{\frac{2}{\alpha}} \pi r^2 \mathcal{A}(\zeta,\alpha)\right], \quad (5)$$

where P_i and P_k are the transmit powers for the $i^{\rm th}$- and $k^{\rm th}$-tier, respectively.

1) UL: As shown in Fig. 1a), we consider the D2D transmission from one UE to another UE using UL radio resources. Let there be $N_{\rm D2D}$ UEs utilizing the UL band in a circular
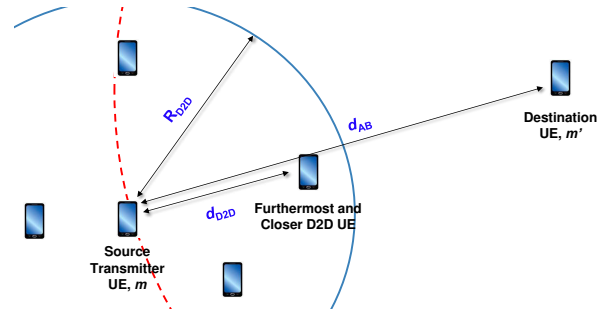

Fig. 2. A D2D transmitter with a coverage range of $R_{\rm D2D}$. For $N_{\rm D2D} - 1$ other potential D2D receivers that are closer to the destination, the furthermost receiver is selected and it is of distance $d_{\rm D2D}$ away.

area of radius R, such that the density of D2D UEs that are in transmission is $\Lambda_{\rm D2D} = N_{\rm D2D}/(\pi R^2)$. As before, we note that there is only one D2D and one CC transmission per band per BS (i.e., $\Lambda_{\rm D2D} = \Lambda_{\rm CC}$). By using (5), the outage probability of a single hop D2D link using UL resources averaged over distances is given by:

$$\begin{aligned}
\mathbb{E}_R({\rm P}_{\rm D2D,out}^{\rm UL}) &= 1 - \int_0^{+\infty} \mathbb{P}(\overline{\gamma}_{\rm D2D}^{\rm UL} > \zeta, r) f_R(r) \, dr \\
&= 1 - 2\pi(N_{\rm D2D} - 1)\Lambda_{\rm D2D} \\
&\quad \times \int_0^{+\infty} r e^{-\left[\Lambda_{\rm D2D}+\Lambda_{\rm CC}\left(\frac{P_{\rm CC}}{P_{\rm D2D}}\right)^{\frac{2}{\alpha}}\right]\pi r^2 \mathcal{B}(\zeta,\alpha)} \, dr \\
&= 1 - \frac{N_{\rm D2D} - 1}{N_{\rm D2D} - 1 + 2\mathcal{B}(\zeta,\alpha)},
\end{aligned} \quad (6)$$

where $f_R(r) = 2\pi r(N_{\rm D2D} - 1)\Lambda_{\rm D2D}e^{-(N_{\rm D2D}-1)\Lambda_{\rm D2D}\pi r^2}$, the $\mathcal{A}()$ function is replaced by $\mathcal{B}(\zeta,\alpha) = \frac{2\pi}{\alpha}\zeta^{\frac{2}{\alpha}} \csc\left(\frac{2\pi}{\alpha}\right)$, considering that the nearest interference source could be closer to the D2D receiver than the D2D transmitter and $\mathcal{B}(\zeta, \alpha = 4) = \pi\sqrt{\zeta}/2$.

2) DL: As shown in Fig. 1b), we consider the D2D transmission from one UE to another UE using DL radio resources. The expected outage probability for a single-hop D2D link using DL resources is obtained following similar steps as (6):

$$\mathbb{E}_R({\rm P}_{\rm D2D,out}^{\rm DL}) = 1 - \frac{N_{\rm D2D} - 1}{N_{\rm D2D} - 1 + \Omega\mathcal{B}(\zeta,\alpha)}, \quad (7)$$

where $\Omega = 1 + \frac{\Lambda_{\rm BS}}{\Lambda_{\rm D2D}} \left(\frac{P_{\rm BS}}{P_{\rm D2D}}\right)^{\frac{2}{\alpha}}$ and $P_{\rm BS}$ is the power of BS.

B. Multi-hop with Shortest-Path-Routing (SPR)

In SPR, each D2D UE knows its own location and that of the final destination UE [6], which is similar to the greedy algorithm in [8]. SPR attempts to minimise the number of hops so as to minimise the outage probability. This is achieved by transmitting to the UE that is closest to the destination UE among those achieving reliable decoding. As shown in Fig. 2, the step-by-step SPR algorithm for a generic D2D source and destination pair is:

1) The transmitting UE identifies the UEs that can decode its transmissions reliably within a coverage radius $R_{\rm D2D}$;

2) The transmitting UE identifies the UEs (from Step 1) that are closer to the destination than itself;

3) The transmitting UE transmits to the UE that is of the longest distance (d_{D2D}) from itself among the UEs identified in Step 2), and this receiving UE becomes the transmitting UE in the next step;

4) Repeat Steps 1)-3) until the destination UE is reached.

When the source-to-destination distance $\overline{d}_{AB}$ is much greater than the maximum range of a single hop R_{D2D}, half of the potential UEs in Step 1) are valid for Step 2).

1) Number of Hops: We analyse the average number of hops J^s required by a multi-hop D2D link (s can be UL or DL). We assume that the UE density is sufficiently high such that the SPR multi-hop route does not deviate significantly. A BS of coverage radius R_{BS} is located at the origin in a polar coordinate system. The average distance $\overline{d}_{AB}$ between two arbitrary points A and B within the coverage is $\overline{d}_{AB} = \frac{128R_{BS}}{45\pi}$.

Using the single-hop outage probability given in (5), we set the reliability constraint as κ:

$$\begin{cases} \mathbb{P}(\overline{\gamma}_{D2D}^{UL} > \zeta) = e^{-(\Lambda_{D2D}+\Lambda_{CC})\pi\left(R_{D2D}^{UL}\right)^2 \mathcal{B}(\zeta,\alpha)} = \kappa, \\ \mathbb{P}(\overline{\gamma}_{D2D}^{DL} > \zeta) = e^{-\left[\Lambda_{D2D}+\Lambda_{BS}\left(\frac{P_{BS}}{P_{D2D}}\right)^{\frac{2}{\alpha}}\right]\pi\left(R_{D2D}^{DL}\right)^2 \mathcal{B}(\zeta,\alpha)} = \kappa. \end{cases}$$
(8)

Solving the above equations for R_{D2D}^{UL} and R_{D2D}^{DL} gives:

$$\begin{cases} R_{D2D}^{UL} = \sqrt{\dfrac{\ln\left(1/\kappa\right)}{(\Lambda_{D2D}+\Lambda_{CC})\pi\mathcal{B}(\zeta,\alpha)}}, \\ R_{D2D}^{DL} = \sqrt{\dfrac{\ln\left(1/\kappa\right)}{\left[\Lambda_{D2D}+\Lambda_{BS}\left(\frac{P_{BS}}{P_{D2D}}\right)^{\frac{2}{\alpha}}\right]\pi\mathcal{B}(\zeta,\alpha)}}, \end{cases}$$
(9)

For a given D2D coverage radius R_{D2D}^s, the number and density of *available* D2D UEs are N_{D2D}^* and $\Lambda_{D2D} = N_{D2D}^*/[\pi(R_{D2D}^s)^2]$, respectively. Since only half of the other D2D UEs are closer to the destination, i.e., $(N_{D2D}^*-1)/2$, the mean distance from the furthermost UE (selected in Step 3) to the current UE is given by (15) in the Appendix:

$$\overline{d}_{D2D}^s = \frac{R_{D2D}^s(N_{D2D}^*-1)}{N_{D2D}^*}.$$
(10)

The average number of hops for the UL and DL cases can be obtained by setting $J^{UL} = \frac{\overline{d}_{AB}}{\overline{d}_{D2D}^{UL}}$ and $J^{DL} = \frac{\overline{d}_{AB}}{\overline{d}_{D2D}^{DL}}$ and by substituting (9) into (10) as follows:

$$\begin{cases} J^{UL} = \left\lceil \dfrac{128 N_{D2D}^* R_{BS}\sqrt{(\Lambda_{D2D}+\Lambda_{CC})\mathcal{B}(\zeta,\alpha)}}{45\sqrt{\pi\ln\left(1/\kappa\right)}(N_{D2D}^*-1)} \right\rceil, \\ J^{DL} = \left\lceil \dfrac{128 N_{D2D}^* R_{BS}\sqrt{\left[\Lambda_{D2D}+\Lambda_{BS}\left(\frac{P_{BS}}{P_{D2D}}\right)^{\frac{2}{\alpha}}\right]\mathcal{B}(\zeta,\alpha)}}{45\sqrt{\pi\ln\left(1/\kappa\right)}(N_{D2D}^*-1)} \right\rceil, \end{cases}$$
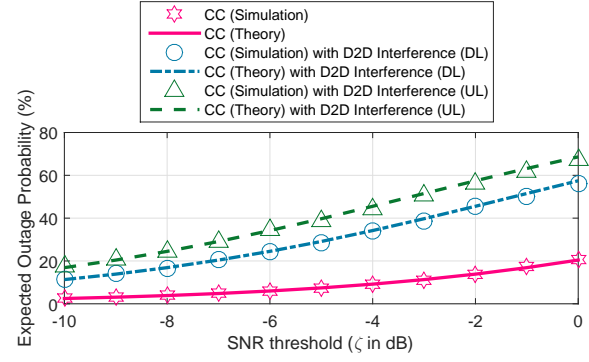(11)

Fig. 3. Outage probability of CC communication network with and without D2D interference for a multi-BS network with Monte-Carlo simulation and theoretical results (4), (13) and (14).

where $\lceil.\rceil$ stands for taking the smallest following integer.

2) Multi-Hop Outage Probability: The outage probability for multi-hop D2D can be obtained as a function of the success probability for each hop as follows:

$$P_{D2D,out}^s = 1 - \prod_{j=1}^{J^s}\left[1 - \mathbb{E}_R\left(P_{D2D,out,j}^s\right)\right],$$
(12)

where the value of J^s is given by (11).

V. CO-EXISTENCE PERFORMANCE

A. Degraded CC Outage Probability

If the UL band is utilised by D2D, the averaged outage probability of a CC link is:

$$\mathbb{E}_R\left(P_{CC,out}^{UL}\right) = 1 - \frac{1}{1+\mathcal{A}(\zeta,\alpha)}\frac{1}{1+2\mathcal{B}(\zeta,\alpha)}.$$
(13)

If the DL band is utilised by D2D, the averaged outage probability of a CC link is:

$$\mathbb{E}_R\left(P_{CC,out}^{DL}\right) = 1 - \frac{\left[1+\mathcal{A}(\zeta,\alpha)\right]^{-1}}{1+\left[1+\frac{\Lambda_{D2D}}{\Lambda_{BS}}\left(\frac{P_{D2D}}{P_{BS}}\right)^{\frac{2}{\alpha}}\right]\mathcal{B}(\zeta,\alpha)}$$
(14)

B. Numerical Results

The theoretical CC outage probability expressions and the validation against simulation results are shown in Fig. 3 for different SNR thresholds ζ. The theoretical result has included a 7 dB antenna gain to account for the difference between Monte-Carlo simulation data and the statistical theory. The correspondence between theory and simulation is very close and it can be seen that for a SNR threshold of -6 dB in LTE, the outage probability for CC communications without D2D interference is approximately 6%.

Fig. 4 plots the outage performance of CC and D2D sharing UL or DL radio resources, as a function of the number of D2D UEs (N_{D2D}^*). The system parameters are given in Table I. The first observation is that the CC outage probability does not change with the number of D2D UEs available for relaying, as at any given time instant there is only one D2D transmission per frequency band. However, the presence of D2D transmissions does increase the CC outage probability.

TABLE I
SIMULATION PARAMETERS

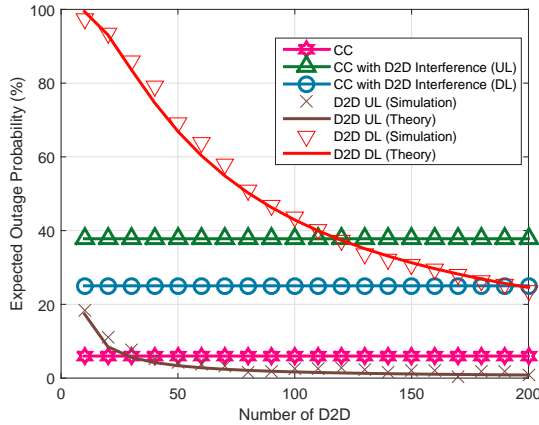| Parameter	Symbol	Value
Data Connectivity Threshold	ζ	-6 dB
CC UE density per BS	Λ_{CC}	1.27 per km^2
D2D sender node density	Λ_{D2D}	1.27 per km^2
D2D nodes available per BS	N_{D2D}^*	10–200
Macro-BS coverage radius	R_{BS}	500 m
Macro-BS density	Λ_{BS}	1.27 per km^2
BS transmit power	P_{BS}	40 W
UE transmit power	P_{D2D}, P_{CC}	0.1 W
Reliability Constraint	κ	0.95

Fig. 4. CC coexisting with D2D (UL and DL) using the SPR algorithm with varying number of available D2D UEs for relaying inside a BS (N_{D2D}^*).

Secondly, the D2D outage probability falls with increasing number of available D2D UEs, because the probability of finding a reliable multi-hop route becomes higher. The D2D sharing the UL band performs the best, achieving an outage probability of 1% for large numbers of D2D UEs. D2D sharing the DL band performs poorly for small numbers of D2D UEs, but the outage probability reduces to 20% at higher numbers of D2D UEs. This is because the DL interference power received from BSs is much stronger than the UL interference power from CC UEs. In this case, it is more desirable for D2D communications to share the UL radio resources. On the other hand, D2D sharing the UL band leads to a much higher CC outage probability than D2D sharing DL resources. Therefore, there is a trade-off between D2D and CC communication reliability while considering whether to use the UL or DL band for D2D communications. Letting D2D transmissions utilise the DL band will favor CC reliability over D2D reliability, whereas letting D2D transmissions utilise the UL band will favor D2D reliability over CC reliability.

VI. CONCLUSIONS

In this paper, we have provided a tractable framework for analysing the performance of multi-hop D2D communications with SPR, where both UL and DL co-channel interference between D2D and CC transmissions is considered. We have shown that D2D transmissions can severely degrade CC communications (25% outage), but D2D links themselves can be reliable ($< 5\%$ outage). There is a trade-off between D2D and CC reliability while considering whether to use UL or DL cellular radio resources for D2D communications. In our future work, we will extend the proposed analytical framework to other routing algorithms for multi-hop D2D communications.

APPENDIX

Consider a D2D transmitter of coverage radius R located at the origin. The location of a D2D receiver within the coverage area can be denoted as (ρ, θ) in the polar coordinate system, where ρ and θ are the distance and the direction from the origin, respectively. We assume the D2D receivers are uniformly distributed in the coverage area, then ρ and θ are random variables according to [9]. Denote $f_\rho(\rho)$ as the PDF of ρ, then the CDF of the distance P from the D2D receiver to the transmitter can be obtained by integral $f_\rho(\rho)$: $F_P(\rho) = \int_0^\rho f_\rho(t)\, dt = \int_0^\rho \frac{2t}{R^2}\, dt = \frac{\rho^2}{R^2}$. Assume there are N receiver points $(X_1, X_2, \ldots, X_N)$ inside the circular area centered at the origin with a radius R. Then the CDF of the distance between each receiver point to the origin was found above. Define Z as the maximum value of all the distances, the CDF of Z is given by: $F_Z(z) = \mathbb{P}(X_1 \leqslant z)\mathbb{P}(X_2 \leqslant z), \ldots, \mathbb{P}(X_N \leqslant z) = \left(\frac{z^2}{R^2}\right)^N$, and the expected value of Z is given by:

$$\mathbb{E}_R(Z) = \int_0^R 1 - \left(\frac{z^2}{R^2}\right)^N\, dz = \frac{2NR}{2N+1}. \qquad (15)$$

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