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Innovation and trade policy coordination: The role of firm heterogeneity[*]

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Abstract

Recent studies have concluded that R&D grants can induce firms to export and that exporting and innovating can be complementary activities at the firm level. Yet the trade literature has paid little attention to the scope of innovation policy as a stimulus to both trade and innovation. To investigate this question we rely on a general work-horse model of trade and firm heterogeneity with firm investments in R&D activities. The interplay of innovation and trade policies uncover novel results. In particular, we show that the effects of either policy depend on the degree of protectionism in a country. Therefore, countries can respond differently to the same policy, and similarly to different policies. In such a context, different governments may face different trade-offs in achieving a given target.

JEL: F12, F13, F15, F61, O32.
Keywords: innovation, innovation policy, heterogeneous firms, technology adoption, trade policy.

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1 Introduction

All industrialized countries use some measures of innovation policy, like R&D grants and tax allowances, to sustain their research activity and economic growth (OECD, 2005). While originally aimed at increasing the productivity of firms, these measures have also contributed to stimulate the exports of firms (Görg et al., 2008). When trading is the *conditio sine qua non* for innovating, trading and innovating become complementary activities (Lileeva and Trefler, 2010). The other side of this complementarity is policy substitution: If innovation policy favors export initiation by strengthening innovation efforts, it renders trade policies with the same objective superfluous, and vice versa.

The importance of policy substitution and, more generally, of policy coordination is underrated in the current models of trade with heterogeneous and monopolistically competitive firms. Motivated by trade liberalization in several countries, these models have offered thorough analysis of the micro effects of these policies, but they have paid little attention to the concomitant role of innovation policy for stimulating both trade and innovation, which is the focus of this paper.

To explore this topic, we opt for a model with technology adoption, which nests in the class of models that have followed the seminal work of Montagna (2001) and Melitz (2003). There are several advantages associated with this approach. First, technology adoption is an important source of industry productivity growth (Griffith et al., 2004). This suggests that innovation policies aiming at promoting technology adoption are relevant from a policy perspective. Second, this approach allows us to study the consequences of these policies while keeping analytical tractability. Finally, this approach is equally well able to encompass the evidence presented in Bustos (2011), featuring a scenario with exporting non-innovators, and the evidence from Castellani and Zanfei (2007), featuring a scenario with innovating non-exporters. The first scenario occurs in quite open countries, where trade is relatively free, so firms can engage in international markets without being innovators. The second scenario is common to relatively closed countries, where trade barriers are relatively high, and it therefore pays to innovate just for the domestic market. For intermediate levels of trade openness, innovation and export activities become complementary, as described in Lileeva and Trefler (2010). These scenarios correspond in our model to three distinct equilibria sustained by different parameter configurations.

In our policy analysis, it is useful to compare the effects of innovation and trade policies on the share of innovators and exporters across all equilibria. This comparison forms the basis of our welfare analysis and reveals a number of interesting results.

First, an innovation policy increases the share of innovating firms in all scenarios, as one would expect. However, it contracts the share of exporting firms except when trading and innovating are complementary activities. Therefore, its scope to support export initiation is limited. On the other hand, a reduction in variable trade costs increases both shares for all countries except for the most protectionist ones. Consequently, its scope to favor innovation is broad.

Second, a given policy has different effects in different equilibria. Hence, the impact of innovation policies and trade policies depends on the degree of protectionism of a country. This means that in different countries, the same policy (e.g., a reduction in variable trade costs, or an R&D grant) produces different aggregate outcomes (in terms of changes in the share of innovators and exporters).

Third, the aggregate outcomes of distinct polices can coincide across equilibria. This implies that liberal countries undergoing reductions in variable trade costs may experience similar aggregate effects to the ones encountered by slightly more protectionist countries introducing R&D grants.

From the perspective of a cross-country comparison, these results suggest that countries can have heterogeneous responses to similar policies, as well as similar responses to heterogeneous policies. But they also indicate that policy makers in different countries will be facing different policy options and, ultimately, different policy-trade offs. We shall argue that the order in which policies are adopted (e.g., policy coordination) can either mitigate or accentuate the occurrence of trade-offs.

To simplify the presentation of our results, we first describe the effects of various policies and analyze the trade-offs that arise with multiple instruments, taking each policy as a *fait accompli* (positive analysis). Then we present the normative implications of these policies to justify government interventions. Finally, we show that the policy conclusions drawn are robust to the presence of spillovers, recognizing that the reasons for resorting to these types of policies originate from market failures caused by spillovers associated with either the innovation or the exporting activity.

Our paper is structured as follows. In section 2 we present our research in the context of the trade literature. We proceed by presenting our model and deriving the equilibrium in a closed and an open economy in section 3. Section 4 briefly outlines the closed economy. In section 5 we then analyze the implications of trade and innovation policies in the open economy and synthesize our results by means of one simple graph, our policy space. This graph illustrates our discussion of the issue of policy coordination in section 6. Sections 7, 8, and 9 round up our policy discussion. They introduce, respectively, the macro-implications of our policies on the industry productivity growth, the welfare analysis of our policies, and the presence of innovation and exporting spillovers in the economy. Section 10 concludes.

2 The background

Given the increasing availability of micro-datasets linked to trade statistics, firm-heterogeneity and its effects have been an important part of recent trade research.[1] The seminal works of Montagna (2001) and Melitz (2003) have been extended to include process innovation besides product innovation. Navas and Sala (2007) and Bustos (2011) consider a firm's technology adoption as a form of process innovation, whereas Atkeson and Burstein (2010) focus on a firm's R&D's investments. The main reason why the literature has focused on process innovation, and in the case of Navas and Sala (2007) or Bustos (2011) on technology adoption, is that both are important sources of industry productivity growth. Doms and Bartelsman (2000) and Foster et al. (2001) provide empirical support that innovation by incumbents accounts for the largest proportion of industrial productivity growth. Akcigit and Kerr (2010), using the US Census of Manufacturing Firms, find that old and large firms mainly undertake innovations whose aim is to encourage productivity improvements, while new and small firms perform product innovation. In the same direction, Griffith et al. (2004) conclude that around 50 per cent of the total contribution of R&D to productivity growth is accounted by technology diffusion in OECD countries. In addition, Bustos (2011) and Bas and Ledezma (2010) find the trade trade liberalization has induced Argentinean and Chilean exporters to upgrade their technology. Greenaway and Kneller (2007) reinforce this result by presenting further evidence from studies in other countries.

However, all these models are hardly reconcilable with the evidence disclosed in Lileeva and Trefler (2010), as they cannot possibly predict the behavior of some Canadian firms that have both upgraded their technologies and simultaneously started to export with the creation of the US-Canada free trade area. Our work shows that the limitation of these models with regard to encompassing these facts rather originates in neglecting that different parameter configurations lead to different types of equilibria.

We consider a simple framework in which firms pay a fixed cost to introduce a new technology that reduces the marginal cost in a fixed proportion. Because the reduction in the marginal cost is proportional, firms will experience heterogeneous innovation gains. While this approach departs from more complex innovation technologies (Atkeson and Burstein, 2010; Long et al., 2011), its analytical tractability allows us to introduce the important issue of policy coordination that arises when a broad range of instruments are available to policy makers. The discussion of this matter is based on a comparative static analysis of our steady states and consequently we do not explore the transitional dynamics as in Costantini and Melitz (2007) of alternative tariff scenarios. Finally, our model suggests that extending the type of counterfactual analysis presented in Corcos et al. (2011) to innovation policy scenarios could be fruitful.

To present our model in the next section, we build on Navas and Sala (2007).

[1] See Greenaway and Kneller (2007) for an extensive review of this literature.

3 The model

Preferences

A continuum of households of measure L have preferences described by a standard C.E.S. utility function,

$$U = \left[\int_{\omega\in\Omega} [q(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1,$$

where σ is the elasticity of substitution across varieties, and Ω is the set of available goods.

Technology

The amount of labor required to produce a quantity $q(\omega)$ of variety ω is

$$l(\omega) = f_D + cq(\omega),$$

where f_D is the fixed labor requirement, and $c \quad 0,\bar{c}$ is the firm-specific marginal labor requirement.

Entry – exit

There is a large (unbounded) pool of prospective entrants into the industry, and prior to entry all firms are identical. Like in Melitz (2003), to enter the industry, a firm must make an initial investment, modeled as a fixed cost of entry $f_E > 0$ measured in labor units, which is thereafter sunk. An entrant then draws a labor-per-unit-output coefficient c from a known and exogenous distribution with cdf $G(c)$ and density function $g(c)$ on the support $c \quad 0,\bar{c}$. Upon observing this draw, a firm may decide to exit or to produce. If the firm does not exit, it bears the fixed overhead labor costs, f_D, and it has the option to improve on its technology. By investing f_I units of labor, it can adopt a more productive technology and produce at a lower cost c $(\gamma < 1)$. Ultimately, the firm faces a choice between a well established "*baseline*" technology - characterized by low implementation costs, normalized to 0, and variable costs of production c - and an *innovative* one - featuring lower variable costs (c), but higher fixed costs of adoption (f_I).[2]

We assume that technological uncertainty and heterogeneity of the Melitz-type relates

[2] This two-step structure allows us to interpret a broader range of innovation processes. It recalls the distinction introduced by Vernon (1966) between a flexible technology adopted by firms in the early stage of the product cycle and a mass production technology adopted in the successive phases of the product cycle to output greater volumes of production. Along similar lines, Lileeva and Van Biesebroeck (2010) describe a shift from flexible technologies for multiple product lines to less flexible mass technologies for narrower product lines as the scale of production increases. We are thankful to an anonymous referee for suggesting this interpretation.

to what we have called a "*baseline*" technology, reflecting that firms have to learn about their market and their productivity before they can plan to improve it. Having found out about their idiosyncratic productivity, all firms can costly adopt an alternative technology, what we have referred to as the "*innovative*" one. While the fixed cost of implementation is the same for each firm, the reduction in variable cost is proportional to the firm's intrinsic marginal cost. Since the Melitz-type entry leads to productivity heterogeneity, the option to adopt is differently attractive to firms with different intrinsic marginal costs. This could be rationalized as some firms being more successful than others in implementing the new technology (i.e. better implementation makes new technologies more productive).[3] Therefore, in this paper process innovation consists in adopting a technology that is "new to the firm" but not to the industry, as both technologies are equally available in all periods. However, adopting firms need to master the new technology and face specific learning curves. This yields equilibria that reproduce the real fact of non-innovating exporters and innovating non-exporters.

Finally, as in Melitz (2003), every incumbent faces a constant (across productivity levels) probability δ in every period of a bad shock that would force it to exit.

Trade

We shall assume that the economy under study can trade with other $\mathrm{n} \geq 1$ symmetric countries. Trade is, however, not free, but involves both fixed and variable costs: the firm has to ship $\tau > 1$ units of a good for each unit to arrive at a destination and has to incur a fixed cost $\mathrm{f_X}$ during the period in which starts exporting.

The symmetry of countries ensures that factor price equalization holds, and all countries share the same aggregate variables.[4]

Prices and profits

Given the CES preferences, the demand of each variety ω is

$$\mathrm{q}(\omega) = \frac{\mathrm{R}}{\mathrm{P}}\left[\frac{\mathrm{p}(\omega)}{\mathrm{P}}\right]^{-\sigma}, \tag{1}$$

where $\mathrm{R} \equiv \int_{\omega\in\Omega} \mathrm{p}(\omega)\mathrm{q}(\omega)\mathrm{d}\omega$ is the aggregate total expenditure, and $P \equiv \left[\int_{\in} \left[p(\quad) \right]^{1} d \quad \right]^{\frac{1}{1}}$ is the price index of the economy.

Facing this demand function, a producer of variety ω with labor output coefficient c

[3] "Technology implementation processes" are in the data the main source of site-to-site variations in the success of the adopter. See Comin and Hobijn (2007) and Bikson et al. (1987). Note that in this framework the productivity ratio of two firms with different intrinsic marginal costs will be constant if both firms adopt the new technology.

[4] This is a technical assumption to preserve the model tractability and a widely used assumption in the literature. The model is therefore most appropriate for trade between similar countries, which still constitutes the major part of total world trade flows. The formulation in Montagna (2001) allows for a non-symmetric *G* but marginal costs follow a specific functional form.

charges the price:

$$\mathrm{p}(\omega) = \frac{\sigma}{\sigma - 1}\mathrm{wc} \equiv \mathrm{p_D(c)}, \tag{2}$$

where w is the common wage rate, hereafter taken as the numeraire ($\mathrm{w} = 1$).[5]

If the firm has opted for the innovative technology, it charges the lower price, $p_I(c) = \ \ p_D(c)$. Therefore, the profits that firm type D (producer with a "traditional" technology) and firm type I (firm with innovative technology) make on the domestic market are, respectively,

$$\pi_{\mathrm{D}}(\mathrm{c}) = \frac{\mathrm{r_D(c)}}{\sigma} - \mathrm{f_D} = \mathrm{Bc}^{1-\sigma} - \mathrm{f_D} \quad \text{and} \tag{3}$$

$$_I(c) = \frac{r_I(c)}{\ } \quad f_D \quad f_I = B(\ \ c)^{1} \quad f_D \quad f_I \tag{4}$$

where $\mathrm{r_s(c)}$ is the revenue of firm type $\mathrm{s} \ \in \{\mathrm{D}, \mathrm{I}\}$, f_I is the per-period amortized investment cost $\mathrm{f_I}$, and $\mathrm{B} = (1/\sigma)\frac{\mathrm{R}}{\mathrm{P}^{1-\sigma}}\left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma}$ is a constant from the prospective of a single producer.

It is worth noting that $r_I(c)/r_D(c) \quad > 1/$, and therefore the income of the firm increases more than proportionally following the introduction of process innovations.

The imported products are more expensive than domestically produced goods due to transportation costs. The effective consumer price for a variety shipped from abroad by a non-innovating exporter is $p_X(c) = \ \ p_D(c)$, and by a firm adopting the innovative technology it is $p_{XI}(c) = \ \ p_X(c)$. Therefore, the profits of an exporter (firm type X) and an innovator-exporter (firm type XI) earned on the foreign market are, respectively,

$$_X(c) = \ ^{1} \ \ Bc^{1} \quad f_X \quad \text{and} \tag{5}$$

$$_{XI}(c) = (\ \ )^{1} \ \ Bc^{1} \quad f_X, \tag{6}$$

where f_X is the amortized per-period fixed cost of the overhead fixed cost $\mathrm{f_X}$ that firms have to pay to export.

As in Melitz (2003), no firms will ever export without also producing for its domestic market, and a firm will either export to all n countries in every period or never export. (5) is therefore the profit from exporting conditional on being a domestic firm and, likewise, (6) is the profit from exporting conditional on being a domestic innovator.

[5] Alternatively, a freely traded homogeneous good produced under constant returns to scale could be introduced as the numeraire good to set the wage to unity in all countries.

4 Equilibrium in a closed economy

The equilibrium entry cost cut-off c_0 and innovation cost cut-off c_I must satisfy

$$_D(c_0) = 0 \Leftrightarrow B(c_0)^{1} = f_D \tag{7}$$

$$_I(c_I) = \;_D(c_I) \Leftrightarrow (\;^{1} \quad 1)B(c_I)^{1} = f_I. \tag{8}$$

To close the model and determine the two equilibrium cost cut-offs c_0 and c_I, as well as B and the number of incumbent firms M, free entry into the market and a stability condition are imposed additionally. Free entry (henceforth *FE*) ensures that firms equate the per-period expected profit from entry to the equivalent amortized per-period entry cost,

$$f_E = \int_0^{c_I} \;_I(c)dG(c) + \int_{c_I}^{c_o} \;_D(c)dG(c).$$

The stationary-equilibrium condition,

$$M_e G(c_0) = \; M,$$

requires the aggregate variables to remain constant over time, as the mass of successful entrants, $M_e G(c_0)$, exactly replaces the mass, M, of incumbents who are hit by the bad shock and exit.

Combining (7) with (8), we have the relation between the innovation and the entry cut-off

$$(c_I)^{1} = \frac{f_I}{\;^{1}} \frac{1}{1\, f_D}(c_0)^{1} = \; (c_0)^{1} \;, \tag{9}$$

where $\frac{f_I}{\;^{1} \quad 1}$ is the cost-to-benefit ratio of innovation. It follows that a necessary and sufficient condition for having selection into the innovation status is $\Psi > 1$, which is assumed to hold throughout since the empirical evidence suggests that only a subset of more productive firms undertakes process innovations.[6]

Finally, we note that the entry productivity cut-off level is higher in our economy than in Melitz (2003).[7] The possibility to innovate allows the most efficient firms that perform process innovation to "steal" market share from the least efficient firms for which it is harder to survive in the market. Consequently, our economy is more efficient, because some varieties are produced at a lower cost, but less varied because some varieties have disappeared. This trade-off has been well emphasized in the growth literature (see Peretto, 1999, and more recently Gustafsson and Segerstrom, 2010).

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[6] See for instance Parisi et al. (2006) for evidence on Italian firms and Baldwin et al. (2004) for evidence on Canada.

[7] The proof of this result has been left to an online appendix.

5 The open economy

To differentiate the entry cut-off and the innovation cut-off from the ones in the closed economy, we denote them in the open equilibrium by c_0^f and c_I^f, respectively.[8] Depending on the relative position of the cut-offs for innovation and exporting, firms select into these activities differently.

Guided by her empirical results, Bustos (2011) focuses only on one possible selection, namely $c_I^f < c_X < c_0^f$, so that the marginal innovating firm is an exporter and responds to tariff cuts with the adoption of a better technology. We label this kind of selection *equilibrium BW* to point to the fact that the growth of the industry productivity has two sources. One, as in Melitz (2003), comes from the reallocation of market shares from low to high productivity firms induced by the selection effect of trade. The second comes from the adoption of better technologies by firms that trade. In its decomposition of the industry productivity growth, Bartelsman et al. (2004) refer to the first source as the *between* variation and to the the second source as the *within* variation. The letters BW (*between-within*) indicate that in this type of equilibrium both sources of variation are present.

However, an equilibrium where both exporters and non-exporters are performing innovation is also plausible and requires $c_X < c_I^f < c_0^f$ (see Castellani and Zanfei, 2007 for some evidence). In this equilibrium, the marginal innovating firm is not an exporter, and therefore it does not respond to a fall in transportation costs with the adoption of innovative technologies. We label this selection *equilibrium B*, as only the *between* variation contributes to the growth of the industry aggregate productivity.

In the limiting case of both these selections, firms engage either in both activities or neither of them ($c_X = c_I^f < c_0^f$). In this equilibrium, trade and innovation become complementary activities (henceforth selection C), consistently with the evidence presented in Lileeva and Trefler (2010).[9]

In what follows, we analyze each equilibrium separately, describing which are the effects of both innovation and trade policies on the innovation and export activities of firms. In particular, the focus of our analysis is on the outcome of these policies, taking them as a "fait accompli" (i.e., positive analysis). Abstracting in the first place from the reasons for why these reforms are implemented permits the simplest discussion of the interactions among these policies. At the end of the paper, we return to the normative implications and show that these policies are welfare enhancing. Therefore, governments aiming at increasing countries' welfare have a rationale for introducing them.

[8] With a slight abuse of notation, we keep denoting the aggregate variables *Q*, *R*, and *B* with the same letter as in the closed economy, although their equilibrium value in the two equilibria will generally differ.

[9] We thank an anonymous referee for highlighting this possibility.

We consider only stationary equilibria in the sense that all aggregate variables are constant over time. We therefore impose on each equilibrium the stationary condition

$$M = M_e G(c_0^f), \tag{10}$$

so that the firms exiting the market are just replaced by the new entrants.

5.1 Selection *BW*

We start by determining the equilibrium cost cut-offs for, respectively, market entry, exporting, and innovating. Given that in equilibrium BW we have $c_I^f \le c_X \le c_0^f$, the cost cut-offs must satisfy the following conditions:

$$ {}_D(c_0) = 0 \Leftrightarrow B(c_0^f)^{1} \quad = f_D \tag{11}$$

$$ {}_X(c_X) = 0 \Leftrightarrow Bc_X^{1} \quad = \frac{f_X}{1} \tag{12}$$

$$ {}_I(c_I^f) + n \quad {}_{XI}(c_I^f) = \quad {}_D(c_I^f) + n \quad {}_X(c_I^f) \Leftrightarrow B\left(c_I^f\right)^{1} \quad = \frac{f_I}{\left(\ ^{1} \quad 1\right)\left(1+n \ ^{1} \ \right)}. \tag{13}$$

The parameter restriction that sustains this equilibrium and will be important for our policy analysis below is

$$\frac{f_I}{\left(\ ^{1} \quad 1\right)} \frac{1}{\left(1+n \ ^{1} \ \right)} \quad f_X \quad ^{1} \quad f_D. \tag{14}$$

The FE condition

$$ {}^{c_0^f}_{c_X} \quad {}_D(c)dG(c) + {}^{c_X}_{c_I^f} \left(\quad {}_D(c) + n \quad {}_X(c)\right)dG(c) + {}^{c_I^f}_{0} \left(\quad {}_I(c) + n \quad {}_{XI}(c)\right)dG(c) = \quad f_E \tag{15}$$

together with the stationary condition (10) close the model to determine B and M.

To compare the share of innovating firms in the trading equilibrium and in the autarky equilibrium, we rewrite the innovation cut-off as a function of the entry cut-off,

$$(c_I^f)^{1} \quad = \frac{f_I}{\left(\ ^{1} \quad 1\right)\left(1+n \ ^{1} \ \right)} \frac{1}{f_D}\left(c_0^f\right)^{1} \quad , \tag{16}$$

and note that this relation differs from equation (9) only by the term $1/(1+\mathrm{n}\tau^{1-\sigma})$. This term represents the increase in variable profit associated with the fact that in free trade the innovation is used in each of the foreign markets that becomes available after autarky is abandoned. Looking at equation (16), free trade affects the innovation cut-off in two opposing ways: It pushes this term below the unity value of the closed economy (e.g., set $\mathrm{n} = 0$ or $\tau \to \infty$), but it also lowers c_0^f below the closed economy threshold c_0, as import competition intensifies and the least productive firms strive to survive in the industry.[10] In the appendix, we prove that the first effect prevails and trade enlarges the share of the innovating firm (i.e., $c_I^f \geq c_I$). Crucial to this result is the selection of firms into exporting activities. In the absence of the fixed costs of trading ($f_X = 0$), all firms would export as in Krugman's (1979) model. With CES preferences, the increased revenue from increased sales abroad induced by trade opportunities would be exactly offset by the loss of domestic revenues due to increased import competition from foreign varieties, so the two opposing effects would exactly equal out. Given that profits would be unchanged, no firms would exit the market. Without exit, there is no reallocation of market share to exporters. With unchanged output and market share the incentive to adopt a more efficient technology in free trade is also unchanged relative to the autarky equilibrium.

While this result is the same as in Bustos (2011), we show that it is more general because it is derived without assuming that G is Pareto.

We now examine "incremental" trade liberalization, intended as a reduction of either the transportation cost, τ, or the regulatory cost of trade, f_X. We summarize the effects of these policies on the export and innovation cut-offs in Table 1. The table reveals that only the reduction of the variable trade costs is comparable to the case of free trade in that it affects the extensive margin of both exporting and innovating positively (see the appendix for a formal proof). The reduction of f_X, on the contrary, contracts the extensive margin of innovation.

INSERT TABLE 1 ABOUT HERE.

This is because only a cut in transportation costs increases the variable profits of all exporting firms, while a fall in f_X cannot alter the profits of incumbent exporters (who have already incurred these costs). So, when transportation costs decline, all exporters can lower prices and increase sales abroad (the intensive margin adjustment). This helps some of the non-innovator exporters to start innovating (higher c_I^f). Moreover, because selling internationally has become cheaper, exporting becomes attractive to some domestic firms (higher c_X). The increase in innovation or export at the extensive margin raises labor demand and the real wage. The firms most hurt are the domestic ones that cannot compensate the increased costs of production with the expansion of foreign activity. The least productive are therefore forced to exit (lower c_0^f), and their market share is redistributed to all incumbent firms. The reallocation process sees high productivity firms expanding, and low productivity firms shrinking. When a reduction of f_X occurs, selling internationally becomes also less expensive and the relatively more productive domestic firms engage in international markets (higher c_X). New exporters bid up the price of the scarce labor input, but this time innovator-exporter and domestic firms alike cannot compensate for higher costs of production

[10] Notice that we can express $\left(c_0^f\right)^1 = (f_D / B)$, where *B* is constant from the firm's point of view. An increase in competition will push this constant down so that the productivity survival threshold will go up.

with an expanding business abroad. Then, some of the innovator-exporters can no longer afford both activities (i.e., innovation cut-off decreases) and some domestic firms exit (i.e., entry cut-off is lower).

Table 1 also emphasizes a similarity in this equilibrium between the effects of a reduction of $\mathrm{f_X}$ and $\mathrm{f_I}$. The latter can be interpreted in our model as a pro-innovation policy (i.e., a R&D grant). The outcomes of these policies are similar in the sense that a reduction of the fixed cost of trade or innovation can just expand the share of firms performing the related activity at the expense of the share of firms performing the other activity.

If the sector-wide productivity is defined as the weighted average of all firms' productivity in the industry, with the weights consisting of the firm's market share, there are two sources of productivity growth at the industry level: changes in the firms' productivity or changes in the firms' market share. In Bartelsman et al.'s (2004) terminology, both the redistribution of market share from low to high productivity firms - the *between-firm* effect - and the technology adoption by some exporters - the *within-firm* effect - are contributing to the industry productivity growth in this equilibrium whenever free trade is introduced or transportation costs are reduced.

5.2 Selection B

In equilibrium B ($\mathrm{c_X \le c_I^f \le c_0^f}$), some innovators are non-exporters. Therefore, the innovating firms are both of the I -type and of the XI -type, and $_D(c_0^f)=0$, $_I\left(c_I^f\right)= \ _D\left(c_I^f\right)$ and $_{XI}\left(c_X\right)=0$ define our cut-off conditions. The necessary and sufficient condition for this equilibrium to hold is

$$f_X \quad {}^{1} \quad \frac{f_I}{(\ {}^{1} \quad 1)} \quad {}^{1} \qquad {}^{1} \ f_D. \tag{17}$$

From Table 1, we deduce that none of the pro-trade policies can stimulate innovation in the sense of leading to a higher share of innovating firms in the economy. Since the marginal innovator is a domestic firm, a reduction of τ or $\mathrm{f_X}$ has no direct effect on its profit, but it makes the profits of exporting firms bigger. Higher profits induce these firms to expand their activity abroad (either at the intensive or extensive margin), which causes labor demand and real wages to increase. The rise of production costs affects the non-exporters, including the marginal innovating firm, more severely. Those firms that cannot break even will be forced out of the market. In the new steady state more domestic firms will stick to the baseline technology (lower $\mathrm{c_I^f}$), and less firms will enter (lower $\mathrm{c_0^f}$). Because market share is redistributed from low to high productivity firms and some firms give up technology adoption, it is clear that industry productivity can grow only because of the *between-firm* component.

Note that in our model technology adoption occurs only at the extensive margin because of the exogeneity of γ. With an endogenous γ and a convex cost of innovation, some of the XI firms, those that expand their activity abroad and enlarge their market shares, would find it optimal in this equilibrium to choose a lower γ and, therefore, promote technology adoption at the intensive margin, too (Vannoorenberghe, 2008).

It is instructive to graph the parameter space which sustains equilibria B and BW. We represent each term of the inequalities (14) and (17) in Figure 1 as a function of an index of variable trade costs ($\tau^{\sigma-1}$). In the graph we omit the term $\gamma^{1-\sigma}\mathrm{f}_{\mathrm{D}}$ as it is irrelevant for equilibrium B provided it lies below the curves $f_X\ \ ^{1}$ and $f_I/(\ ^{1}\ \ 1)$. Condition (14) holds in the region labeled with BW for transportation costs below $\tau_{\mathrm{BW}}^{\sigma-1}$ and if n is not too large. Likewise, condition (17) is satisfied in the region labeled with B for transportation costs above $\tau_{\mathrm{B}}^{\sigma-1}$. Therefore, these two equilibria are not contiguous in this space, as no equilibrium is defined for intermediate values of τ. Note that the size of this middle region crucially depends on the magnitudes of the fixed costs of trading and innovating as well as on the number of trading partners.

INSERT Figure 1 ABOUT HERE

As transportation costs increase beyond $\tau_{\mathrm{BW}}^{\sigma-1}$, firms in region BW in Figure 1 regard exporting without innovating as unsustainable, but they cannot yet justify being domestic innovators until transportation costs have increased to at least $\tau_{\mathrm{B}}^{\sigma-1}$. Likewise, as transportation costs decrease below $\tau_{\mathrm{B}}^{\sigma-1}$, firms in region B in the graph can no longer justify being innovators without exporting, but becoming exporters without innovating is equally no option until the transportation costs have fallen below $\tau_{\mathrm{BW}}^{\sigma-1}$. Therefore, when transportation costs fall in this intermediate region, there is a profitable deviation from the strategy adopted in region BW or B. In Figure 1, we label the region between $\tau_{\mathrm{BW}}^{\sigma-1}$ and $\tau_{\mathrm{B}}^{\sigma-1}$ with the letter C, because it corresponds to the equilibrium described in Lileeva and Trefler (2010), where innovation and exporting are complementary activities in the sense that no firm will ever start one activity without performing the other, too.

To develop the intuition, consider a domestic firm willing to both export and adopt the innovative technology. Such a strategy is wise if

$$[n\ \ _{XI}(c)+\ \ _{I}(c)]\quad\ _{D}(c)=[n\ \ _{XI}(c)+\ \ _{I}(c)\quad n\ \ _{X}(c)+\ \ _{D}(c)]+n\ \ _{X}(c)\quad 0,$$

where the first equation is the profit differential from implementing both options at the same time compared to the profit earned on the domestic market. Note that for a firm with $\mathrm{c}\leq\mathrm{c}_{\mathrm{X}}$ in equilibrium B, the first equation is necessarily positive as $\mathrm{n}\pi_{\mathrm{XI}}$ and $\pi_{\mathrm{I}}-\pi_{\mathrm{D}}$ are positive. Likewise, for a firm with $\mathrm{c}\leq\mathrm{c}_{\mathrm{I}}$ in equilibrium BW this condition holds as the squared bracket and the last addend in the second equation are both positive. This is not surprising as we know that this kind of firm performs both activities in each equilibrium, respectively. Yet this condition could more generally hold in either equilibrium for other firms too, although all terms may not necessarily be positive. Indeed, a domestic firm would want to pursue a complementary strategy if the double option is also more profitable than just either exporting or innovating, so if

$$\mathrm{n}\pi_{\mathrm{XI}}(\mathrm{c})+\pi_{\mathrm{I}}(\mathrm{c})-\pi_{\mathrm{D}}(\mathrm{c})\geq \mathrm{n}\pi_{\mathrm{X}}(\mathrm{c})+\pi_{\mathrm{D}}(\mathrm{c})-\pi_{\mathrm{D}}(\mathrm{c}) \tag{18}$$

and

$$\mathrm{n}\pi_{\mathrm{XI}}(\mathrm{c})+\pi_{\mathrm{I}}(\mathrm{c})-\pi_{\mathrm{D}}(\mathrm{c})\geq \pi_{\mathrm{I}}(\mathrm{c})-\pi_{\mathrm{D}}(\mathrm{c}). \tag{19}$$

Equation (18) implies $Bc^{1} \quad (\; f_I)/(\; ^{1} \quad 1) * 1/(1+n \; ^{1} \;)$, which evaluated at the marginal exporting firm with $\mathrm{c}=\mathrm{c}_{\mathrm{X}}$ in equilibrium BW translates into $f_X \quad ^{1} \quad (\; f_I)/(\; ^{1} \quad 1) * 1/(1+n \; ^{1} \;)$. For this parameter range the marginal exporting firm finds in the double option a profitable deviation. And so will all other firms with higher productivity.

Equation (19) implies $Bc^{1} \quad f_X \quad ^{1} \quad ^{1}$, or for the marginal innovating firm at $\mathrm{c}=\mathrm{c}_{\mathrm{I}}^{\mathrm{f}}$ in equilibrium B, $f_I(\quad)^{1} \quad /(\; ^{1} \quad 1) \quad f_X$. Under these circumstances, the marginal innovating firm finds the double option more profitable, and so will all other firms with higher productivity.

Therefore, only when transportation costs are sufficiently low - below $\tau_{\mathrm{BW}}^{\sigma-1}$ - some firms find exporting to be more profitable than undertaking both investments simultaneously. Likewise, only for relatively high transportation costs - above $\tau_{\mathrm{B}}^{\sigma-1}$ - some firms innovate exclusively for the domestic market without seeking participation in foreign markets. In the intermediate range of transportation costs, when converging to this limiting zone from BW, no firms would ever export without innovating and would ever innovate without exporting like in B. Likewise, when converging to this limiting zone from B, one does not fall straight into BW, as no firm would ever export without innovating.

We consider next the equilibrium where this double strategy or complementary strategy between innovating and exporting is optimal.

5.3 Export and Innovate: complementary activities (selection *C*)

The innovator and the exporter types coincide ($\mathrm{c}_{\mathrm{X}}=\mathrm{c}_{\mathrm{I}}^{\mathrm{f}}$) because exporting and innovating are complementary activities.[11] Rearranging (14) and (17), the parameter space that is complementary to both equilibria BW and B can be expressed as

[11] We thank an anonymous referee for pointing this limiting case out to us as the Lileeva and Trefler (2010) equilibrium.

$$\frac{f_I}{(\ ^{1}\quad 1)}\frac{1}{(1+n\ ^{1}\quad)}\quad f_X\ ^{1}\quad \frac{f_I}{(\ ^{1}\quad 1)}\ ^{1}\ .$$

We prove in the appendix that only under this parameter restriction can the strategy of innovating and exporting simultaneously be optimal. Not surprisingly, being the limiting case of both BW and B, this condition is defined for transportation costs between the two other equilibria.

Referring to Table 1 again, we note that because the activity of exporting and innovating are complementary, every pro-trade policy in this equilibrium is also a pro-innovation policy, and vice versa every pro-innovation policy is a pro-trade policy, too. In other words, innovation and trade policies become substitutes, in the sense they can used interchangeably to achieve the same policy aim. This feature distinguishes this equilibrium from the other two.

6 The Policy Space

Combining the information of Figure 1 and Table 1 is useful to provide a unified discussion of the effects of various policies, and to introduce the concept of policy space.

Each equilibrium slices Figure 1 in three regions whose boundaries are entirely determined by the regulatory cost of trade and innovation. Therefore, differences in $\mathrm{f_X}$ or $\mathrm{f_I}$ across countries result in regions of different sizes. The variable costs of trade determine the region in which each country falls and, ultimately, the effects that trade and innovation policies have on the different cost cut-offs (Table 1). Because any changes to a "policy-triple" $(\tau, \mathrm{f_X}, \mathrm{f_I})$ can be mapped into our parameter space, we refer to Figure 1 as the "policy space" of a country.

A first implication is that a given policy impacts different economies differently: For example, a reduction of tariff lines increases the share of exporting and innovating firms in countries with low initial tariffs (region BW), but reduces the share of innovators in countries with higher tariff levels (region B). This is because the extensive margin of innovation ($\mathrm{c_I^f}$) reacts positively in the first case and negatively in the second case (see Table 1). Likewise, an innovation policy reducing $\mathrm{f_I}$ raises $\mathrm{c_X}$ only for intermediate levels of trade protection (region C), but lowers it in high or low trade cost situations (regions BW and B). This same policy has a differentiated outcome also for countries with similar levels of transportation costs. To illustrate this point, let us consider countries located in region BW at the boundary with region C. A reduction of $\mathrm{f_I}$ in Figure 1 has the effect of squeezing region BW in favor of region C, as $\tau_{\mathrm{BW}}^{\sigma-1}$ shifts to the left. This change, if sufficiently large, may push a country into region C, all else equal. In the new steady state, this policy raises the share of exporters. In a cross-country perspective, only countries which saw a change large enough to fall in the new region experience a pro-trade policy; for the other countries, the movement of c_X goes in the opposite direction.

Applying a time series logic to this example brings us to a second implication, namely that the effects of a given policy mutate with time. If we assume that the reduction of f_I happens in two distinct periods of time rather than across two countries, and that only in the second period of time the equilibrium crossing occurs, the outcome of the policy will be pro-trade in the second period but not in the first period.

A third implication is that policy makers face different policy trade-offs in the three regions. It is apparent from Table 1 that equilibrium C is trade-off free, because both innovation and trade policies increase the share of firms that export and innovate in the economy. In equilibrium BW this is only true for a reduction of the level of tariffs, since all other instruments increase the share of firms performing one activity, but reduce the share of firms performing the other activity. Finally, all policy instruments in equilibrium B involve some sort of trade-off in terms of the activities they promote and harm.

We now discuss the extent to which policy makers can alleviate these trade-offs by introducing policy coordination among different instruments. Let us consider in Figure 1 a country in region B, at the border with region C, which aims at incentivizing the adoption of the more productive technology. In this region, the only means to achieve this goal is a reduction of f_I. This policy, however, also has the secondary effect of pushing the country deeper into region B, as the boundary to region C moves to the left. At each attempt of increasing the share of innovating firms by resorting repeatedly to this type of policy, the share of exporting firms will drop even further. Unfortunately, counterbalancing this effect with the subsequent use of trade policy would not yield the desired effect either, but would only nullify the effects obtained with the innovation policy.[12] The graph suggests that a different strategy centered on a different policy mix could prove to be more beneficial in this circumstance. Reducing first either the tariff or the regulatory costs of trade would tend to push this country toward region C instead. After the transition to the new region, these same trade policies would also favor innovation. Furthermore, the implementation of innovation policies would no longer entail any trade-off in terms of exporting activities.

A similar policy dilemma concerns a country that in Figure 1 is located at the left border of region BW. Such a country has exhausted its possibility to resort to tariff cuts to sustain both trade and innovation, as it approaches $\tau = 1$. Without the tariff instrument at its disposal, policy makers can either sustain innovators (by reducing f_I) but hurt exporters, or sustain exporters (by lowering f_X) and hurt innovators. And using the two policies sequentially is only detrimental as they tend to cancel each other out. If instead this country were at the other extreme of the same region, bordering on region C, it would not face any immediate policy

[12] Note that we cannot here discuss the effects of using innovation and trade policies simultaneously. Table 2 indeed summarizes only the partial derivatives, not the cross-derivatives.

trade-offs. Tariff cuts could indeed be used to increase both the proportion of firms that adopt new technologies and the proportion of firms that export. However, if this country were to pursue this policy repeatedly, it would slide toward the left end of this region and progressively exhaust its degrees of freedom. Starting instead by lowering f_I, it could find itself in region C, as this region expands and region BW contracts. And in the new steady state of region C, pro-innovation policies are also pro-trade policies, and vice versa.

In conclusion, the graph clearly shows two results. First, the boundaries and the size of each region potentially differ across countries. For this reason alone, countries with similar levels of transport costs may nevertheless be in different regions, thus experiencing different outcomes from the same policies. Second, depending on the current level of trade costs, the order in which policies are implemented matters for their final outcomes. Interestingly, putting both these results together, it is possible to reach two apparently antithetic conclusions: The same tariff cut may impact countries with similar levels of tariff protection differently and countries with different levels of tariff protection similarly. Indeed, think of two countries in Figure 1 having different region boundaries and the same level of transportation costs. It is then possible that they fall into two different regions and face heterogeneous responses to the same policy. Likewise, it is possible that two countries with different transportation costs and different region boundaries fall in the same equilibrium and have identical responses to the same policy.

Moreover, when policy makers can resort to multiple instruments, the choice they make in the first place affects the effectiveness of future policy options and the prospective policy trade-offs. In the examples above, some types of policies had undesirable effects which could be avoided with an alternative sequence of policies. In this sense, the model justifies the recent OECD emphasis on the desirability of coordination among trade and innovation policies.[13] But it also highlights the fact that the level of coordination needed is heterogeneous and depends upon the level of trade costs. The graph and the discussion above suggest that in low trade-cost situations innovation policies, eventually followed by trade policies, are beneficial to avoid "policy trade-offs". In high trade-cost situations, the opposite order of policies seems more appropriate. Finally, with an intermediate level of trade costs, coordination is not stringent.

Our discussion so far has abstracted from institutional, administrative, or budget constraints that may limit policy makers' actions. For instance, it can be argued that the institutional context for the approval of innovation and trade policies is radically different in most European countries. Provided they comply with European laws, innovation policies are largely a national matter, whereas changes in trade policies fall under the competence of the European Union or the WTO. Considering in Figure 1 a country positioned in region B at the border with region C, we have argued that pursuing trade policies in the first place was preferable to pursuing innovation policies. However, this conclusion does not take into account whether this strategy was institutionally feasible, nor whether it was the most cost-effective. While we feel that all these elements elevate the discussion of policy coordination, analyzing

[13] See Onodera (2008) for a recent OECD discussion, and OECD (2009).

them in this framework requires the additional introduction of an endogenous public sector, which is beyond the scope of the present paper. However, in the next section we shall briefly discuss the conditions under which the types of innovation and trade policies considered are welfare enhancing, thus providing governments with a rationale for introducing them.

Finally, our results are based on a comparative static analysis between three different steady states and three different policies considered one at a time. It is undeniable that analyzing multiple policy scenarios and/or characterizing the transitional dynamic between different states should also become a prerogative for future research on the comparative analysis of different policy mixes across countries.

7 Trade and the moments of the productivity distribution

The redistribution of market share from exiting firms to incumbent firms (the "between" component) contributes to increase the average productivity in the industry. Indeed, to a higher c_0^f corresponds a higher truncation point of the lower tail of the productivity distribution $G(\cdot)$ and therefore a higher mean. But whether the average productivity will increase also depends on the "within" component. Only when a higher share of firms adopts the innovative technology can the first moment admittedly increase. This is a different prediction from the Melitz's (2003) model where the "within" effect is absent and trade unambiguously increases the average productivity. Likewise, the prediction of our model for the second moment of the distribution differs from Melitz (2003). While in the latter a higher productivity truncation point associated with trade necessarily translates into a lower variance of the productivity distribution, in our model this effect is counterbalanced in some equilibria by the *within-firm* effect. Firms introducing the new technologies indeed contribute to widening the variance of the distribution.

This means that the different policies we have analyzed will also have different effects on the moments of the productivity distribution, and these effects will depend on the current "mix" of policies adopted.

8 Welfare implications of policies

So far, we have considered policies as a *fait accompli* and focused on their outcomes. In this section, we briefly analyze the normative implications of adopting these policies to legitimize their adoption by governments.

We take the indirect utility function $U = R/P$ as the measure of a country's welfare. This expression can be conveniently reduced to

$$U = (\quad 1) f_D \; \frac{L}{f_D} \div^{\overline{\quad 1}} \frac{1}{c_0}, \tag{20}$$

which shows that the entry cost cut-off c_0 is a sufficient statistic to evaluate the welfare impact of a policy. Given that all trade liberalization and innovation policies considered above lower the entry cut-off (Table 1), they are all welfare enhancing.

The main reason behind this result is that these policies reduce the price index (i.e., greater efficiency).

While all policies are welfare-enhancing, without further assumptions on the productivity distribution, we are unable to offer a proper welfare ranking of these policies. This ranking is influenced by the outcome of each policy on the different cost cut-offs and will change across the different regions of Figure 1. Consequently, the discussion above about the different trade-offs faced by the policy makers in terms of the share of firms performing a given activity is still relevant from a policy perspective. Moreover, even if all policies are welfare enhancing, our discussion of the policy space emphasizes that these policies have distinct distributional consequences, which, of course, translate into policy makers' welfare trade-offs.[14]

Discussing the policy space above, we have interpreted reductions in transportation costs as comparable to tariff cuts. This comparison is appropriate as far as the positive effects of tariffs are concerned. In fact, an iceberg transport cost affects production costs exactly as the imposition of a *comparable tariff* would,[15] but it does not generate any additional government revenues. However, when we consider that these revenues are redistributed to consumers in a lump-sum fashion, as is standard in the literature, consumers' and firms' optimal choices are not distorted by this redistribution. In this case it can be shown that the effects of trade and innovation policies on the productivity cut-offs (and indirectly on the share of exporting and innovating firms) are exactly the same as in the case of iceberg transport costs.

9 Technological Spillovers

The existence of technological spillovers constitutes an important element in standard innovation and technology adoption models (Romer, 1986; Romer, 1990; Rivera-Batiz and Romer, 1991; Aghion and Howitt, 1992; and more recently Acemoglu et al., 2006; Keller, 2004; Impulliti and Licandro, 2011).[16] In addition, the "new new trade" literature has found empirical support for the existence of export spillovers and these have recently been incorporated in

[14] To properly account for the impact of these distributional effects we should extend our model to a heterogeneous agent environment. This is a fruitful avenue for further research.

[15] See Schroder and Sorensen (2014) for a specific definition of this type of tariff and the effects of different tariff policies on welfare in the Melitz's (2003) model.

[16] Teece (1977) finds that the cost of technology transfer across countries for multinational firms clearly declines with measures of experience with the technology transferred in the industry.

theoretical models (Rauch and Watson, 2003; Krautheim, 2012). In this section, we show that the main results about the inter-connection between trade and innovation policies are robust to the existence of these technological spillovers.

Following the paper by Krautheim (2012) we consider the notion that current exporters and innovators benefit from the investment in export/innovation that other firms are undertaking at the moment. More precisely, the fixed cost of either innovation or exporting is given by the following functional form:

$$f_i = \lambda_i \left(\theta_i \frac{M_i}{M}\right)^{-\eta}, i = X, I \quad \text{and} \quad \eta < \frac{1}{k}$$

where the fixed cost for each activity depends on an invariant cost which varies across activities (λ_{i}) and the technological spillover (the element in parentheses in the right-hand side of the equation). Notice that this cost declines with the proportion of firms in the current activity status; that is, the larger the proportion of exporters (innovators) in an industry the smaller is the fixed cost of exporting (innovating).[17] θ_{i} is a technological constant, reflecting differences in the effectiveness of technological spillovers across activities, and η is the speed of learning which is common across activities. We assume that η is sufficiently low as in Krautheim (2012) to focus on interior solutions.

Consider the case in which the economy is already open to trade. Solving for each type of equilibria, we can again distinguish three different cases, which are characterized by the conditions provided in Table 2.

INSERT TABLE 2 ABOUT HERE.

If we recall $\widetilde{\mathrm{f}}_{\mathrm{i}} = (\theta_{\mathrm{i}})^{-\eta}\lambda_{\mathrm{i}}$, the equations displayed in Table 2 are very similar to the corresponding ones in a model without spillover (see Table A1 in the appendix). The regions in which each of the different types of equilibria occur clearly do not depend on the exponent associated with the relative productivity cut-offs, and consequently Figure 1 remains unaltered.[18] It could also be shown that the sign of each of the derivatives computed in the new specification exhibits exactly the same sign, provided that these derivatives are computed with respect to $\widetilde{\mathrm{f}}_{\mathrm{i}}$.[19] Knowledge spillovers in our economy affect our conclusions in quantitative terms but not in qualitative terms.[20]

[17] We assume that the technological spillover element depends on the proportion rather than the number of firms in the current activity status to avoid the traditional scale effect problem found in early innovation models: The possibility that population size affects the industry's average productivity.

[18] The additional necessary conditions for equilibrium *C* to exist also holds in this context.

[19] Results available upon request.

[20] For example the existence of technological spillovers clearly reduces the distance between the different productivity cut-offs across equilibria $(c_I/c_X),(c_X/c_0)$. It is, however, challenging to offer an idea on the magnitude in each of the productivity cut-offs without considering a specific functional form for the productivity distribution of firms.

10 Conclusion

To examine the scope for innovation policy as a stimulus to both innovation and trade, we include the option of technology adoption in a workhorse trade model with firm heterogeneity. Leaving aside any discussion of whether the notion of productivity-enhancing investment is the same thing as innovation, this model is capable of unifying several empirical scenarios, in which innovating non-exporters and exporting non-innovators are represented. These scenarios can be ordered along an increasing measure of countries' degree of protectionism.

We propose a comparative analysis of policies aiming at reducing the variable costs of trade, the regulatory costs of trade (fixed cost of exporting), and the fixed costs of innovation. The outcomes of interest are the shares of both innovating and exporting firms. Whenever the share of the latter declines in response to an innovation policy, this policy cannot stimulate trade. This is the case for the most liberal or protectionist countries. The mirror channel is the influence of trade liberalization on innovation.

The interplay between the two channels leads to a number of results. First, the policy that stimulates one activity (e.g., trade policy) can be counter-productive for the other activity (e.g., innovation) under certain circumstances (e.g., high trade costs). Second, the "initial condition" in a country (i.e., trade openness) determines the impact of a given policy. Third, the order in which policies are implemented matters for the future efficacy of policies and is important in order to mitigate policy trade-offs.

The paper has abstracted from the realistic consideration that policies are typically costly to implement. Costly could mean "institutionally" costly, in the sense that legal constraints increase the cost of a policy change; or, "administratively" costly, so that the choice of a policy is dictated by cost-efficiency reasons. Incorporating these considerations requires a framework with an endogenous public sector, certainly an interesting and relevant avenue for a future research agenda, that is beyond the scope of the present paper.

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Appendix

.1 Appendix A - Definitions

In this section we focus only on the open economy scenario. For a description of the closed economy scenario see the online appendix pubblished at: (*forthcoming*).

Open economy
For deriving the aggregate properties of the model, the cost distributions should be defined for each type of equilibria, since innovators and exporters price differently than domestic firms and the sorting process changes with each type of equilibria.

Equilibrium BW
Cost distributions and productivity indexes

Let $\mu_{\mathrm{D}}(\mathrm{c}) = \left\{ \begin{matrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_0^f) - G(c_X)}}, & \mathrm{c_X < c \le c_0^f} \\ 0 & \text{otherwise} \end{matrix} \right\}$,

$\mu_{\mathrm{XN}}(\mathrm{c}) = \left\{ \begin{matrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_X) - G(c_I^f)}}, & \mathrm{c_I^f < c \le c_X} \\ 0 & \text{otherwise} \end{matrix} \right\}$,

$\mu_{\mathrm{I}}(\mathrm{c}) = \left\{ \begin{matrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_I^f)}}, & \mathrm{0 \le c \le c_I^f} \\ 0 & \text{otherwise} \end{matrix} \right\}$

denote the cost distributions in each subgroup prior to innovation.

Let $\left(\tilde{c}_D^f\right)^{1-\sigma} = \int_{c_X}^{c_0^f} c^{1-\sigma}\mu_D(c)dc$, $\left(\tilde{c}_I^f\right)^{1-\sigma} = \int_0^{c_I^f} c^{1-\sigma}\mu_I(c)dc$ and $\left(\tilde{c}_{XN}\right)^{1-\sigma} = \int_{c_I^f}^{c_X} c^{1-\sigma}\mu_{XN}(c)dc$ be the respective average productivities for domestic firms, innovators, and exporting non-innovators prior to innovation.

Aggregate variables

Analogous to the closed economy version we obtain $P^{1-\sigma} = \left(\dfrac{\sigma}{\sigma-1}\right)^{1-\sigma} M^f\left(\tilde{c}^f\right)^{1-\sigma}$, where $\left(\tilde{c}^f\right)^{1-\sigma} = \dfrac{1}{M^f}\left[M_D\left(\tilde{c}_D^f\right)^{1-\sigma} + \left(1+n\tau^{1-\sigma}\right)\left(M_{XN}\left(\tilde{c}_{XN}\right)^{1-\sigma} + M_I\gamma^{1-\sigma}\left(\tilde{c}_I^f\right)^{1-\sigma}\right)\right]$ is again the weighted average productivity index of the economy. As in the closed economy it can be shown that

$$R = M^f r(\tilde{c}^f) = M^f \bar{r}^f \tag{21}$$

$$-^f = \frac{\bar{r}^f}{\quad} \quad f_D \quad \frac{G(c_I^f)}{G(c_0^f)} \; f_I \quad \frac{G(c_X)}{G(c_0^f)} n \;\; f_X. \tag{22}$$

The latter equations together with

$$\bar{\pi}^{\mathrm{f}} = \frac{\delta \mathrm{f_e}}{\mathrm{G(c_0^f)}} \tag{23}$$

and

$$\mathrm{c_X} = \left(\frac{\mathrm{f_D}}{\delta \mathrm{f_X}}\right)^{\frac{1}{\sigma-1}} \frac{\mathrm{c_0^f}}{\tau} = \mathrm{f}(\mathrm{f_D}, \mathrm{f_X}, \delta, \mathrm{c_0^f}, \tau)$$

and

$$\mathrm{c_I^f} = \left(\frac{(\gamma^{1-\sigma}-1)(1+\mathrm{n}\tau^{1-\sigma})\,\mathrm{f_D}}{\delta \mathrm{f_I}}\right)^{\frac{1}{\sigma-1}} \mathrm{c_0^f} = \mathrm{g}(\mathrm{f_D}, \mathrm{f_I}, \delta, \mathrm{c_0^f}, \tau, \mathrm{n}, \gamma) \tag{24}$$

determine the unique equilibrium. For the latter proofs it is useful to express the productivity cut-off as

$$\mathrm{c_I^f} = \left(\frac{(\gamma^{1-\sigma}-1)(1+\mathrm{n}\tau^{1-\sigma})\,\mathrm{f_X}}{\mathrm{f_I}}\right)^{\frac{1}{\sigma-1}} \tau \mathrm{c_X} = \mathrm{h}(\mathrm{f_X}, \mathrm{f_I}, \mathrm{c_X}, \tau, \mathrm{n}, \gamma)$$

Equilibrium B

In equilibrium B all exporters are innovators, but not all innovators are exporters. There is a subset of domestic firms which also innovate. To derive the aggregate properties of the model, let us define the following cost functions:

$$\mu_{\mathrm{D}}(\mathrm{c}) = \begin{Bmatrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_0^f)} - \mathrm{G(c_I^f)}}, & \mathrm{c_I^f} < \mathrm{c} \le \mathrm{c_0^f} \\ 0 & \text{otherwise} \end{Bmatrix},$$

$$\mu_{\mathrm{NI}}(\mathrm{c}) = \begin{Bmatrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_I^f)} - \mathrm{G(c_X)}}, & \mathrm{c_X} < \mathrm{c} \le \mathrm{c_I^f} \\ 0 & \text{otherwise} \end{Bmatrix},$$

$$\mu_{\mathrm{X}}(\mathrm{c}) = \begin{Bmatrix} \dfrac{\mathrm{g(c)}}{\mathrm{G(c_X)}}, & 0 \le \mathrm{c} \le \mathrm{c_X} \\ 0 & \text{otherwise} \end{Bmatrix}.$$

Then we have

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M^{f}\left(\tilde{c}^{f}\right)^{1-\sigma},$$

where $\left(\tilde{c}^{f}\right)^{1-\sigma} = \dfrac{1}{M^{f}}\left[\gamma^{1-\sigma} M_{NI}\left(\tilde{c}_{NI}\right)^{1-\sigma} + M_{D}\left(\tilde{c}_{D}^{f}\right)^{1-\sigma} + (1+n\tau^{1-\sigma}) M_{X}\gamma^{1-\sigma}\left(\tilde{c}_{X}\right)^{1-\sigma}\right)\right],$

where $\left(\tilde{c}_{D}^{f}\right)^{1-\sigma} = \int_{c_I^f}^{c_0^f} c^{1} \quad {}_{D}(c)dc$ and $\left(\tilde{c}_{NI}\right)^{1-\sigma} = \int_{c_X}^{c_I^f} c^{1-\sigma}\mu_{NI}(c)dc$. As in the equilibrium BW equations (21), (22), (23) can be derived. These together with

$$c_X = \left(\frac{f_D}{\delta f_X}\right)^{\frac{1}{\sigma-1}} \frac{c_0^f}{\tau\gamma} = f^*(f_D, f_X, \delta, c_0^f, \tau, \gamma)$$

and

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)\, f_D}{\delta f_I}\right)^{\frac{1}{\sigma-1}} c_0^f = g^*(f_D, f_I, \delta, c_0^f, \tau, \gamma)$$

determine the unique equilibrium. For latter proofs it is also useful to express the productivity cut-off as

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)\, f_X}{f_I}\right)^{\frac{1}{\sigma-1}} (\tau\gamma) c_X = h^*(f_X, f_I, c_X, \tau, \gamma).$$

Equilibrium C

In equilibrium C all exporters are innovators, and all innovators are exporters. The conditional productivity distributions of exporters and innovators prior to innovation are the same. The expression for the aggregate price index now becomes:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M^f \left(\tilde{c}^f\right)^{1-\sigma},$$

where $\left(\tilde{c}^f\right)^{1-\sigma} = \frac{1}{M^f}\left[M_D\left(\tilde{c}_D^f\right)^{1-\sigma} + \left(1+n\tau^{1-\sigma}\right)\left(M_I\gamma^{1-\sigma}\left(\tilde{c}_I^f\right)^{1-\sigma}\right)\right]$, and the relationships

between the export and innovation productivity cut-offs are given by

$$c_X = \left(\frac{\delta(f_I + nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)\, f_D}\right)^{\frac{1}{1-\sigma}} c_0^f = f^{**} = g^{**}(f_X, f_I, c_0^f, \tau, n, \gamma, f_D, \delta)$$

and since $c_X = c_I \Rightarrow h^{**} = 1$. As in equilibrium BW (21), (22), (23) are easily derived. For the next propositions we are able to express the ZP condition in all three equilibria as

$$f_D\, j_0(c_0^f) + n\delta f_X\, j_X(c_X) + \delta f_I\, j_I(c_I^f) = \delta f_E, \tag{25}$$

where

$$j_i(c_i) = \left(\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} - 1\right) G(c_i), c_i = c_0^f, c_X, c_I^f \text{ and } i = 0, X, I$$

and $\left(\tilde{c}_0^f\right)^{1-\sigma} = \int_0^{c_0^f} c^{1-\sigma}\mu(c)\,dc$ and $\mu(c) = \begin{cases} \dfrac{g(c)}{G(c_0^f)}, & 0 \le c \le c_0^f \\ 0 & \text{otherwise} \end{cases}$.

This productivity distribution corresponds to the incumbent productivity distribution prior to innovation. The $j_i(c_i)$ functions are continuous. In the online appendix we show that

$$j_i^{'}(c_i) = \frac{(\sigma - 1)}{c_i}\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} G(c_i) > 0,$$

and therefore these functions are monotonically increasing in their respective arguments. We also show that the elasticities are given by:

$$\frac{j_i^{'}(c_i)}{j_i(c_i)} c_i = \frac{(\sigma - 1)\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma}}{\left(\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} - 1\right)}. \tag{26}$$

These results will be useful in the following section. A summary of the results are displayed in Table A1

Equilibrium	$\frac{c_I}{c_X}$	$\frac{c_X}{c_0}$
BW	$\left(\frac{f_I \tau^{1-\sigma}}{f_X(\gamma^{1-\sigma} - 1)(1 + n\tau^{1-\sigma})}\right)^{\frac{1}{1-\sigma}}$	$\frac{f_X}{{}^{1}\ f_D} \div^{\frac{1}{1}}$
C	1	$\left(\frac{\delta(f_I + nf_X)}{((1 + n\tau^{1-\sigma})\gamma^{1-\sigma} - 1)f_D}\right)^{\frac{1}{1-\sigma}}$
B	$\left(\left(\frac{f_I(\tau\gamma)^{1-\sigma}}{f_X(\gamma^{1-\sigma} - 1)}\right)\right)^{\frac{1}{1-\sigma}}$	$\frac{f_X}{(\ )^{1}\ f_D} \div \div^{\frac{1}{1}}$

Table A1: Equilibria without technological spillovers

.2 Appendix B - Parameter restriction for equilibrium C

In equilibrium C, firms consider the joint option of exporting and innovating. According to what is derived in section 3.3, no firm will innovate without being an exporter, and no firm will export without being an innovator. The firm evaluates whether to export and innovate is better than not doing both and remain local. The partition in this equilibrium is between exporting and innovating firms, and local firms. Let us call c_I w.l.o.g. the marginal cost associated with the firm which is indifferent between both options.

Then this marginal firm will satisfy the following condition:

$$(1 + n\tau^{1-\sigma})\gamma^{1-\sigma} B(c_I^f)^{1-\sigma} - \delta(f_I + nf_X) - f_D - \left(B(c_I^f)^{1-\sigma} - f_D\right) = 0$$

$$((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)B\left(c_I^f\right)^{1-\sigma}=\delta(f_I+nf_X).$$

The marginal firm being indifferent between staying in the market or not satisfies

$$Bc_0^{1-\sigma}=f_D.$$

Then a necessary condition for this equilibrium to exist is that

$$\left(\frac{c_I^f}{c_0}\right)^{1-\sigma}>1\Rightarrow\frac{\delta(f_I+nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)\,f_D}>1.$$

The previous equation guarantees that not all firms innovate and export. However, for equilibrium C to exist, the marginal firm must be indifferent between innovating and exporting or being local. This implies that the income of the marginal firm in equilibrium C must be larger than the income of this firm when it innovates, provided that the firm will be an exporter, or when it exports, provided that the firm will be an innovator. In other terms:

$$B\left(c_I^f\right)^{1-\sigma}\geq\frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}}\tag{27}$$

$$B\left(c_I^f\right)^{1-\sigma}\geq\delta f_X\gamma^{\sigma-1}\tau^{\sigma-1}.\tag{28}$$

The marginal firm in equilibrium C has the following income:

$$B\left(c_I^f\right)^{1-\sigma}=\frac{\delta(f_I+nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}.\tag{29}$$

Substituting (29) in (27), we have

$$\frac{\delta(f_I+nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\geq\frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}}.$$

Rearranging terms in the latter equation, we have

$$\delta f_I\gamma^{1-\sigma}\left(1+n\tau^{1-\sigma}\right)-\delta f_I\left(1+n\tau^{1-\sigma}\right)+n\delta f_X\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right)\geq\delta f_I\gamma^{1-\sigma}\left(1+n\tau^{1-\sigma}\right)-\delta f_I.$$

Rearranging terms, we have

$$-\delta f_I n\tau^{1-\sigma}+n\delta f_X\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right)\geq 0,$$

and then this implies

$$\delta f_X\tau^{\sigma-1}\geq\frac{\delta f_I}{\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right)}.$$

This is one of the restrictions needed to be satisfied if equilibrium C holds.
Now substituting (29) in (28) we have

$$\frac{\delta(f_I + nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma} - 1)} \geq \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1}.$$

Rearranging terms, we have

$$\delta f_I + n\delta f_X \geq \delta f_X \tau^{\sigma-1} + n\delta f_X - \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1},$$

and this implies

$$\delta f_I \geq \delta f_X \tau^{\sigma-1} - \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1},$$

which implies

$$\delta f_I \geq \delta f_X \tau^{\sigma-1}(1-\gamma^{\sigma-1}).$$

Multiplying both sides by $(\tau\gamma)^{1-\sigma}$ and rearranging terms, we have

$$\frac{\delta f_I}{(\gamma^{1-\sigma} - 1)}\gamma^{1-\sigma} \geq \tau^{\sigma-1}\delta f_X$$

which is the other requirement for equilibrium C to hold.

.3 Appendix C - Proof of Propositions.

Proposition 1 *(Trade Liberalization: Equilibrium BW): Trade Liberalization yields the following results:*

1. Tariff policy: $\frac{\partial c_0^f}{\partial \tau} > 0, \quad \frac{\partial c_X}{\partial \tau} < 0, \quad \frac{\partial c_I^f}{\partial \tau} < 0$
2. Export regulation costs (f_X): $\frac{\partial c_0^f}{\partial f_X} > 0, \quad \frac{\partial c_X}{\partial f_X} < 0, \quad \frac{\partial c_I^f}{\partial f_X} > 0$

Proof. 1. A tariff reduction:

Totally differentiate (25) with respect to τ:

$$f_D\, j_0'(c_0^f)\frac{\partial c_0^f}{\partial \tau} + n\delta f_X\, j_X'(c_X)\frac{\partial c_X}{\partial \tau} + \delta f_I\, j_I'(c_I^f)\frac{\partial c_I^f}{\partial \tau} = 0, \tag{30}$$

where, as described above, $c_X = f(c_0^f, \tau, f_D, f_X, \delta)$, $c_I^f = g(c_0^f, \tau, f_D, f_I, n, \delta, \gamma)$, $c_I^f = h(c_X, f_I, \tau, f_X, n, \gamma)$. Applying the chain rule and applying the following results: $\frac{\partial f}{\partial c_0^f} = \frac{c_X}{c_0^f}, \frac{\partial f}{\partial \tau} = -\frac{c_X}{\tau}, \frac{\partial g}{\partial c_0^f} = \frac{c_I^f}{c_0^f}, \frac{\partial g}{\partial \tau} = -\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right),$ we obtain

$$\frac{\partial c_0^f}{\partial \tau} = \frac{n\delta f_X\, j_X'(c_X)\frac{c_X}{\tau} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)}{\left(f_D\, j_0'(c_0^f) + n\delta f_X\, j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

Taking into consideration that

$$\frac{\partial c_I^f}{\partial \tau} = \frac{\partial h}{\partial c_X}\frac{\partial c_X}{\partial \tau} + \frac{\partial h}{\partial \tau}$$

in (30) and applying the following results: $\frac{\partial h}{\partial c_X} = \frac{c_I^f}{c_X}$ $\quad \frac{\partial h}{\partial \tau} = \frac{\partial h}{\partial \phi}\frac{\partial \phi}{\partial \tau} = \frac{c_I^f}{\tau}\frac{1}{1+n\tau^{1-\sigma}}$ where $\phi = \tau^{1-\sigma}$, then:

$$\frac{\partial c_X}{\partial \tau} = \frac{-\left(f_D\, j_0'(c_0^f)\frac{\partial c_0^f}{\partial \tau} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{\tau}\frac{1}{1+n\tau^{1-\sigma}}\right)}{\left(n\delta f_X\, j_X'(c_X) + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{c_X}\right)} < 0.$$

To get the sign of the derivative of $\frac{\partial c_I^f}{\partial \tau}$ we apply the chain rule

$$\frac{\partial c_I^f}{\partial \tau} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau} + \frac{\partial g}{\partial \tau}.$$

Then substituting the expressions for $\frac{\partial g}{\partial c_0^f}$, $\frac{\partial g}{\partial \tau}$ we get

$$\frac{\partial c_I^f}{\partial \tau} = \frac{c_I^f}{c_0^f}\frac{\partial c_0^f}{\partial \tau} - \frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Then showing $\frac{\partial c_I^f}{\partial \tau} < 0$ implies

$$\frac{\partial c_0^f}{\partial \tau} < \frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Substituting the expression for $\frac{\partial c_0^f}{\partial \tau}$ in the previous equation, we obtain

$$\frac{n\delta f_X\, j_X'(c_X)\frac{c_X}{\tau} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)}{\left(f_D\, j_0'(c_0^f) + n\delta f_X\, j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} < \frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Manipulating the expression we arrive at the following condition

$$f_D\, j_0'(c_0^f)\frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) - n\delta f_X\, j_X'(c_X)\frac{c_X}{\tau}\left(\frac{1}{1+n\tau^{1-\sigma}}\right) > 0.$$

Simplifying, we get

$$f_D\, j_0'(c_0^f)\, c_0^f\, \tau^{1-\sigma} > \delta f_X\, j_X'(c_X)\, c_X$$

rearranging

$$\left(\frac{\tau^{1-\sigma} f_D}{\delta f_X}\right) j_0'(c_0^f)\, c_0^f > j_X'(c_X)\, c_X,$$

where the first element is

$$\left(\frac{c_0^f}{c_X}\right)^{1-\sigma} j_0'(c_0^f)\, c_0^f > j_X'(c_X)\, c_X.$$

Substituting the expressions $j_D'(c_0^f)$, $j_X'(c_X)$, we have that

$$\left(\tilde{c}_0^f\right)^{1-\sigma} G\left(c_0^f\right) > \left(\tilde{c}_X\right)^{1-\sigma} G\left(c_X\right),$$

and substituting the expressions $\left(\tilde{c}_0^f\right)^{1-\sigma}, \left(\tilde{c}_X\right)^{1-\sigma}$, we get

$$\int_0^{c_0^f} c^{1-\sigma} g(c)\,dc > \int_0^{c_X} c^{1-\sigma} g(c)\,dc$$

since $c_X < c_0^f$.

2. A decrease in f_X:

Totally differentiating (25) with respect to f_X, we find that

$$f_D\, j_0'(c_0^f)\frac{\partial c_0^f}{\partial f_X} + n\delta j_X\left(c_X\right) + n\delta f_X\, j_X'(c_X)\frac{\partial c_X}{\partial f_X} + \delta f_I\, j_I'(c_I^f)\frac{\partial c_I^f}{\partial f_X} = 0. \qquad (31)$$

Again applying the chain rule and the following results: $\frac{\partial f}{\partial f_X} = \frac{-1}{\sigma-1}\frac{c_X}{f_X}$, $\frac{\partial h}{\partial f_X} = \frac{1}{\sigma-1}\frac{c_I^f}{f_X}$, $\frac{\partial g}{\partial f_X} = 0$, we have that

$$\frac{\partial c_0^f}{\partial f_X} = \frac{n\delta\left(j_X'(c_X)\frac{c_X}{\sigma-1} - j_X\left(c_X\right)\right)}{\left(f_D\, j_0'(c_0^f) + n\delta f_X\, j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I\, j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

The latter is positive. To see this, notice that the numerator is positive iff

$$\frac{j_X'(c_X)}{j_X(c_X)} c_X > \sigma - 1. \qquad (32)$$

Using (26) in (32):

$$\left(\frac{\tilde{c}_X}{c_X}\right)^{1-\sigma} > \left(\left(\frac{\tilde{c}_X}{c_X}\right)^{1-\sigma} - 1\right),$$

which is always satisfied.

To get the sign of $\frac{\partial c_X}{\partial f_X}$, we use (31) and the fact that

$$\frac{\partial c_I^f}{\partial f_X} = \frac{\partial h}{\partial c_X}\frac{\partial c_X}{\partial f_X} + \frac{\partial h}{\partial f_X}$$

to obtain

$$\frac{\partial c_X}{\partial f_X} = \frac{-\left(f_D\, j_0^{'}(c_0^f)\frac{\partial c_0^f}{\partial f_X} + \delta f_I\, j_I^{'}(c_I^f)\frac{1}{\sigma - 1}\frac{c_I^f}{f_X} + n\delta j_X(c_X)\right)}{\left(n\delta f_X\, j_X^{'}(c_X) + \delta f_I\, j_I^{'}(c_I^f)\frac{c_I^f}{c_X}\right)} < 0.$$

Note that from the chain rule we can derive

$$\frac{\partial c_I^f}{\partial f_X} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_X} + \frac{\partial g}{\partial f_X} > 0.$$

■

Proposition 2 *Trade Liberalization (Equilibrium* B*) : Trade Liberalization experiments in this equilibrium yields the following results:*

1. Tariff policy: $\frac{\partial c_0^f}{\partial \tau} > 0$, $\frac{\partial c_X}{\partial \tau} < 0$, $\frac{\partial c_I^f}{\partial \tau} > 0$

2. Export regulation costs (f_X): $\frac{\partial c_0^f}{\partial f_X} > 0$, $\frac{\partial c_X}{\partial f_X} < 0$, $\frac{\partial c_I^f}{\partial f_X} > 0$

Proof. The Proof follows the same scheme as the previous proof. However, we know that now the equations determining the cut-offs have changed and then we have that

$$c_X = f^*(c_0^f, \tau, f_X, \delta), \quad c_I^f = g^*(c_0^f, \tau, f_I, \delta, \gamma, f_D), \quad c_I^f = h^*(c_X, \tau, f_X, f_I, \gamma).$$

1. Tariff policy:

Applying the chain rule and using (30) and the following results: $\frac{\partial f^*}{\partial \tau} = \frac{-c_X}{\tau}$ $\frac{\partial f^*}{\partial c_0^f} = \frac{c_X}{c_0^f}$ $\frac{\partial g^*}{\partial \tau} = 0$ $\frac{\partial g^*}{\partial c_0^f} = \frac{c_I^f}{c_0^f}$, we obtain

$$\frac{\partial c_0^f}{\partial \tau} = \frac{n\delta f_X\, j_X^{'}(c_X)\frac{c_X}{\tau}}{\left(f_D\, j_0^{'}(c_0^f) + n\delta f_X\, j_X^{'}(c_X)\frac{c_X}{c_0^f} + \delta f_I\, j_I^{'}(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

Applying the chain rule in (30) and using $\frac{\partial f^*}{\partial \tau} = \frac{-c_X}{\tau}$ $\frac{\partial f^*}{\partial c_0^f} = \frac{c_X}{c_0^f}$, we obtain

$$\frac{\partial c_X}{\partial \tau}=\frac{-\left(f_D\, j_0^{'}(c_0^f)\dfrac{c_0^f}{\tau}+\delta f_I\, j_I^{'}(c_I^f)\dfrac{c_I^f}{\tau}\right)}{\left(f_D\, j_0^{'}(c_0^f)\dfrac{c_0^f}{c_X}+n\delta f_X\, j_X^{'}(c_X)+\delta f_I\, j_I^{'}(c_I^f)\dfrac{c_I^f}{c_X}\right)}<0$$

and for the innovation cut-off we just apply the chain rule:

$$\frac{\partial c_I^f}{\partial \tau}=\frac{\partial g^*}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau}+\frac{\partial g^*}{\partial \tau}>0,$$

but we know that: $\frac{\partial g^*}{\partial \tau}=0$ and $\frac{\partial g^*}{\partial c_0^f}>0,\ \frac{\partial c_0^f}{\partial \tau}>0.$ So $\frac{\partial c_I^f}{\partial \tau}>0.$

2. Export regulation costs:

Notice that $\frac{\partial f^*}{\partial f_X}=\frac{-1}{\sigma-1}\frac{c_X}{f_X}$, $\frac{\partial h^*}{\partial f_X}=\frac{1}{\sigma-1}\frac{c_I^f}{f_X}$, $\frac{\partial g^*}{\partial f_X}=0$, and $\frac{\partial f^*}{\partial c_0^f}=\frac{c_X}{c_0^f},\frac{\partial g^*}{\partial c_0^f}=\frac{c_I^f}{c_0^f},\frac{\partial h^*}{\partial c_0^f}=\frac{c_I^f}{c_X}$. Since $j_i^{'}(c_i)>0 \quad \forall i=0,X,I$ and $c_i=c_0^f,c_X,c_I^f$. The partial derivatives mentioned above are the same in both equilibria BW and B; therefore the same proof applies. ■

Proposition 3 *(Trade Liberalization. Equilibrium C): Trade Liberalization experiments in this equilibrium yield the following results:*

1. Tariff policy: $\frac{\partial c_0^f}{\partial \tau}>0,\ \frac{\partial c_X}{\partial \tau}<0,\ \frac{\partial c_I^f}{\partial \tau}<0$

2. Export regulation costs (f_X): $\frac{\partial c_0^f}{\partial f_X}>0,\ \frac{\partial c_X}{\partial f_X}<0,\ \frac{\partial c_I^f}{\partial f_X}<0$

Proof. The proof is analogous to the other type of equilibria. It is important to remember however that in this equilibrium: $c_I^f=c_X \Rightarrow \tilde{c}_I^f=\tilde{c}_X \Rightarrow j_X(c_X)=j_I(c_I^f)$. Also $c_X=f^{**}(c_0^f,\tau,f_X,f_D,n,\delta,\gamma,f_I)$, $c_I^f=g^{**}(c_0^f,\tau,f_X,f_D,n,\delta,\gamma,f_I)$, $c_I^f=h^{**}(c_X)$, where $f^{**}(c_0^f,\tau,f_X,f_D,n,\delta,\gamma,f_I)=g^{**}(c_0^f,\tau,f_X,f_D,n,\delta,\gamma,f_I)$. Then $c_I^f=h^{**}(c_X)=c_X$.

1. Tariff policy:

Totally differentiating (25) we get

$$f_D\, j_0^{'}(c_0^f)\frac{\partial c_0^f}{\partial \tau}+n\delta f_X\, j_X^{'}(c_X)\frac{\partial c_X}{\partial \tau}+\delta f_I\, j_I^{'}(c_I^f)\frac{\partial c_I^f}{\partial \tau}=0.$$

Applying the chain rule and the following results, $\frac{\partial f^{**}}{\partial \tau}=\frac{\partial g^{**}}{\partial \tau}=\frac{-c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{\left((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1\right)}\right)$ $\frac{\partial f^{**}}{\partial c_0^f}=\frac{\partial g^{**}}{\partial c_0^f}=\frac{c_X}{c_0^f}$, we arrive at the following result:

$$\frac{\partial c_0^f}{\partial \tau}=\frac{\delta(f_I+nf_X)\,j_X^{'}(c_X)\dfrac{c_X}{\tau}\left(\dfrac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{\left((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1\right)}\right)}{\left(f_D\,j_0^{'}(c_0^f)+\delta(f_I+nf_X)\,j_X^{'}(c_X)\dfrac{c_X}{c_0^f}\right)}>0.$$

Remember that in this equilibrium $\frac{\partial c_I^f}{\partial \tau}=\frac{\partial c_X}{\partial \tau}$. To obtain the sign of this derivative, we first apply the chain rule to get

$$\frac{\partial c_X}{\partial \tau}=\frac{\partial f^{**}}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau}+\frac{\partial f^{**}}{\partial \tau}.$$

Substituting $\frac{\partial c_0^f}{\partial \tau}$ into the previous expression, we get

$$\frac{\partial c_X}{\partial \tau}=\frac{\delta(nf_X+f_I)\,j_X^{'}(c_X)\dfrac{c_X}{\tau}\left(\dfrac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{\left((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1\right)}\right)}{\left(f_D\,j_0^{'}(c_0^f)\dfrac{c_0^f}{c_X}+\delta(f_I+nf_X)\,j_X^{'}(c_X)\right)}-\frac{c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{\left((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1\right)}\right).$$

Rearranging terms it can be shown that this expression is negative whenever the following holds:

$$f_D\,j_0^{'}(c_0^f)\frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{\left((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1\right)}\right)>0.$$

2. Export regulation costs:
Note that

$$\frac{\partial f^{**}}{\partial f_X}=\frac{-c_X}{\sigma-1}\left(\frac{n}{f_I+nf_X}\right).$$

Then totally differentiating and applying the latter results yields the following expression:

$$\frac{\partial c_0^f}{\partial f_X}=\frac{\delta(f_I+nf_X)\,j_X^{'}(c_X)\dfrac{c_X}{\sigma-1}\left(\dfrac{n}{(f_I+nf_X)}\right)-n\delta j_X(c_X)}{\left(f_D\,j_0^{'}(c_0^f)+\delta(f_I+nf_X)\,j_X^{'}(c_X)\dfrac{c_X}{c_0^f}\right)}>0.$$

Note that the sign of this derivative is determined by the sign of the numerator. Simplifying we obtain

$$\frac{j_X^{'}(c_X)c_X}{j_X(c_X)}>\sigma-1,$$

which clearly holds as we have shown in previous sections.

To get the sign for $\frac{\partial c_X}{\partial f_X}=\frac{\partial c_I^f}{\partial f_X}$, we totally differentiate expression (25) and we apply

$\frac{\partial h}{\partial c_X} = 0$. Then we obtain

$$\frac{\partial c_X}{\partial f_X} = \frac{-\left(f_D\, j_0^{'}(c_0^f)\frac{\partial c_0^f}{\partial f_X} + n\delta j_X(c_X) \right)}{\delta(f_I + nf_X)\, j_X^{'}(c_X)} < 0$$

since $\frac{\partial c_0^f}{\partial f_X} > 0$. ■

Proposition 4 *(Innovation policies). A reduction in the costs of innovation reduces the export cut-off*(c_X) *and the domestic cut-off* (c_X). *It increases the innovation cut-off* $\left(c_I^f\right)$.

Proof. Equilibrium BW

We show that $\frac{\partial c_0^f}{\partial f_I} > 0, \frac{\partial c_X}{\partial f_I} > 0, \frac{\partial c_I^f}{\partial f_I} < 0.$

Differentiating (25) with respect to f_I we get

$$f_D\, j_0^{'}(c_0^f)\frac{\partial c_0^f}{\partial f_I} + n\delta f_X\, j_X^{'}(c_X)\frac{\partial c_X}{\partial f_I} + \delta j_I(c_I^f) + \delta f_I\, j_I^{'}(c_I^f)\frac{\partial c_I^f}{\partial f_I} = 0.$$

Totally differentiating the previous conditions, we have

$$\frac{\partial c_X}{\partial f_I} = \frac{\partial f}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial f}{\partial f_I}$$

$$\frac{\partial c_I^f}{\partial f_I} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial g}{\partial f_I}$$

and also applying the following results: $\frac{\partial f}{\partial f_I} = 0,\quad \frac{\partial g}{\partial f_I} = \frac{1}{1-\sigma}\frac{c_I^f}{f_I}$

$$\frac{\partial c_I^f}{\partial f_I} = \frac{\delta\left(\frac{j_I^{'}(c_I^f)c_I^f}{\sigma-1} - j_I(c_I^f) \right)}{\left(f_D\, j_0^{'}(c_0^f) + n\delta f_X\, j_X^{'}(c_X)\frac{c_X}{c_0^f} + \delta f_I\, j_I^{'}(c_I^f)\frac{c_I^f}{c_0^f} \right)}. \tag{33}$$

This condition is positive provided that

$$\frac{j_I^{'}(c_I^f)c_I^f}{\sigma-1} - j_I(c_I^f) > 0.$$

Rearranging terms and using (26), for the latter expression to be positive the following must hold:

$$\left(\frac{\tilde{c}_I^f}{c_I^f}\right)^{1-\sigma} > \left(\left(\frac{\tilde{c}_I^f}{c_I^f}\right)^{1-\sigma} - 1\right),$$

which obviously holds. Moreover, since

$$\frac{\partial c_X}{\partial f_I} = \frac{\partial f}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial f}{\partial f_I}$$

we then have $\frac{\partial c_X}{\partial f_I} > 0$, since $\frac{\partial f}{\partial f_I} = 0$. To get the sign for $\frac{\partial c_I^f}{\partial f_I}$, we use the following expression:

$$\frac{\partial c_I^f}{\partial f_I} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial g}{\partial f_I}.$$

We want to show that $\frac{\partial c_I^f}{\partial f_I} < 0$. Substituting (33) and rearranging terms, we arrive at the following expression:

$$\frac{\delta\left(\dfrac{j_I^{'}(c_I^f)c_I^f}{\sigma - 1} - j_I(c_I^f)\right)}{\left(f_D\, j_0^{'}(c_0^f)\dfrac{c_0^f}{c_I^f} + n\delta f_X\, j_X^{'}(c_X)\dfrac{c_X}{c_I^f} + \delta f_I\, j_I^{'}(c_I^f)\right)} - \frac{1}{\sigma - 1}\frac{c_I^f}{f_I} < 0.$$

Rearranging terms we arrive at the following expression:

$$f_D\, j_0^{'}(c_0^f)\frac{c_0^f}{(\sigma - 1)f_I} + n\delta f_X\, j_X^{'}(c_X)\frac{c_X}{(\sigma - 1)f_I} + \delta j_I(c_I^f) > 0,$$

which clearly holds.

Equilibrium B

Differentiating f^*, g^*, and h^* with respect to f_I, we get $\frac{\partial f^*}{\partial f_I} = 0$, $\frac{\partial g^*}{\partial f_I} = \frac{1}{1-\sigma}\frac{c_I^f}{f_I}$.

Together with this we have $\frac{\partial f^*}{\partial c_0^f} = \frac{\partial f}{\partial c_0^f}, \frac{\partial g^*}{\partial c_0^f} = \frac{\partial g}{\partial c_0^f}$. Then the results are analogous to the ones in equilibrium BW. The proof is also analogous.

Equilibrium C

Differentiating f^{**}, g^{**}, and h^{**} with respect to f_I, we get $\frac{\partial f^{**}}{\partial f_I} = \frac{\partial g^{**}}{\partial f_I} = \frac{-c_X}{\sigma - 1}\left(\frac{1}{f_I + nf_X}\right)$.

Remember that in Equilibrium C $j_X(c_X) = j_I(c_I^f)$. Differentiating (25) with respect to f_I, we get

$$f_D\, j_0^{'}(c_0^f)\frac{\partial c_0^f}{\partial f_I} + \delta j_X(c_X) + \delta\left(f_I + nf_X\right) j_X^{'}(c_X)\frac{\partial c_X}{\partial f_I} = 0.$$

Applying the chain rule, we have

$$\frac{\partial c_0^f}{\partial f_I} = \frac{\delta\left(\frac{j_X^{'}(c_X)c_X}{(\sigma-1)} - j_X(c_X)\right)}{f_D\, j_0^{'}(c_0^f) + \delta\left(f_I + nf_X\right) j_X^{'}(c_X)\frac{c_X}{c_0^f}} > 0$$

since, as previously shown, $\frac{j_X^{'}(c_X)c_X}{(\sigma-1)} - j_X(c_X) > 0$. To get $\frac{\partial c_X}{\partial f_I} = \frac{\partial c_I^f}{\partial f_I}$, we apply the chain rule to get

$$\frac{\partial c_X}{\partial f_I} = \frac{\partial f^{**}}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial f^{**}}{\partial f_I}.$$

Substituting the values for $\frac{\partial c_0^f}{\partial f_I}$ and $\frac{\partial f^{**}}{\partial f_I}$, we have

$$\frac{\partial c_X}{\partial f_I} = \frac{\delta\left(\frac{j_X^{'}(c_X)c_X}{(\sigma-1)} - j_X(c_X)\right)}{f_D\, j_0^{'}(c_0^f)\frac{c_0^f}{c_X} + \delta\left(f_I + nf_X\right) j_X^{'}(c_X)} - \frac{c_X}{\sigma-1}\left(\frac{1}{f_I + nf_X}\right).$$

Rearranging terms we have

$$\frac{\partial c_X}{\partial f_I} = \frac{-\left(j_X(c_X) + f_D\, j_0^{'}(c_0^f)\frac{c_0^f}{(\sigma-1)\left(f_I + nf_X\right)}\right)}{f_D\, j_0^{'}(c_0^f)\frac{c_0^f}{c_X} + \delta\left(f_I + nf_X\right) j_X^{'}(c_X)} < 0.$$

■

Table 1: The Policy Matrix

BW	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} < 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} > 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} > 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$
B	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} > 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} > 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} > 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$
C	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} < 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} < 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} < 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$

Var./Equilib.	Equilibrium BW	Equilibrium C	Equilibrium B
$\frac{c_I}{c_X}$	$\left(\frac{\left(\frac{\theta_I}{\theta_X}\right)^{-\eta}\frac{\lambda_I}{\lambda_X}\tau^{1-\sigma}}{\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right)}\right)^{\frac{1}{1-\sigma+k\eta}}$	1	$\left(\left(\frac{(\theta_I)^{-\eta}\lambda_I(\tau\gamma)^{1-\sigma}}{(\theta_X)^{-\eta}\lambda_X\left(\gamma^{1-\sigma}-1\right)}\right)\right)^{\frac{1}{1-\sigma+k\eta}}$
$\frac{c_X}{c_0}$	$\left(\frac{\delta(\theta_X)^{-\eta}\lambda_X}{\tau^{1-\sigma}f_D}\right)^{\frac{1}{1-\sigma+k\eta}}$	$\left(\frac{\delta\left((\theta_I)^{-\eta}\lambda_I+n(\theta_X)^{-\eta}\lambda_X\right)}{\left(\left(1+n\tau^{1-\sigma}\right)\gamma^{1-\sigma}-1\right)f_D}\right)^{\frac{1}{1-\sigma+k\eta}}$	$\left(\frac{\delta(\theta_X)^{-\eta}\lambda_X}{(\tau\gamma)^{1-\sigma}f_D}\right)^{\frac{1}{1-\sigma+k\eta}}$

Table 2: Equilibria with technological spillovers

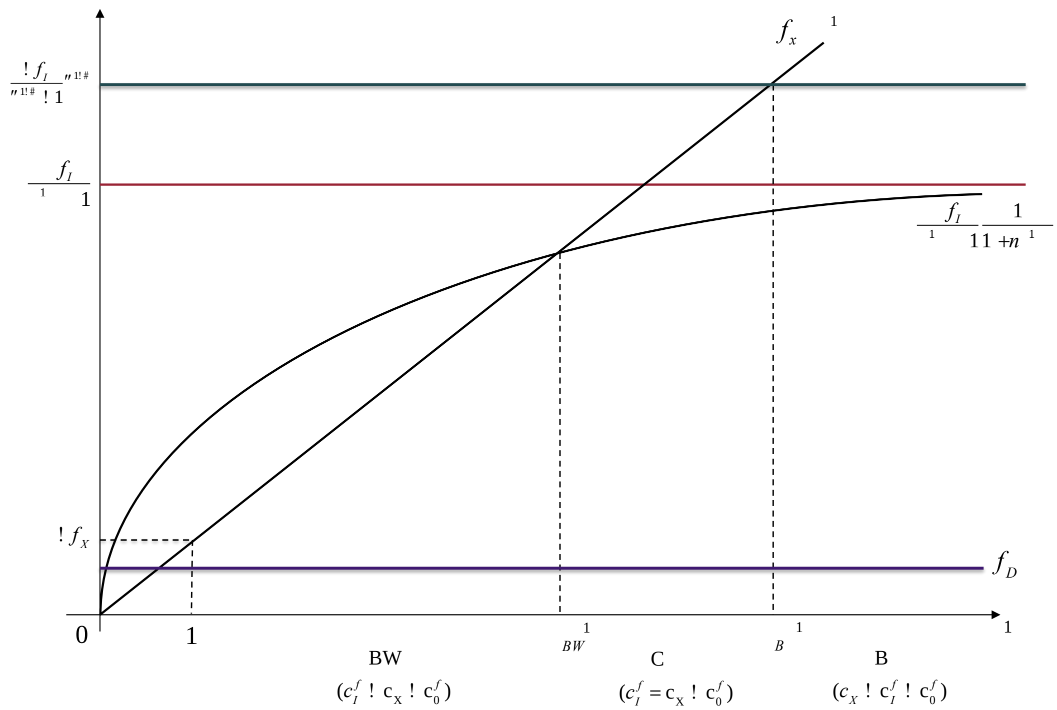

Figure 1: The Policy Space