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A STUDY OF THE COMPOSITE BEHAVIOUR OF REINFORCEMENT AND CONCRETE IN TENSION

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ABSTRACT

The paper aims to further the understanding of the interaction between reinforcement in tension and the surrounding cracked concrete**.** This is achieved using the elastic analysis of axi-symmetric prisms reinforced with a single central bar. As a preliminary to the analyses, the behaviour of axially reinforced prisms is described based on previous experiments. This preliminary analysis confirms that the elastic analysis adopted in this investigation is reasonable.

Two analytical exercises are described; the first assumes no slip, plasticity or internal cracking at the interface between the steel and the concrete while the second introduces internal cracking and de-bonding between ribs. The first analysis indicates that shear deformation of the surrounding concrete accounts for a substantial proportion of the surface crack width and therefore that this form of deformation cannot be ignored in crack prediction formulae. The second analytical exercise shows that the internal cracking model described by Goto is appropriate.

Keywords: cracking mechanisms; finite element modelling; axi-symmetric tension specimens; crack width calculations; cover; shear distortions

INTRODUCTION

The objective of this study is to gain greater understanding of the interaction of reinforcement and concrete in tension. The analysis models used have been kept as simple as possible; the approach has been limited to pure elastic behaviour and an assumed internal cracking pattern, based on the work by Goto (7), has been adopted (rather than use a non-linear finite analysis software based on, for instance, a smeared cracking approach where cracks are predicted regions of damaged material with degraded properties). This keeps difficulties in interpretation to a minimum, though it is recognised that concrete does not necessarily behave in a perfectly elastic manner.

Use of elastic modelling, where the cracks being studied are open, is not so unreasonable as might be thought by some, as the extensive data obtained by Scott and Gill (1) and Beeby and Scott (2, 3, 4,) suggest that much of the behaviour revealed during tension tests is close to what would be expected from an assumption of elastic-brittle behaviour for concrete in tension. This is effectively what will be assumed in the study.

RESEARCH SIGNIFICANCE

In reinforced concrete, the interaction between reinforcement in tension and the surrounding concrete is still not fully understood. Two internal failure mechanisms; pure slip and internal cracking, form the basis of three approaches which exist in the codes to model the tension zone behaviour under service loads. The models presented in this investigation are based only on Goto's internal cracking mechanism. These models predict the experimental behaviour of tension members quite effectively. The simpler model proposed here further confirms the concept that crack width is a function of the shear deformation of the concrete cover.

BASIC BEHAVIOUR AS REVEALED BY TESTS

The information used here is taken from reference (3) above. Initially, strain data for specimen 100T12 will be presented as this gives a convenient illustration of a number of aspects of behaviour. Figure 1 shows the load – average reinforcement strain response for this specimen.

It will be seen that the response is not a continuous smooth curve as is commonly plotted but is made up of a series of linear segments separated by a sudden increase in strain on the occurrence of each crack. Up to a load of about 7.868 kips [35 kN], these linear segments, extrapolated backwards, can be seen to pass through the origin. The behaviour of the tension specimen with a given number of cracks is thus elastic. Using the computer to produce 'best fit' lines for each segment enables the stiffness of the specimen to be established for each crack configuration. Figure 2 shows this compliance plotted against the number of cracks. It will be seen that there is a linear relationship between stiffness and number of cracks. This implies that the formation of each crack reduces the stiffness of the element by a constant amount. The final point for 4 cracks does not quite fit the linear relationship. This point is obtained from the behaviour immediately after formation of the fourth crack. Figure 1 shows that at higher loads there are two further sudden increases in strain. These increases were not related to the formation of visible surface cracks and it may be speculated that they arise from some form of internal failure. It should be noted that both these increases in strain occurred only at very high levels of stress in the reinforcement (> 58.02 ksi [400 MPa]).

Figures 1 and 2 show that the behaviour between cracking events is generally elastic and this is the assumption which will effectively be made in the finite element analyses.

The next aspect of behaviour which needs to be considered is the variation of steel stress or concrete stress with distance from a crack. Figure 3 shows the variation of strain along a reinforcing bar for various levels of axial load.

Various pieces of information can be gleaned from this figure.

Firstly, over a considerable part of the distance from a crack or the free end of the specimen, the variation in strain is very close to linear. This can possibly be seen better from Figure 4 which shows the variation in concrete stress over the end 11.81 in. [300 mm], enlarged for two levels of load. At the lower load level there is a clear curve over the part of the bar where the stress is close to that for uncracked concrete. This is not really clear for the higher load where it would not be unreasonable to consider the relationship to be linear over the whole distance to mid-way between cracks.

Secondly, even at the low level of load, which is below the cracking load for the specimen, the strain in the reinforcement over most of the length affected by the end of the specimen (S_o) is considerably greater than the cracking strain of the concrete, which can be assumed to be in the range 100 to 150 x 10^{-6}. This implies that some form of internal failure must occur over the whole length S_o as soon as cracking occurs.

A critical factor in crack prediction theories is the definition of the transfer length. This is the distance on one side of a crack over which the stress in the reinforcement is affected by the crack. In References (2), (3) and (4), this is represented by the symbol S_o. In some papers, the symbol l_{tr} is used. There are various ways of estimating S_o from experimental results. Most of these are indirect and assume a relationship, for example, between S_o and crack width. However, the work reported in (2), (3) and (4), recorded the variation in strain at closely spaced intervals along the reinforcing bar, permit the direct measurement of S_o. Even with these tests there are, however, difficulties as can be seen from the strain variation for the 4.496 kips [20 kN] level of load shown in Figure 4. It is difficult to define exactly the point where the crack no longer influences the strain. The method used to establish a value for S_o has been illustrated in Figure 4 where S_o is defined as the distance from the crack (or free end of the bar) and the point where a 'best fit' line through the strains intersects the strain in the uncracked concrete. This is clearly an imperfect procedure but is consistent and seems to agree well with the calculation of S_o by indirect means. A good straight line relationship was found between S_o and cover. This is illustrated in Figure 5, which also includes the equation for the straight line.

An issue which has been studied by a number of researchers but is generally ignored by those developing theories of cracking is the shape of the crack (i.e. how the width of a crack varies between the bar surface and the surface of the concrete). An assumption generally seems implicit in cracking formulae based on the classical theory that the crack width at the concrete surface is the same as that at the bar surface. There is now ample evidence that this is not the case. The research evidence has been reviewed in (5) and this shows that the cracks are tapered, being much smaller near the bar surface than at the concrete surface. Though the results are highly variable, it can be concluded that the width at the concrete surface is at least twice that near the bar surface.

SIMPLE ELASTIC ANALYSIS WITHOUT INTERNAL CRACKING

Initially, a simple 2-D analysis was performed on an axially reinforced circular cross-section prism to give some idea of the stress and deformation conditions around a bar. The axial symmetry greatly simplified the analysis. A fully elastic analysis of the area surrounding a bar, assuming no slip between the bar and the concrete was considered. The situation analysed is illustrated in Figure 6 in which w_s is the surface crack width.

Analyses were carried out using axi-symmetric elements in GSA (General Structural Analysis – an OASYS package). For the initial analysis, a 0.79 in. [20 mm] diameter (ϕ) bar was considered with 1.97 in. [50 mm] cover (c). Five mm square elements were used and a length from the free face up to the fixed end of 5.91 in. [150 mm] was assumed. A uniform stress of 14.51 kips / in^2 [100 MPa] was applied to the free end of the bar. Figure 7 shows the stress distribution in the concrete along the specimen obtained in two ways; the stress in the concrete on the outer face (i.e. the concrete surface) and the average stress in the concrete. The average stress was calculated by taking the force in the reinforcing bar at each 0.20 in. [5mm] section, deducting this from the total applied force at the free end and dividing this difference by the area of the concrete. In algebraic terms, if T is the tension force applied at the free end, the bar area is A_s and the concrete area is A_c, the stress in the bar at any section a distance x from the crack is σ_{sx} and the average stress in the concrete is σ_{cx} then, by equilibrium, since the force at section x must be T, the stress in the concrete is given by:

$$\sigma_{cx} = (T - A_s\sigma_{sx})/A_c \qquad (1)$$

The average stress in the concrete has been calculated in this way because it corresponds to the method of calculating the concrete stress used in References (2), (3) and (4). It is also effectively what is used in many theoretical derivations of crack prediction formulae.

There are several interesting points which arise from Figure 7. Firstly, and entirely as one might expect, the surface stress is not the same as the average stress but is considerably lower over almost the whole of the length analysed. S_o for the surface is thus different to S_o for the average stress, with the surface value being considerably longer. Straight lines have been drawn in passing through the origin and the point where the calculated curves reach 2/3 of the homogeneous stress. This is simply done to permit an easy visual comparison of the curves to be made. It may also be noted that the surface is actually in compression for the area closest to the crack face. Secondly, the stress in the concrete does not actually reach the stress calculated for a homogeneous section. There is thus no absolutely clear definition of S_o, as is assumed in all theoretical equations for predicting cracking. This is not necessarily a critical point but it may be worth remembering that S_o is an effective value rather than an absolute one.

Figure 8 shows the variation in the deformed shape of the free end over the height of the crack from the bar surface to the specimen surface. Quantitative comparisons cannot directly be made in this case as the geometries of the available experimental specimens are somewhat different to that analysed. The deformation at the surface in Figure 8 corresponds to a crack width of 0.0019 in. [0.047 mm].

Analyses have been carried out for different covers and Figure 9 shows the calculated crack widths as a function of cover.

It will be seen that crack width decreases with a decrease in cover. The decrease is not linear as suggested from the experimental data (2, 3 and 4), however, certain factors need to be borne in mind. The finite element analyses are elastic and therefore any specimen having a geometrically similar cross-section to the one for which the crack width has been calculated will give a crack width which can be calculated by direct scaling from the previously calculated width. Thus, for example, the crack width for any specimen with a value of c/ϕ of 2.5 will lie on a straight line joining the point for $c/\phi =$ 2.5 to the origin. This applies for any other value of c/ϕ. Thus, all results for any specimens with c/ϕ in the range 1 to 2.5 will lie between the two dashed lines drawn in Figure 9. If a relatively random set of tension specimens are analysed, the result, when plotted on a graph such as Figure 9 will, to the engineering eye, be accepted as giving a linear relationship between cover and crack width with some relatively small level of scatter.

Figure 10 aims to make an approximate quantitative comparison between calculated and experimental crack widths. The test specimens, from Farra and Jacccoud (6) were 3.94 in. [100 mm] square and reinforced with a single axial 0.79 in. [20 mm] bar. The cover was thus 1.57 in. [40 mm] and results from an analysis for 1.57 in. [40 mm] cover have been used in the comparison. It should be remembered, however, that the experimental specimens had a square cross-section whereas this analysis considered a circular cross-section. It will be seen that the analysis underestimates the maximum crack width by about 30%. This is to be expected as no account has been taken in this analysis of internal failure (slip or internal cracking) which, as has been discussed earlier, must occur and which will reduce the stiffness of the concrete in tension. This will be considered further later.

It seems likely that this initial simple analysis gives a lower bound indication of the deformation of the tensile concrete and hence the estimate of the crack width. In reality, concrete in tension is not absolutely elastic-brittle but will undergo some plastic deformation before rupture. This will result in the actual deformation of the concrete being greater than that calculated on the assumption of elasticity. Additionally, a short term value has been used for E_c. There is actually likely to be some creep during the test which would result in further deformation of the concrete and steel and hence higher calculated crack widths. Depending on the effect of these two factors, the calculated width could be closer to the experimental values.

Overall, the analysis seems to have been very successful in predicting the general qualitative behaviour of axially reinforced specimens.

ANALYSIS OF THE TENSION ZONE INCLUDING INTERNAL CRACKS

As mentioned earlier, some form of internal failure must take place over the whole of the length S_o from a very early load stage. There are two mechanisms which are commonly proposed for this internal failure: slip along the steel-concrete interface and internal cracks forming at an angle to the axis of the bar. Slip is the most common mechanism invoked and has been used as the basis for many crack prediction theories. The internal cracking mechanism was first illustrated by Goto (7) and has recently been elaborated somewhat by Beeby and Scott (3). It is Beeby and Scott's model which will be investigated in this paper (it is believed that there are plenty of advocates of the slip model who can, if they wish carry out further modelling of this option). It needs to be noted that the type of modelling which will be attempted here will not prove that a particular model is the actual behaviour; at best it can simply show that the particular model gives a reasonable simulation of reality. There may exist other models which are as good or better. It could, however, show that a particular model was unreasonable.

As before, an axially symmetrical specimen has been chosen for the analysis. Initially, the analysis will be carried out on a specimen the same basic size as that used to produce the results detailed in Figures 7 and 8. However, it will now also model a number of internal cracks. The length of the model specimen has now been doubled to 11.81 in. [300 mm] partly because the length S_o was expected to be greater than in the case with no internal cracks and partly because it was felt that the length used in the earlier analysis may have been slightly short for the largest cover. The number of cracks, the angle of the cracks to the bar axis and the length of the cracks is somewhat arbitrary but is based on photographs from Goto (7) and Otsuka and Ozaka (8) and the analysis presented in Beeby and Scott (3).

Experimental work by Goto and others suggest that internal cracks form at each rib on the bar. The spacing used in the model analysed here is rather larger than the rib spacing for reasons of practicality. An angle of 60^{o} to the axis of the bar was chosen, although this angle could only be approximately maintained as the basic grid of 5 mm did not permit exactly 60^{o} to be maintained for all cracks. Furthermore, the aim was to use a linear decrease in crack height with distance from the crack face. Again, the 0.20 in. [5mm] grid meant that this could only be achieved approximately. The elastic finite element model used is illustrated in Figure 11 and 12. Others have carried out analyses aimed at studying internal cracking (for example, Gerstle and Ingraffea (9)). The difference between these and this analysis is that most of the other analyses have attempted to study the development of the internal cracking as a function of applied load whereas, in this study, in order to be able to study a larger range of variables, a crack pattern has been assumed.

The results from the analysis are shown in Figures 13 to 18.

Figure 15 shows a number of interesting changes from the stress results shown in Figure 7 resulting from the element without internal cracks. Firstly, the relationship between the average stress and distance from the crack is much closer to linear. It now models more closely the experimental result shown in Figure 4.

The deformed shape of the free end (Figure 16), which is actually a tracing of the FEA graphical output (Figure 14) with the elements and nodes removed for clarity, is now possibly less similar to that obtained for the specimen without internal cracks and to that of the experimental specimen but it is still reasonable. The surface deformation corresponds to a crack width of 0.0031 in. [0.08 mm]; 60% greater than that obtained for the specimen without internal cracks. This agrees closely with the experimental results, as can be seen from Figure 17 which shows the same data as used for Figure 10 but with the calculated line shown for the analysis with internal cracks.

The predicted effect of cover is shown in Figure 18, where it can be seen that a reasonably linear relationship is predicted between cover and surface crack width. However, there is some scatter in these results. This is thought to be due to the difficulty of modelling absolutely geometrically similar internal cracks in the analyses for the various covers.

A further assessment of the performance of the model can be performed by considering the work by Broms (10) and Taylor and Beeby (11). Broms (10) carried out a series of tests on short prisms and measured the longitudinal extension at various stress levels in the reinforcement. Results are presented in (10) for a circular cross-section specimen, 6 in. [152 mm] in diameter and 8 in. [203 mm] long with a central 1 inch diameter bar. An elastic analysis has been carried out for this specimen and Figure 19 shows the experimental and calculated results for two levels of stress. The experimental results have been scaled from a figure in Broms' paper. It will be seen that, in this case, the experimental results exceed the calculated results by about 20%. However, the general trend of the results is well reflected by the calculations. This is not in absolute agreement but it is probably within the range that could be covered by judicious adjustment of the model.

A cylindrical specimen was tested by Taylor and Beeby (reported in (11)) where the overall extension of a 5.91 in. [150 mm] diameter specimen with an axial 0.87 in. [22 mm] bar at various distances from the bar surface were measured. This has been analysed and the extensions scaled off the figure presented in (11). Figure 20 shows the measured extensions compared with the FE calculations for two levels of steel stress. Agreement between experiment and calculation is slightly better than for Broms in Figure 19, though the calculation, again, tends to underestimate the measured results.

The introduction of the internal cracks (Figures 12 to 14) has, in general, resulted in an improved agreement with the experimental behaviour and does suggest that the internal cracking model is a reasonable model for the behaviour of tension zones. The analysis is clearly capable of further refinement and has only been carried far enough to demonstrate its inherent reasonableness.

DISCUSSION

Elastic analyses

Two basic analyses have been described in this paper. In the first the concrete is considered to remain elastic and uncracked and complete bond is assumed between the reinforcement and the concrete. In the second, a pattern of internal cracking has been assumed, based on the findings of Goto (7). In neither of the analyses is any form of bond-slip relationship assumed and so bond-slip can have no influence on the results obtained.

In the first analysis (without internal cracking) it was expected that the predicted crack widths would be less than obtained experimentally, and this proved to be the case. Nevertheless, the analyses were not trivial and the results illustrate a significant point which has commonly been ignored. If a shear stress is applied to a material then shear strains and displacements must occur. Bond stress is simply a shear stress and therefore the concrete surrounding a bar in the region of a crack must undergo shear deformations. Though we have not seen this explicitly stated, the classical theories of cracking which lie behind many crack prediction equations implicitly assume that this shear deformation is negligible. The analyses show that this is not so; the elastic shear deformation of the concrete in the analyses reported here account for around two thirds of the crack width. Had other material factors such as creep or inelasticity of the concrete in tension been taken into account in the analyses, the shear deformations and hence their contribution to the crack widths would have been even greater. This substantial contribution of the shear deformation of the cover concrete seems inescapable and suggests that any approach to the prediction of crack widths which ignore this are fundamentally flawed.

In the second analysis, the output from the model with internal cracks generally agreed with the experimental data, where comparisons could be made. This suggests that the internal cracking model of behaviour can provide a good model of cracking behaviour. It does not prove that the mechanism accommodating excess tensile strains above those which the concrete can support in tension is internal cracking; it shows that it is a viable alternative to the bond-slip model and should not be dismissed.

The analyses carried out are somewhat limited and can be considered to make a prima facie case for the reasonableness of the internal cracking model rather than a fully developed analytical study. Some of the more obvious limitations of the model are given below.

- Circular cross-sections are analysed not square or rectangular ones.

  Due to difficulties in manufacture, very few circular specimens have been made and tested so it is not possible to compare the analytical results rigorously with test results which are almost all from specimens with square or rectangular cross-sections.
- Location and size of internal cracks is somewhat arbitrary.

  As mentioned in an earlier section, no attempt has been made to refine the form of the internal cracking. From the existing experimental evidence, the pattern assumed seems reasonable but it cannot be said to be rigorously justified.
- Rib pattern

  By it's nature, the axi-symmetric analysis assumes that the ribs are perpendicular to the bar axis and extend round the full circumference of the bar. This is not normally so for modern ribbed bars where the ribs tend to be staggered. There is also frequently a longitudinal rib.

The result of this is that, in reality, the pattern of internal cracks may be considerably more complicated than is modelled in this analysis.

- The spacing of the internal cracks, which would be expected to follow the spacing of the ribs, is rather too large in the model.

Issues requiring further study.

What is measured when crack widths are being investigated is the crack width on the surface. It is found that the surface crack width is strongly dependent on where the cracks are measured relative to the position of the reinforcement. If the cracks are measured at points on the surface directly over the bar, they are found to be substantially smaller than if they were measured, say, close to the corner of an axially reinforced prism. The variation is less in situations where there are multiple bars and the crack width over the bars is compared with that at mid-spacing. This behaviour is illustrated in Table 1, containing data from (12).

This effect seems perfectly rational for crack widths resulting from shear deformation of the concrete; the shear displacement will increase with increasing distance from the bar in any direction. Since the corner of the specimens used in Table 1 are further from the bar than a point directly over the bar, the deformation will be greater and the crack width larger. This effect was recognised by Broms and in the work carried out at the Cement and Concrete Association. It is implicitly included in the ACI code formula and taken into account directly in the UK code.

The analyses performed here have been exclusively concerned with members subjected to pure tension. There is evidence that flexural members behave rather differently. Studies by Beeby (13) showed that, in shallow members (such as slabs) the depth of the tension zone has a significant effect on the crack width. This is explicitly taken into account in the UK code (see below) and is also recognised in Eurocode 2 (14).

Although it is hoped to do so in the future, these effects have not been investigated in this study because the finite element package used did not permit the analysis of three-dimensional specimens (with the exception of axi-symmetric situations).

Issues relating to the development of a valid design formulae for crack width prediction.

It will help further discussion if a brief outline is given of the development of crack theories and code provisions.

The earliest developed theory of cracking assumed that the widths of cracks accommodated slip between the bar and the concrete. The theory ignored any contribution to the crack width from the shear deformation of the cover concrete. To develop equations it required assumptions to be made about the development of the bond stresses as a function of slip. Many different assumptions were considered but all resulted in a basic equation of the form:

$$w = k\phi f_{ct}\varepsilon/\tau\rho \qquad (2)$$

where w = crack width (variously defined)

k = a constant

f_{ct} = the tensile strength of the concrete

ε = strain (variously defined)

ϕ = the bar diameter

ρ = reinforcement ratio (variously defined)

τ = bond strength

Many design provisions have been based directly on this equation, including those in the CEB Model Code 90 (15).

In 1965 Broms (16) and Broms and Lutz (17) published a radically different theory which assumed that the crack width arose entirely from the shear deformation of the cover concrete. He developed the formula:

$$w_{av} = 2t\varepsilon_s \qquad (3)$$

or: $$w_{max} = 4t\varepsilon_s$$

where w_{av} = the average crack width

w_{max} = the maximum crack width

t = distance from the centre of the bar to the point on the surface where the crack width is considered.

ε_s = average strain of the steel.

For multiple bars, t was modified to t_e , an effective distance, which is defined, for bottom cracks, as $\sqrt[3]{(t_b A)}$ where t_b is the distance from the bottom of the beam to the centre of the lowest layer of bars and A is the area of concrete immediately surrounding the tension reinforcement. This formula, along with many others was tested against the available crack width data by Gergely and Lutz (18) and shown to be the best available at the time. The formula has formed the basis of the ACI code crack width control provisions ever since.

At the same time as the work being carried out at the University of Illinois by Broms, Lutz and Gergely, a major series of tests were carried out at the Cement and Concrete Association (C and CA) in the UK. The first series of tests consisted of 105 beams and the results were with the publishers at the time that Broms' paper appeared. The paper (19) concluded that crack width could be predicted by the formula:

$$w_{max} = 3.3 a_{cr} \varepsilon_m \qquad (4)$$

where w_{max} = the maximum crack width

a_{cr} = the distance from the point where the crack width is being considered to the surface of the nearest bar.

ε_m = the average strain at the level where the width is considered.

It will be seen that, while a_{cr} is slightly larger than t in Broms' equation and that the equation aims to predict the maximum width rather than the average, the C&CA and Broms' basic formula are almost the same. Like Broms' theoretical approach, the formula assumes that there is no bond failure or slip at the bar-concrete interface and that the crack width results entirely from the deformation of the cover concrete. The C&CA work was extended over the following few years by Beeby (20), (21) and (22) resulting in modifications to deal with bar spacing and the effect of the depth of the tension zone. It was recognised that some mechanism was necessary to accommodate the strains in the concrete in excess of the tensile strain capacity of the concrete and it was proposed that the form of cracking identified by Goto provided that mechanism. The resulting formula was simplified somewhat and has been used in UK codes since 1972 (23). This formula is:

$$w = 3a_{cr}\varepsilon_m/\{1 + 2(a_{cr} - c)/(h - x)\} \qquad (5)$$

where c = the cover to the face of the member where the crack width is being considered.

h = the overall depth of the section

x = the neutral axis depth (depth of the compression zone)

Also at a similar time, Ferry-Borges (24) developed a formula which combines the theoretical ideas behind the classical bond-slip model and the shear deformation models. His formula is:

$$s_{av} = k_1 c + k_2 \phi/\rho \qquad (6)$$

$$w_{av} = s_{av}\varepsilon_m$$

where s_{av} = the average crack spacing

The term k_1c is justified in (24) by the following statement:

> "The need to consider the influence of the thickness of the cover, c, is easy to understand. In fact, even if perfect bonding between concrete and steel existed, the mean distance between cracks would not be zero but proportional to c."

This formula was adopted in the CEB Model Code of 1978 (25) and also by Eurocode 2 (14).

The object of this survey of approaches to crack width calculation is to point out that there are three basic approaches used in codes:

(a) The assumption that the crack width arises purely from slip. This is used in a number of codes and, notably, in MC90 (15).
(b) The assumption that there is no significant slip and that the crack width arises entirely from the deformation of the concrete around the bar. Bond and slip do not feature in formulae derived on this basis. Strains beyond those supportable by concrete in tension are accommodated by a reduction in the stiffness of the cover concrete by internal cracking. This is the case for the UK and ACI codes and the codes of any country that basically follow either British or American practice.
(c) The assumption that the crack width arises from a combination of slip and deformation of the concrete. This is true of MC78 (25) and Eurocode 2 (14) and will become the case for all countries which either adopt Eurocode 2 or base their national codes on Eurocode 2.

From this investigation, it is apparent that a significant proportion of the crack width must be due to the deformation of the concrete surrounding the bar. Therefore, (a) above is not tenable as a basis for a rational crack prediction formula; (b) or (c) are tenable depending on whether or not slip plays a significant role. The paper does not aim to show which of these possibilities is closest to the truth, merely that the concept of crack width being a function of the deformation of the cover concrete is reasonable.

CONCLUSIONS

In this paper, a number of simple elastic finite element analyses of the concrete in tension surrounding tensile reinforcement, in members subject to pure tension, are described and the results compared with the behaviour of actual tension specimens, as revealed by experiment. The study leads to the following conclusions.

1. The analyses show clearly that cover must be an important factor in any approach to the calculation of crack widths. This effect arises from the shear distortion of the concrete between the bar surface and the concrete surface.

2. Experimental results show clearly that there must be some form of internal failure in the region of the bar over the whole length over which the crack influences the stress distribution. Two mechanisms have been proposed for this internal failure: slip along the bar-concrete interface and internal cracks of the form proposed by Goto (7) and elaborated by Beeby and Scott (3). In this paper Beeby and Scott's model is analysed and is shown to be able describe the experimental behaviour of tension members effectively.

3. The results from this study and those described in References (1) to (4) suggest the possibility of a model for tension zone behaviour under service loads which is, in principle, simpler and more all embracing than current models. This can be described by the following two assumptions:

   (a) Concrete in tension behaves in an elastic-brittle manner
   (b) Force is transferred between ribbed reinforcing bars and concrete by the mechanical action of the ribs. It is assumed that there is no bond between the ribs and no slip past the ribs.

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TABLES AND FIGURES

List of Tables:

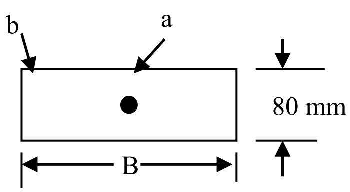
List of Figures:

Fig. 18 – Predicted maximum crack widths as a function of cover for analyses including internal cracks

Fig. 19 – Comparison of calculated and measured crack widths for Brom's specimen T-C-5 (10)

Fig. 20 – Comparison of calculated and measured overall extension for specimen reported in (11)

Table 1-Comparison of crack widths at centre and near corner of axially reinforced tension specimens (from (12))



Specimen	B	Mean values of w/ε			5% values of w/ε		
		a	b	b/a	a	b	b/a
Z2	80	76	113	1.48	174	203	1.17
Z6	130	104	171	1.64	253	354	1.40
Z7	180	91	254	2.79	231	535	2.32
Z9	230	101	259	2.56	282	580	2.06

Note: B, a and b are in mm; 1 mm = 0.0394 in.
w/ε = average crack width / average surface strain.

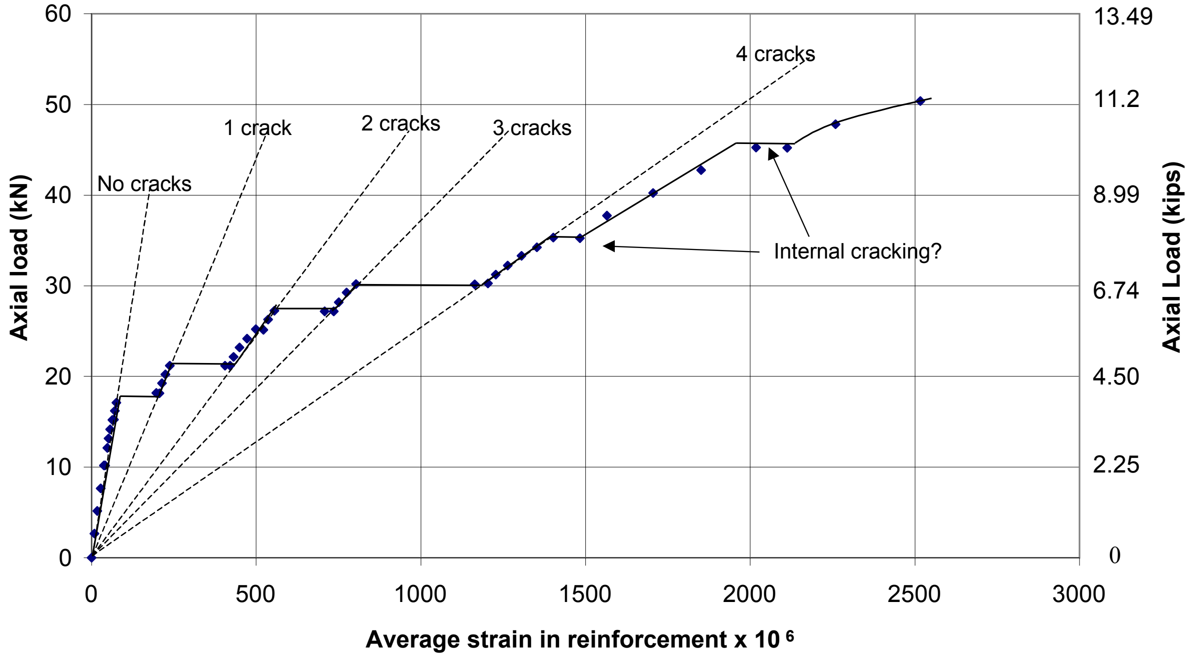


Fig. 1-Load – Deformation response of Specimen 100T12 (Figure taken from (3)).

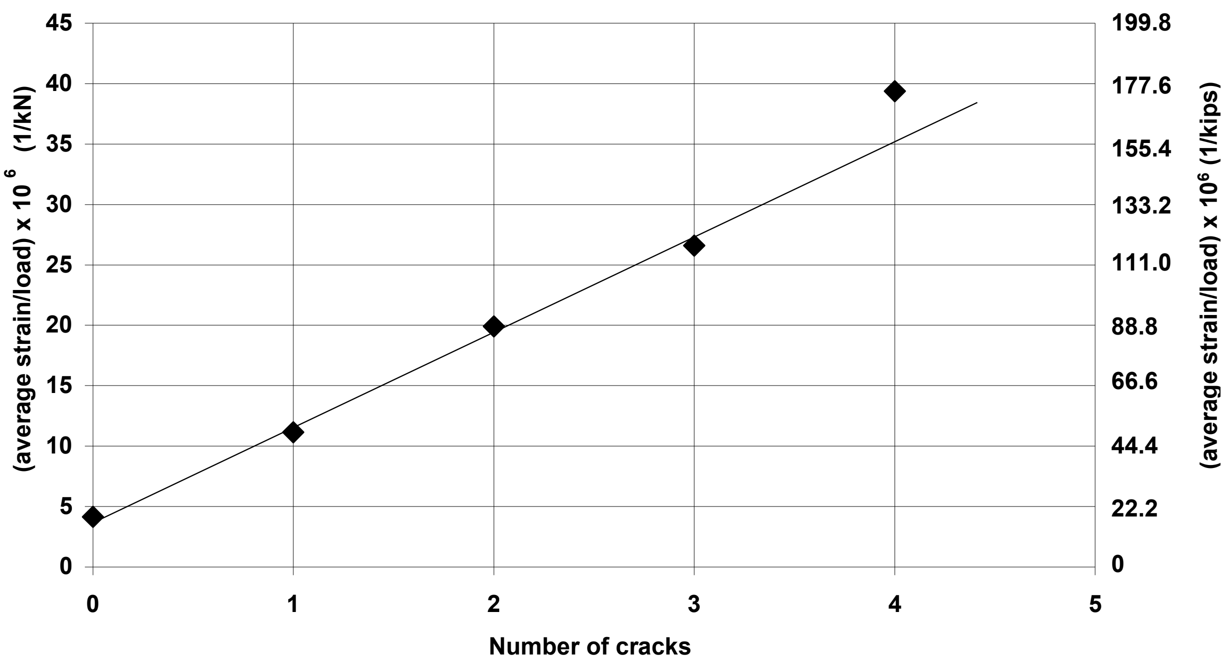


Fig.2-Stiffness of element as a function of number of cracks for Specimen 100T12.

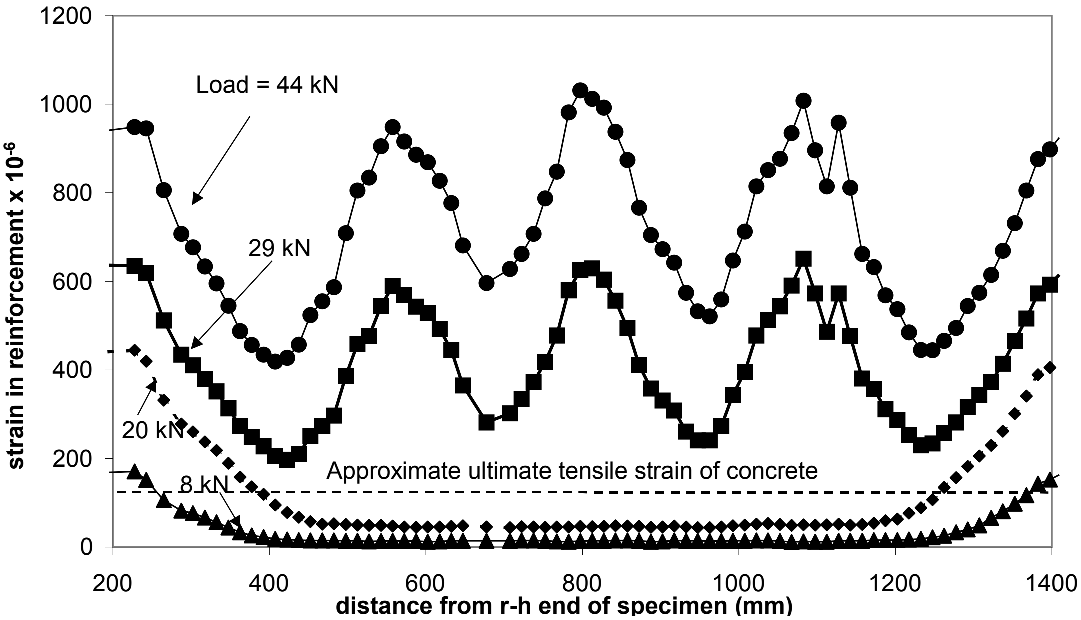


Fig. 3-Specimen T16B1 - Reinforcement strains along the bar at various loads.
(1 kN = 0.225 kips: 1 mm = 0.0394 in.)

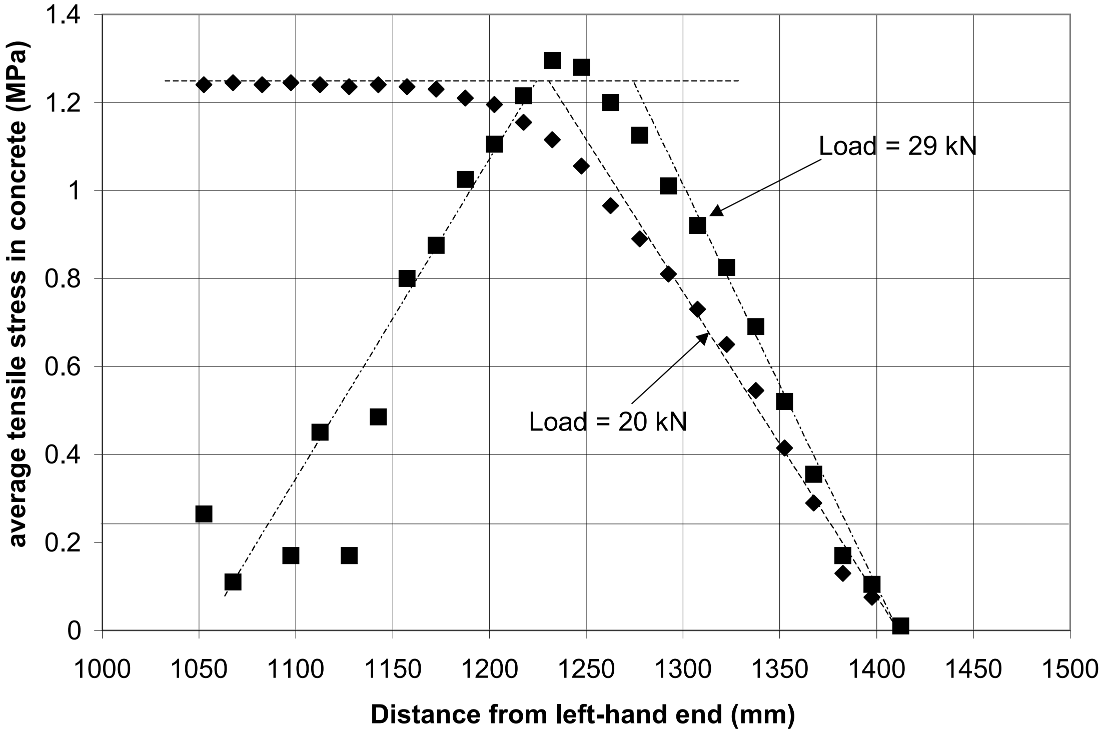


Fig. 4-Concrete stresses at right-hand end of Specimen T16B1.
(1 MPa = 145 psi: 1 kN = 0.225 kips: 1 mm = 0.0394 in.)

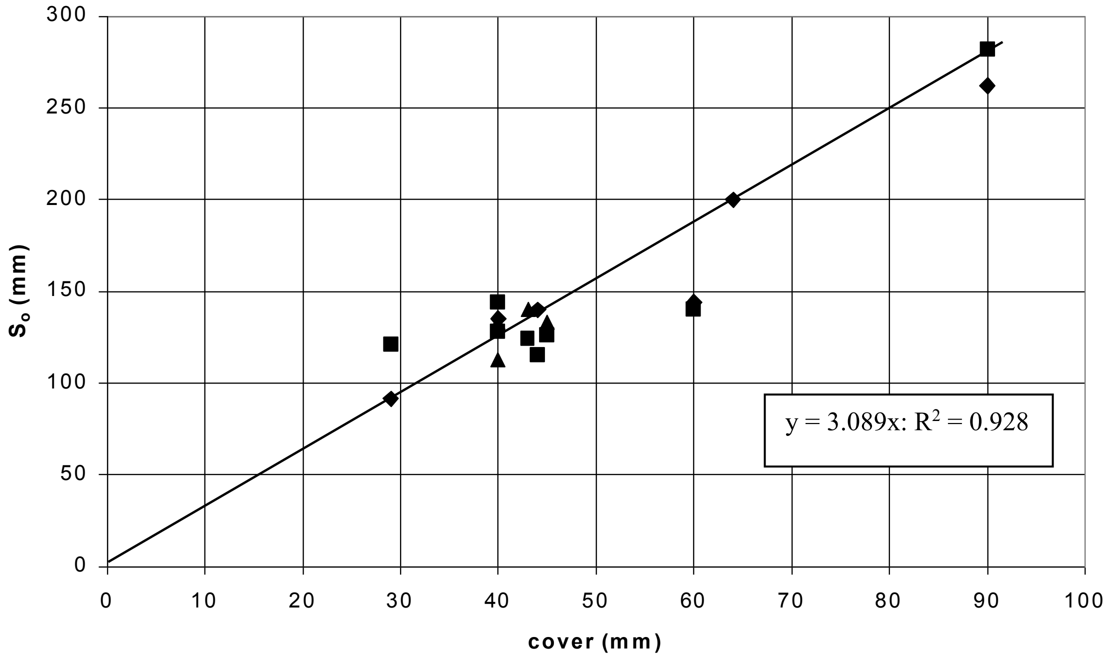


Fig. 5-Relationship between S_o and cover.
(1 mm = 0.0394 in.)

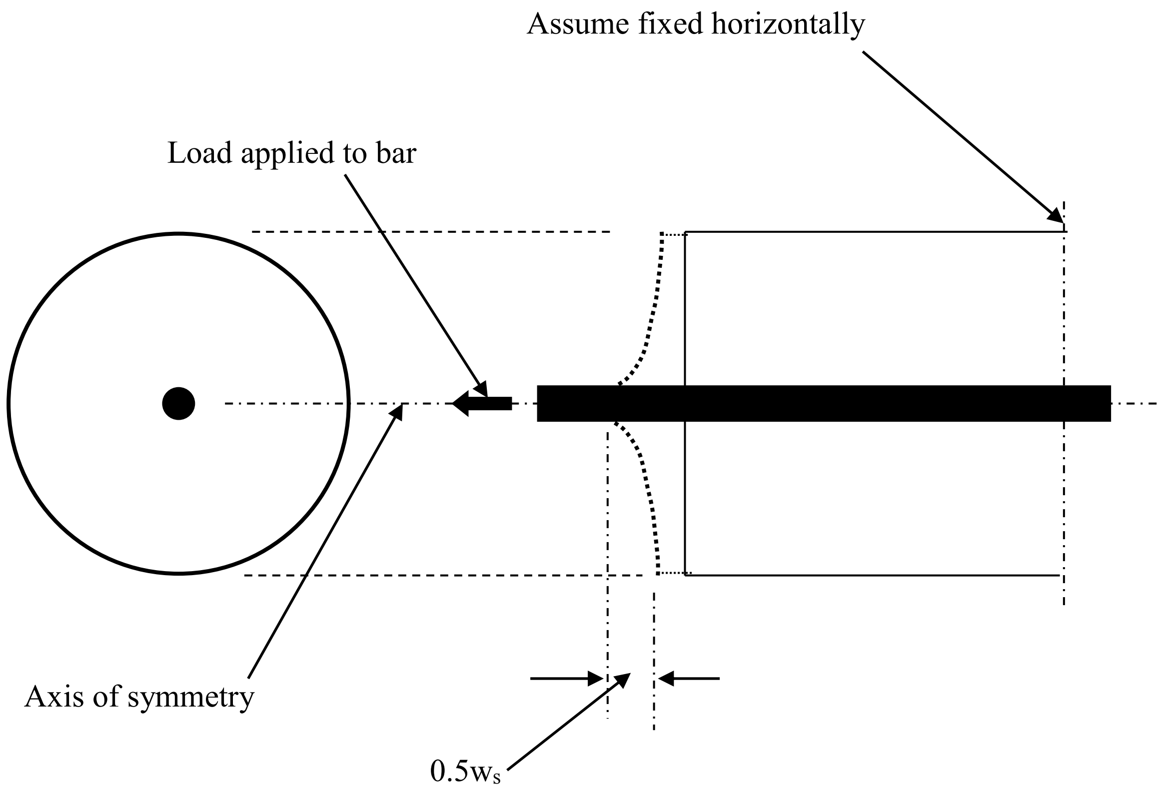


Fig. 6-Schematic illustration of situation analysed.

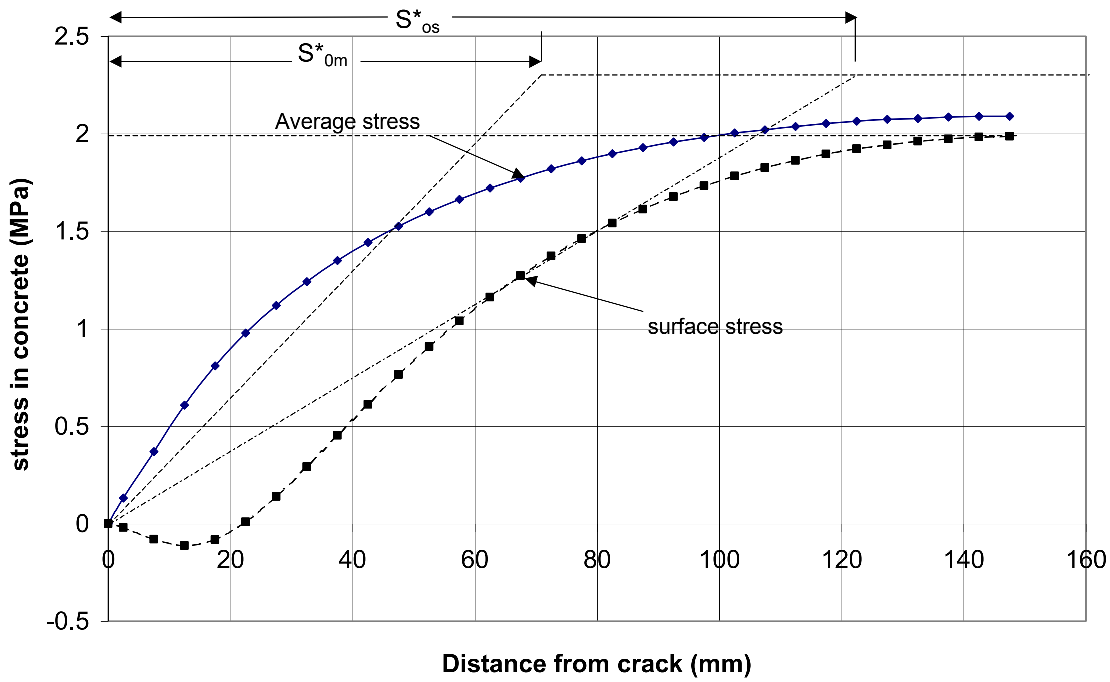


Fig. 7-Variation in calculated stresses in concrete with distance from free end.
(1 MPa = 145 psi: 1 mm = 0.0394 in.)
Note: S^*_{0s} = surface spacing; S^*_{0m} = mean spacing.

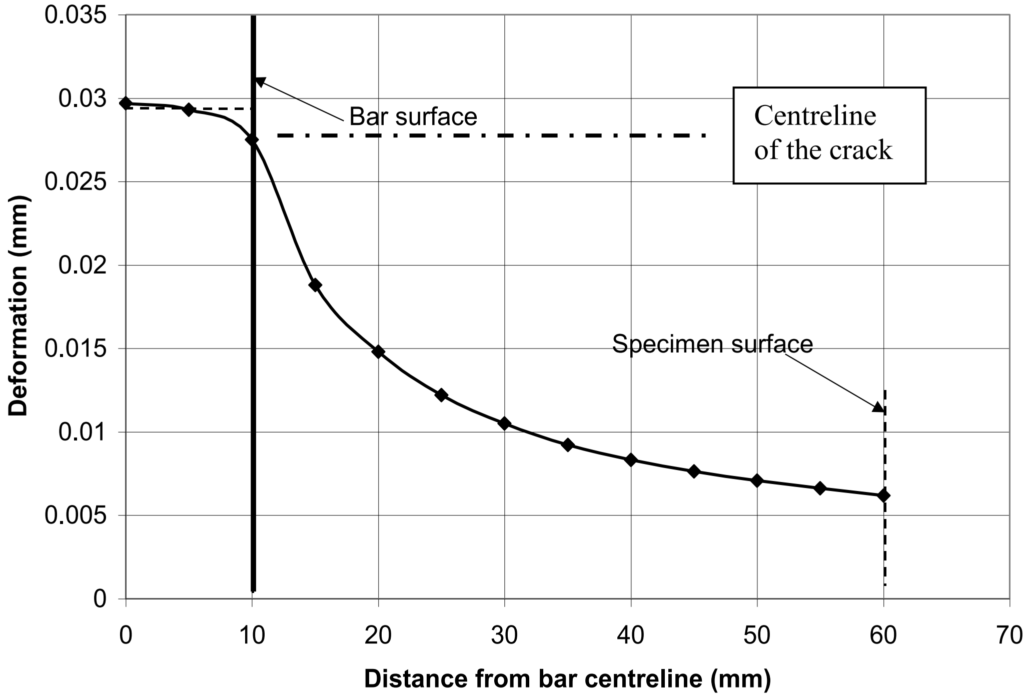


Fig. 8-Variation in calculated crack width with distance from bar surface.
(1 mm = 0.0394 in.)

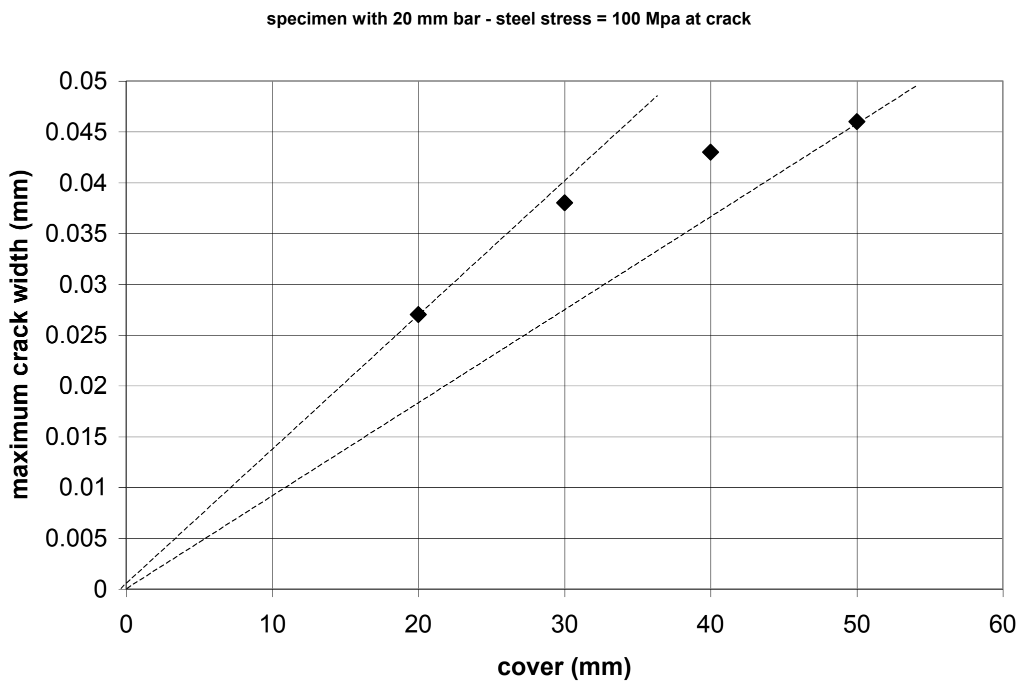


Fig. 9-Variation in maximum surface crack width with cover.
(1 MPa = 145 psi: 1 mm = 0.0394 in.)

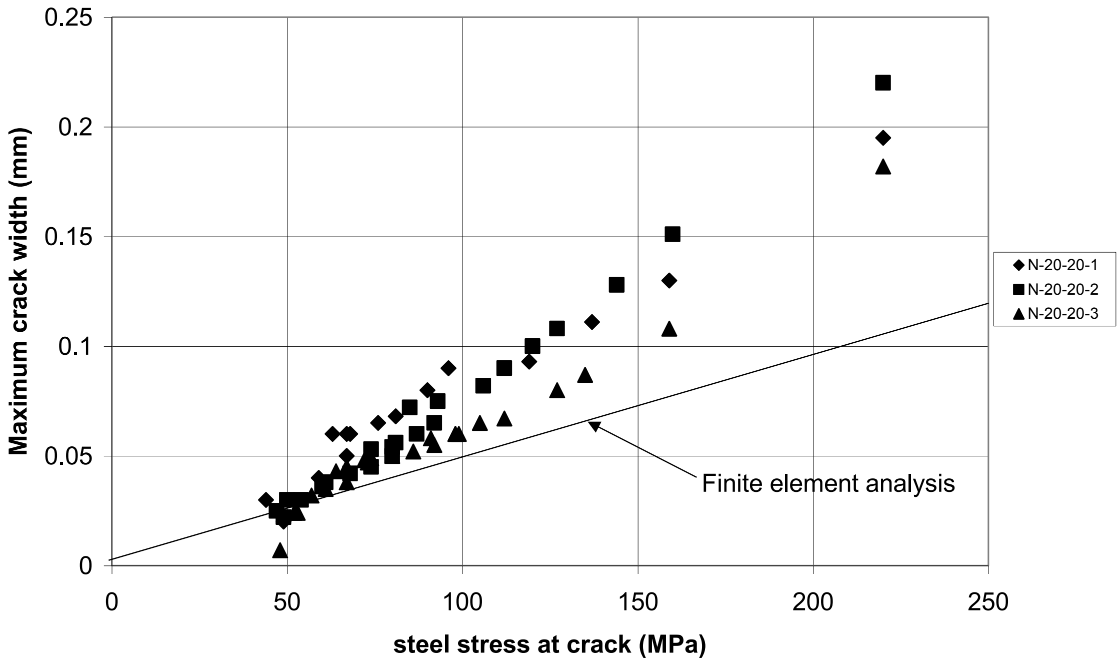


Fig. 10-Maximum crack widths from Farra and Jaccoud specimens N-20-20 (6) compared with FE analysis. (1 MPa = 145 psi: 1 mm = 0.0394 in.)

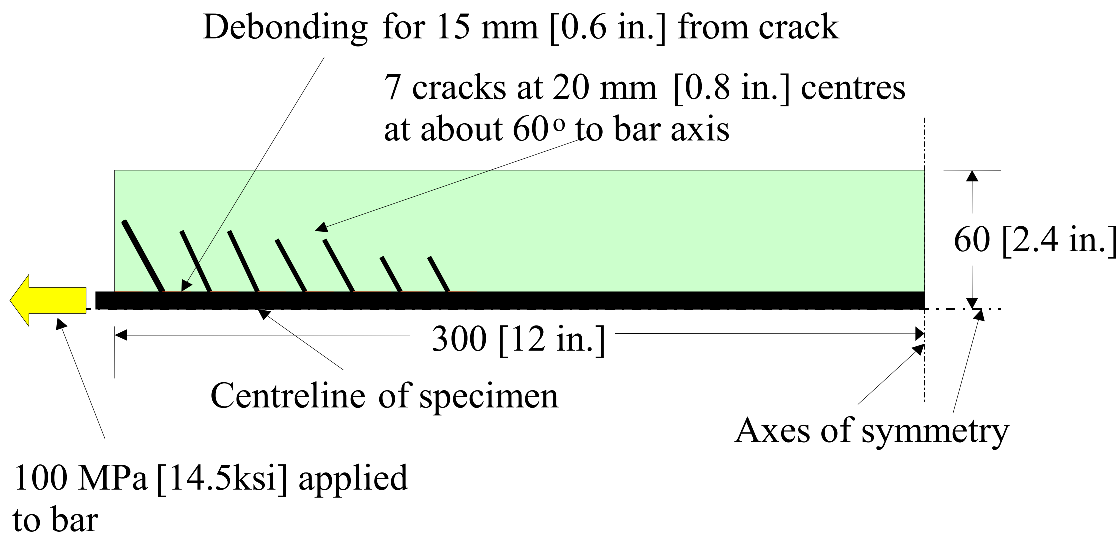


Fig. 11-Finite element model of axially reinforced tension specimen with internal cracks.

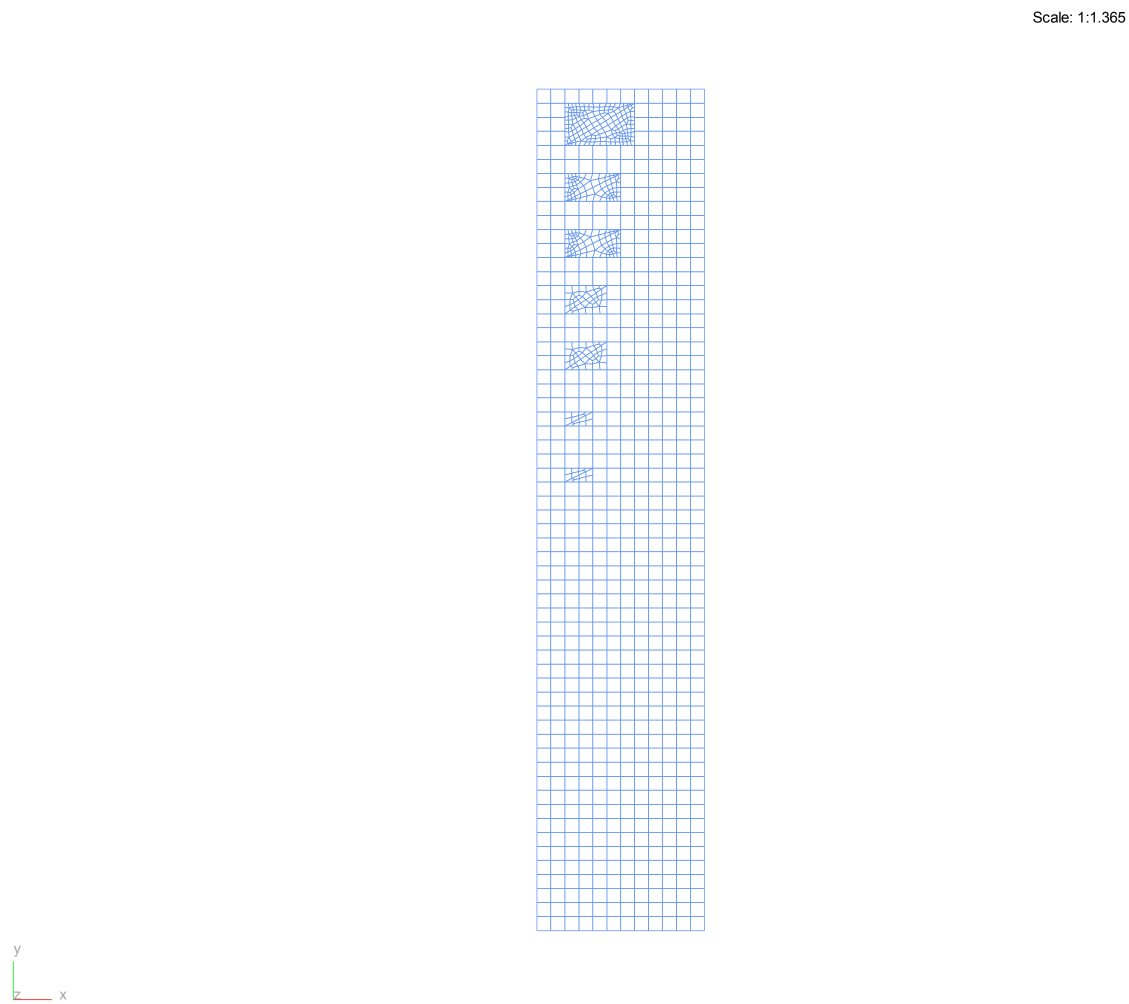


Fig. 12-Axi-symmetric model created using OASYS – GSA software.

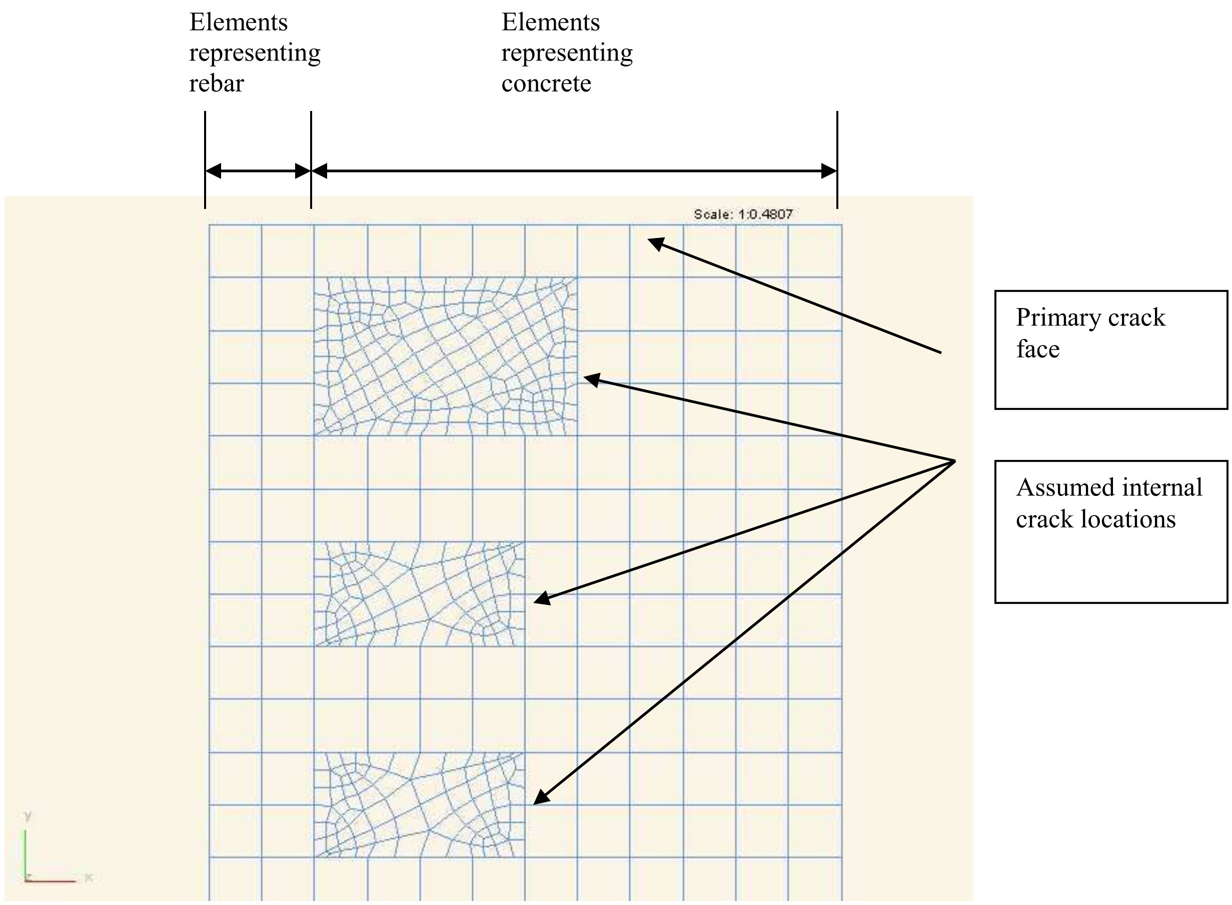


Fig.13-Axi-symmetric model - detail of assumed crack pattern.

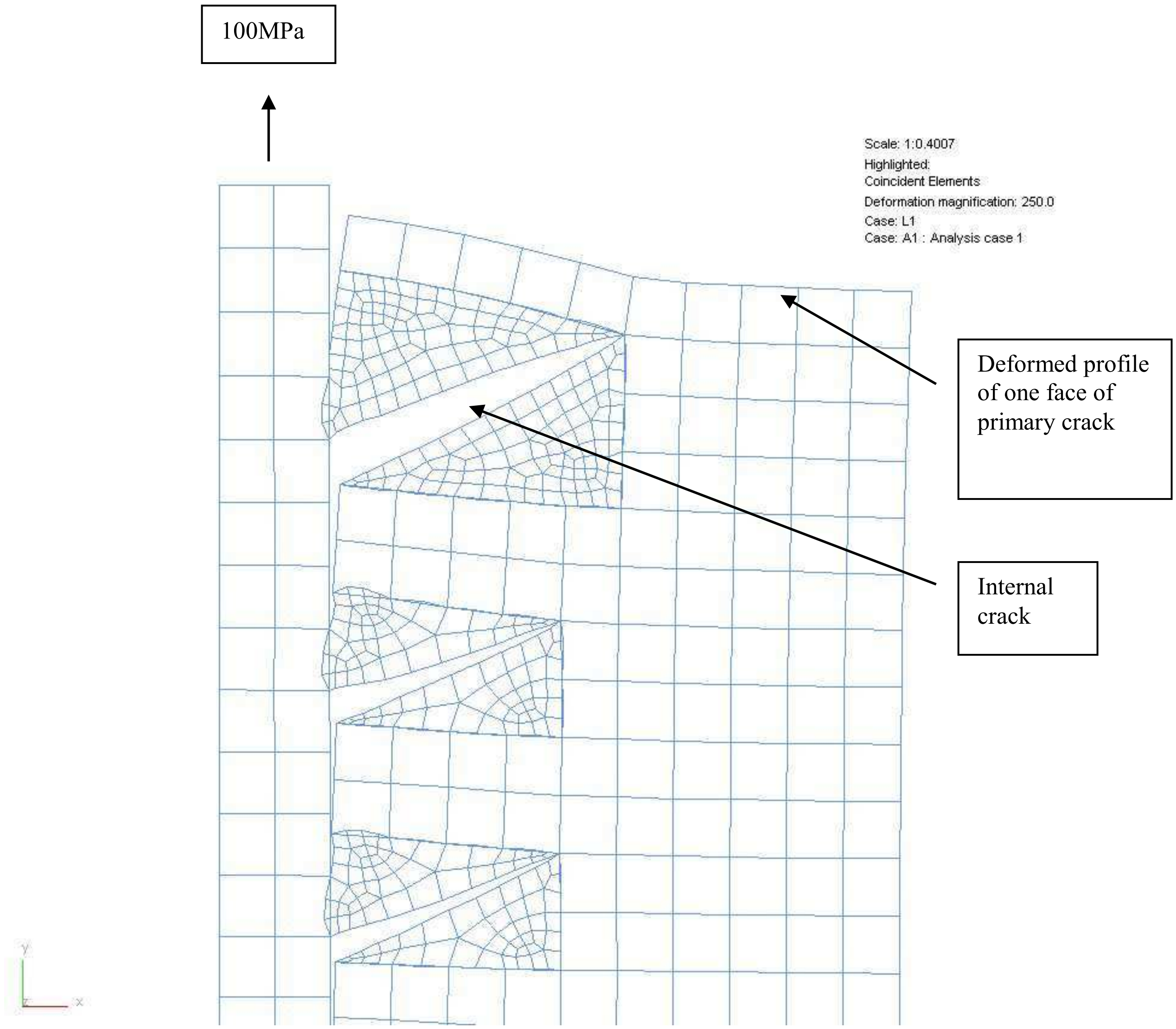


Fig.14-Axi-symmetric model – deformed state.

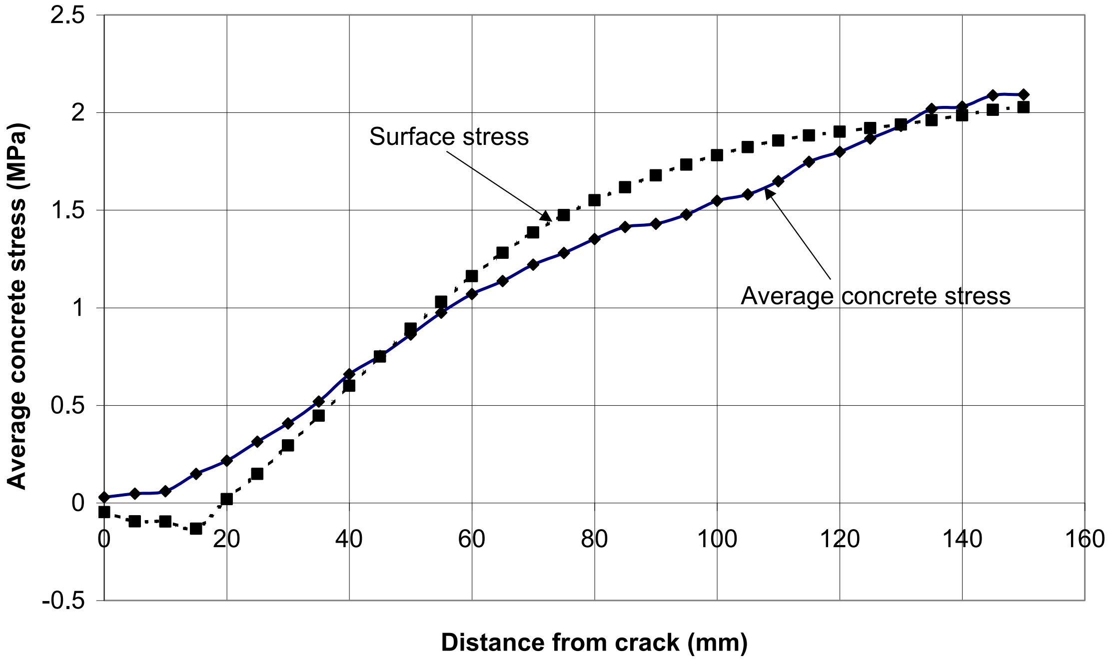


Fig. 15-Concrete stresses calculated by finite element analysis for specimen with 7 internal cracks. (1 MPa = 145 psi: 1 mm = 0.0394 in.)

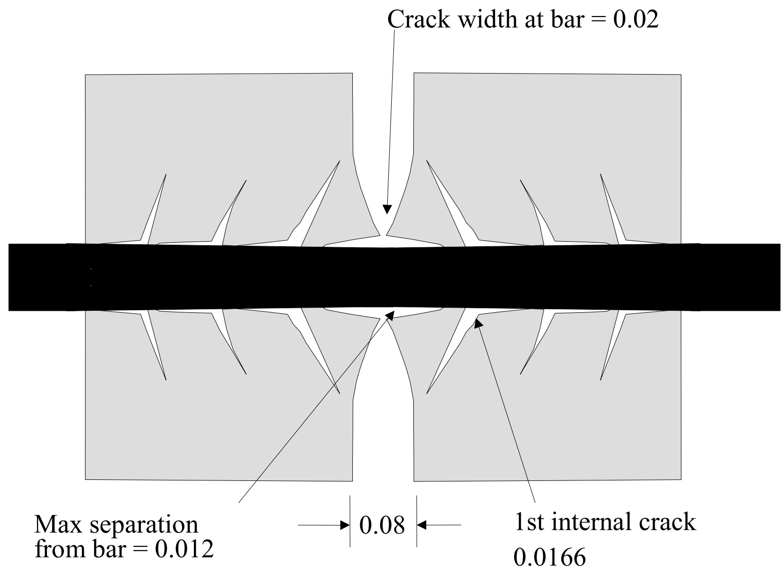


Fig. 16-Calculated deformation of specimen with 7 internal cracks.
(1 mm = 0.0394 in.)

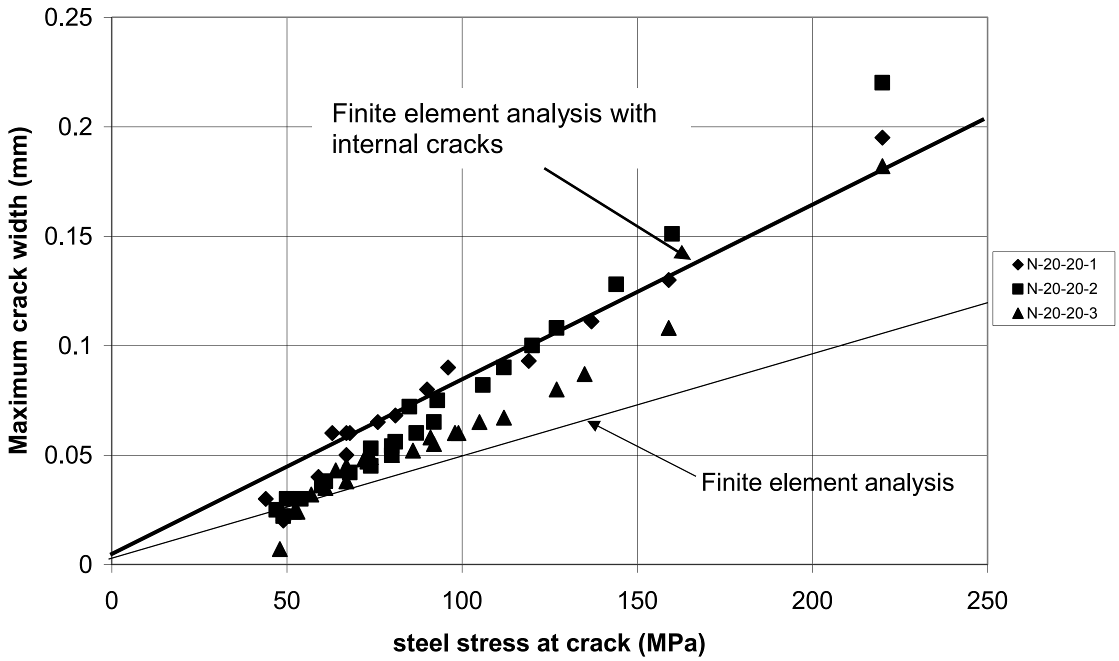


Fig. 17-Maximum crack widths from Farra and Jaccoud specimens N-20-20 (6) compared with finite element analyses with- and without internal cracks. (1 MPa = 145 psi: 1 mm = 0.0394 in.)

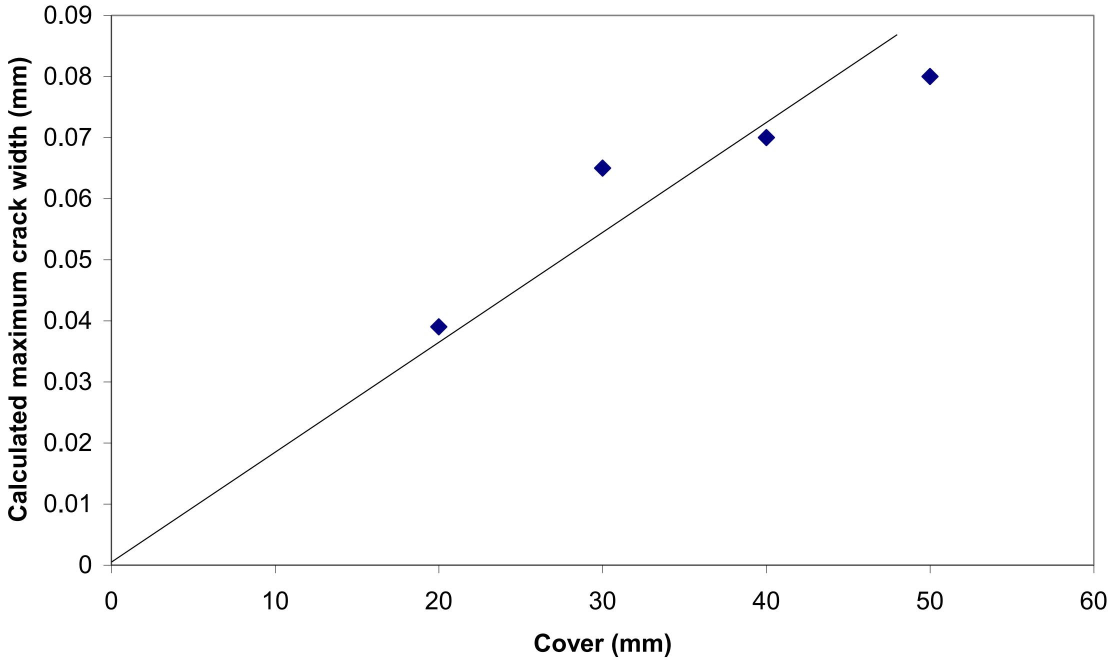


Fig. 18-Predicted maximum crack widths as a function of cover for analyses including internal cracks. (1 mm = 0.0394 in.)

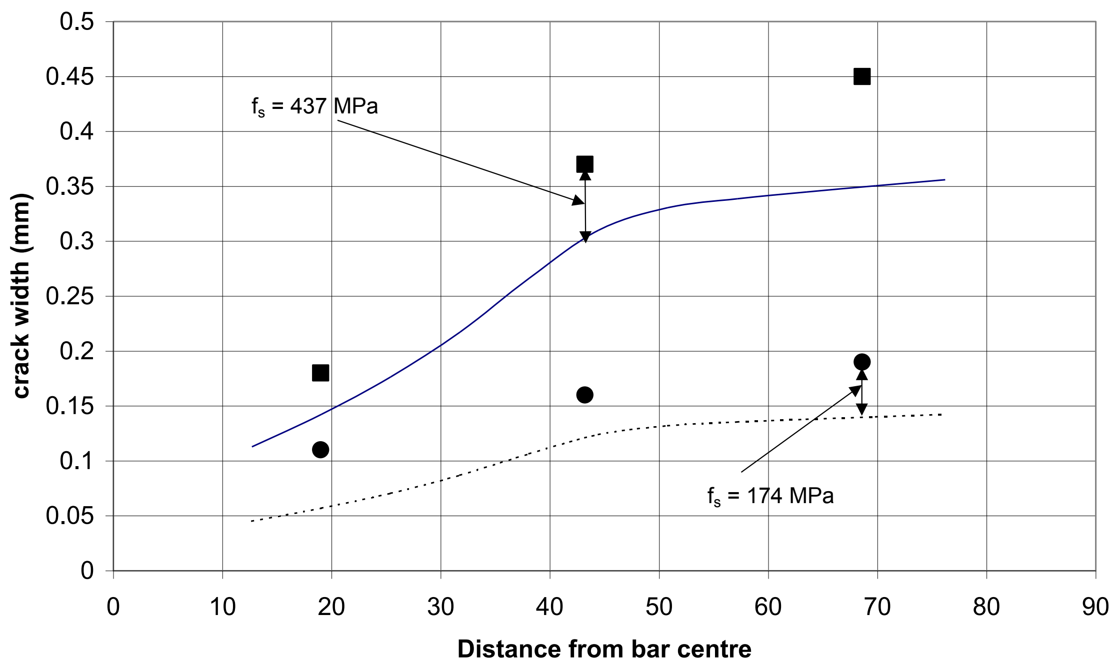


Fig. 19-Comparison of calculated and measured crack widths for Broms' specimen T-C-5 (10).
(1 MPa = 145 psi: 1 mm = 0.0394 in.)

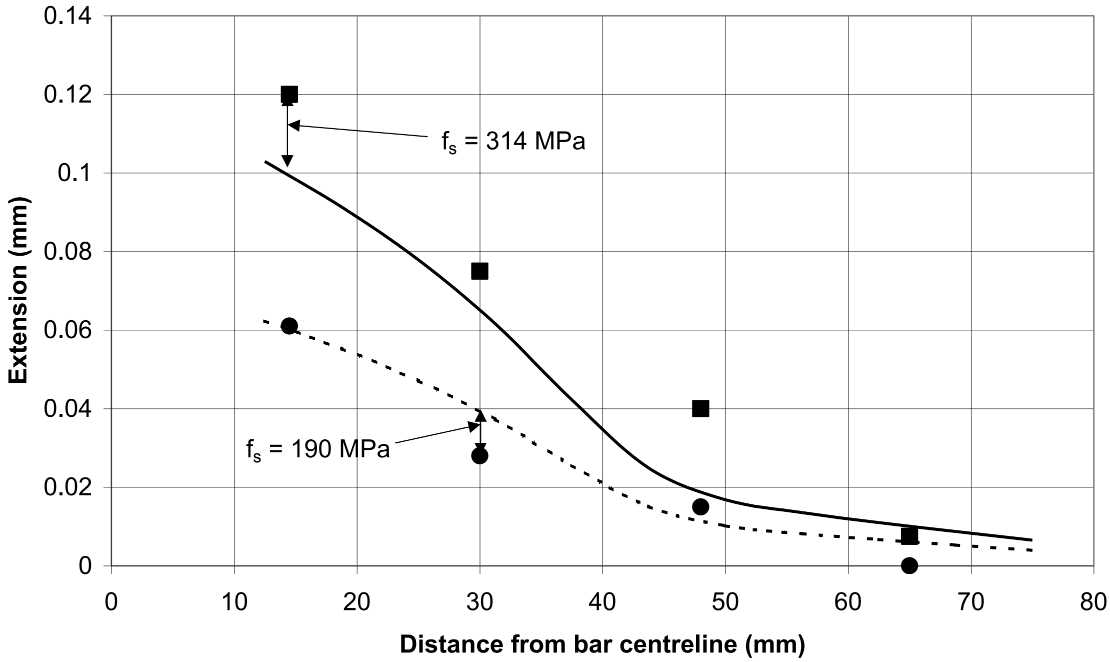


Fig. 20-Comparison of calculated and measured overall extension for specimen reported in (11).
(1 MPa = 145 psi: 1 mm = 0.0394 in.)