Analysis of Anisotropic Bonded NdFeB Halbach Cylinders Accounting for Partial Powder Alignment

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***Abstract*—An analytical technique is developed for predicting the performance of a bonded Halbach oriented anisotropic magnet, with due account of partial alignment of the NdFeB powder during injection molding. The predicted performance of a 12-pole injection molded, Halbach oriented magnet is compared with measurement.**

***Index Terms*—Bonded NdFeB, electrical machine, Halbach magnetization, permanent magnet.**

I. Introduction

BRUSHLESS permanent magnet machines employing Halbach magnetized rotors offer several potentially attractive features [1]–[3]. For example, due to their "self-shielding" property the rotor back-iron is not essential, whilst due to their inherent sinusoidal airgap field distribution, they exhibit negligible cogging torque and an essentially sinusoidal emf waveform. Thus, skew is not required and a nonoverlapping stator winding can be employed, which is conducive to low cost manufacture. The paper describes the analytical modeling of the powder aligning system which is used to impart a Halbach orientation during the injection molding of an anisotropic bonded NdFeB Halbach cylinder for use in a brushless machine, and the subsequent prediction of its performance, with due account of the partial alignment of the NdFeB powder, which inevitably occurs in practice.

II. Analytical Magnet Model Accounting for Partial Powder Alignment

A commercial grade of anisotropic NdFeB injection molding compound, NDA502E, which contains 65% by volume of MQE anisotropic NdFeB powder, compounded with a Nylon binder and proprietary lubricants, was used in the investigation. For simplicity, it was assumed that, after injection molding, the impulse magnetizing field could fully saturate the injection molded ring magnet throughout its entire volume.

The relationship between the remanence which results after exposure of the NDA502E compound to a specified level of aligning field during injection molding and its subsequent impulse magnetization to saturation has been measured, Fig. 1, and curve-fitted to

$$B_{rem} = F(B_a) \tag{1}$$

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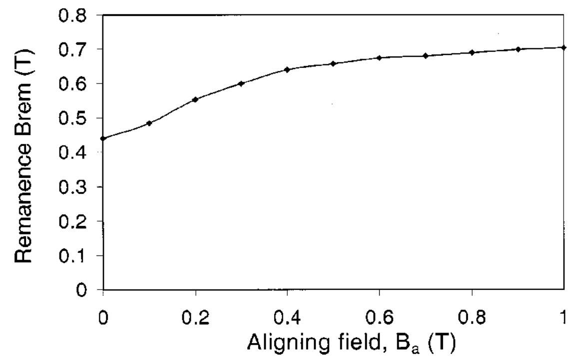


Fig. 1. Remanence versus aligning field for NDA-502E Compodic NdFeB injection molding compound.

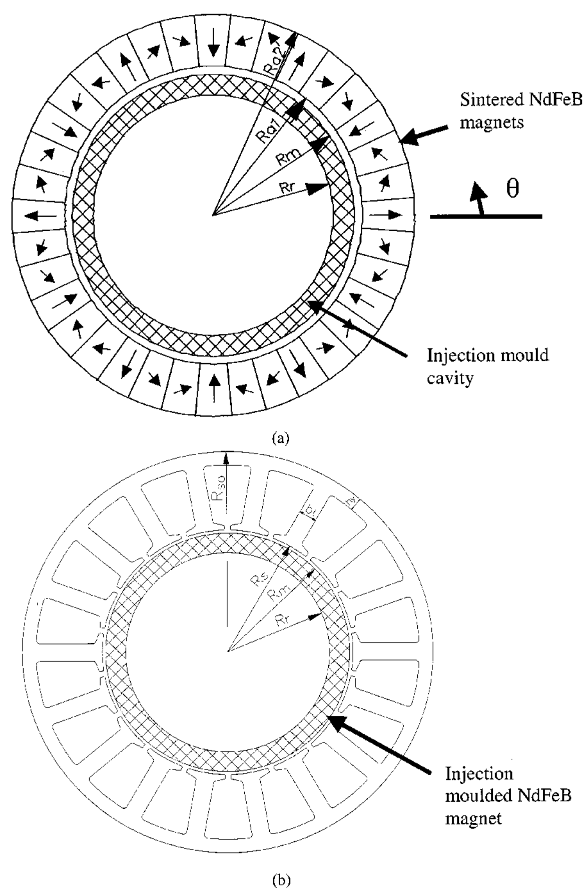


Fig. 2. (a) Twelve-pole Halbach powder aligning system. (b) Brushless permanent magnet motor.

Fig. 2(a) shows a schematic of a 12-pole Halbach cylinder powder aligning system, which is fabricated from pre-magnetized sintered anisotropic NdFeB magnet segments and used to orientate the NDA502E compound during the injection molding of a ring magnet having inner and outer radii of R_r and R_m, respectively.

In an ideal Halbach magnetized magnet, the magnetization distribution, in polar coordinates, can be expressed as $\vec{M} = M_r\vec{e}_r + M_\theta\vec{e}_\theta$, where $M_r = M\cos(p\theta)$ and $M_\theta = \pm M\sin(p\theta)$, p is the number of pole-pairs. However, when a Halbach magnetized magnet comprises discrete magnet segments, such as in the segmented powder aligning magnet shown in Fig. 2(a), the above field distribution is approximated, since the direction of magnetization is fixed in each individual magnet segment by: $\theta_m = (1 \pm p)\theta_i$, where θ_i is the angle between $\theta = 0$ and the center of the ith magnet segment, "+" being for an internal field Halbach cylinder [such as the aligning magnet shown in Fig. 2(a)] and "$-$" for an external field Halbach cylinder [such as the injection molded magnet employed in Fig. 2(b)].

The amplitude of the flux density which is produced by the powder aligning system is given by [1]:

$$|B_a(r)| = B_{ra}\sum_{v=0}^{\infty}(-1)^v\left(\frac{r}{R_{a1}}\right)^{n-1}\left(\frac{n}{n-1}\right) \cdot \left[1-\left(\frac{R_{a1}}{R_{a2}}\right)^{n-1}\right]\frac{\sin\frac{(n+1)\pi}{N}}{\frac{(n+1)\pi}{N}} \tag{2}$$

where

- B_{ra} is the remanence of the sintered NdFeB magnets from which the powder aligning system is fabricated,
- N is the total number of magnet segments, $n = p + vN$, and
- R_{a1} and R_{a2} are the inner and outer radii of the Halbach cylinder, respectively.

Fig. 3 shows the powder aligning system which employs 3 sintered NdFeB magnet segments per pole, and for which $R_{a1} = 29$ mm and $R_{a2} = 39$ mm, together with the field distribution. Fig. 4 compares the analytically predicted radial flux density distribution, deduced from equation (2), with the finite element predicted distribution, the internal and external radii of the injection mold being $R_r = 17$ mm and $R_m = 27$ mm, respectively.

As will be evident, the amplitude of the aligning field B_a varies throughout the radial thickness of the mold, the variation depending on the pole number. However, the radial and circumferential components of the aligning field vary essentially cosinusoidally and sinusoidally, respectively, with angular position, and the amplitude of the resultant field is constant at any given radius. In order to determine the flux density distribution and the flux per pole which results when such an injection molded Halbach oriented magnet has been fully magnetized and is subsequently employed in a brushless motor, as in Fig. 2(b), an analytically-based magnet model is employed. As shown schematically in Fig. 5, the Halbach magnetized magnet is sub-divided into a number, n, of concentric annular rings, each having an elemental radial thickness Δ. For the ith annular ring,

$$r(i) = R_r + (i-1)\Delta + \frac{\Delta}{2} \tag{3}$$

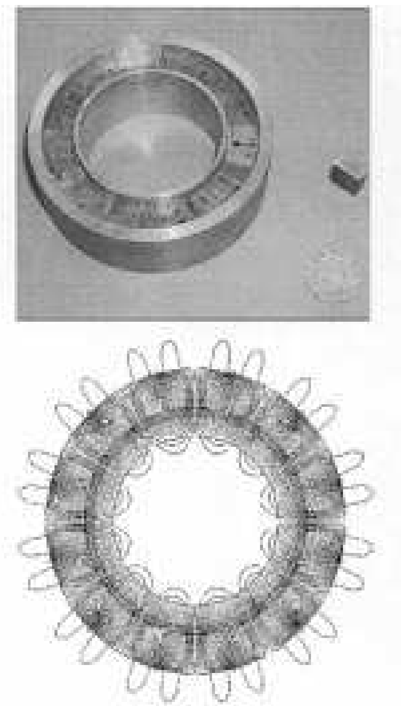
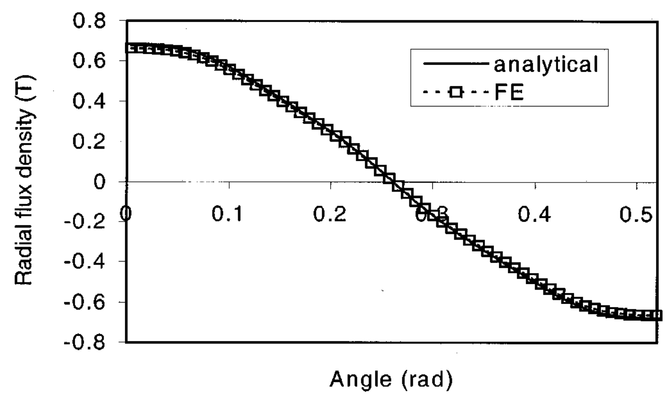
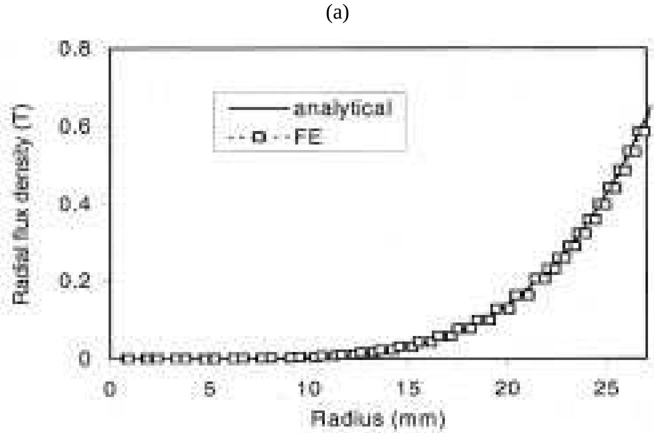


Fig. 3. Twelve-pole Halbach powder aligning system and field distribution.



(a)



(b)

Fig. 4. Comparison of analytical and finite element predicted flux density distribution. (a) Radial component at $r = 27$ mm. (b) Peak value as a function of radius.

where $\Delta = (R_m - R_r)/n$, $r(i)_i = R_r + (i-1)\Delta$, and $r(i)_o = R_r + i\Delta$. Thus, during injection molding within the Halbach

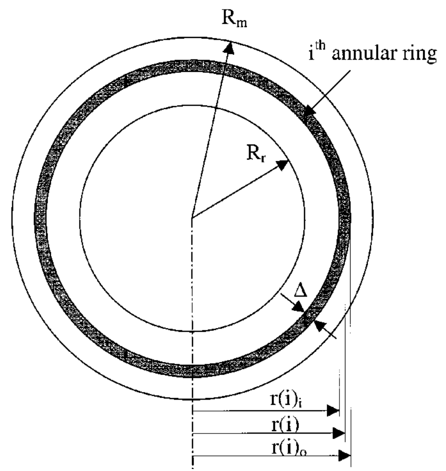


Fig. 5. Annular ring magnet model.

cylinder powder aligning system, the aligning field B_{ai} which results in each elemental ring is given by (2), whilst the effective remanence, following subsequent magnetization to saturation, is obtained from (1), i.e. $B_{rem}(i) = F(B_{ai})$. When the magnet is subsequently used for the rotor of a permanent magnet brushless motor [Fig. 2(b)] the contribution of each aligned and fully magnetized elemental ring to the airgap flux density can be derived. At the stator bore, R_s:

$$B_r(i) = \frac{2p}{p+1} B_{rem}(i) K \left[1 - \left(\frac{r(i)_i}{r(i)_o}\right)^{p+1}\right] \left(\frac{r(i)_o}{R_s}\right)^{p+1} \tag{4}$$

The radial and circumferential components of airgap flux density vary cosinusoidally and sinusoidally, *viz.* $B_r(i)\cos(p\theta)$ and $B_r(i)\sin(p\theta)$. The corresponding contribution to the flux per pole is:

$$\Phi_r(i) = 2\int_0^{\pi/2p} B_r(i) R_s L_a \cos(p\theta)\, d\theta = \frac{2B_r(i) R_s L_a}{p} \tag{5}$$

where L_a is the active axial length, and

$$K = \begin{cases} 1 & \text{for an air-cored rotor} \\ \left[1 - (R_r/R_s)^{2p}\right]^{-1} & \text{for an iron-cored rotor} \end{cases} \tag{6}$$

Thus, the airgap flux density and the flux per pole which result from a Halbach magnetized magnet comprising a number of such elemental rings is obtained by superimposition, *viz.*:

$$B_{rtot} = \sum_{i=1}^{n} B_r(i) \quad \text{and} \quad \Phi_{rtot} = \sum_{i=1}^{n} \Phi_r(i) \tag{7}$$

III. Results and Validations

The derived analytical technique has been used to predict the field distribution produced by an injection molded anisotropic Halbach magnetized magnet, with due account of the partial alignment of the NdFeB powder. The dimensions of the powder aligning system and the injection molded magnet are $R_r = 22.4$ mm, $R_m = 27$ mm, $R_{a1} = 29$ mm, R_{a2} =39 mm, and $L_a = 13.5$ mm. Fig. 6 compares the distribution of the radial flux density produced by a 12-pole injection molded magnet, following magnetization to saturation throughout its volume [2] and insertion in an iron cylinder having a bore radius R_s =27.5 mm, which is identical to the stator bore radius of the brushless motor shown in Fig. 2(b), a) without the application of the Halbach aligning field (which results in an isotropic magnet with $B_{rem} = 0.45$ T), b) assuming an ideal Halbach alignment (which results in an anisotropic magnet with $B_{rem} = 0.7$ T), and c) with the actual powder alignment which is achieved with the Halbach powder aligning system of Fig. 2(a) [which results in an anisotropic magnet with $B_{rem} = F(B_a)$], respectively. For cases a) and b), the analytically predicted results are also compared with finite element calculations, whilst for c), the analytical prediction is compared with the measured flux density distribution. As can be seen, good agreement is achieved. It will also be seen that a) the field distributions are essentially sinusoidal; b) the amplitude of the airgap field is larger than the remanence of the magnet, due to flux focusing which results from the Halbach magnetization; c) the field is significantly enhanced by orienting the powder; and d) the developed analytical technique accounts for the reduction in the airgap field which results from partial alignment of the powder.

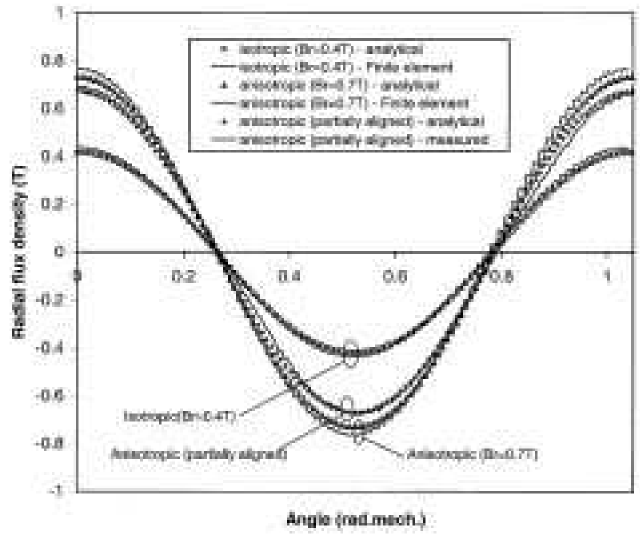


Fig. 6. Comparison of radial field distribution at stator iron bore surface, $R_s = 27.5$ mm.

IV. Conclusion

An analytical technique has been developed for modeling a bonded anisotropic magnet having a Halbach orientation imparted during the injection molding process, with due account of the degree of alignment of the NdFeB powder. Predictions have been validated by finite element calculations and measurements.

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