Re-Parametrizing the Dilemmas
Comment on "Universal Scaling the Dilemma Strength in Evolutionary Games", by Z. Wang *et al.*

Mauro Mobilia

Department of Applied Mathematics, School of Mathematics, University of Leeds, Leeds, LS2 9JT, United Kingdom

Keywords: Cooperation Dilemmas, Evolutionary Games, Nonlinear Dynamics, Fixation

Understanding the mechanisms at the origin of cooperation is a major challenge to Darwin's natural selection theory [1]: If only the fittest individuals survive, how to explain the ubiquity of cooperation? Among many examples of altruistic behaviors, one can mention insects that coordinate their efforts for the benefit of their "queen" [2].

In this context, Evolutionary Game Theory (EGT), see *e.g.* [3, 4, 5, 6, 7], provides a suitable framework to address this question by means of parsimonious models in which each agent's reproductive potential (expected payoff or fitness) depends on the others' fitness. In infinitely large populations, the abundance of the competing species/strategies is described by their "replicator (nonlinear differential) equations" and, in such a mean field setting, the evolutionary stable strategies of symmetric two-strategy games can readily be found in terms of two parameters directly obtained by rescaling the entries of the payoff matrix. In what follows, we will refer to this operation as the "replicator parametrization". Interestingly, the fitness optimization at an individual level may cause the reduction of the population's average fitness yielding a "social dilemma", as in the celebrated two-strategy prisoner's dilemma, snowdrift and stag-hunt games [3, 4, 5, 6, 7]. At mean field level, these cooperation dilemmas can easily and comprehensively be analysed by using the replicator parametrization. However, when the size of the population is finite, stochastic effects play an important role and the notion of evolutionary stability needs to be refined: In the simple case of two-strategy games, a strategy is evolutionary stable (ESS_N) in a well-mixed finite population if it has a higher fitness than the alternative strategy, *and* if a single player adopting the mutant-strategy has a lower fixation probability than in the absence of selection. The latter condition accounts for the selection to oppose the replacement by the alternative strategy [9]. Due to these two conditions, whether a strategy is an ESS_N cannot inferred by simply rescaling the payoff matrix and a comprehensive analysis of the dilemmas can no longer be carried out by using only the replicator paramatrization.

In their review article [10], Wang *et al.* propose to describe the classical two-player social dilemmas in terms of a new set of parameters, obtained by generalizing the approach used in Ref. [8] for the so-called donor-and-recipient prisoner's dilemma game. By focusing chiefly on the case of well-mixed finite populations, the authors of Ref. [10] show that the parametrization that they put forward is both a natural and efficient choice to analyze the properties of symmetric two-player games. Wang *et al.* make their point by revisiting five popular reciprocity mechanisms [8] and the necessary replacement condition for a strategy to be an ESS_N under weak selection [9]. They show that their new parametrization allows to re-derive non-trivial results in a simple and general manner, and is therefore superior to the replicator parametrization. As a further application, Wang *et al.* also demonstrate that the use of their set of parameters allows to resolve an apparent paradox in some two-strategy game with positive assortment [11]. Moreover, Wang *et al.* present a series of numerical investigations on various topologies indicating that the parametrization that they introduce is also particularly suited to describe the properties of social dilemmas in spatial settings.

In addition to what the authors of [10] have outlined in their interesting review article, the following open questions may also be considered for possible future research: (i) Is there a natural generalization of the proposed parametrization for games with more than three pure strategies, and/or for asymmetric games. (ii) Can the proposed parametrization help reveal in a simple and transparent way what are the conditions for a strategy to be an ESS_N under selection of arbitrary strength? (iii) Would the proposed parametrization help describe the properties of cooperation dilemmas in the presence of “facilitators” or “zealots” considered in Refs. [12, 13, 14]?

References

[1] E. Pennisi, How Did Cooperative Behavior Evolve?, Science 2005; 309: 93.

[2] E. O. Wilson, The Insect Societies. Harvard: Harvard University Press; 1971.

[3] J. Maynard Smith. Evolution and the Theory of Games. Cambridge: Cambridge University Press; 1982.

[4] J. Hofbauer and K. Sigmund. Evolutionary Games and Population Dynamics. Cambridge: Cambridge University Press; 1998.

[5] H. Gintis. Game Theory Evolving. Princeton: Princeton University Press; 2000.

[6] M. A. Nowak. Evolutionary Dynamics. Cambridge: Belknap Press; 2006.

[7] G. Szabó, G. Fáth, Evolutionary games on graphs, Phys. Rep. 2007; 446: 97 – 216.

[8] M. A. Nowak. Five rules for the evolution of cooperation. Science 2006; 314:1560 – 1563.

[9] M. A. Nowak, A. Sasaki, C. Taylor, and D. Fudenberg. Emergence of cooperation and evolutionary stability in finite populations. Nature (London) 2004; 428: 646 – 650.

[10] Z. Wang, S. Kokubo, M. Jusup, and J. Tanimoto. Universal Scaling the Dilemma Strength in Evolutionary Games Physics of Life Reviews 2015; 14: xxx-yyy, this issue.

[11] A. Németh and K. Takác. The paradox of cooperation benefits. J. Theor. Biol. **11**, 264 (2010).

[12] M. Mobilia. Stochastic dynamics of the prisoner’s dilemma with cooperation facilitators. Phys. Rev. E 2012; 86: 011134-1 – 011134-9.

[13] N. Masuda, Evolution of cooperation driven by zealots. Scientific Reports 2012; 2, 646: 1–5.

[14] M. Mobilia. Evolutionary games with facilitators: When does selection favor cooperation? Chaos, Solitons & Fractals 2013; 56, 113–123.