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## TOPOLOGY OPTIMIZATION FOR THE SEISMIC DESIGN OF TRUSS-LIKE STRUCTURES

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### ABSTRACT

A practical optimization method is applied to design nonlinear truss-like structures subjected to seismic excitation. To achieve minimum weight design, inefficient material is gradually shifted from strong parts to weak parts of a structure until a state of uniform deformation prevails. By considering different truss structures, effects of seismic excitation, target ductility and buckling of the compression members on optimum topology are investigated. It is shown that the proposed method could lead to 60% less structural weight compared to optimization methods based on elastic behaviour and equivalent static loads, and is efficient at controlling performance parameters under a design earthquake.

**Keywords:** Truss structure, Topology optimization, Weight minimisation, Seismic excitation, Nonlinear behaviour

### 1- INTRODUCTION

Structural optimization techniques are effective tools that can be used to obtain lightweight, low-cost and high performance structures. In the absence of efficient practical topology design tools, the selection of element topology in current design practice is usually based on the engineering judgment and past experience. Optimal design of truss structures has always been a challenge as they present a common and complex category of engineering structures. Various techniques based on classical optimisation methods such as feasible direction method [1], primal-dual method [2], dynamic programming [3], random search method [4] and sequence of linear programs [5] have been developed to find optimum design of elastic truss structures. These conventional methods are usually gradient-based solution strategies that require

the satisfaction of some specific mathematical conditions. Due to the high computational effort needed to calculate derivatives of the objective functions at each load step, these methods are not appropriate for optimum design of non-linear truss structures subjected to dynamic loads. Kirsch [6], studied the effect of compatibility conditions on optimal topology of truss structures. His studies showed that using classical numerical search algorithms for particular geometries or loading conditions may lead to an optimal topology that represents an unstable structure.

Several methods have been proposed for topology optimization of trusses by using continuum structures, including evolutionary structural optimization [7], homogenization-based optimization [8], fully stressed topology design [9], rule-based optimization [10] and soft-kill optimization [11]. However, by utilizing these methods, in general, it is not possible to find the optimal cross sectional properties of truss members. This may lead to suboptimal results as the size of the structural elements has a significant effect on the weight and structural performance of the final design. Besides, the application of these methods is limited to elastic structures.

Genetic algorithms (GAs) have been widely used in topology optimization of truss structures [12-15]. These methods represent general adaptive search techniques that simulate natural inheritance by adapting appropriate models based on genetics and natural selection. Computational effort that is associated with GAs seems to be high even for optimizing large elastic truss structures under static loads [16]. Therefore, it is not realistic to use these algorithms for optimum seismic design of truss structures in the practice.

By considering the nonlinear dynamic behaviour of structures, most of the available optimization techniques become very complicated. However, it is well known that many of the structural systems experience nonlinear deformations during medium to strong earthquakes. Therefore, the employment of conventional optimization techniques based on the elastic behaviour of structures may not lead to the optimum use of materials in the nonlinear range of behaviour. This paper presents a new practical method for optimum design of nonlinear truss-like structures subjected to dynamic excitations. This simplified method is based on the concept of uniform distribution of deformation demands as it is introduced in Section 2. Section 3 explains the developed optimization algorithm in detail. The proposed algorithm is applied on a simple truss structure in section 4, and the effects of target ductility demand, seismic exaction and buckling of compressive members on the optimum solution are investigated. A practical method for optimum seismic design of truss structures subjected to a group of earthquakes is also introduced in this section. Section 5 presents application of the proposed method for optimum design of more complex truss structures subjected to gravity and lateral seismic loads. Concluding remarks are given in section 6.

## 2- PROPOSED METHODOLOGY

Seismic design of structures is currently based on force rather than displacement as the seismic design codes generally use lateral inertia forces to account for seismic ground motion effects. The distribution of these static forces (and therefore, stiffness and strength) is based implicitly on the elastic vibration modes [17]. Optimization of non-linear truss-like structures for dynamic excitations is more challenging and associated with high computational effort and numerical difficulties due to the complexity of the problem.

In severe earthquakes, as structures exceed their elastic limits, the use of inertia forces corresponding to elastic modes may not lead to the optimum distribution of structural properties [18]. Therefore, during strong earthquakes the deformation demand in some parts of the structures does not utilize the allowable level of seismic capacity. If the strength of these underused elements (inefficient material) is decreased incrementally, it is expected to eventually obtain a status of uniform deformation demand. In such a condition, the dissipation of seismic energy in each structural element is maximized and the material capacity is fully exploited. Previous studies showed that the seismic performance of such a structure is near optimal, and it undergoes less damage in comparison with code-based designed structures having similar structural weight [18, 19]. Therefore, in general, it can be concluded that a status of uniform deformation demand is a direct consequence of the optimum use of material [19]. It should be mentioned that the optimum structure with minimum structural weight may not necessarily be the one with uniform ductility demand. Nevertheless, the proposed method has capability to decrease the required structural weight by exploiting all the material capacity.

The concept of the proposed optimization method (i.e. exploiting all the material capacity) is close to the classic optimization methods (such as fully-stress method [9, 20]) which are sometimes used in engineering practice. However, the application of conventional methods are usually limited to structures with elastic behaviour. The optimization method developed in this study is one of the few methods that is capable of optimizing non-linear structures subjected to dynamic excitations. While the basic philosophy of the proposed procedure is traditional, the developed optimization algorithm is capable of converging to the optimum solution in a few steps and without any fluctuation.

## 3- OPTIMISATION ALGORITHM

Assuming that the cost of a member is proportional to its material weight, the least-cost design can be interpreted as the least-weight design of the structure. Therefore, the cost objective function  $f$  to be minimized can be formulated as:

$$\text{Minimize: } f(A) = \sum_{i=1}^n \rho_i l_i A_i \quad (1)$$

where the design variables are the cross-sectional areas of truss members (denoted as  $A$ ) and  $\rho_i$  and  $l_i$  are material density and length of  $i^{\text{th}}$  member respectively.

In the performance-based design methods, design criteria are expressed in terms of performance objectives. Recent design guidelines, such as FEMA 356 [21], place limits on acceptable values of response parameters, implying that exceeding these acceptable values represents violation of performance objectives. The proposed method can optimise the design for different types of performance objectives such as deformation, acceleration and velocity. In the nonlinear range of response, member ductility ratio is a good criterion for assessing seismic performance of truss-like structures. Therefore, here the design variables are chosen to satisfy design constraints as follows:

$$\text{Subject to: } \mu_i \leq \mu_t \quad (i=1,2,\dots,n) \quad (2)$$

where  $\mu_i$  and  $\mu_t$  are maximum ductility ratio of  $i^{\text{th}}$  member and target ductility ratio, respectively.

In this study, the iterative optimization procedure developed by Hajirasouliha and Moghaddam [18] for optimum design of shear-building models is extended for topology optimization of non-linear truss structures. The proposed optimization algorithm is explained in detail in the following steps.

1- To design a truss-structure, certain nodes are considered to be important and must exist in any feasible design, while some optional nodes are just added for load sharing. The important nodes are usually the ones that carry external loads or support the truss, and are usually specified by the designer. The proposed algorithm assumes a ground structure, which is a complete truss with all possible (or reasonable) member connections among all nodes in the structure. The initial cross-sectional areas of the members are allowed to have arbitrary different values. In this study, an initial ground structure is assumed with all elements having uniform cross-sectional area. It is shown later that the initial cross-section areas do not have any effect on the final optimum solution.

2- The structure is subjected to design loads (e.g. gravity loads and seismic excitations), and maximum ductility demand for each structural element ( $\mu_i$ ) is calculated, and compared with the target value ( $\mu_t$ ). If the calculated ductility demands are close enough to the target ductility (e.g. 5% tolerance), the truss structure is considered to be practically optimum. Otherwise, the optimization algorithm is continued.

3- Positions where the ductility demand is less than the target values are identified and the inefficient material is reduced until an optimum structure is obtained. To obtain good convergence in numerical

calculations, this alteration is applied incrementally [22]. To accomplish this, the following equation is used:

$$[A_i]_{m+1} = [A_i]_m [SR_i]^\alpha \quad (3)$$

$$SR_i = \frac{\varepsilon_i}{\varepsilon_t} = \frac{\mu_i}{\mu_t} \quad (4)$$

where  $[A_i]_m$  is the cross-sectional areas of the  $i^{\text{th}}$  member at  $m^{\text{th}}$  iteration.  $\varepsilon_i$  and  $\varepsilon_t$  are maximum strain of the  $i^{\text{th}}$  member and target strain, respectively, and  $\alpha$  is the convergence parameter ranging from 0 to 1. The proposed equations are valid for both elastic and inelastic structures, since they are based on strain rather than stress. The convergence parameter  $\alpha$  plays an important role in the convergence of the problem as will be discussed in the following section. In this study,  $\alpha$  was set to be 0.2.

4- The need of specific structural members is determined by comparing the cross-sectional area of each member with a user-defined minimum value of critical cross-sectional area,  $a_0$ . If the element area is smaller than  $a_0$ , that member is removed from the regenerated truss structure. Subsequently, the optimization procedure is repeated from step 2 until the maximum ductility demand of all truss elements reaches the target ductility (i.e.  $SR_i=1$ ). At this stage a status of uniform ductility demand is achieved; and therefore, based on the concept of uniform distribution of deformation demands a truss structure with minimum required structural weight is obtained.

The user-defined parameter of  $a_0$  plays an important role in the stability of the final solution. If  $a_0$  is small enough, the optimisation algorithm always leads to stable results, since the dynamic behaviour and stability of the structure is controlled at each iteration. On the other hand, to find a solution with the minimum number of truss members,  $a_0$  should be greater than a minimum value that is a function of the cross sectional area of the elements. Numerous analyses carried out on different truss-like structures showed that, to obtain the best results,  $a_0$  should be between 5 to 10% of the average cross sectional area of all remaining elements [22]. In this case,  $a_0$  is adaptive and always results in a stable design with the minimum number of structural elements. In this study,  $a_0$  has been taken equal to 10% of the average cross sectional area of all remaining elements.

The proposed method can be a very useful tool for optimum design problems encompassing both the selection of optimal topology and the determination of optimal sizing of structural members. While using this method can rapidly lead to an optimum design, the quality of the solution depends on the location of the nodes and the connectivity of bars of the initial ground structure. However, to obtain a design that is close to the global optimum solution there is no need to consider a dense initial ground structure [22, 23].

The proposed optimisation method applies for members with continuous variation of the cross-sectional area. However, in the final design, the cross-sectional area of truss elements could be replaced by the first higher available section. It should be mentioned that the minimum weight design is not always the least-cost design as other factors such as the cost of the joints could be considerable. It was explained before that the proposed optimisation method removes very lightly loaded elements, and therefore, tends to minimise the number of elements. As the cost of joints is to some extent proportional to the number of elements and the force they carry, their cost is indirectly taken into account in the optimisation process.

#### **4- ELASTIC DESIGN EXAMPLE**

To investigate the effectiveness and validity of the proposed optimisation method, a problem for which the optimal solution is known is considered in this section. The optimization objective is to find the optimum topology for a truss bridge with 30 m span (Figure 1-a) that leads to a minimum structural weight design. The design criterion adopted limits the compressive and tensile stress of all truss elements to 60% of the yield stress (i.e.  $SR_i=0.6$ ) when the structure is subjected to a 400 kN concentrated static load at the mid-span. The initial ground structure is chosen by considering all possible member connections, as shown in Figure 1-a. Using the proposed optimisation algorithm, the optimum topology is determined by removing all inefficient elements. Figure 2 shows the variation of total structural weight from initial structure to the optimum design. The result indicates that, using the proposed method, the optimum solution was obtained in less than 10 steps. It should be mentioned that the optimum topology in this example is not dependent on the magnitude of the applied load and maximum allowable stress, since the structure is within the elastic range of behaviour. Figure 1 compares the results of this study with the optimum topology reported by Liang [9] based on the fully stressed method using continuum structures. The results confirm the validity of the optimization method proposed in this study as it was capable of reaching the known optimum solution. However, unlike most of the conventional optimization methods such as the one used by Liang [9], the application of the proposed method is not limited only to elastic structures subjected to static loads.

#### **5- NON-LINEAR DESIGN EXAMPLE**

The efficiency of the proposed optimization method for non-linear structures is demonstrated through a conceptually simple example shown in Figure 3a. The objective is to design a truss structure for sustaining three masses  $M_1$  to  $M_3$  (which are assumed to be 10, 20 and 10 tons, respectively) by using any number of members connecting these masses to each other and to the supports. Seismically induced

inertia forces are the only forces that act on points  $M_1$  to  $M_3$ . This structure should not exceed a member ductility demand of 4 when subjected to the horizontal component of the Northridge Earthquake of 1994 (CNP196). As a starting point, a ground structure with members having identical cross-sectional area is chosen by considering all possible member connections, as shown in Figure 3b. It is assumed that the yield strength of each member is equal in tension and compression. Nonlinear dynamic analyses were performed using DRAIN-2DX program [24]. A stiffness and mass proportional Rayleigh damping model with a constant damping ratio of 0.02 was assigned to the first and second modes of vibration. Buckling at the global level is considered by taking into account the P- $\Delta$  effects. An 18-bar truss shown in Figure 3b with uniform cross-sectional area equal to  $10 \text{ cm}^2$  was considered as the initial ground structure. By applying the proposed optimization method, the optimum design was obtained. Table 1 shows the cross-sectional area of truss members for the optimum solution. The results indicate that the proposed method was able to recognize and eliminate the redundant and inefficient members. Out of the 18 members in the primary arrangement in Figure 3b, only 8 members remained in the final step as shown in Figure 4a. In the optimum design structure, the maximum ductility demands for all of the remaining truss elements are very close to the target ductility of 4. The effect of the convergence parameter, initial cross sectional area, target ductility demand, seismic excitation and buckling of the compressive members on the optimum design is investigated in the following sections.

### **5-1- Efficiency of the proposed method**

The preliminary seismic design of most truss-like structures is commonly based on equivalent static forces that are distributed proportional to the weight (or mass) of different parts of the structure (Figure 5a). The optimum topology of the above example (Figure 3a) was calculated for the equivalent static forces shown in Figure 5a, by utilizing the fully stress design method [20]. Figure 5b, shows that the optimum topology in this case is different with the optimum topology based on the non-linear dynamic behaviour of the structure (Figure 4a). Table 1 compares the cross-sectional area of truss members for the optimal and conventionally design structures with the target ductility demand of 4. The proposed optimization method resulted in a truss structure with 535 kg structural weight, while using the optimum topology based on the equivalent static loads led to a structure with 748 kg structural weight (40% more), to obtain the same maximum member ductility demand (i.e.  $\mu_t = 4$ ). This implies that optimizing a structure based on the equivalent static loads and elastic behaviour does not necessarily lead to a minimum weight design solution when the structure is subjected to seismic excitations. This conclusion is in agreement with the results obtained by further analysis on different models and seismic ground motions [22]. It should be emphasized that the proposed optimization method leads to a more efficient seismic resistance design as it is based on the nonlinear dynamic behaviour of the structures.



## 5-2- Effect of Convergence Parameter

In order to study the effect of convergence parameter,  $\alpha$ , the previous example was solved for different values of  $\alpha$ . Figure 6 shows how the total structural weight varies from the initial ground structure (with uniform cross-sectional area) toward the optimum design using values of 0.05, 0.1, 0.2 and 0.5 for  $\alpha$ . It is evident that as  $\alpha$  increases from 0.05 to 0.2, the convergence speed increases without any fluctuation. However, for  $\alpha$  equal to 0.5, the method is not stable and the problem does not converge to the optimum solution. It is concluded that an  $\alpha$  value of 0.2 results in the best convergence for this design problem. Numerous analyses carried out on different structures indicate that, for non-linear truss structures, an acceptable convergence is usually obtained by using  $\alpha$  values of 0.1 to 0.2.

## 5-3- Effect of initial cross-sectional area

An initial cross-sectional area is always needed to begin the optimization algorithm. To investigate the effect of this initial cross-sectional area on the final results, the previous example was solved using two different initial ground structures with uniform cross-sectional area equal to 10 and 1000 mm<sup>2</sup>. The variations of total strength from each initial ground structure toward the final solution are compared in Figure 7. It is shown in this figure that the optimum design is not dependent on the initial cross-sectional area, as two different ground structures converged to the same total structural weight level. This conclusion has been confirmed by analysis of different structures and ground motion records [22].

The speed of convergence also appears to be dependent on the initial material distribution. The results indicate that using a more educated estimate for the initial cross-sectional area (i.e. relatively close to the final solution) results in a faster convergence. It is shown in Figure 7 that a ground structure with uniform cross-sectional area equal to 1000 mm<sup>2</sup> practically converged to the optimum solution in less than 15 steps.

## 5-4- Effect of buckling of members under compression

In practical applications, compressive members tend to buckle before they reach the yield point. In addition, local buckling can significantly change the hysteresis behaviour of axially loaded steel members, thus influencing optimum design. To address this issue, the hysteretic model suggested by Jain et al. [25] is utilized in this study. In this model, the axial compressive strength ( $P_c$ ) of truss members is reduced by using the following equation to consider the influence of buckling on their hysteretic behaviour and energy dissipation capacity:

$$P_c = \frac{25}{\left(\frac{KL}{r}\right)} P_y \quad (5)$$

where  $(KL/r)$  and  $P_y$  are effective slenderness ratio and yield strength of truss members, respectively. This hysteresis model, in general, compares well with the inelastic behaviour observed in experimental investigations [25]. In this study, the cyclic behaviour of axially loaded members after buckling has been considered at element level, and local buckling has not been considered directly. However, local buckling of individual elements could be accounted in exactly the same way, but with a different hysteretic model. To eliminate the effect of discrete section sizes, auxiliary tube sections have been artificially developed by assuming a continuous variation of section properties based on those of ASTM.

In order to achieve better convergence, when buckling behaviour of compressive members is considered in the optimization procedure, the criterion for selection of structural members should be based on the effective slenderness ratio of truss members rather than their cross-sectional area. Therefore, in this case, the proposed optimization algorithm eliminates truss members with relatively high slenderness ratio (e.g. over five times of the average slenderness ratio of all remaining elements). After eliminating less efficient members, the buckling length of the remaining elements is modified automatically. This may increase the second order effects (i.e. P- $\Delta$  effects) for some members that remain active in the structure. If the new design solution requires more structural weight, it is more cost effective to keep intersecting elements. In this case, the elimination process is reversible and a minimum cross-sectional area is chosen for the intersecting elements. These intersecting elements are just required to provide lateral stability for long length elements, and their cross-sectional areas do not change during the optimization process. Table 1 shows the cross-sectional area of truss members for the optimum solution in this case.

The optimum topology with and without considering the buckling of compressive members are compared in Figure 4. It is shown that buckling of compressive members influences the optimum topology of the truss structure by avoiding long elements that are more vulnerable to buckling.

### **5-5- Effect of target ductility demand**

To study the effect of non-linear behaviour of members on the optimum topology of truss structures, the proposed optimization algorithm was applied on the previous example (without buckling) for different values of target ductility demands. The cross sectional areas of optimum design structures for target ductility demands of 1, 1.5, 2, 3, 4, 5, 6 and 8 are given in Table 2. As expected, Table 2 shows that cross-sectional area of optimum designed structures, in general, was decreased by an increase in target ductility demand. The optimum topology for different target ductility demands are compared in Figure 8. It is shown in this figure that optimum topology depends on target ductility demand; and therefore, a fixed arrangement of truss members is not appropriate for different performance levels. This confirms that conventional optimization methods which are usually based on elastic vibration modes may not lead to an optimum design for structures in the nonlinear range of behaviour.

## 5-6- Effect of seismic excitation

Four strong ground motion records were used to evaluate the effect of seismic excitation on the optimum topology of non-linear truss structures, including EQ1: the 1994 Northridge earthquake CNP196 component with a PGA (Peak Ground Acceleration) of 0.42g, EQ2: the 1994 Northridge earthquake STC180 component with a PGA of 0.48g, EQ3: the 1979 Imperial Valley earthquake H-E04140 component with a PGA of 0.49g, and EQ4: the 1979 Imperial Valley earthquake H-E05140 component with a PGA of 0.52g. All of these excitations correspond to the sites with soil profile similar to the  $S_D$  type of UBC-97 [26] and were recorded in a low to moderate distance from the epicentre (less than 45 km) with rather high local magnitudes (i.e.,  $M_L > 6.7$ ). Acceleration response spectra of these records are relatively close to each other as shown in Figure 9.

The normalized cross sectional area (with respect to maximum member cross-sectional area) of optimum structures determined for the above mentioned seismic excitations are compared in Table 3 for target ductility demands of 1 and 4. It is shown that every seismic excitation has a unique optimum distribution of structural materials. While the optimum topologies for elastic truss structures ( $\mu_t = 1$ ) seem to be very similar in the four earthquake excitations, the results indicate that they could be very different in the non-linear range of behaviour ( $\mu_t = 4$ ).

To obtain required performance criteria (such as maximum ductility demand) during the design earthquake, a structure should be designed based on an appropriate load level. In the conventional force-based seismic design methods, a force reduction factor (behaviour factor  $q$ ) is generally considered in the calculation of design seismic loads that accounts for inherent ductility of a structure.

$$q = \frac{V_e}{V_y} \quad (6)$$

where  $V_e$  and  $V_y$  are lateral strength for elastic and inelastic structures, respectively. For optimised truss-like structures designed mainly to resist earthquake loads, total structural weight could be considered proportional to the design seismic load and lateral strength. Therefore, for this type of structural systems, the behaviour factor  $q$  could be expressed by the following equation:

$$q = \frac{W_e}{W_\mu} \quad (7)$$

where  $W_e$  and  $W_\mu$  are total structural weight of elastic and inelastic truss structures, respectively.

For each target ductility demand, the required structural weight is calculated for the optimum designed truss structures subjected to different seismic excitations. Figure 10 compares the ratio of

required structural weight for elastic ( $\mu_t = 1$ ) to inelastic structures with the results of Equation 7. The results indicate that for low to medium target ductility demands (i.e  $\mu_t \leq 4$ ); the structural weight ratio was close to the target ductility demand. This implies that the required structural weight for optimum elastic truss structure was almost  $\mu_t$  times higher than a similar inelastic structure. Therefore, in practical applications where  $\mu_t$  is usually less than 4, the behaviour factor  $q$  for truss structures resisting earthquake loads could be considered equal to the target ductility demand. Similar results have been obtained by using different truss structures and seismic ground motions [22]. However, to propose an appropriate behaviour factor for truss-like structures, further analysis should be done, something that is not within the scope of this study.

### **5-7- Design for a group of earthquakes**

The seismic load that a structure may experience during its lifetime is an important source of uncertainty. While each of the future seismic events will have its own signature, it is generally accepted that they will have relatively similar characteristics. Previous studies by Moghaddam and Hajirasouliha [27] showed that, for shear building models, a better seismic design could be found by averaging the optimum patterns corresponding to a number of earthquakes representing a design spectrum. To verify the validity of this idea for seismic design of truss structures, the average of optimum material distribution pattern corresponding to the four selected earthquakes were calculated for the previous example using different target ductility demands. Subsequently, these average patterns were used to design an example truss structure, similar to the one shown in Figure 3. For each seismic excitation, the cross-sectional areas of the truss structure were scaled, without changing the primary pattern, until the maximum member ductility reached the target ductility demand. As expected, the required structural weight for this structure was higher compared to the optimum design corresponding to each specific seismic excitation. Figure 11 compares the ratio of required to optimum structural weight for truss structures designed using the equivalent static loads (Figure 5a) and the average of the optimum material distribution patterns. This figure shows the average of the results obtained in the four selected earthquakes (EQ1 to 4). It is shown that using equivalent static loads and elastic analysis resulted in truss structures with up to 60% more structural weight compared to the optimum designed models based on non-linear dynamic behaviour of structures. It should be noticed that truss structures designed using the average of optimum patterns and the equivalent static loads have similar member ductility demand (performance objective), and therefore, are of similar safety margin. This implies that using extra structural material did not provide higher safety for the conventionally designed structures.

The results indicate that, for the same target ductility demand, the required structural weight for truss structures designed according to the average of the optimum material distributions is on average 20% less than those optimised based on the equivalent static loads. Figure 11 shows that the efficiency of using the average of the optimum distributions patterns is relatively higher for elastic and near-elastic structures

(i.e.  $\mu_t \leq 1.5$ ) where optimum distribution patterns for different earthquakes are almost similar. However, using this average pattern always leads to a reduction in the required structural weight compared to the conventional design methods. Therefore, the average of the optimum distribution patterns corresponding to a group of similar earthquakes, representing a design spectrum, seems to be appropriate for seismic design of non-linear truss structures. For engineering practices, a set of synthetic earthquakes having a close approximation to the code-based elastic design spectrum could be utilized.

## 6- OPTIMUM DESIGN FOR MULTIPLE LOAD CASES

The proposed optimization algorithm, can be easily extended to consider multiple load cases for practical design applications. To achieve this, in the proposed optimization algorithm, Equation (4) should be substituted with the following equation:

$$SR_i = \text{Max} \left[ \frac{(\varepsilon_i)_1}{(\varepsilon_t)_1}, \frac{(\varepsilon_i)_2}{(\varepsilon_t)_2}, \dots, \frac{(\varepsilon_i)_j}{(\varepsilon_t)_j} \right] \quad (8)$$

where  $(\varepsilon_i)_j$  and  $(\varepsilon_t)_j$  are maximum strain and target strain of the  $i^{\text{th}}$  member for  $j^{\text{th}}$  load case, respectively. Using Equation (8), different load cases can be considered in the design process for both elastic and inelastic range of behaviour. This leads to an optimum design that satisfies all the design requirements.

The application of the proposed optimization method for a truss structure subjected to a multiple load case (seismic and gravity loads) is demonstrated by considering an illustrative example shown in Figure 12. The objective is to design an optimum truss structure to sustain seven 100 ton-force equipment located in a 7-storey 28 m height building (Figure 12). The maximum member ductility demand of 1.5 (i.e.  $\varepsilon_t=1.5\varepsilon_y$ ) is considered as the design criterion when the truss structure is subjected to the horizontal component of the Northridge Earthquake of 1994 (CNP196). At the same time it is required that the truss elements are capable of resisting gravity loads with maximum axial stress ( $\sigma_{\text{max}}$ ) less than  $0.5f_y$  (i.e.  $\varepsilon_t=0.5\varepsilon_y$ ). In this example the strength of truss members is considered to be similar in tension and compression (buckling restrained members). The Rayleigh damping model with a constant damping ratio of 0.05 was used for the first mode and for any mode at which the cumulative mass participation exceeded 95%. Symmetry constraints are considered by applying the design seismic excitation in two opposite directions (two different load cases).

A highly connected initial ground structure with a reasonable member distribution was initially selected using basic engineering judgement. The cross-sectional area, axial stress ratio (due to gravity loads) and maximum member ductility demand of the optimum structure are shown in Table 4. The

optimum topology of the design example is illustrated in Figure 12-b. It is shown that out of more than 800 truss members in the initial ground structure, only 26 members (13 different types as shown in Figure 12-b) remained in the final solution. While maximum axial stress due to the gravity loads is less than  $0.5f_y$ , member ductility demand for all remaining elements is equal to the target ductility of 1.5. Therefore, the material distribution is regarded as being optimum.

Surprisingly, the optimum solution results in a rather simple structural configuration preferring triangular arrangements to carry all individual masses. It is shown that all truss elements of the optimum designed structure are only tied at the ground level and two more levels. Since the mass distribution of a structure is normally more complex than the one shown, the optimum topology found for this structure is not likely to be the same for a more uniformly distributed load.

To investigate the effect of mass distribution on the optimum topology, the above mentioned example was solved by considering two 50 ton-force masses located at each storey, as shown in Figure 13. Figure 13-b shows the optimum topology of the design example in this case. The cross-sectional area, axial stress ratio (due to gravity loads) and maximum member ductility demand of the optimum structure are shown in Table 5. The results indicate that this optimum structure is capable of resisting seismic and gravity loads without violating any design constraints. Although the resulting topology is more complex compared to the previous structure, the structural configuration follows a similar trend. The masses are still carried by simple triangles which are tied at the ground and two more levels. It is anticipated that if the floor elements were maintained at each level, since they are required to carry the uniform dead-loads anyway, a different optimum topology will be obtained.

It should be mentioned that in the performance-based design methods, different multiple limit states (e.g. service event, rare event, very rare event) are usually considered. Optimum design for a specific limit state does not guarantee the optimum behaviour in other conditions. In this case, it is usually accepted to consider the very rare event as the governing criterion for preliminary design, and control the design for other events. Based on the above discussions, the proposed optimization method seems to be reliable and should prove useful in practical performance-based seismic design of truss like structures. However, more research needs to be done to extend this method to include a penalty factor to account for the cost of joints and incorporate more complex multi-criteria objectives.

## 7- CONCLUSIONS

In this study a practical optimization method is presented for optimum seismic design of non-linear truss structures. Based on the results, the following conclusions can be drawn:

- The concept of uniform deformation can be used efficiently for topology optimization of nonlinear truss structures subjected to gravity loads and seismic excitations. It is demonstrated that there is a unique optimum distribution of structural properties, which is independent of the initial cross-sectional area of the ground structure.
- The results indicate that optimal topology is influenced by the variation of target ductility demand; and therefore, a fixed arrangement of truss members cannot be appropriate for different performance levels. In addition, buckling of compressive members influences the optimum topology by avoiding long slender elements.
- It is shown that using conventional optimization methods based on elastic behaviour and equivalent static loads could lead to a design with up to 60% more structural weight compared to the optimum designed model. It was concluded that non-linear dynamic behaviour of truss structures should be considered in optimum topology design of truss structures subjected to seismic excitations.
- While there is a specific optimum topology corresponding to each seismic excitation, it is shown that using the average of optimum distribution patterns for a group of earthquakes leads to acceptable results for practical applications. The results indicate that, for the same target ductility demand, truss structures designed according to the average of the optimum material distributions have on average 20% less structural weight compared to those designed based on the conventional elastic optimisation methods.

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**Figure 1.** (a) Preliminary arrangement of members in ground structure; (b) Optimum topology reported by Liang [9]; (c) Optimum topology using the proposed optimisation method

**Figure 2.** Variation of total structural weight from initial structure to the optimum design

**Figure 3.** (a) Position of masses and supports; (b) Joint numbers and preliminary arrangement of members in ground structure

**Figure 4.** Optimum topology (a) Similar behaviour in tension and compression; (b) Considering buckling of compressive members,  $\mu_t=4$

**Figure 5.** (a) Distribution of equivalent static seismic forces; (b) Optimum topology based on the equivalent static loads

**Figure 6.** Variation of total structural weight using different convergence parameters,  $\mu_t=4$

**Figure 7.** Convergence to the optimum solution from two different initial ground structures with uniform cross-sectional area equal to 10 and 1000 mm<sup>2</sup>,  $\mu_t=4$

**Figure 8.** Optimum topology for different target ductility demands, Northridge 1994 (CNP196)

**Figure 9.** Acceleration response spectra of selected seismic excitations

**Figure 10.** Required structural weight ratio of elastic to inelastic structure as a function of target ductility demand in different earthquakes

**Figure 11.** The ratio of required to optimum structural weight for truss structures designed using equivalent static loads and average of optimum material distribution patterns (average for 4 earthquakes)

**Figure 12.** (a) Geometry and potential nodes of initial ground structure; (b) Optimum topology and identification number of truss elements, Northridge 1994 (CNP196)

**Figure 13.** (a) Geometry and potential nodes of initial ground structure; (b) Optimum topology and identification number of truss elements, Northridge 1994 (CNP196)

**Table 1.** Definition of truss members and optimum answer for target ductility demand of 4

Member	First joint	Second joint	Cross-sectional area (cm <sup>2</sup> )		
			Based on the non-linear dynamic behaviour	Based on the equivalent static loads	Considering buckling effects
1	1	6	10.7	7.2	21.9
2	2	6	0	0	0
3	3	6	0	0	18.1
4	4	6	10.5	2.9	0
5	5	6	0	7.3	0
6	7	6	0	0	0
7	8	6	5.0	0	6.9
8	1	7	0	7.3	0
9	2	7	10.5	2.9	0
10	3	7	0	0	18.1
11	4	7	0	0	0
12	5	7	10.7	7.2	21.9
13	8	7	5.0	0	6.9
14	1	8	6.3	19.4	0
15	2	8	0	0	0
16	3	8	0	0	0
17	4	8	0	0	0
18	5	8	6.3	19.4	0

**Table 2.** The optimum arrangement of truss members for different target ductility demands

Member	Cross-sectional area (cm <sup>2</sup> )							
	$\mu_t=1$	$\mu_t=1.5$	$\mu_t=2$	$\mu_t=3$	$\mu_t=4$	$\mu_t=5$	$\mu_t=6$	$\mu_t=8$
1	34.0	26.0	17.9	12.7	10.7	11.8	4.8	3.8
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	4.9	10.5	10.5	0	0
5	35.5	33.0	20.9	6.2	0	0	5.9	4.7
6	0	0	0	0	0	0	0	0
7	11.2	5.4	5.9	7.5	5.0	7.0	2.2	1.4
8	35.5	33.0	20.9	6.2	0	0	5.9	4.7
9	0	0	0	4.9	10.5	10.5	0	0
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
12	34.0	26.0	17.9	12.7	10.7	11.8	4.8	3.8
13	11.2	5.4	5.9	7.5	5.0	7.0	2.2	1.4
14	63.7	59.2	31.2	4.2	6.3	0	4.7	4.6
15	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0
18	63.7	59.2	31.2	4.2	6.3	0	4.7	4.6

**Table 3.** Comparison of normalised cross-sectional area corresponding to optimum topology in four different seismic excitations,  $\mu_t=1$  and 4

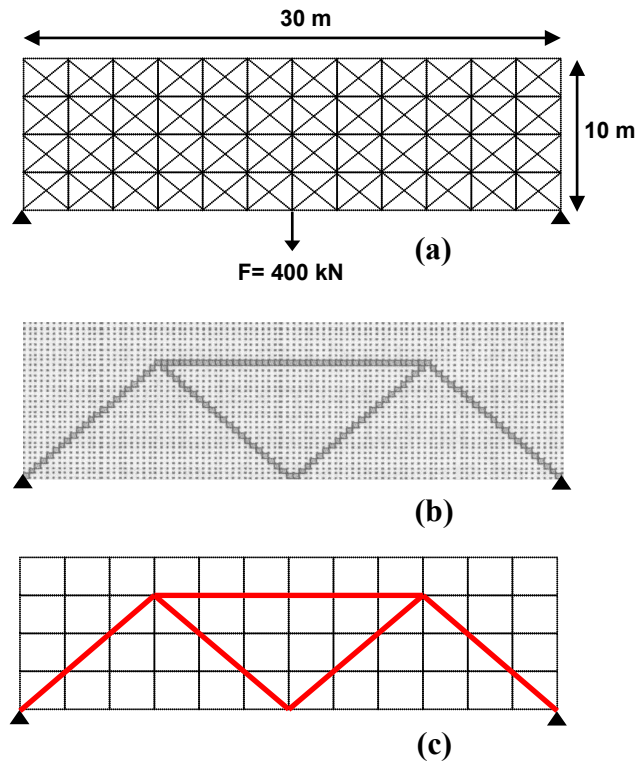
Member	Scaled Cross-Sectional Area ( $\mu_t=1$ )				Scaled Cross-Sectional Area ( $\mu_t=4$ )			
	EQ 1	EQ 2	EQ 3	EQ 4	EQ 1	EQ 2	EQ 3	EQ 4
1	0.53	0.49	0.44	0.60	1	0.93	1	0.87
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0.98	0	0	0
5	0.56	0.63	0.51	0.76	0	1	0.90	0.97
6	0	0	0	0	0	0	0	0
7	0.18	0.15	0.11	0.23	0.47	0.47	0.68	0.36
8	0.56	0.63	0.51	0.76	0	1	0.90	0.97
9	0	0	0	0	0.98	0	0	0
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
12	0.53	0.49	0.44	0.60	1	0.93	1	0.87
13	0.18	0.15	0.11	0.23	0.47	0.47	0.68	0.36
14	1	1	1	1	0.59	0.73	0	1
15	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0
18	1	1	1	1	0.59	0.73	0	1

**Table 4.** Cross-sectional area, axial stress ratio (due to gravity loads) and maximum member ductility demand of optimum design truss structure shown in Figure 10 (b)

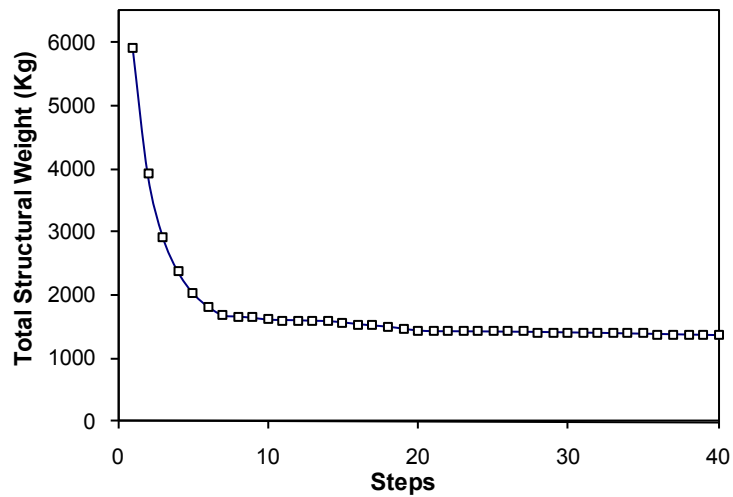
Member type	Cross-sectional area (cm <sup>2</sup> )	$\sigma_{max} / f_y$	Ductility
1	167.2	0.19	1.50
2	485.0	0.07	1.49
3	33.4	0.37	1.50
4	552.4	0.17	1.50
5	361.3	0.12	1.49
6	176.4	0.18	1.50
7	125.8	0.30	1.50
8	13.1	0.38	1.49
9	222.6	0.17	1.49
10	12.7	0.41	1.50
11	303.7	0.18	1.50
12	106.7	0.32	1.49
13	185.0	0.18	1.49

**Table 5.** Cross-sectional area, axial stress ratio (due to gravity loads) and maximum member ductility demand of optimum design truss structure shown in Figure 11 (b)

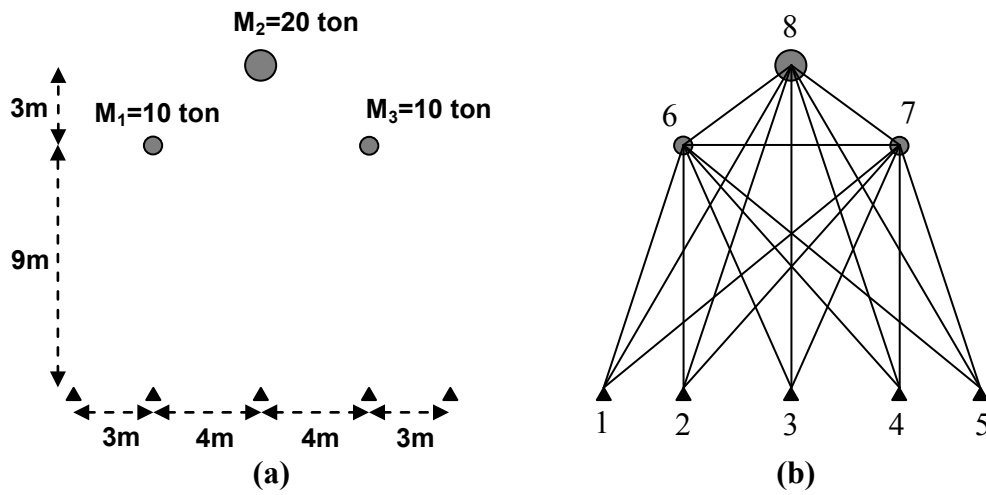
Member type	Cross-sectional area (cm <sup>2</sup> )	$\sigma_{\max} / f_y$	Ductility
1	344.2	0.08	1.50
2	513.8	0.14	1.50
3	132.8	0.13	1.50
4	298.9	0.10	1.49
5	153.4	0.14	1.50
6	216.8	0.05	1.49
7	294.3	0.04	1.51
8	436.1	0.04	1.50
9	321.8	0.15	1.50
10	213.4	0.07	1.50
11	214.5	0.05	1.50
12	181.9	0.03	1.50
13	254.4	0.07	1.51
14	98.4	0.08	1.50
15	130.6	0.13	1.51
16	106.8	0.07	1.50
17	173.5	0.09	1.49
18	41.1	0.46	1.50
19	160.9	0.14	1.49
20	285.3	0.09	1.51



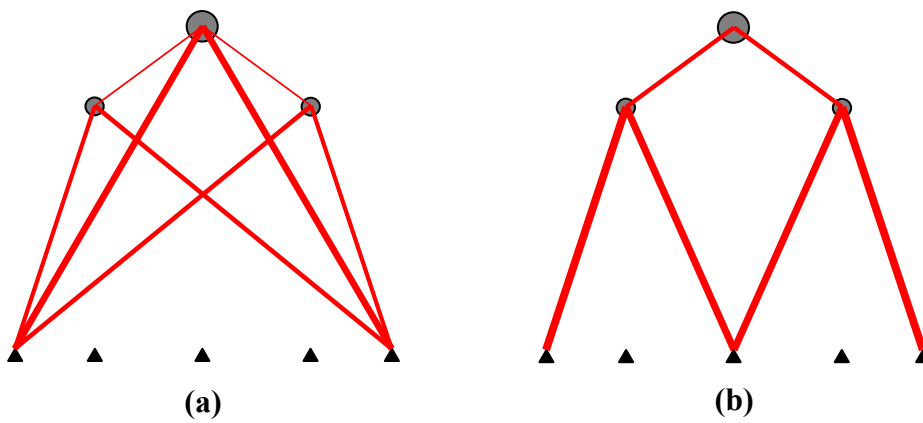
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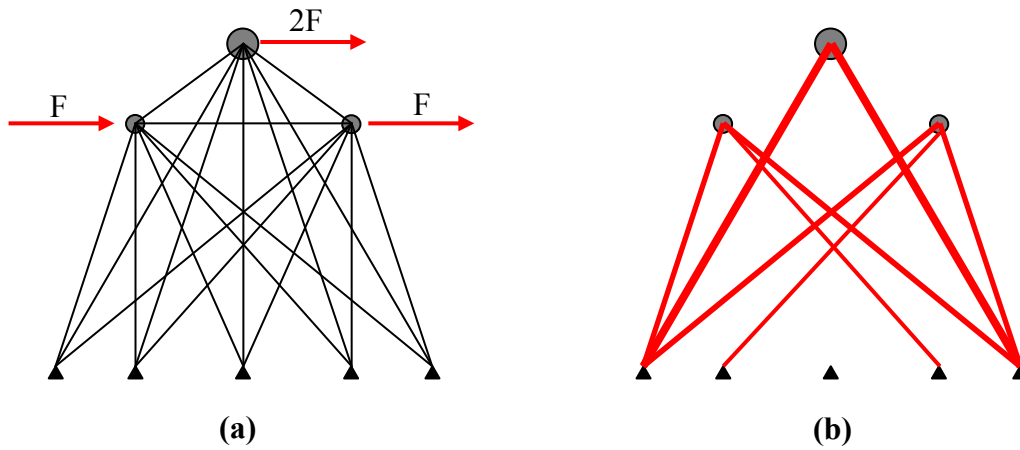
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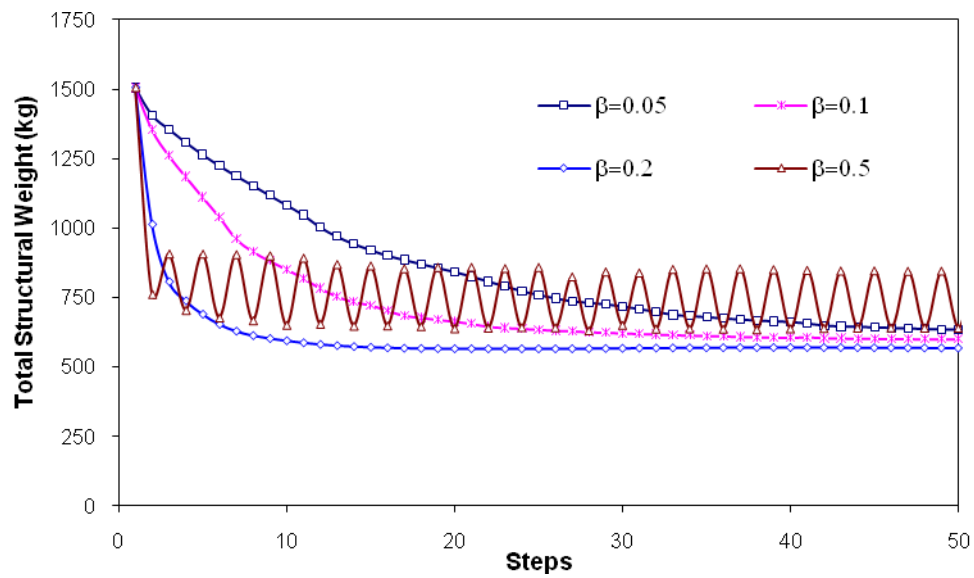
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**Figure 4.** Optimum topology (a) Similar behaviour in tension and compression; (b) Considering buckling of compressive members,  $\mu_4 = 4$

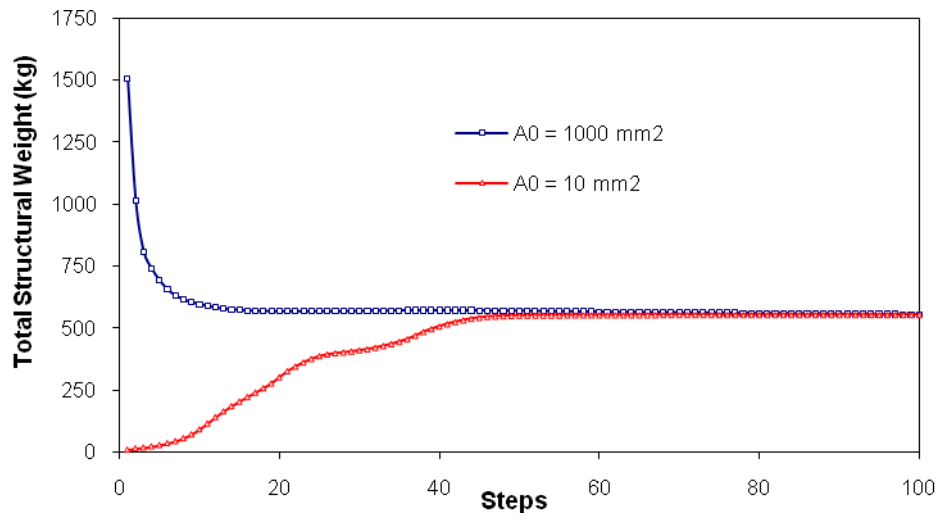


**Figure 5.** (a) Distribution of equivalent static seismic forces; (b) Optimum topology based on the equivalent static loads

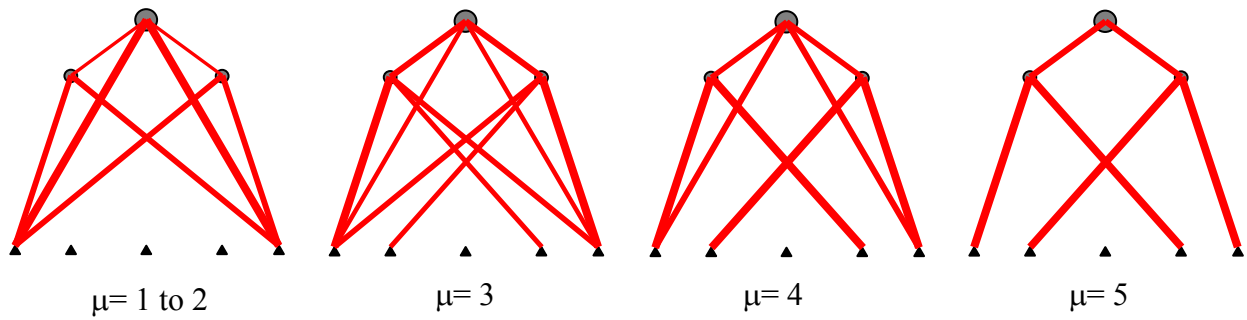


**Figure 6.** Variation of total structural weight using different convergence parameters,  $\mu_4=4$

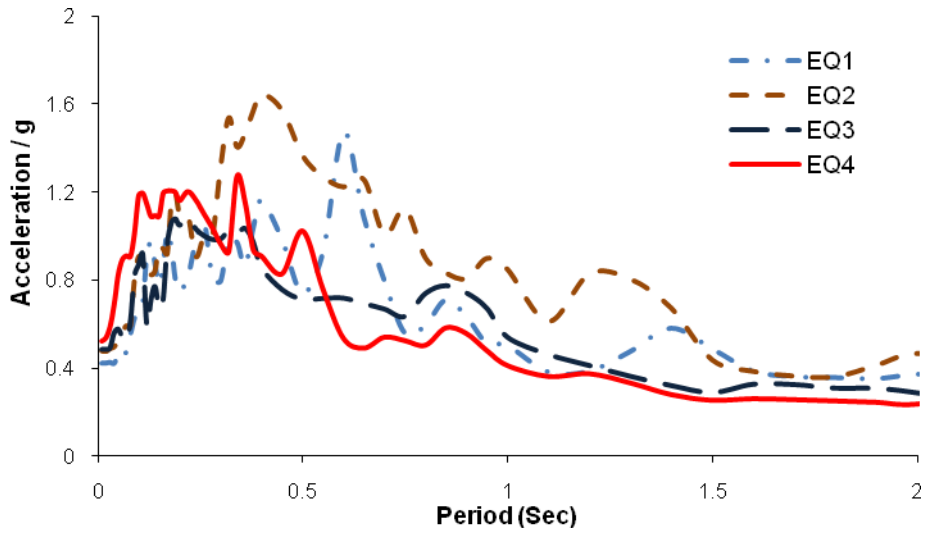




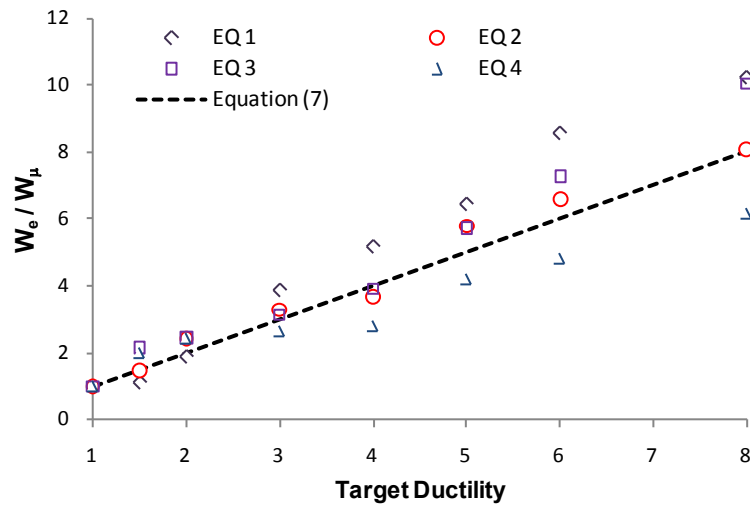
**Figure 7.** Convergence to the optimum solution from two different initial ground structures with uniform cross-sectional area equal to 10 and 1000 mm<sup>2</sup>,  $\mu_t = 4$



**Figure 8.** Optimum topology for different target ductility demands, Northridge 1994 (CNP196)



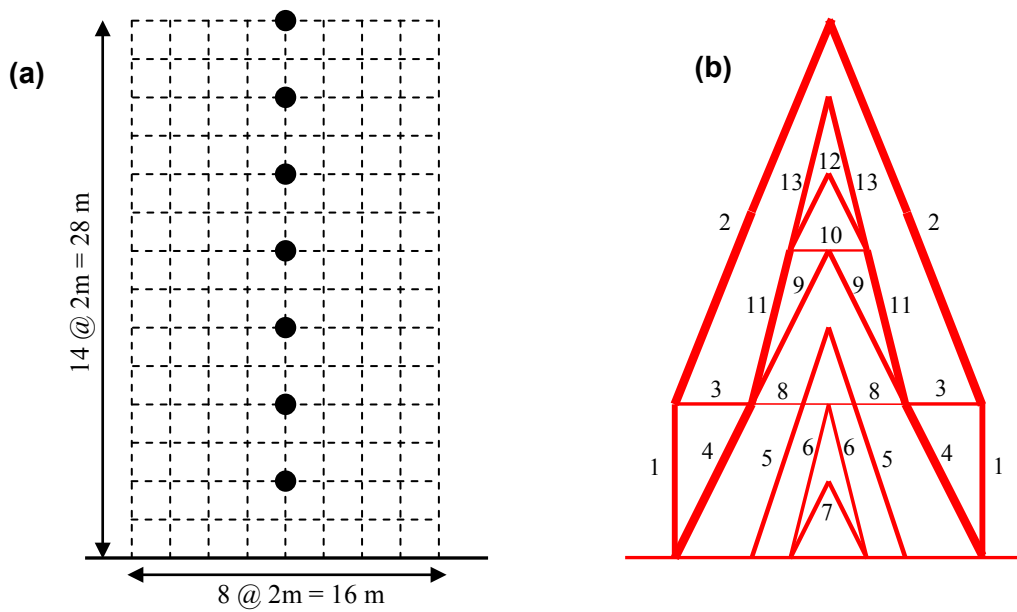
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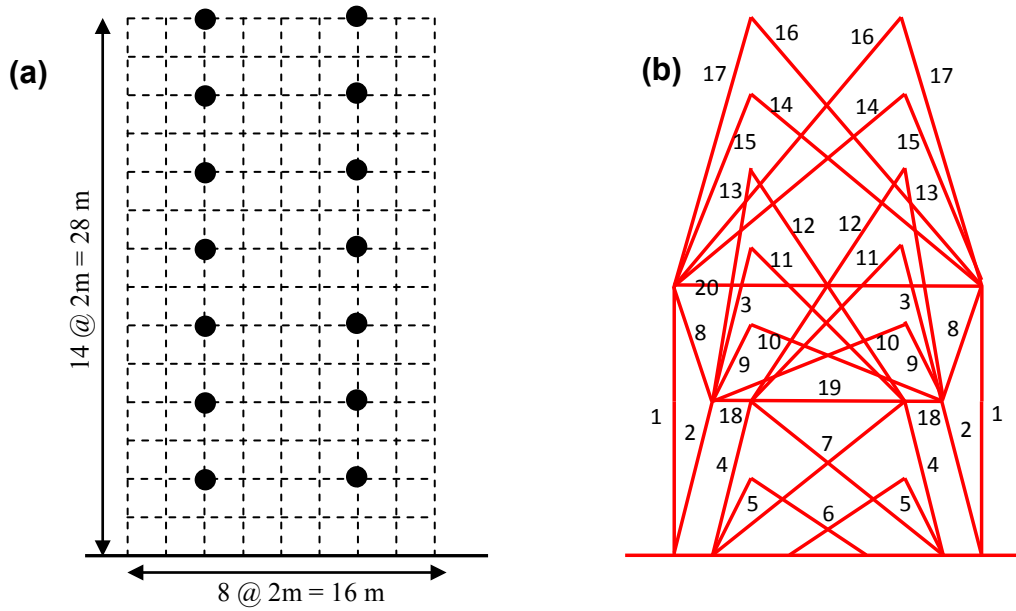
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