DYADIC MODIFICATION TO THE SEQUENTIAL TECHNIQUE FOR MULTIVARIABLE CONTROL SYSTEMS DESIGN

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Research report No. 18

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December 1973

ABSTRACT

A technique is suggested for easing the loop ordering difficulty inherent in Mayne's sequential method for multivariable feedback control system design, by combining the concept of loop addition with that of dyadic approximation. It is shown that if the plant matrix is dyadic, the problem is completely removed.

In a recent paper[1] Mayne describes a technique for the design of linear multivariable systems which enables the feedback loops to be designed sequentially using single-loop design concepts. As each loop is designed, the system transfer function matrix is updated to ensure that the interactive effect of previous design decisions are included in the analysis of the new loop. Although the procedure described seems highly practical, it does not completely solve several basic multivariable problems[1,2], and in particular, it is not evident in general what is a good order of loop closing[1]. In some physical systems[3], there is no a priori optimum choice of ordering. An arbitrary choice of ordering may limit the achievable performance, or the choice of compensation coefficients at any stage may introduce difficulties into the design of subsequent loops.

It is the purpose of this letter to suggest a technique which could ease this difficulty by noting that, for certain practical control configurations[3,4], a modal or dyadic[4] description of the system input-output dynamics simplifies the feedback control design analysis. For example, consider a system described by the NxN dyadic transfer function matrix[4]

$$G_p(s) = \sum_{j=1}^{N} h_j(s) w_j v_j^+ \qquad (1)$$

where $\{w_j\}$ and $\{v_j\}$ are linearly independent frequency independent sets of vectors, $\{h_j(s)\}$ are rational scalar transfer functions, and, for convenience, $h_j(o) = 1$, $1 \leqslant j \leqslant N$. It has been shown[4] that if

$$S = [w_1, w_2, \ldots, w_N] \qquad (2)$$

then

$$S^{-1} G_p(s) G_p^{-1}(o) S = \mathrm{diag}\{h_1(s), h_2(s), \ldots, h_N(s)\} \qquad (3)$$

That is, the use of a precompensator $G_p^{-1}(o)$ and the transformation to the basis set S reduces the feedback control problem to N non-interacting single-loop designs, for which there is no ordering problem.

The above analysis suggests that the problem of loop ordering may be reduced in certain cases by combining Mayne's method with the theory of dyadic transfer function matrices and the technique of dyadic approximation[4]. The following procedure is suggested for the control design analysis of a general plant $G_p(s)$:-

STEP 1

Compute the eigenvectors $\{w_j\}$ of the matrix[4]

$$H_2 = \lim_{s \to o} \frac{1}{s} \{G_p(o)G_p^{-1}(s) - I\} \tag{4}$$

and, assuming that they are complete, set up the similarity transformation S (eqn 2).

STEP 2

Apply Mayne's technique to the transformed system $S^{-1}G_p(s)G_p^{-1}(o)S$ to obtain a compensator matrix $G_c(s)$ and a diagonal matrix K(s) which produce the required closed-loop system properties in the transformed basis. This modified formulation is equivalent to sequentially designing feedback loops for the vector modes $\{w_j\}$.

STEP 3

Transform back to the original basis to obtain the controller matrix $G_p^{-1}(o)S\ G_c(s)K(s)S^{-1}$. The following identity illustrates that the stability of the feedback configuration is invariant under this transformation,

$$\begin{aligned} &|I + S^{-1}G_p(s)G_p^{-1}(o)SG_c(s)K(s)| \\ &= |I + G_p(s)\{G_p^{-1}(o)SG_c(s)K(s)S^{-1}\}| \end{aligned} \tag{5}$$

The advantages of following this procedure are as follows:-

(a) If $G_p(s)$ is a dyadic transfer function matrix, then the above analysis (with $G_c(s) = I$) indicates that the problem is reduced to N non-interacting single-loop designs, and consequently the ordering problem does not occur. Mayne's example[1] is easily shown to be dyadic so the loop ordering problem is removed by the above procedure.

(b) If $G_p(s)$ is approximately dyadic[4], then, although $G_p(s)$ may not be diagonally dominant, the transformed plant $S^{-1}G_p(s)G_p^{-1}(o)S$ will be diagonally dominant on the Nyquist contour. If the transformed plant is nearly diagonal or triangular then the ordering problem disappears[1] as in (a), but in the more general case, physical interpretation of the form of the vectors $\{w_j\}$ can lead to a physically meaningful ordering. For example, in certain nuclear reactor spatial problems[3,4], the $\{w_j\}$ have a direct physical interpretation in terms of spatial eigenmodes of the underlying partial differential equations. These eigenmodes have a well-defined order of importance[5].

(c) For a more general system, the transformation will not produce a diagonal or diagonally dominant matrix. However[4], the off-diagonal modal-interaction terms[4] are of order s^2 and hence, the matrix is diagonally dominant over some frequency interval about the origin. The reasoning of (b) could then be applied to order the feedback loops, but the ordering will be strictly valid only at low frequencies. Alternatively, the precompensator $G_c(s)$ could be chosen to extend the diagonal dominance to the whole of the Nyquist contour.

To illustrate the procedure consider the system

$$G_p(s) = \frac{1}{d(s)} \begin{bmatrix} 1 + 0.5s + s^2, & 6 + 6.5s + 6s^2 \\ 2 + 2.5s + 4s^2, & 5 + 4.5s + 3s^2 \end{bmatrix} \tag{6}$$

where d(s) is a polynomial in s of degree greater than two. Hence

$$G_p(s)G_p^{-1}(o) = \frac{d(o)}{d(s)} \begin{bmatrix} 1 + 1.5s + s^2 & -0.5s \\ -0.5s - 2s^2 & 1 + 1.5s + 3s^2 \end{bmatrix} \tag{7}$$

and (eqn. 4)

$$H_2 = \begin{bmatrix} -1.5 & 0.5 \\ 0.5 & -1.5 \end{bmatrix} \tag{8}$$

from which (eqn 2)

$$S = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad (9)$$

and the transformed plant $S^{-1}G_p(s)G_p^{-1}(o)S$ is

$$\frac{d(o)}{d(s)} \begin{bmatrix} 1 + s + s^2 & 2s^2 \\ 0 & 1 + 2s + 3s^2 \end{bmatrix} \quad (10)$$

which is upper triangular. Although the original system $G_p(s)$ does not have an a priori optimal loop ordering, the transformed system is upper triangular and hence the loop ordering problem inherent in Mayne's technique disappears[1] in the basis S. Choosing $G_c(s) = I$, say, and $K(s) = \mathrm{diag}\{k_1(s), k_2(s)\}$ as required in the transformed basis, the inverse transformation yields the controller

$$d(o) \begin{bmatrix} 1 & 6 \\ 2 & 5 \end{bmatrix}^{-1} \tfrac{1}{2} \begin{bmatrix} k_1(s) + k_2(s) & k_1(s) - k_2(s) \\ k_1(s) - k_2(s) & k_1(s) + k_2(s) \end{bmatrix} \quad (11)$$

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