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# Calculation of $d$ - and $q$ -Axis Inductances of PM Brushless ac Machines Accounting for Skew

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**A hybrid two-dimensional (2-D) finite-element/analytical technique is described for predicting the  $d$ -axis and  $q$ -axis inductances of permanent magnet (PM) brushless ac machines, with due account for the influence of skew. Predicted inductances are compared with measured values for two machines having identical stators, which are skewed by one slot-pitch, but which have different rotor topologies, one having surface-mounted magnets and the other having interior magnets.**

**Index Terms**—Brushless machine, permanent magnet (PM), permanent magnet (PM) machine, winding inductance.

## I. INTRODUCTION

THE  $d$ -axis and  $q$ -axis inductances have an important influence on both the steady-state and dynamic performance of permanent magnet (PM) brushless ac machines [1]. Thus, their accurate prediction is essential, not only for predicting performance aspects such as the torque and flux-weakening capabilities, but also for designing control systems, in order to maximize the efficiency, power factor, etc. The influence of magnetic saturation is usually accounted for by determining current-dependent inductances [1], [2], i.e.,  $L_d = f(I_d)$  and  $L_q = f(I_q)$ . However, since high local saturation may occur due to the combined influence of the PMs and the armature reaction field, especially in interior-magnet rotor machines, analytical methods for determining the winding inductances are often inadequate, and a numerical method must be employed. When the stator slots or rotor magnets are skewed, however, three-dimensional (3-D) calculations are required, which remain time consuming.

In this paper, the influence of skew on the  $d$ -axis and  $q$ -axis inductances is investigated by employing a hybrid two-dimensional (2-D) finite-element/analytical technique, with due account for skew and end effects. Predictions are validated by measurements, using a dc inductance bridge technique [2], on two brushless ac machines having identical stators, which are skewed by one slot-pitch but different rotor topologies.

## II. CALCULATION OF $d$ -AXIS AND $q$ -AXIS INDUCTANCES

Various techniques have been employed to predict the winding inductances using the finite-element method [3]–[5]. When the magnetic circuit can be assumed to be linear, the self- and mutual-winding inductances can be evaluated by energizing the appropriate stator windings, with the PMs replaced by air [5], and determining the stored energy or winding flux linkage, such that the  $d$ -axis and  $q$ -axis inductances can subsequently be deduced. However, if magnetic saturation is significant, both the self- and mutual inductances, and the  $d$ -axis and  $q$ -axis inductances, are current dependent. The incremental and

apparent self- and mutual inductances should then be calculated by either the energy perturbation method or the current perturbation method, and the influence of the PMs should be considered [3], [4]. However, although generally applicable, the method described in [4] requires multiple finite-element field solutions, for a range of current perturbations in each winding, and, therefore, demands significant computational effort.

The  $d$ -axis and  $q$ -axis inductances,  $L_d$  and  $L_q$ , respectively, which are used in the flux-weakening control of PM brushless ac machines [6], can be calculated from [2], as follows:

$$L_d = \frac{\psi_d - \psi_m}{I_d} \quad (1)$$

$$L_q = \frac{\psi_q}{I_q} \quad (2)$$

where  $I_d$ ,  $I_q$ ,  $\psi_d$ , and  $\psi_q$  are the  $d$ -axis and  $q$ -axis currents and flux-linkages, respectively; and  $\psi_m$  is the flux-linkage due to the PM rotor.

However, while 2-D finite-element analysis can account for the airgap, slot leakage, tooth-tip leakage and magnetomotive force (MMF) harmonic leakage components of inductance to a high accuracy, it cannot account for the end-winding component of leakage inductance or the influence of skew.

While skew is effective in reducing the harmonic content in the flux linkage and back-electromotive force (EMF) waveforms, as well as in reducing the cogging torque, skewing either the stator teeth or the rotor magnets results in a 3-D magnetic-field distribution in the active region of a machine. However, since the axial variation of saturation due to skew is generally small in most PM machines, the relationship between the  $d$ -axis and  $q$ -axis inductances,  $L_d^{2D}$  and  $L_q^{2D}$ , which are calculated by 2-D finite-element analysis of the active region of a machine without skew and the  $d$ -axis and  $q$ -axis inductances,  $L_d^{\text{sk}}$  and  $L_q^{\text{sk}}$ , for the same machine with skew can be derived analytically, as will be described.

The self-inductance  $L$  and the mutual inductance  $M$  can be expressed as functions of the rotor electrical angle  $\theta$  [7], as follows:

$$L = L_0 + L_1 + L_2 \cos 2\theta \quad (3)$$

$$M = M_0 + M_2 \cos(2\theta - 120^\circ) \quad (4)$$

where  $L_0$  and  $M_0$  are the components of the self- and mutual inductances due to the space-fundamental airgap flux,  $L_l$  is the additional leakage flux component, and the second harmonic components  $L_2$  and  $M_2$  result from saliency of the rotor.

Skew changes the relative position between the stator and rotor along the axial length of a machine. Hence, the magnetic-field distribution also varies. However, the self-inductance  $L^{\text{sk}}$  and the mutual inductance  $M^{\text{sk}}$  of a machine with skew can be deduced from the inductances of the same machine without skew by subdividing the active length of the machine into  $n$  axial slices, with each slice having a skew angle of  $\alpha/n$ . Hence,

$$L^{\text{sk}} = \frac{1}{\alpha} \int_{\theta-\alpha/2}^{\theta+\alpha/2} (L_0 + L_l + L_2 \cos 2\theta) d\theta \quad (5)$$

$$= L_0 + L_l + L_2^{\text{sk}} \cos 2\theta \quad (5)$$

$$M^{\text{sk}} = M_0 + M_2^{\text{sk}} \cos(2\theta - 120^\circ) \quad (6)$$

where  $\alpha$  is the electrical skew angle, and

$$L_2^{\text{sk}} = K_{\text{sk}} L_2 \quad (7)$$

$$M_2^{\text{sk}} = K_{\text{sk}} M_2 \quad (8)$$

where the skew factor  $K_{\text{sk}}$  is given by

$$K_{\text{sk}} = \sin \alpha / \alpha. \quad (9)$$

Thus, the constant components of the winding inductances remain unchanged when skew is employed, while the second harmonic components are reduced by the skew factor  $K_{\text{sk}}$ , which is  $\leq 1$ , i.e., the inductance variation due to saliency is reduced by skew.

In general, from the Park transformation, the relationship between the phase self- and mutual inductances,  $L$  and  $M$ , and the  $d$ -axis and  $q$ -axis inductances,  $L_d$  and  $L_q$ , are given by [7]

$$L_d = L_l + (L_0 - M_0) + \left( \frac{L_2}{2} + M_2 \right) \quad (10)$$

$$L_q = L_l + (L_0 - M_0) - \left( \frac{L_2}{2} + M_2 \right). \quad (11)$$

Thus, from (5)–(9), the  $d$ -axis and  $q$ -axis inductances, accounting for the effect of skew, can be obtained as

$$L_d^{\text{sk}} = L_l + (L_0 - M_0) + K_{\text{sk}} \left( \frac{L_2}{2} + M_2 \right) \quad (12)$$

$$L_q^{\text{sk}} = L_l + (L_0 - M_0) - K_{\text{sk}} \left( \frac{L_2}{2} + M_2 \right). \quad (13)$$

Combining (11)–(13), the relationship between the inductances  $L_d^{2D}$  and  $L_q^{2D}$ , which are deduced from 2-D finite-element analyses of the active region of a machine neglecting skew, and the inductances  $L_d^{\text{sk}}$  and  $L_q^{\text{sk}}$ , which account for skew, can be derived as

$$L_d^{\text{sk}} = L_d^{2D} + \frac{1 - K_{\text{sk}}}{2} (L_q^{2D} - L_d^{2D}) \quad (14)$$

$$L_q^{\text{sk}} = L_q^{2D} - \frac{1 - K_{\text{sk}}}{2} (L_q^{2D} - L_d^{2D}). \quad (15)$$

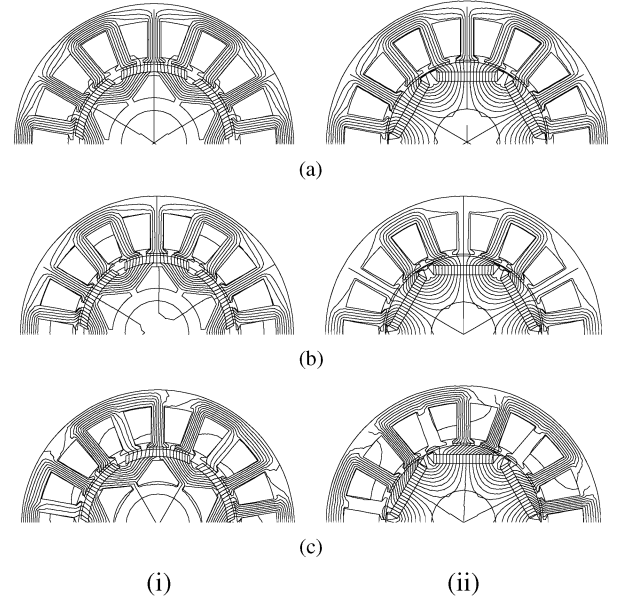


Fig. 1. Magnetic field distributions. (i) Surface-mounted magnet rotor: (a) Open circuit; (b) rated  $d$ -axis current; and (c) rated  $q$ -axis current. (ii) Interior-magnet rotor: (a) Open circuit; (b) rated  $d$ -axis current; and (c) rated  $q$ -axis current.

The end-winding component of inductance  $L_e$  may be estimated using a well-proven method that has been applied extensively to induction motors [8]. Thus, the resultant  $d$ -axis and  $q$ -axis inductances are

$$L_d = L_d^{\text{sk}} + L_e \quad (16)$$

$$L_q = L_q^{\text{sk}} + L_e. \quad (17)$$

### III. COMPARISON AND EXPERIMENTAL VALIDATION

Predicted  $d$ -axis and  $q$ -axis inductances have been compared with measured values for two 1.3-kW, three-phase PM brushless ac motors having identical stators, with six poles, 18 slots, an overlapping winding, and a skew of one slot pitch, but different rotors, viz. one with surface-mounted magnets and another with interior magnets. The main design parameters are as follows: stator outside diameter = 106.6 mm, active length = 32 mm, airgap length = 0.75 mm, rotor diameter = 60.5 mm, and width of stator slot openings = 2.5 mm. Both rotors have sintered NdFeB magnets with a remanence of 1.17 T and a relative recoil permeability of 1.07. It will be noted that the rotor back-iron of the machine with the surface-mounted magnet rotor has air spaces to reduce the inertia and minimize the influence of  $q$ -axis magnetic saturation, and thereby maximize the overload torque capability.

Fig. 1 shows field distributions when the two motors are on open circuit and on load when rated current (4 A at peak) is applied to the  $d$  and  $q$  axes. Fig. 2 compares measured and predicted  $d$ - and  $q$ -axis inductances, from which the influence of magnetic saturation can be clearly observed. The predictions include the end-winding component of inductance, viz. 6.9 mH

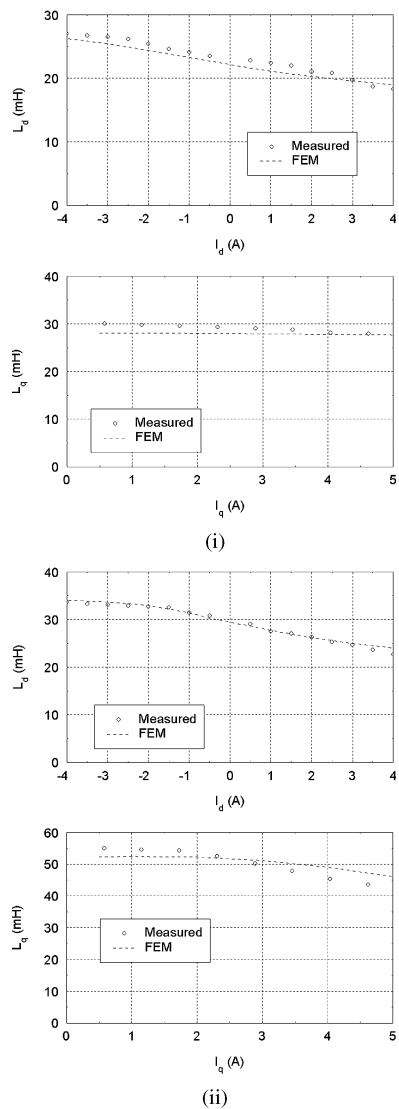


Fig. 2. Measured and predicted  $d$ -axis and  $q$ -axis inductances: (i) Surface-mounted magnet rotor motor and (ii) interior-magnet rotor motor.

[8]. The  $d$ -axis inductance  $L_d$  was measured by locking the rotor along the  $d$  axis, which was determined by applying a  $d$ -axis current  $I_d$  ( $I_a = I, I_b = -I/2$  and  $I_c = -I/2$ ) and allowing the rotor to rotate to the position at which the torque was zero, and using a dc inductance bridge [2]. The  $q$ -axis inductance  $L_q$  was similarly measured. In general, very good agreement is obtained. As will be seen, due to the different rotor topologies, the variation of  $L_d$  and  $L_q$  with  $I_d$  and  $I_q$ , respectively, is significantly different for the two motors.

Fig. 3 shows the influence of skew on the predicted  $d$ -axis and  $q$ -axis inductances for both motors. It will be noted, both from Fig. 3 and the foregoing equations, that irrespective of  $L_d$  and  $L_q$ , the smallest inductance component is increased as a result of skew, while the other component is reduced. Hence, the use of skew reduces the influence of any rotor saliency, and both  $L_d^{sk}$  and  $L_q^{sk}$  will approach the value  $0.5(L_d + L_q)$  as the skew angle is increased. For a machine without saliency, i.e.,  $L_d \approx L_q$ , the influence of skew will generally be negligible.

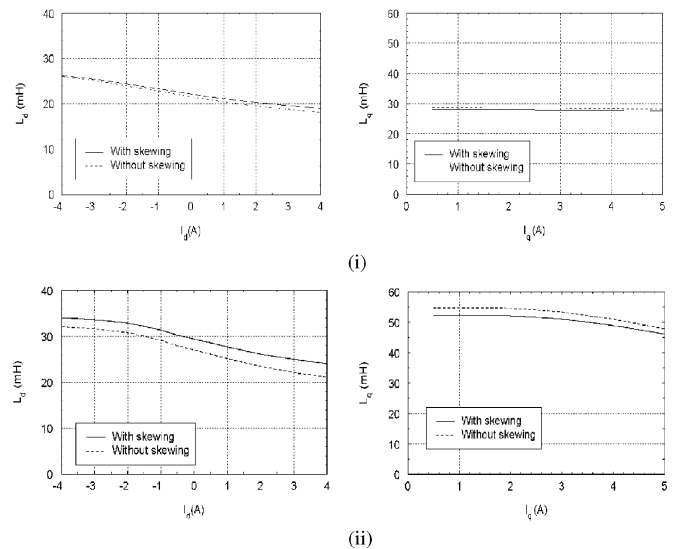


Fig. 3. Predicted  $d$ -axis and  $q$ -axis inductances with and without skew: (i) surface-mounted magnet rotor motor and (ii) interior-magnet rotor motor.

#### IV. CONCLUSION

A hybrid two-dimensional (2-D) finite-element/analytical technique has been proposed for predicting the  $d$ -axis and  $q$ -axis inductances of permanent magnet (PM) brushless ac machines with skew. Predicted  $d$ -axis and  $q$ -axis inductances have been compared with measurements in two machines having identical stators, which are skewed by one slot pitch, but different rotor topologies, and good agreement has been achieved.

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