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Analysis of Cogging Torque in Brushless Machines Having Nonuniformly Distributed Stator Slots and Stepped Rotor Magnets

Z. Q. Zhu, *Senior Member, IEEE*, S. Ruangsinchaiwanich, D. Ishak, and D. Howe

Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield S1 3JD, U.K.

A simple analytical technique is proposed for synthesizing the cogging torque waveform of a permanent magnet brushless machine from the cogging torque waveform that is associated with a single stator slot. The machine may have either uniformly or nonuniformly distributed stator slots and/or a skewed rotor, in which the skew is realized by circumferentially displacing the magnets of each pole. The technique is validated by finite element analysis and measurements.

***Index Terms*—Analytical modeling, brushless machines, cogging torque, finite element, permanent magnet.**

I. INTRODUCTION

NUMEROUS methods for reducing the cogging torque, which results from the interaction of the rotor permanent magnets and the stator slots, such as skewing the slots and/or magnets, shaping the magnets, employing auxiliary slots or teeth, optimizing the magnet pole-arc to pole-pitch ratio, using a fractional number of slots per pole, etc., have been proposed [1]–[3]. Analytical techniques for determining the cogging torque waveform remain useful and aid in the understanding of the fundamental cause of cogging [2].

In this paper, a simple analytical technique is proposed to synthesize the cogging torque waveform that results in a permanent magnet brushless machine from the cogging torque waveform that is associated with a single stator slot. It is used to analyze the cogging torque waveforms that result in 12-slot 10-pole machines having both uniformly and nonuniformly distributed stator slots, and to demonstrate the utility of incorporating skew simply by circumferentially displacing the two magnet segments of each pole (Fig. 1).

II. SYNTHESIS OF COGGING TORQUE

The cogging torque waveform that results with a single stator slot may be predicted either by finite element analysis or from an analytical model, and can be expressed by a Fourier series as

$$T_{sc} = \sum_{i=1,2,3,\ldots}^{\infty} T_{sci} \sin 2pi\theta \tag{1}$$

where T_{sc} is the cogging torque due to the single stator slot, T_{sci} is the amplitude of the ith harmonic, and $2p$ is the pole number. For a machine with N_s uniformly distributed stator slots, the resultant cogging torque can be derived by summing the contributions of each individual slot, with due account for the phase shift, i.e.,

$$T_{\text{cog}} = \sum_{n=1,2,3,\ldots}^{\infty} N_s T_{sci}\big|_{i=\frac{N_s}{C}n} \sin N_c n\theta \tag{2}$$

where $C = 2pN_s/N_c$ and N_c is the least common multiple between $2p$ and N_s.

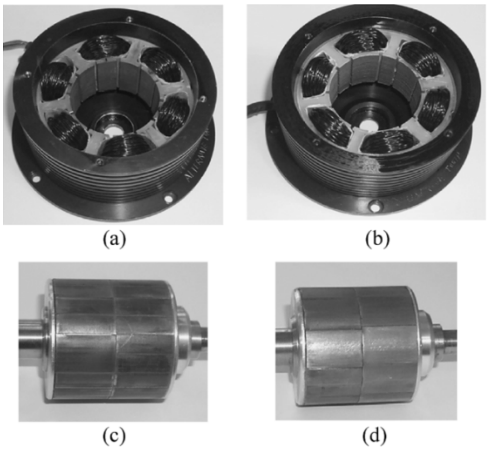

Fig. 1. Twelve-slot ten-pole permanent magnet brushless machines. (a) Stator with uniformly distributed slots. (b) Stator with nonuniformly distributed slots. (c) Rotor with two magnet segments/pole aligned. (d) Rotor with two magnet segments/pole displaced by 3° mech.

For a machine with nonuniformly distributed stator slots, the resultant cogging torque can be similarly obtained by summation. For example, for a machine in which alternate teeth are wound, the unwound teeth are narrower than the wound teeth, in order to maximize the flux linkage per coil [4], the resultant cogging torque can be obtained as

$$T_{\text{cog}} = \sum_{n=1,2,3,\ldots}^{\infty} 2N_s' T_{sci}\big|_{i=\frac{N_s'}{C'}n} \times \cos\left(N_c' n \frac{\Delta_s}{2}\right) \sin\left(N_c' n\theta + \frac{\Delta_s}{2}\right) \tag{3}$$

where $C' = 2p(N_s/2)/N_c' = pN_s/N_c'$, Δ_s is the angular displacement between two adjacent slots, and N_c' is the least common multiple between $2p$ and N_s', which is equal to $N_s/2$.

Similarly, for a machine having a skewed rotor, for which the circumferential displacement between the two magnet segments of each pole is Δ_m, the resultant cogging torque can be obtained as

$$T_{\text{cog}} = \sum_{n=1,2,3,\ldots}^{\infty} N_s T_{sci}\big|_{i=\frac{N_s}{C}n} \times \cos\left(N_c n \frac{\Delta_m}{2}\right) \sin\left(N_c n\theta + \frac{\Delta_m}{2}\right). \tag{4}$$

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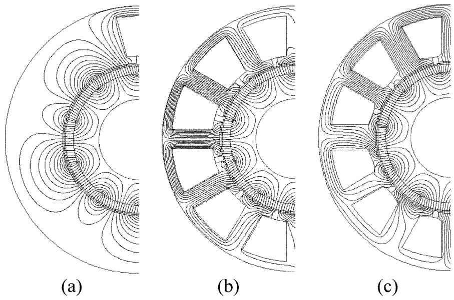


Fig. 2. Open-circuit field distributions for ten-pole machines. (a) One slot. (b) Twelve uniformly distributed slots. (c) Twelve nonuniformly distributed slots.

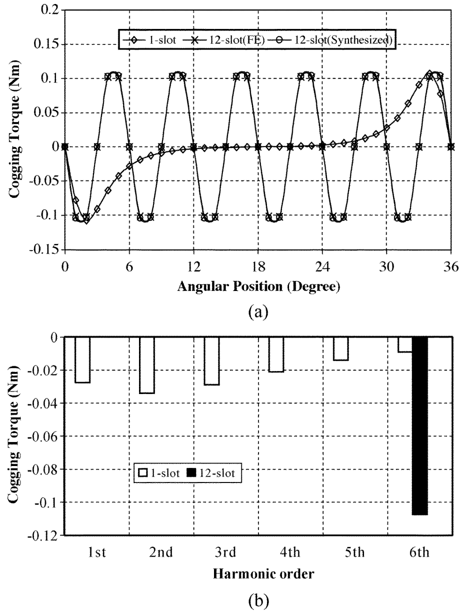


Fig. 3. Cogging torque of 1-slot and uniformly distributed 12-slot 10-pole machines. (a) Waveform. (b) Harmonic spectrum.

For machines having both nonuniformly distributed stator slots and similarly skewed rotor magnets, the same procedure can be applied to derive the analytical expression

$$T_{\text{cog}} = \sum_{n=1,2,3,\ldots}^{\infty} 2N_s' T_{sci}\big|_{i=\frac{N_s'}{C'}n} \cos\left(N_c' n \frac{\Delta_s}{2}\right) \times \cos\left(N_c n \frac{\Delta_m}{2}\right) \sin\left(N_c' n\theta + \frac{\Delta_s}{2} + \frac{\Delta_m}{2}\right). \quad (5)$$

Equations (1)–(5) show that not all the harmonics that exist in the cogging torque waveform due to a single slot are present in the resultant cogging torque, and that the most significant components can be identified analytically.

III. APPLICATION TO 12-SLOT 10-POLE MACHINES

The synthesis technique has been applied to the 12-slot 10-pole machines shown in Fig. 1. The outer stator diameter and axial length are 100 and 50 mm, respectively, while the airgap length and magnet thickness are 1 and 3 mm, respectively. The magnets are parallel magnetized NdFeB having a remanence of 1.2 T. Fig. 2 shows open-circuit field distributions for a 1-slot 10-pole motor and 12-slot 10-pole machines having uniformly and nonuniformly distributed slots, respectively.

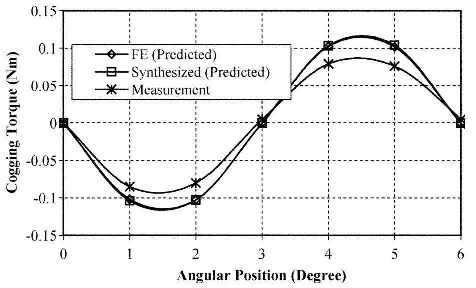


Fig. 4. Comparison of predicted and measured cogging torque waveforms for 12-slot 10-pole machine having uniformly distributed slots.

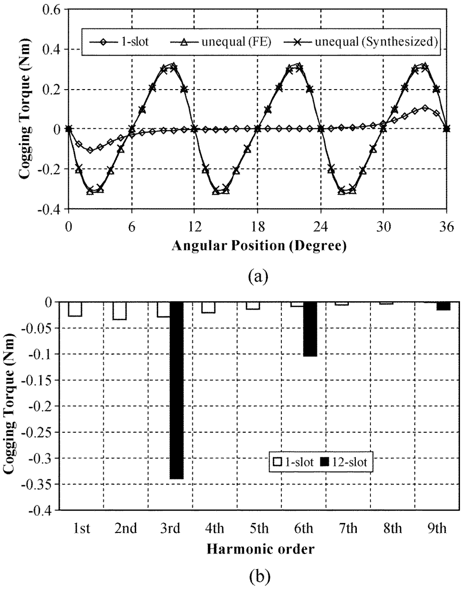


Fig. 5. Cogging torque for 1-slot and nonuniformly distributed 12-slot 10-pole machines. (a) Waveform. (b) Harmonic spectrum.

A. Uniformly Distributed Stator Slots

Fig. 3 compares the cogging torque for the machine with 12 uniformly distributed slots with that which results with only 1 slot. Since the least common multiple N_c of the 1-slot 10-pole motor is 10, the cogging torque periodicity is 36°mech, while for the 12-slot 10-pole motor, $N_c = 60$, $C = 2$, and the cogging torque periodicity is only 6°mech, i.e., its frequency is six times higher. Only the sixth harmonic and multiples thereof in the cogging waveform torque for the 1-slot motor contribute to the resultant cogging torque waveform of the 12-slot motor. Fig. 4 compares predicted and measured cogging torque waveforms.

B. Nonuniformly Distributed Stator Slots

For the machine having 12 nonuniformly distributed slots, the stator can be considered to comprise two groups of six uniformly distributed slots with 23°mech angular displacement between each group. Fig. 5 compares finite element calculated and analytically synthesized cogging torque waveforms and the harmonic spectra for the 1-slot and 12-slot machines, while measured and predicted cogging torque waveforms are compared in Fig. 6. Due to the nonuniform distribution of the slots, the periodicity of the cogging torque waveform is half that for the 12-slot motor with uniformly distributed slots. However, the amplitude of the cogging torque is increased significantly due to the fact that the tooth-tips of the wider teeth are approximately

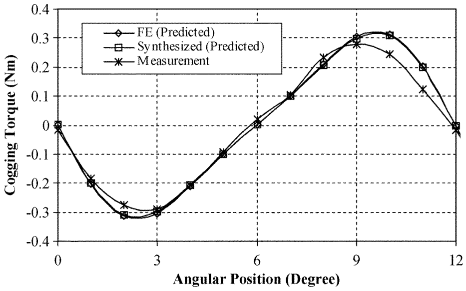
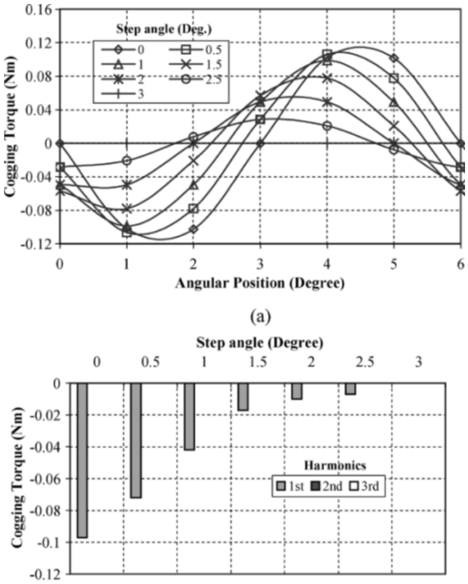


Fig. 6. Comparison of predicted and measured cogging torque waveforms for 12-slot 10-pole machine having nonuniformly distributed slots.



(a)

(b)

Fig. 7. Variation of cogging torque with magnet step angle for 12-slot 10-pole machine having uniformly distributed slots. (a) Waveform. (b) Amplitude of harmonics.

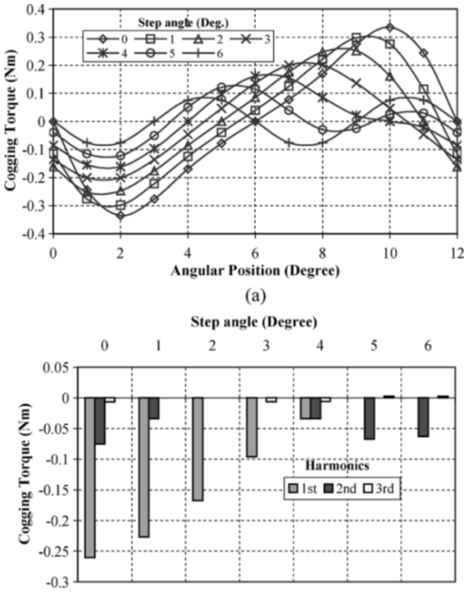
equal to the pole-pitch in order to maximize the flux linkage per coil [4].

C. Stepped Rotor Magnets

When the two magnet segments of each pole are displaced circumferentially, i.e., stepped, the cogging torque can be synthesized according to (4) and (5). Figs. 7 and 8 show the variation of the cogging torque waveform and the amplitude of the harmonics with the skew angle, while Fig. 9 shows the effect of displacing the magnet segments by 3°mech on the predicted and measured cogging torque waveforms. As will be evident, the optimum step angle of 3°mech essentially eliminates cogging in the machine with uniformly distributed slots, and a significant residual cogging torque remains in the machine with nonuniformly distributed slots even with the optimum step angle of 4°mech.

IV. CONCLUSION

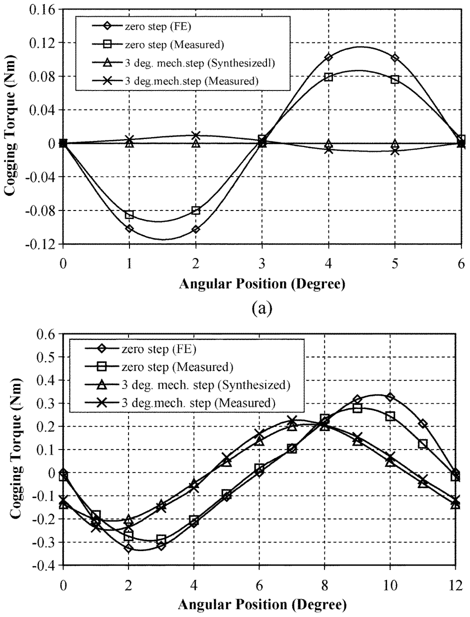
The cogging torque waveform that results in a permanent magnet brushless machine having either uniformly or nonuniformly distributed stator slots and/or stepped rotor magnets can be synthesized from the analysis of the cogging torque waveform associated with a single stator slot, as confirmed by finite element analysis and measurements.



(a)

(b)

Fig. 8. Variation of cogging torque with magnet step angle for 12-slot 10-pole machine having nonuniformly distributed slots. (a) Waveform. (b) Amplitude of harmonics.



(a)

(b)

Fig. 9. Comparison of predicted and measured cogging torque waveforms for 12-slot 10-pole motors with/without 3°mech step. (a) Uniformly distributed slots. (b) Nonuniformly distributed slots.

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