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A MACROSCOPIC TRANSPORT PROBLEM

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A MACROSCOPIC TRANSPORT PROBLEM

Abstract

An analytical approach is used to investigate the process of iron ore sintering which represents a problem in macroscopic transport and as such, the approach used may be of value to other workers concerned with modelling processes involving gas-solid and gas-liquid energy and mass transfer.

The paper discusses the formulation and simulation of the process equations. The results of a simulation exercise are compared with those obtained from tests on pilot plant to establish the validity of the model. The effects of ignition time, ignition temperature, size of mix, coke content, water content and ore density on heat wave propagation are then investigated.
**NOTATION**

- $a$: Specific surface area ($m^2/m^3$ (bed)). In the case of spherical particles in a packed bed $a = 6(1-\varepsilon)/d_p$.
- $C_{CO_2}$, $C_{N_2}$, $C_{O_2}$: Concentrations of $CO_2$, $N_2$, and $O_2$ in gas (mol/m$^3$).
- $C_{CO_2}^*$, $C_{N_2}^*$, $C_{O_2}^*$: Concentrations of $CO_2$, $N_2$, and $O_2$ in gas at $0^\circ C$ (mol/m$^3$).
- $C_p$, $C_p^*$: Specific heats of gases and solid particles (J/Kg·°C).
- $D$: Diffusion coefficient ($m^2/s$).
- $D_{ON}$: Diffusion coefficient of $O_2$ through $N_2$ ($m^2/s$).
- $d_p$: Average diameter of solid particles (m).
- $F_u$: Heat available for fusion (J).
- $G$: Mass velocity of gas (Kg/m$^2$s).
- $H_C$: Heat of combustion of coke (J/g·atom(c)).
- $H_V$: Latent heat of vaporization of water (J/Kg).
- $h$: Depth of bed (m).
- $h_P$: Convective heat transfer coefficient between solid particles and gas (J/m$^2$s·°C).
- $K$: Overall combustion rate constant (m/s).
- $K_c$: Chemical combustion rate constant (m/s).
- $K_g$: Thermal conductivity of the gas (J/m·s·°C).
- $K_m$: Mass transfer coefficient (m/s).
- $K_P$: Thermal conductivity of solid particles (J/m·s·°C).
- $n_c$: Number of coke particles per unit volume of the bed (1/m$^3$ (bed)).
- $N_u$: Nusselt number (-).
- $Pr$: Prandtl number (-).
- $R$: Universal gas constant (J/mol·°K).
- $R_C$: Overall reaction rate of coke in the bed (g·atom(C)/s·m$^3$(bed)).
- $Re$: Reynolds number (-).
- $r_c$: Radius of coke (m).
- $S_c$: Schmidt number (-).
- $Sh$: Sherwood number (-).
- $t$: Temperature of solid particles (°K).
- $T_F$: Fusion temperature (°K).
W  Humidity of the gas (Kg ./Kg (dry gas))
Δz  Width of zones into which the bed is divided during simulation (m)
z  Downward distance from the surface of the bed (m)
ε  Voidage (-)
θ  Time (s)
λ  Thermal conductivity of gas (J/m s°C)
μ  Viscosity of gas (Kg / ms)
ρ_p  Density of sinter materials (Kg /m^3)
ρ_g  Density of gas (Kg /m^3)
τ  Temperature of gas (°K)
Δφ  Update period for simulation (s)
A common method of producing sinter from iron ore involves the use of a Dwight-Lloyd sinter strand. The sinter strand may be regarded as a continuous moving grate which is loaded with a raw mixture of ore, coke and water to form a bed. Situated below the bed there is a series of windboxes which allows a large fan to suck air downwards through the bed. The material is ignited as it passes under an ignition hood at the beginning of the strand and a heat wave gradually passes down through the bed at a rate which causes burnthrough to occur at the output end of the strand. The effect of the heat wave is to drive off the volatiles and agglomerate the mixture to form sinter. For optimum working conditions both sinter quality and plant output must be maximised. The maximum plant output for any given suction across the bed occurs when the burnthrough point of the heat wave is at the end of the strand. Thus the influence of the process variables on sinter quality and the rate of heat wave propagation should be known so that satisfactory operating conditions may be chosen. The sinter bed may be regarded as a series of vertically divided sections each moving horizontally along the strand. The temperature history of one such section from ignition to burnthrough is representative of the whole bed and is obtained in practice by the use of pilot plant in the form of a sinter test-pot. A test-pot provides a realistic but limited facility for the practical analysis of the sintering process, since a change in a controlled variable simultaneously changes a number of fundamental process variables. It is therefore difficult to establish from test-pot data the significance of individual process variables. A digital simulation of a test-pot process is not, however, similarly limited as it enables the influence of individual process variables to be readily investigated.

Initially, the paper briefly outlines the physical relationships on which sintering is dependant. Obviously, it is impracticable to include all the relationships within an analytical model and so only those relationships
which in the authors' views are important are included in the model discussed here. The next part of the paper explains the technique used to solve the non-linear partial differential and algebraic equations describing the process. A comparison is then made between the results of the digital simulation and actual test-pot data to show that the simulation results are satisfactory.

The final part of the paper is devoted to an investigation of the effects of individual process variables on sinter quality and heat wave propagation.

**FUNDAMENTAL SINTER BED EQUATIONS**

**Mass and Energy Balance Equations**

Although the aim is to derive a set of analytical equations describing the sintering process, several empirical assumptions must be made where insufficient information exists for an analytical approach to be made. The sintering process may be studied by assuming a zonal structure, Fig. 1. The zones chosen are fairly arbitrary and in practice overlap. The approach adopted is to solve all equations simultaneously, permitting combustion to die out naturally as all the carbon particles react. The 'zones' thus develop from the equations rather than being imposed as a prerequisite of the model.

The overall combustion rate of coke particles is dependant upon both chemical and mass transfer because before combustion of coke can occur oxygen has to diffuse up to the coke particle surface. The overall combustion rate is given by:

\[
R_c = 4\pi r_c^2 n_c K C_{O_2} \\
\text{where } 4\pi r_c^2 = \text{surface area of coke particles} \\
n_c = \text{number of coke particles per unit volume} \\
\text{and } K = \frac{K_c K_m}{K_c + K_m}
\]  

\[ \text{........................ (1)} \]
\( K_m \) is the coefficient of mass transfer,

\( K_c \) is the chemical combustion rate constant.

Using basic chemical engineering principles, the following equations for the heating, cooling and combustion zones have been derived \( \{1, 2\} \):

**Mass velocity of gas through bed:**

\[
G = \varepsilon \rho g \left( \frac{\partial G}{\partial z} \right) \quad \cdots \cdots (2)
\]

The energy balance equation for the gas passing through the bed is:

\[
G \left( \frac{\partial (C_t)}{\partial z} \right) + \varepsilon \rho g \left( \frac{\partial (C_t)}{\partial \theta} \right) + \varepsilon h_p a(t-t) = 0 \quad \cdots \cdots (3)
\]

**bulk convection** accumulation in gas convective exchange to solid

An equivalent equation for the solid is:

\[(1-\varepsilon) \rho_p \left( \frac{\partial (C_t)}{\partial \theta} \right) + h_p a(t-t) + \frac{H_c}{R_c} = 0 \quad \cdots \cdots (4)
\]

accumulation in solid convection exchange heat from reactions in solid

The following equations represent the mass balance of \( O_2 \) in the gas phase:

\[
\frac{\partial (G_{O_2})}{\partial z} + \frac{\partial (G_{CO_2})}{\partial \theta} + \frac{\partial (G_{N_2})}{\partial \theta} + R_c = 0 \quad \cdots \cdots (5)
\]

**bulk convection** accumulation net change in gas \( O_2 \) by reaction

where

\[
C_{O_2} + C_{CO_2} + C_{N_2} = 44.6 \left( \frac{273}{r} \right) \quad \cdots \cdots (6)
\]

and

\[
\rho g = (32 C_{O_2}^* + 44 C_{CO_2}^* + 28 C_{N_2}^*) 10^{-3} \left( \frac{273}{r} \right) \quad \cdots \cdots (7)
\]

The assumptions made in formulating equations (1) to (7) are:

(1) Gas flow is one-dimensional (If the diameter of the particles in a stable granular bed is less than thirty times the bed diameter, piston flow can be assumed).

(2) Gas species are ideal.

(3) Eddy diffusion effects are small and can be ignored.

(4) System is adiabatic.
(5) Radiative effects within the gas and between the gas and solid are insignificant.

(6) The reaction of coke with oxygen takes place on the surface of the coke particle and not in the gas.

(7) The inside temperature of a solid particle is identical to its surface temperature.

Preliminary investigations show that convective heat transfer is the major factor promoting heat wave propagation and that for the gas the axial heat conduction and enthalpy of gas from the solid can be ignored.

Similarly, axial heat conduction and radiation from solid to solid may be left out of the solid energy balance equation.

Drying and Condensation

Although it is possible to formulate energy and mass balance equations for the drying zones, both falling and constant rate, it proved to be impractical to implement the equations and a simpler approach was used. The main reasons for the impracticality of the analytical approach were:

(1) No data could be found for the proportions of water dried during the constant rate period and the first falling rate period.

(2) The rate constant for drying at the front edge of the heat wave was of the same order of magnitude as the update period of the simulation, hence a detailed investigation would have necessitated the use of smaller time increments and hence slowed down the computation of the solution. The removal of water during sintering accounts for only about 10% of the available energy, hence a less accurate representation does not significantly affect the overall solution.

Representation of the drying phase was therefore implemented by assuming that all evaporation occurred at 100°C, the expression for evaporation being:

\[ \text{heat from gas} = (\text{water evaporated}) \times (\text{latent heat of vaporization}) \]

As the gas passes down the bed laden with moisture from the drying zone it cools until it reaches the dewpoint. At the dewpoint, the moisture
condenses and the bed temperature rises. Using this approach, the raw mix below the combustion and heating zones gradually rises in temperature until it is all at the dewpoint temperature. Using a least squares curve-fitting technique and graphical data given by Perry (4) the relationship between dewpoint temperature $T_{DEW}$ and moisture content was found to be approximately

$$T_{DEW} = 293.4 + 324.6 W - 594.1 W^2 + 292.1 W^3$$

(8)

The equation for the heat gained by the bed due to condensation is:

$$\text{heat to bed} = (\text{mass of water condensed}) \times (\text{latent heat of condensation})$$

**Fusion and Limestone Reduction**

The set of equations (2-7) excluded limestone reduction (limestone is contained in self-fluxing sinters) and fusion, both of which need to be included in a realistic model of the sintering process.

Purely analytical modelling of the fusion process is extremely complex, particularly for self-fluxing sinter. Work reported by Nyquist (5) and Price et al (6) on practical investigations illustrates the difficulty of predicting mineralogy changes due to fusion. To avoid this problem the assumption is made that when a bed temperature is reached corresponding to the maximum temperature obtained from test rig results, the excess energy is used in fusion. This assumption is founded on the fact that the maximum temperature achieved in practice is reasonably independent of coke content for high quality self-fluxing mixes containing hematite and magnetite. The majority of sintering mixes are of this type. In the model the heat is stored until cooling occurs behind the combustion zone. The results of a simulation exercise in which the above assumptions were made, agree with the results of tests on pilot plant.

Limestone reduction accounts for only about 5% of the available energy and therefore has little effect on the temperature profile. At present, limestone reduction is built into the model as a simple endothermic reaction.
The expressions for most of the important terms used in the previous 
equations, e.g. chemical combustion rate of coke, specific heats of solids 
and gas, are given in Appendix I.

METHOD OF SIMULATION

The energy balance equation for the gas phase passing through the bed 
and the equivalent equation for the solid are:

\[ G \left( \frac{\partial (C_g \cdot \tau)}{\partial z} \right) + \varepsilon_p \frac{\partial (C_g \cdot \tau)}{\partial \theta} + h_p a (\tau - t) = 0 \] .......... (9)

and

\[ (1 - \varepsilon) \frac{\partial (C_t \cdot \tau)}{\partial \theta} + h_p a (t - \tau) + H_c R_c = 0 \] .......... (10)

Now for the case of convective heat transfer only, without limestone, 
drying, combustion and fusion. The method of solution is to assume that the 
test-pot may be vertically divided into \( n \) zones. The thickness of the \( \text{th} \) 
zone is \( \Delta z_i \) and the total bed thickness is therefore \( \frac{\Delta z}{1} \).

The above equations may be simplified by substituting for \( C_g \) and \( C_p \) 
from equations given in Appendix I.

\[ \text{. \ . } G \cdot B_g \cdot \partial \tau/\partial z + \varepsilon_p B_g \cdot \partial \tau/\partial \theta + h_p a (\tau - t) = 0 \] .......... (11)

and

\[ (1 - \varepsilon) \frac{B_g \cdot \partial \tau}{\partial \theta} + h p a (t - \tau) = 0 \] .......... (12)

where

\[ B_g = 881.0 + 0.62 \tau - 2.37 \cdot 10^{-4} \cdot \tau^2 \] .......... (13)

and

\[ B_g = 753.0 + 0.26 \tau \] .......... (14)

Consider a narrow band of hot gas passing through the bed. Due to the 
very large difference in density of the solid and gas (approx. 3000 :1), 
the change in temperature of the solid during the period which the gas takes 
to pass through it is negligible. The gas temperature, however, changes quite 
significantly. The temperature of a band of gas of thickness \( \delta z \), where \( \delta z \) 
is very small, may be repeatedly calculated as the band passes through the bed.
During transit time the solid temperatures \( t_1, \ldots, t_n \) are assumed constant.

The transit time through the \( i^{th} \) zone of thickness \( \Delta z_i \) is given by:

\[
\Delta t_i = \frac{G \Delta z_i}{\varepsilon \rho} \tag{15}
\]

The gas temperature leaving the \( i^{th} \) zone and entering the \( i+1^{th} \) zone is given by the equation:

\[
\int_{t_{i-1}}^{t_i} \frac{\varepsilon \rho}{h_p(a(t_i-t))} \, dt = \int_0^{\Delta t_i} \, d\theta \tag{16}
\]

Thus by considering the \( n \) zones in turn, the temperature history of a band of gas passing through the bed may be calculated, from a knowledge of the solid temperature at each level and the temperature of the gas as it enters the bed.

It may be assumed that the values \( t_1, \ldots, t_n \) represent the instantaneous gas temperature profile because the total transit time for a band of gas is small compared with the total simulation period.

The solid temperatures \( t_1, \ldots, t_n \) may now be updated after a period \( \Delta \theta \) which greatly exceeds the gas transit period for the bed, using the heat balance equation:

\[
t_{i, t_{i+1}}^{\Delta \theta} = t_{i, t_i} + \frac{G \rho(1-\varepsilon)}{B_p^2} \frac{(t_{i-1} - t_i) \cdot \Delta \theta}{\Delta z_i} \tag{17}
\]

The repeated use of the above procedure thus enables gas and solid temperatures to be predicted from the initial condition data on the temperature distribution of the solid material forming the bed and the applied gas temperature.

This method was expanded to include fusion, limestone, combustion, drying, the growth of particles during fusion and oxygen concentration. The difference equations used when simulating the test-pot are given in Appendix II.
The accuracy of the digital simulation was verified by comparing test-rig results with simulation results.

The input information for the simulation was mix size, mix density, voidage, coke content, coke size, water content, limestone content, ignition time, ignition temperature and the history of the gas velocity from ignition to burnthrough.

Fig. 2 compares the heat wave leading edge profiles generated by the model and the actual test-pot data. Fig. 2 shows the profiles at 0.076, 0.129 and 0.182 m for a hematite mix with 4.5% coke. The profiles from the model match quite well with the test-pot profiles. Any discrepancies are probably caused by the omission of minor terms from the simulation.

**EFFECT OF PROCESS VARIABLES ON HEAT WAVE PROPAGATION AND THE QUALITY OF SINTER PRODUCED**

The model provides a useful means for investigating the effects of manipulating process variables. The manipulation of variables such as air-flow and voidage will have two basic effects; one is to change the heat wave propagation velocity, the other is to change the width of the fusion zone.

The velocity of the heat wave through a sinter bed should be as high as possible if a sinter plant is to produce a maximum output, however if the fusion zone does not produce the right degree of agglomeration there may be a large quantity of rejected sinter and hence a reduced productivity. Too wide a fusion zone will cause large lumps of sinter to form, which will tend to block the passage of air through the bed and hinder the advancement of the heat wave. Too narrow a fusion zone results in a weak fretted structure and a high return fines ratio. The desirable agglomeration conditions probably differ from ore-type to ore-type and also depend upon the lime/silica ratio, however the following simulation results give an insight into the effect of manipulating individual process variables. The authors simulated a 'typical' 0.25m bed. The input data used for calculating the results shown in figures 3-6 are given in Table 1.
Table 1: Input data used for calculation of results shown in figures 3-6

Temperature of Ignition Gases

Fig. 3 shows the temperature distribution within the bed after 130 seconds, for three different ignition temperatures.

The main points arising from these results are:

(i) Propagation of the heat wave is only slightly affected by the ignition temperature.

(ii) The bed would not ignite at 980°C whilst at temperatures above 1030°C sintering was successfully commenced.

Ignition Period

The effect of longer ignition periods is to slightly widen the fusion zone, Fig. 4. Also the point at which ignition commences is dependent on the ignition time.

Density of Mix, Voidage and Gas Velocity

Fig. 5 shows the effect of changes in voidage, gas velocity and density on burnthrough time. Changes in gas velocity have a very marked effect on heat...
wave propagation velocity, whilst voidage and density changes make less
though still significant changes to the burnthrough time. Changes to these
variables appear to have little effect on the width of the fusion zone.

**Coke and Water Content of Mix**

Variations in coke and water content whilst not altering the burnthrough
time have a marked effect on the width of the fusion zone Fig. 6. When
simulating low coke contents (<3.5%) a longer ignition period proved
necessary.

**Diameter of Mix**

Variations in the diameter of the raw mix particles do not affect either
the width of the combustion zone or the burnthrough time. However, if the mix
is larger than 5mm. ignition will not occur unless the ignition time is increased.

**CONCLUSIONS**

A model of a sinter bed, based mainly on fundamental relationships, has been
formulated and successfully simulated. The model has been validated by comparing
heat wave profiles with those from a test-pot. Some initial investigations into
the effect of manipulating process variables have been carried out. It was
found that variations in burnthrough time (heat wave propagation velocity)
during sinter plant operation are mainly caused by changes in gas velocity
through the bed. Voidage and density of ore mix are contributary factors.
The quality of sinter output will be affected by the proportion of coke in
the mix and also by any large deviations in the water content. Ignition time
and temperature do not appear to be critical once the minimum time and
temperature for ignition have been exceeded.

The methods used are applicable to other problems in macroscopic transport
and as such it is hoped that the report will find application outside "sintering".

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REFERENCES


APPENDIX I

Formulas that may be used to obtain the important coefficients used in
the digital simulation of a sinter test-pot.

Heat and Mass Transfer

The principle mechanism of heat transfer in a sinter bed is by convection.
Of the formulas available, (1,8,9,10) for the convective heat transfer coe-
efficient, the following was chosen.

\[ \varepsilon(N_u) = 2.0 + 0.71 \left( \frac{P_r}{R_e} \right)^{1/3} \left( R_e \right)^{1/2} \]

where

\[ \frac{N_u}{P} = \frac{h d}{\lambda} \quad P_r = \frac{C}{\mu} \quad R_e = \frac{d}{\mu} \frac{G}{h} \]

A similar formula also exists for the coefficient of mass transfer \( (K_m) \)
between a gas and particles, in a packed bed.

\[ \varepsilon(S_{h*}) = 2.0 + 0.71 \left( \frac{S_c}{R_e} \right)^{1/3} \left( R_e \right)^{1/2} \]

where

\[ S_{h*} = \frac{K_m \cdot d}{D} \quad S_c = \frac{\mu}{\rho} \frac{D}{g} \]

Specific Heat

The specific heat of 'sinter type materials' is given by

\[ C_p = 753 + 0.13t \]

A similar relationship exists for sintering gases

\[ C_g = 881 + 0.31t - 7.98 \times 10^{-5} t^2 \]

Changes in the ratio of \( O_2 \) to \( C_0 \) have a negligible effect.

Viscosity

The viscosity of sintering gases is given by:

\[ \mu = \mu_0 \left( \frac{t}{273} \right)^{3/2} \sqrt{\frac{386.1}{113+\tau}} \]

where \( \mu_0 = 1.72 \times 10^{-5} \) Kg/ms.

Diffusion Coefficient of \( O_2 \) through \( N_2 \)

The diffusion coefficient is:

\[ D_{ON} = 1.8 \times 10^{-5} \cdot \left( \frac{t}{273} \right)^n \text{m}^3/\text{s} \]
where \( n \) lies between 1.5 and 2.0

Parker et al (11) suggest a value of 1.5

**Coke Combustion**

Parker et al (11) have shown that \( K_c \), the chemical combustion rate, is given by:

\[
K_c = 6.52 \times 10^5 \cdot e^{-18500/RT} \cdot \sqrt{\frac{\tau}{c}}
\]

From equation (b) the coefficient of mass transfer is:

\[
\text{Mass transfer } K_m = \frac{D_{ON}}{e} \cdot \frac{d}{dp} \left[ 2.0 + 0.71 \left( \frac{S_c}{S_c} \right)^{1/3} \left( \frac{R_e}{R_e} \right)^{1/2} \right]
\]

Experimental analyses of sinter bed exhaust have shown that quantities of carbon monoxide are present in the windboxes (9,12,13). The formulae presented above are based on the assumption of complete conversion of coke to carbon dioxide. R. Schluter et al (13) introduce a factor \( \phi \) to modify the mass transfer equation, i.e.

\[
K_m = \phi \cdot \frac{D_{ON}}{e} \cdot \frac{d}{dp} \left[ 2.0 + 0.71 \left( \frac{S_c}{S_c} \right)^{1/3} \left( \frac{R_e}{R_e} \right)^{1/2} \right]
\]

Where \( \phi \) is a factor equal to unity for carbon conversion to CO\(_2\) and two for conversion to CO. The CO\(_2\): CO ratio in the windbox gas is approximately 4:1 where \( \phi = 1.11 \).

It is possible that a layer of ash could surround the particle and hinder combustion but S. Yagi et al (14) suggest that turbulence in the sinter bed would prevent such a layer forming.
A cross-section of a partly sintered test-pot yields, from the top to the bottom, the following layer structure. (Fig.1)

(a) A cooling layer. In this layer the sintered material is convectively cooled by the inlet air.

(b) A layer of solidification and agglomeration. The combustion of coke lower in the bed causes a certain amount of fusion. In this layer the combustion has ceased and the liquid iron oxides solidify to form sinter.

(c) A layer in which fusion occurs. The bed is at $T_F$, the fusion temperature. Any remaining heat from uncombusted coke is used for fusion of the iron ore.

(d) A layer of convective heating and coke combustion. The temperature range in this layer is $1000^\circ K$ to $T_F$. $1000^\circ K$ is the point at which coke combustion commences.

(e) A convective heating layer. The temperature range of this layer is $373^\circ K$ - $1000^\circ K$.

(f) Limestone reduction layer. This narrow layer occurs at approximately $870^\circ K$. The assumption made by the authors is that the bed temperature remains constant ($870^\circ K$) within this layer, all available heat being used for the endothermic reduction of limestone.

(g) A layer of drying. This layer has a temperature of $373^\circ K$ (boiling point of water) and all available heat is used for vaporisation of water.

(h) The bottom of the bed is a layer of condensative and convective heating. The technique outlined in the paper for convective heating, may be expanded to simulate sintering. The bed is divided into a number of vertical zones (typical number 200 zones/m of bed) of width $\Delta z$.

The gas transit time through each zone is

$$\Delta \theta_i = \frac{\epsilon \cdot \rho \cdot \Delta z_i}{G}$$
The gas temperature is given by the same equation for all parts of the bed:

\[
\frac{\Delta \theta_i}{\Delta t} = \int_0^\theta \frac{g B}{h_p a (t_i - \tau)} \, d\theta
\]

The bed temperature for layers (a) and (e) is:

\[
t_{i, \theta + \Delta \theta} = t_{i, \theta} + \frac{g B (t_{i-1} - t_i) \Delta \phi}{B_s (1 - \varepsilon_p) \Delta \phi_i}
\]

The bed temperature is \( T_F \) in layer (b). The heat available for fusion \( F_u \) is given by:

\[
F_{u_{i, \theta + \Delta \phi}} = F_{u_{i, \theta}} + g B (t_{i-1} - t_i) \Delta \phi
\]

Similarly for layer (c) the bed temperature is \( T_F \) and \( F_u \) is given by:

\[
F_{u_{i, \theta + \Delta \phi}} = F_{u_{i, \theta}} + g B (t_{i-1} - t_i) \Delta \phi + \Delta \phi_i \frac{R_c c_i}{c}
\]

In layer (d) the bed temperature is given by:

\[
t_{i, \theta + \Delta \phi} = t_{i, \theta} + \frac{g B (t_{i-1} - t_i) \Delta \phi}{B_s (1 - \varepsilon_p) \Delta \phi_i} + \frac{(1 - \varepsilon_p) \rho_i \Delta z_i}{B_s (1 - \varepsilon_p) \Delta \phi_i}
\]

Layer (f), limestone reduction. The total heat needed for limestone reduction \( L_R \) is given by the product of the mass of limestone in a zone and the heat needed to reduce a unit mass of limestone. The following equation may be used to simulate limestone reduction.

\[
L_{R_{i, \theta + \Delta \phi}} = L_{R_{i, \theta}} - g B (t_{i-1} - t_i) \Delta \phi
\]

Layer (g). The rate of removal of water is given by

\[
X_{w_{i, \theta + \Delta \phi}} = X_{w_{i, \theta}} - g B (t_{i-1} - t_i) \Delta \phi / H_v
\]

where \( H_v \) is the latent heat of vaporization of water, \( X_w \) is the mass of water.

Layer (h). The bed temperature is given by

\[
t_{i, \theta + \Delta \phi} = t_{i, \theta} + \frac{B_s (1 - \varepsilon_p) \rho_i \Delta z_i}{B_s (1 - \varepsilon_p) \Delta \phi_i} + \frac{B_s (1 - \varepsilon_p) \rho \Delta \theta_i \Delta z_i}{B_s (1 - \varepsilon_p) \Delta \phi_i}
\]
When the bed reaches the dew point no heat is supplied by the condensation of water vapour. \( w_{g_{i'}} \) is the mass of water condensed from the gas during gas transit time \( \Delta \theta_{i'} \). The oxygen content at any point in the bed is:

\[
C_{O_2, i} = C_{O_2, i-1} - R_i \Delta \theta_{i'} \cdot \frac{\tau_i}{273.3}
\]

\( \tau_i/273.3 \) is included so that \( C_{O_2} \) is kept at N.T.P. The authors have assumed that there is an increase in the diameter of the input mix caused by the fusion process. This may be justified by the observation, that the gradient of the heating edge is steeper than the gradient of the cooling edge of the heat wave temperature profile. This formula has been adopted for the growth of particles during fusion.

\[
d_{F_i} = d_{i} \cdot (1 + b \cdot F_{u(max)}^i)
\]

where

- \( d_{i} \) is diameter of raw mix particles
- \( d_{F} \) is final diameter of mix
- \( F_{u(max)} \) is maximum heat available for fusion at that point in the bed
- \( b \) is a constant.
Fig. 1: SINTER TEST-POT AND ZONAL STRUCTURE
Fig. 4: EFFECT OF IGNITION PERIOD ON HEAT WAVE PROFILE
Fig. 5: EFFECT OF DENSITY, VOIDAGE AND GAS VELOCITY ON BURNTHROUGH
Fig. 6: EFFECT OF COKE AND WATER CONTENT ON WIDTH OF FUSION ZONE