Do maximum waiting times guarantees change clinical priorities for elective treatment? Evidence from Scotland.

Silviya Nikolova\textsuperscript{a,1,}\textsuperscript{*}, Arthur Sinko\textsuperscript{b,1}, Matt Sutton\textsuperscript{c,1}


\textsuperscript{b}Economics, School of Social Sciences, Arthur Lewis Bld., Oxford Rd., University of Manchester, Manchester, M13 9PL U.K.

\textsuperscript{c}Manchester Centre for Health Economics, Institute of Population Health, Jean McFarlane Bld., Oxford Rd., University of Manchester, Manchester, M13 9PL U.K.

Abstract

The level and distribution of patient waiting times for elective treatment is a major concern in publicly funded health care systems. Strict targets, which have specified maximum waiting times, have been introduced in the NHS over the last decade and have been criticised for distorting existing clinical priorities in scheduling hospital treatment. We demonstrate the usefulness of Conditional Density Estimation (CDE) in the evaluation of the reform using data for Scotland for 2002 and 2007. We develop a modified goodness of fit test to discriminate between models with different numbers of bins. We document a change in prioritisation between different patient groups with longer waiting patients benefiting at the expense of those who previously waited less. Our results contribute to understanding the response of publicly funded health systems to enforced targets for maximum waiting times.

Keywords: health care, waiting times, conditional density estimation

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\textsuperscript{*}Corresponding author, phone: (+44)(0)1133430578

Email addresses: s.k.nikolova@leeds.ac.uk (Silviya Nikolova), arthur.sinko@manchester.ac.uk (Arthur Sinko), matt.sutton@manchester.ac.uk (Matt Sutton)
1. Introduction

Waiting times are of public concern in state healthcare systems because they are a key determinant of satisfaction with public services (Sanmartin et al., 2007; Cutler, 2002), a perceived indicator of public sector inefficiency (Cullis and Jones, 1983, 1985; Oliver, 2005; Smith, 2002), and a source of discomfort and anxiety (Lindsay and Feigenbaum, 1984; Propper, 1995; Siciliani and Hurst, 2005). It is possible also, though the evidence is very limited, that delays in treatment may have negative health consequences (Siciliani and Gravelle, 2008; Appleby et al., 2003; Noseworthy et al., 2005; Garbuz et al., 2006; Escobar et al., 2009; Oudhoff et al., 2007; Nikolova et al., 2014). Also of concern are variations in waiting times across geographical areas and personal characteristics since such variations may represent a source of inequity (Dimakou et al., 2009; Askildsen et al., 2011). In a number of OECD countries, individuals with higher socioeconomic status (as measured by income or educational attainment) tend to wait less for publicly funded hospital care than those with lower socioeconomic status (see Siciliani and Verzulli, 2009; Cooper et al., 2009; Laudicella et al., 2012) for England, Johar et al. (2010); Sharma et al. (2013) for Australia, Monstad et al. (2014); Carlsen and Kaarboe (2010) for Norway.)

The setting for this paper is Scotland where high-profile, political guarantees on waiting times for elective hospital admissions were introduced in 2003. We compare the structure of waiting lists for elective surgery before and after the reform. We find that prioritisation between different patient groups changed with longer waiting patients benefiting at the expense of those who previously waited less. Our results contribute to understanding the response of publicly funded health systems to enforced targets for maximum waiting times.

1.1. Institutional Background

Long waiting times for NHS treatment were a significant source of public and policy concern across the United Kingdom throughout the 1990s. Prior to devolution in 1999, targets for
elective wait were set at 12 months for Scotland by Patients’ Charter (1995). However, this Charter was not rigorously enforced.

The first attempt in earnest by Scottish Executive (the new devolved administration responsible for health policy) to reduce waiting times was announced in 2000 (Scottish Executive, 2000). The maximum waiting time for elective patients was to be reduced from 12 months to nine months in December 2003. In addition, patients were to wait no more than 12 weeks for angiography or 24 weeks for revascularisation by end of 2002. Finally, the target of two months from urgent referral to treatment for all cancers (and one month for breast cancer) was set, to be achieved by the end of 2005.

A more ambitious target of six months for 2005 was announced in a 2002 press release (Audit Scotland, 2006). This press release also announced targets of eight weeks for angiography or 18 weeks for revascularisation by end of 2004. However, the strongest signal that health policy in Scotland was changing was when the White Paper issued in February 2003 offered patients guarantee that “waiting times targets will be met ...[and] monitored”. It also emphasised the way waiting lists were managed expressing concern that clinical activity response to waiting time pressure has “not resulted in sustained service improvement and ... sometimes distorted clinical priorities”. A further White Paper in 2004 pledged to reduce waiting times to 18 weeks for inpatients by 2007 (Scottish Executive, 2004). The targets for cardiac surgery was shortened to 16 weeks, and new targets were set for cataract surgery of 18 weeks from referral to treatment and nine weeks for eight key diagnostic tests, to be achieved by the end of 2007.

Hospitals in Scotland did not incur any economic penalty if waiting times were violated. However, the regional health boards were monitored on a monthly basis on their complete achievement of the maximum waiting times targets. Individual “breaches” of the waiting times targets had to be reported to the Scottish Executive and were rigorously investigated. This monitoring regime was similar in approach to the “targets and terror” regime that had been adopted in England some years earlier (Propper et al., 2008), and the dissolution of one regional
health board in 2006 was credited to its poor performances on waiting times and finances.

Elective patients in Scotland are those that are pre-booked for treatment. Thus elective waiting times reflect the time that elapses between the hospital specialist’s decision that a patient needs treatment to the date at which this treatment episode begins. This is only a part, and sometimes less than a majority, of the total delay between when a patient initially seeks and receives treatment. The Scottish NHS operates a gatekeeping system under which, for elective treatments, patients must first seek the advice of a General Practitioner (GP) (with a trivial wait), second receive a referral from the GP to a hospital specialist (often a more substantial wait, that may involve waiting for diagnostic test results), and third a decision by the specialist that hospital treatment is necessary (which may also involve waiting for diagnostic test results). Direct access by patients to hospital specialists is only possible for emergency care through hospital Accident and Emergency departments.

Patients at risk of breaching the targets were diverted to a national waiting times centre, a dedicated hospital that the NHS had bought from the private sector. It has been estimated that about GBP 116 million was spent on tackling waiting times in 2004/05. Approximately 40% (GBP 45.7 million) of this was spent on the national waiting time centre (Scottish Parliament, 2006). This additional expenditure on reducing waiting times was made at a time of substantial growth in the general resources spent on the hospital sector in Scotland. Annual growth rates in expenditure in the hospital sector in Scotland were 6.0% in 2000/01, 7.4% in 2001/02, 9.6% in 2002/03, 7.0% in 2003/04, 11.1% in 2004/05, 7.2% in 2005/06 and 4.8% in 2006/07.

1.2. Patient prioritisation

There has been widespread concern that the policy of waiting time guarantees would result in fraudulent statistics and distortion of clinical priorities. The National Audit Office (2001) reported that 20% of consultants surveyed in three specialties claimed that they changed the ordering of patients for treatment in order to meet the 18-month target in England. Given the similarity of reforms in England and Scotland, the behaviour of consultants is likely to be
similar. Siciliani and Hurst (2005) suggested that maximum waiting time guarantees in theory may be effective in reducing long waiting times, but might distort the incentives for hospitals: “they are not very effective in reducing mean or median waiting times, if the provider simply gives higher priority to less severe patients (who have waited longest), as they approach the maximum” (p.212). Appleby et al. (2003) conducted “before-and-after” comparison of waiting times distributions for English trauma and orthopaedic patients to evaluate the implications of the reform on patient prioritisation. They calculated that the number of admissions around the 15-month target at the time increased by 2.2% of all orthopaedic admissions in the post-reform period. While they could not unambiguously establish whether additional admissions had lead to delayed treatment for other patients, there was no evidence that very short wait patients suffered. Askildsen et al. (2011) compared actual waiting times to the recommended maximum waiting times in Norway. They found that the reduction in waiting times favoured patients who had longer waits.

Evaluating the impact of government targets for waiting times on patterns of average waits for different patient groups is related to the broader literature of patient ordering for treatment in health care. One principle for decision-making is the “rule of rescue” (Härdor, 1991). This implies that patients with most serious conditions are treated first. Thus, severity of a patient’s illness establishes priority for health care treatment. Cullis et al. (2000) argue that one of the criteria for determining waiting times should be the severity of the condition. This point of view concurs with NICE Citizens Council view on clinical need (NICE Citizens Council, 2002). Using a Cox proportional hazard model, Arnesen et al. (2001) showed that perceived or verified severity of patient health condition is the strongest predictor of a physician’s decision regarding wait for inpatient treatment. They also find that age is not a significant predictor although a tendency to longer waiting times for patients age 70 or older was present. This is consistent with the public’s preference to assign higher utility of health to younger, rather than older, patients (Busschbach et al., 1994; Cropper et al., 1994).

Despite the fact that the reform towards reduction in waiting times was carried out over the
course of the previous decade, it is unclear whether patient prioritisation changed. This paper examines whether the ordering for elective hospital treatment changed after the introduction of maximum waiting time guarantees. We compare waiting times distribution before the implementation of the political guarantees (2002) with distribution after their implementation (2007). Characteristics of patient’s health (age, disease severity, disease type) influence doctors’ perception of urgency for treatment (Cullis et al., 2000). For example, an increase in disease severity might not reduce the waiting times for elderly patients older than 85 years who already have several health problems, but speed up treatment for younger adults. The conditional density estimation (CDE) allows for such variations in covariate effects. In addition, its flexible specification of conditional probability functions and, hence, conditional expectations of the outcome of interest avoids restrictive assumptions about error distribution and functional form.

1.3. Methods overview

The purpose of this subsection is twofold; namely the comparison of the different methods used in prioritisation policy analysis and review of the results in related literature. It compares different methods that have been applied to the analysis of the prioritisation reform and contrasts the ability of these methods to incorporate skewness and point masses in the data. The subsection also reviews the contributions of Donald et al. (2001) and Gilleskie and Mroz (2004) to analysing outcomes characterised by skewed, multi-peaked distributions.

The ultimate goal of waiting times modelling has been to recover a functional relationship between waiting time \( Y \) and relevant covariates \( X \) and construct correct statistical inference. Jamulevicuï te et al. (2013) studied the change in waiting times in Scotland and Norway using ordinary least squares (OLS) of the logarithm of waiting time on a set of covariates with matching to risk adjust the groups of individuals before and after the reform. Their results show that the aggressive waiting time targets in Scotland contributed to shorter waiting times for patients in low-priority disease categories while leaving patients in the high-priority groups unaffected. Assuming the model specification is correct, logarithmic transformation of the
dependent variable can overcome skewness in some cases. However this approach is likely to face a new set of problems if the effect of interest is the level of the outcome variable. Moreover, when the error term \( u \) is heteroscedastic in covariates \( X \), the retransformation procedure to consistently recover \( \hat{Y} | X \) becomes significantly more complicated (Manning, 1998; Mullahy, 1998).

Propper et al. (2010) adopted a difference-in-difference approach. They considered only a limited range of possible distributional consequences of the waiting times targets. The authors compared various points in the waiting times distributions of England and Scotland. They found a small increase in waiting times at the lower end of the distribution. They also addressed possible patient reprioritisation concerns by examining urgent cases and complication rates. However, their analysis did not satisfy the difference-in-differences assumption of no difference in trends prior to the reform and the findings were therefore inconclusive.

One approach that deals with skewed, multi-peaked distributions is based on survival or failure time models. Dimakou et al. (2009) used the Kaplan-Meier estimator to study the distributional changes in waiting times in the English NHS. They showed that the introduction of waiting time targets produced shifting spikes in the hazard rate just below the waiting time limit. They also showed that there was wide variation in waiting times distribution by hospital, specialty, and procedure. Using parametric Proportional Hazard (PH) and Accelerated Failure Time (AFT) models they evaluated covariate effects on the respective dependent variable\(^1\) and found no statistically significant difference in characteristics such as age and sex.

An alternative approach to recover conditional expectations is to estimate a conditional probability density function itself. Efron (1988) proposed approximating unconditional distributions by a sequence of hazard rates. It was extended to conditional cases by Donald, Green and Paarsch (2001) and Gilleskie and Mroz (2004). The former approximates

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\(^1\)For the AFT models, the dependent variable is waiting time until admission; for the PH model it is the hazard rate.
conditional density as a continuous function with structural shifts to allow for varying
dependence of the covariates, while the latter uses sequences of logit hazard rates to reconstruct
a discrete approximation of the density function. Both approaches rely on pre-defined data
partitioning. Gilleskie and Mroz call their approach conditional density estimation (CDE).
The CDE uses flexible functional forms when defining sequences of conditional probabilities.
This means that we have flexible representations of the conditional density functions, and
consequently flexible representations of the expected value of the outcome conditional on
covariates. This makes the method appropriate for modeling waiting time distributions. Since
it estimates the conditional distribution function directly, it avoids the well-known issues of
transformation and subsequent retransformation of a dependent variable with strictly positive
and possibly multi-peaked distribution. The model framework of Gilleskie and Mroz relies on
the assumption that the number of observations per bin in a data partition is fixed across bins.
The structure of waiting times data requires a generalisation as the number of observations
varies from bin to bin. For a detailed explanation of the method and its modification please see
Appendix A.

2. Data

2.1. Scottish Morbidity Record 01 (SMR01)

We use the Scottish Morbidity Record 01 (SMR01) data set. It records detailed information
on all admissions to acute hospitals including patient characteristics such as waiting time, age,
number of co-morbidity conditions, and disease type. This is information on the distribution
of waiting times only for patients “admitted for treatment from the waiting list”. Patients who
are still waiting for treatment are not included. This information is collected by census data at
the end of every month. The former measures the full duration of waiting for patients who were
treated and contains rich data on their personal characteristics. The latter dataset provides an
incomplete measure of the realised waiting time and has only monthly frequency. We leave the
combined analysis of the two distributions for future research.

We construct a disease severity index (DSI), which is the sum of the Charlson index and number of co-morbidities. The latter accounts for the presence of medical conditions other than the primary diagnosis. We do not use the Charlson index by itself since there is insufficient variation in this index in some disease categories. Our covariates include all powers up to the fourth degree and cross-products between age (in years) and the DSI.

We compare the waiting times distributions before the implementation of the political waiting time guarantees (2002) with distribution after its implementation (2007). This is in contrast to Propper et al. (2010). They use 1998–2003 data to test the impact of maximum waiting time targets on patient prioritisation for elective treatment using a difference-in-differences method for England and Scotland. The policy was common to both countries prior to 1999 which was the start of the English reform. The goal of this study is different. We aim to assess the impact of the maximum waiting times reform on patient prioritisation in Scotland only. To capture the full effect of the reform we focus on years 2002 and 2007.

There is a concern, however, that since there is a five year distance between the two samples, it is possible that any change in prioritisation patterns is the result of factors other than the reform. We explore the possible validity behind this claim by comparing the difference between unconditional distributions for each year in 1998–2006 period and year 2007. The unconditional distributions were relatively stable over 1998–2002 period and started to shift significantly following the beginning of waiting times reform in 2003. Thus it is unlikely that other hypothetically confounding factors could account for the observed change in prioritisation as they should have had exactly the same timing as the reform.

We extract a subset of patients from the full-year population who were admitted for elective procedures. We next restrict our attention to only the first hospital stay for each patient in each year. We lose, respectively, 33.3% and 35% of the sample for years 2002 and 2007. We disregard observations with missing data on waiting times. This omits 4.3% and 2.6% from the original
2002 and 2007 samples. We also exclude observations where the waiting time is longer than two years as these are most likely coding errors. This results in additionally constricting the sample by 0.5% and 0.4% for years 2002 and 2007. Finally, we exclude from analysis pregnancy and conditions originating in the perinatal period (ICD-10 chapters 15 and 16) because of small numbers of observations. We have omitted external causes of morbidity and mortality and codes for special purposes (ICD-10 chapters 20 and 22) because the same ICD-10 code can be used to describe more than one medical condition with different severity and, hence, priority for treatment. As a result, our analytical sample has 657,443 observations in total with 321,929 for 2002 and 335,514 for 2007. In preparing our data we follow Janulevicute et al. (2013).

The data show that patients do not leave the waiting list monotonically (Figure 1). We observe peaks at the 1st and 7th day in 2002, 63rd and 126th day in 2007 data, and 365th day in both years. We also observe a strong weekly pattern (Figure 2). In particular, the number of patients accepted for treatment on the day they see a hospital specialist is significantly smaller than the number of people who wait for treatment 1 – 65 days. The first and most pronounced peak in the data occurs on the first day. It is followed by peaks at 7, 14, 21, etc. days. Minimums, respectively, are around at 3, 10, 17, etc. days. Such a pattern can be generated by the following three conditions: (1) almost no consultant appointments or elective treatments take place on weekends; (2) the probability of consultant appointment or treatment is approximately the same on all workdays; (3) the probability of being non-treated is not increasing over time. (1) and (2) jointly create spikes every seven days, while (3) creates downward sloping trend.

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2 We include in this category patients with waiting times value of 999 days.
3 Cullis et al. (2000) argue that one of the criteria for determining waiting times should be severity of the condition.
4 Figures 1 and 2 are on the log scale.
3. Results

3.1. Descriptive Analysis

We first discuss the features of the unconditional and marginal distributions of waiting times based on observed data in 2002 and 2007 (Table 1). The table consists of five panels. Panel 1.1 reports annual data averages in days. Panel 1.2 reports percentiles in bins (see Appendix A.3 for partition selection procedure). Panel 1.3 presents estimated and observed average waiting times for different values of the DSI, and Panel 1.4 shows estimated and observed average waiting times for eight age categories. Panel 1.5 is for the observed and estimated waiting time averages for different disease categories.

Average waiting time decreased from 79.4 to 63 days (Panel 1.1). Panel 1.2 shows that patients below the median waited, on average, longer in 2007. Although there were fewer patients on the waiting list in 2002 the number of patients treated within the first two weeks is larger (Figure 2). Waiting times declined significantly for patients above the median with patients at the 75th and 90th percentiles waiting, respectively, 2 and 13 bin units less.

Panels 1.3 and 1.4 present the pre- and post-reform means of waiting times and counts for different patient groups depending on patient’s age category and DSI. The mean waiting times declined for all age groups except for children age 1–6 years. The patient count for all categories below the age of 40 is larger in 2002. This clearly shows that waiting times for children under the age of 6 are shorter in 2002 in relative as well as in absolute terms. The average waiting times for all disease severity groups declined uniformly. The number of patients increased for all DSI categories except for DSI 0 which is consistent with aging of the population.

Panel 1.5 presents the pre- and post-reform averages of waiting times and counts for different ICD-10 disease chapters. It shows that mean waiting times declined for the majority of disease types except for patients with infectious and metabolic diseases, mental health patients\footnote{These patients are excluded if in psychiatric care.}, and
diseases of the nervous system (Ch. I, IV, V, VI). The table also shows that, in 2007, there were fewer patients taken off from the waiting lists for elective treatment in a number of categories with the largest declines in infectious, metabolic, and mental diseases.

To further investigate the results in Panel 1.2, we plot differences in survival functions in bin units between post- and pre-reform years using real data for different disease categories. The difference measures the change in the proportion of non-treated people between these two years up to date $t$. We define it as

$$\Delta S(t) = S^{07}(t) - S^{02}(t)$$

where $S^{year}(t) = 1 - CDF^{year}(t)$, $CDF^{year}(t)$ is the cumulative distribution function of waiting times.

The upper left graph in Figure 3 shows the difference in the share of all non-treated patients up to a certain day between 2007 and 2002. The share of non-treated patients within the first 8 weeks was larger in 2007. The dynamics changes for waiting times above 8 weeks. In particular, for any time period longer than 8 weeks, the survival function is larger in 2002. The shape of the graph can be best understood by analysing Figure 1. The rate of taking patients off the waiting lists between weeks 9 and 18 decays at a slower rate in 2007 compared to 2002. Within the first 18 weeks almost all patients are treated in 2007. As a result, we observe decreasing difference between survival functions after this date. The waiting time targets for 9 and 18 weeks in 2007 create two spikes in the distributions. The spike in the probability of treatment in Scotland at 18 weeks is consistent with the findings of Dimakou et al. (2009) and Appleby et al. (2003) based on data for the English NHS. In contrast, Janulevicuite et al. (2013) find no similar pattern using Scottish data for August 2002/July 2003 and August 2005/July 2006 periods. The graph supports the summary statistics in Panel 1.2 which shows decreases in waiting times below the median and increases above it.

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6Please note that the absolute number of patients is different in 2002 and 2007.
Exploring the survival difference for each disease category, we find similar patterns to those based on all disease chapters\(^7\) (the rest of Figure 3). However, even though the pattern is the same across different ICD-10 groups, the magnitude of the effect is very different. We are able to identify three sets of disease categories based on the difference in the magnitude of the effects.\(^8\)

The first group comprises cancer (chapter 2), eye (chapter 7), and cardiovascular (chapter 9) patients. The common feature is that the initial increase in the share of non-treated patients is small (\(\sim 2\) percentage points). In addition, these three categories were subject to explicit 9-week waiting time target in 2007. The second group includes respiratory conditions (chapter 10), skin (chapter 12), congenital (chapter 17), abnormalities not diagnosed elsewhere (chapter 18), health status factors (chapter 21), external sources injuries (chapter 19), genitourinary (chapter 14), musculoskeletal (chapter 13), digestive (chapter 11), and infections (chapter 1). The common characteristic between the conditions in this big group is that the magnitude of the initial increase is between 3-7 percentage points. The third group includes disease categories for which the initial increase in the share of non-treated patients was 10 percentage points or larger. These are blood (chapter 3), endocrine and metabolic (chapter 4), mental (chapter 5), nervous system (chapter 6), and ear (chapter 8) diseases. We focus on circulatory system diseases, digestive system diseases, and diseases of the nervous system. These are large disease categories selected to represent each group described above.

The mean waiting time for diseases of the circulatory system declined from 88.3 to 56.2 days. There are approximately 24,000 patients on the waiting lists in both years. The plot for cardiovascular patients (lower left part of Figure 3) shows that a smaller fraction of cardiovascular patients are treated within the first 3 weeks in 2007. The initial decline is 2 percentage points. However, the share of patients treated within the first 9 weeks in 2007 is 10.7 percentage points larger than in 2002. The difference in cumulative distributions peaks at the 18-week and stands at 13.9 percentage points which corresponds to treating 92.5% of

\(^7\)The results for all individual disease categories are available upon request

\(^8\)These results are available upon request.
cardiovascular patients in 2007 and 78.6% in 2002\[9\].

The mean waiting time for diseases of the digestive system declined from 69.7 to 54.5 days. The number of patients on the waiting list is approximately the same. The upper right part of Figure 3 shows that the share of patients waiting between 1 and 42 days is larger in 2002. In particular, the share of patients treated within the first 7 days in 2002 is 7.5 percentage points higher. However, by the end of the 9th week, the share of patients treated in 2007 exceeds by 4.8 percentage points the share in 2002. The difference in shares between the two years peaks at the 18-week target at 8.9 percentage points and relates to treating 93.5% and 84.6% of patients on the waiting list\[10\].

The mean waiting time for diseases of the nervous system declined slightly from 65.9 to 64.7 days. The number of patients added to the waiting list increases from 6823 to 8444. The lower right part of Figure 3 shows that the share of patients treated around the time of the 9th week is 10 percentage points smaller in 2007. However, by the end of the 18-week target, the share of patients accepted for treatment in 2007 is 5.2 percentage points larger. This difference corresponds to 87% and 92.5% of all nervous system patients treated in 2002 and 2007 respectively\[11\].

3.2. Model Fit

The goal of this subsection is to assess the ability of the CDE model to fit the data. For comparison purposes we report results from gamma GLM model which is well established in the literature on skewed outcomes in health care.

Panel 1.2 compares CDE- to data-implied percentiles before and after the reform. CDE estimates a probability distribution function for each combination of patient characteristics. We recover an unconditional distribution by averaging these conditional probability functions.

\[9\] The last two numbers are not shown on the graph
\[10\] Not shown on the graph
\[11\] Not shown on the graph
across all individuals in the data set. As probability distribution functions are defined on
discrete bins, there is no unique way to return to weekly/daily frequency. Thus, CDE-implied
and data-implied percentiles are reported in bin units. Based on this metric the model fits the
data well. It tends to underestimate the upper 10% of the waiting time distribution in 2002.
The difference between CDE- and data- implied values for the 90th percentile in this case is two
bin units (four weeks). There is no difference between estimated and data-implied percentiles
in 2007.

Panels 1.3 and 1.4 show the estimated and observed average waiting times for two marginal
distributions: age category and disease severity. The model provides good fit for both
distributions for the two years of our analysis. Most of the model results are within 3 days of
the observed values.

Panel 1.5 presents the observed and estimated averages across different disease categories. The
difference between the two averages, except for infection and mental diseases, is within 3 days.
These two chapters are the smallest in size. In addition, the model correctly captures the signs
of all changes in average waiting times between 2002 and 2007 for all categories considered.

We observe a slight improvement in model prediction between 2002 and 2007. The average
difference between the data and model estimates across disease categories, are, respectively,
0.67 for 2002 and 0.57 for 2007 with standard deviations of 1.43 and 1.30 and square root of
mean square errors 1.54 and 1.39 correspondingly. We find similar results for the other marginal
comparisons.

While gamma GLM directly fits the conditional mean for daily frequency, CDE maximises the
probability of fitting observations in particular weekly, bi-weekly, and four-weekly bins. Thus
we might expect that GLM model would perform better for values which are closer to the full
sample averages and worse for values that are relatively far from the sample means. Table 1
supports our intuition. For individual disease chapters and DSI categories GLM model performs
better, while results for aggregated age demonstrate better performance for CDE in 2002 and
comparable performance in 2007. There is smaller variability in waiting times across different age groups in 2007 which might contribute to the above outcome.

3.3. Main results

We next fit the data in our model and compare prioritisation schemes before and after the introduction of the waiting time targets. Results in Tables 2, 3, 4 come from the estimation of 1000 conditional expectations and corresponding marginal effects for each disease chapter and year using all combinations of patients’ ages and disease severity index. We report results for mid-age for eight age categories defined in Panel 1.4. Standard errors for all estimation procedures come from the standard deviation of 1000 bootstrap replications. We assume independence between individual admissions. Since we construct the sample by including only the first admission for each patient in 2002 and 2007 and individual admissions over time are not traced, the standard errors reflect only patient-level variation. Statistically significant results for the 95% two-sided confidence interval are in bold.


We study how the change in conditional mean estimates of waiting times vary across different subsets of individuals. Results in Table 2 show that the mean waiting times for all cardiac patients declined in 2007. Our estimates point to statistically significant declines for most patients. Waiting times for children age 3 with DSI 2 or 3 and 89 years old with DSI 5+ experience increases, although these results are not statistically significant.

Table 3 presents the mean waiting time estimates and their change for patients with diseases of the digestive system. We find that the waiting times for most patients declined. The waiting times for children age 3 with DSI 0 or 1 and children age 12 with DSI 0 increased. They wait,

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12 We have obtained an alternative set of results for averaged effects within age categories. The results are quite similar and available upon request.
13 For brevity, in the rest of this subsection, we use “increase/decrease” for “statistically significant increase/decrease”, “no change” for “no statistically significant change for 5% confidence level”.

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respectively, 13.3, 5.7, and 3.6 days more. The rest of the 3 year old, 12 and 28 year old patients with DSI 5+ have no change in their waiting times.

The conditional means and their changes over time for diseases of the nervous system in Table 4 show that the conditional waiting times increased for all 3 year old and 12 year old children with DSI 4–5+. We observe that the mean waiting times for patients age 47 or 59 with DSI 0 declined. The remaining groups of patients did not experience any change in their waiting times.

The changes in conditional means suggest that, on average, the patient groups that waited more in 2002 tended to wait less in 2007, while the shorter waiting groups in 2002 tended to wait more in 2007. Sinko et al. (2014), in a related study on England, develop a theoretical framework which suggests that, to meet the waiting times targets, hospitals are likely to abandon prioritisation by treating instead on a ”First Come, First Serve” basis. This approach implies a decrease in variability in expected waiting times across different patient groups. Their empirical results largely support this model.

3.3.2. Effects for change in the disease severity index

We consider the effects for change in disease severity index. We focus on cardiovascular patients first (Panel 4). The results show that the overall reduction in waiting times is accompanied by only one change in the pattern for treatment. In particular, the marginal effects for patients age 79 and 89 with DSI 3 or 4 are positive and significant in 2007. We do not observe this pattern in 2002.

We next study the marginal effects for patients with diseases of the digestive system (Panel 5). We uncover four changes in priorities depending on patient’s DSI. First, in 2002 patients age 3, 12 and 28 with DSI 5+ wait significantly less for treatment than patients with DSI 4. The sign of the respective results for 2007 varies and the estimates are not statistically significant. Second, marginal effects for patients age 47, 59, and 69 with DSI 3 are positive in 2007. In
2002 the marginal effect for these categories of patients are negative, although not all estimates are statistically significant. Third, the marginal effects for patients age 12, 28, 47, and 59 with DSI 4 are positive in 2007, although these estimates are not statistically significant. Marginal effects for the same ages are negative in 2002. Finally, marginal effects for patients age 69 and 79 with DSI 5+ are negative in 2007. No such pattern is observed in 2002 where these effects are positive but not statistically significant. To summarise, we find that most patient groups with change in prioritisation waited relatively longer in the post-reform year.

We focus next on patients with diseases of the nervous system (Panel 4.3). After the introduction of the waiting time targets the marginal effects for all patients with DSI 3 are positive, although only the estimates for 3 and 12 year old are statistically significant. In contrast, in 2002, all marginal effects for DSI 3 are negative, although none of the 2002 estimates are statistically significant. Second, marginal effects for patients age 3, 12, 28, and 47 with DSI 4 are positive but insignificant in 2007. In 2002 marginal effects for the same patients groups are negative and insignificant. Thus, similar to the digestive system, most patient groups within the nervous system disease chapter which were re-prioritised waited relatively longer in 2007.

3.3.3. Effects for change in age

We report results for change in age in Panels 2.4, 3.4, 4.4. These results are a numerical approximation of the derivative of the conditional expectation with respect to age (marginal effect for age) for the mid-age of each category defined in Panel 1.3.

Our findings point to no change in ordering for treatment for patient with cardiovascular and digestive system diseases (Panels 2.4, 3.4). The patterns of prioritisation for treatment based on patient’s age changed for nervous system patients (Panel 4.4). In 2002 marginal effects for all patients aged 3, 12, or 28 are positive. In contrast, in 2007, most of these marginal effects are negative but not statistically significant.
3.3.4. Re-prioritisation results. Aggregate evidence.

The push to reduce the maximum waiting times to 18 weeks in 2007 could have been achieved in three ways. First, it could have been done through faster processing of all patients on the waiting lists. However, Figures 1 and 2 show that the number of patients treated within the first two weeks is larger in 2002 compared to 2007. Alternatively, providers could have decided to reduce the number of patients within a patient category that used to be treated within the first several weeks and instead treated faster patients who waited longer applying exactly the same scheme to each patient category. In this case we would expect that, for all groups of patients of a given age and DSI, the share of those treated is smaller below the median waiting time and larger above it. Finally, providers could substitute short-wait patients from one group of patients with long-wait patients from another. The third scenario is also consistent with a decrease in variability in average waiting times for different patient groups. The last two scenarios are consistent with Figure 3. In this subsection we will further investigate these alternatives.

To extend the results reported in Figure 3 to conditional model-implied survival function $S$ we adopt the following approach. Using the estimation results we compute conditional survival functions ($S$) for each patient category $i$ before and after the reform. There are 10908 such groups for each year, which is the total number of combinations between 101 age years, 6 disease severity indices and 18 disease chapters. Next, we take a difference between them over time and count the number of times when the difference is larger than a threshold level $\delta$, i.e.:

$$n_{2002}^\delta(t) = \sum_{i=1}^{N} I((S_{07}^i(t) - S_{02}^i(t)) > \delta)$$

$$n_{2007}^\delta(t) = \sum_{i=1}^{N} I((S_{02}^i(t) - S_{07}^i(t)) > \delta)$$

where $n$ (later referred to as count) is the variable of interest, $\delta$ – threshold level, $N$ – total number of categories. In other words, 2002 count with threshold level $\delta$ at a particular point
in time $t$ corresponds to the number of patient groups for which the cumulative probability of treatment exceeds $\delta\%$ compared to the same patient groups at the same $t$ in 2007.

Results are reported in Figure 4 for $\delta = 1\%, 5\%, 10\%, 20\%$. We observe that across all threshold levels for short waiting time the number of groups that have a larger proportion of patients treated in 2002, or 2002 count (red line), constitute a significant part of the sample. The 2007 count (blue line) for short waiting times is significantly lower, but not zero. The pattern changes with time. At the 9th week target, the 2002 count is approximately equal to the 2007 count. By week 18, the 2002 count is virtually 0, while the 2007 count reaches its maximum over all patient categories.

Within this framework the second scenario is characterised by a close-to-zero blue line and a close-to-maximum red line for waiting times below group-specific medians. The third scenario is characterized by non-zero group counts for 2002 and 2007 years (red and blue lines) from the very beginning. The fact that, for short waiting times, different groups are affected differently by the prioritisation reform implies that re-prioritisation follows the third scenario of substitution between different patients or a mixture of scenarios 2 and 3.

4. Conclusions

The paper compares waiting times for elective treatment before and after the maximum waiting times reform in Scotland. The paper investigates two aspects of the change. First, it compares estimated average waiting times before and after the reform. Second, the paper documents a change in prioritisation between patients with different characteristics. While the former has been covered in previous literature \cite{Propper2008, Dimakou2009, Januleviciute2013}, the latter, to the best of our knowledge, is a novel result.

For our analysis we adopt the CDE method \cite{Gilleskie2004}. It allows us to estimate directly a discrete approximation of conditional densities of waiting times for different patient categories. We generalise this method to the case for arbitrary number of observations per bin
in a partition. We modify the model selection procedure accordingly.

To analyse CDE performance, we compare (i) quantiles and conditional averages directly derived from the data, (ii) quantiles and conditional averages implied by CDE, and (iii) conditional averages implied by Gamma GLM model. Both data-implied and CDE-implied quantiles indicate that after the reform patients in the upper tail of the waiting time distribution tend to wait less, while patients in the lower tail of the distribution tend to wait more. The values of the conditional means evaluated using the three approaches above are close to each other.

Overall, the maximum waiting times reform and increase in funding achieved the goal of decreasing the waiting times for the longer waiting patients. Average waiting times decreased across all disease severity groups, for all patients but children age 1–6 years, and for all but four disease chapters. We find that all patient groups with heart problems which were subject to explicit lower waiting times experienced declines in their average waits over the period. Moreover, their relative ordering for treatment remained largely the same. For nervous and digestive systems we find that all children groups waited longer post reform. We detect changes in their prioritisation based on disease severity in 2007. Thus it appears that the target was achieved at the expense of some patient groups. This finding is reinforced by the fact that conditional probability of not being treated up to a particular date significantly differs between 2002 and 2007 with more untreated patients in the first several weeks of 2007. Despite this, the majority of patients were treated within the 18-week target in 2007.

The question arises as to whether other factors could account for the prioritisation effects we uncover. We find this unlikely. We compare distributions of waiting times for each year separately for the period 1998–2007 and their changes over time are consistent with the timing of the reform. Any technological advance would be more likely to change the waiting times for the entire disease chapter rather than affect patient groups within a chapter differently. Over the period there were no hospital mergers in Scotland, which might contribute to the change in waiting times (Gaynor et al., 2012). We cannot control for any changes in the way GPs admit
into the system. However, the population of treated patients became older and sicker in 2007, which, given that the number of treated patients slightly increased, suggests that changes in the age composition of the population or patients do not drive the results.
Bibliography


Figure 1: Waiting times frequency

Graphs by year

Figure 2: Waiting times frequency less than 63 days

Graphs by year
Figure 3: Differences in Survival Functions between 2002 and 2007.
Dependence of $S_{t}^{07} - S_{t}^{02}$ on waiting time for aggregate and disease-specific distributions for cardiovascular, digestive, and nervous system diseases (ICD-10 Chapter 9, Chapter 11, and Chapter 6). Results are constructed using real data. The two vertical red lines correspond to 63 days (9 weeks) and 126 days (18 weeks). These are the waiting time targets in 2007.
Figure 4: Group count for different threshold levels

Number of groups of patients where $S_{07}^t - S_{02}^t > \delta$ (red line) and $S_{02}^t - S_{07}^t > \delta$ (blue line), $t$–number of adjusted time units. Vertical red dotted lines are at the 9th and 18th week of the sample.
Table 1: Actual and Implied Average Waiting Times.
Comparison between average waiting times computed directly from the data and implied by CDE and GLM models. Note: For the first part of the table, waiting times are reported in adjusted units (see p. 39). Below each panel as we report mean prediction error (MPE), standard deviation (SD) and root mean square predicted error (RMSPE) for CDE and Gamma models are reported.

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<th>CDE</th>
<th>Gamma N</th>
<th>2007 Data</th>
<th>CDE</th>
<th>Gamma N</th>
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<td>32,378</td>
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<td>7 7</td>
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<td>-</td>
<td>242,839</td>
<td>15 15</td>
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| MPE | 0.22 | -0.03 | - | 0.03 | -0.13 | - |
| Standard Deviation | -0.98 | 0.99 | - | 0.44 | 0.34 | - |
| RMSPE | -0.80 | 0.79 | - | 0.35 | 0.29 | - |
| **MPE** | -0.67 | -0.56 | - | 0.03 | -0.20 | - |
| Standard Deviation | -1.43 | 1.94 | - | 1.36 | 1.29 | - |
| RMSPE | -1.34 | 1.90 | - | 1.27 | 1.22 | - |

**Comparison between average waiting times computed directly from the data and implied by CDE and GLM models. Note: For the first part of the table, waiting times are reported in adjusted units (see p. 39). Below each panel as we report mean prediction error (MPE), standard deviation (SD) and root mean square predicted error (RMSPE) for CDE and Gamma models are reported.**
Table 2: Diseases of the Circulatory System

CDE-implied conditional mean waiting times for different patient groups that vary by age category and number of co-morbidity conditions in 2002 and 2007. Statistically significant results for the 95% two-sided confidence interval are in bold font.

| Age (yrs) | Disease Severity Index 2002 | | Disease Severity Index 2007 | |
|----------|-----------------------------| |-----------------------------| |
|          | 0  | 1  | 2   | 3  | 4 | 5+ | 0 | 1  | 2   | 3  | 4 | 5+ |
| 3 yrs    | 86.1 | 54.6 | 39.7 | 32.8 | 29.3 | 26.9 | 72.1 | 49.9 | 41.8 | 36.6 | 30.0 | 17.7 |
| 12 yrs   | 112.4 | 73.2 | 55.1 | 46.9 | 42.4 | 37.8 | 85.1 | 57.7 | 48.2 | 43.0 | 36.9 | 26.4 |
| 28 yrs   | 136.4 | 96.2 | 77.8 | 69.8 | 64.3 | 55.8 | 94.7 | 66.1 | 56.0 | 50.9 | 45.1 | 35.8 |
| 47 yrs   | 126.6 | 81.9 | 70.3 | 67.2 | 62.4 | 56.5 | 85.6 | 61.8 | 54.2 | 51.7 | 40.4 | 30.3 |
| 59 yrs   | 106.0 | 82.0 | 73.4 | 70.7 | 67.3 | 58.4 | 69.0 | 52.6 | 48.0 | 47.1 | 45.4 | 39.6 |

2.1 Conditional Means

| Age (yrs) | | | | | | | | | | | | |
| 3 yrs     | (1.5) | (1.5) | (1.3) | (1.8) | (1.9) | (1.8) | (1.1) | (0.9) | (0.7) | (0.9) | (1.0) | (1.0) |
| 12 yrs    | (1.6) | (1.3) | (1.2) | (1.5) | (1.6) | (1.4) | (1.0) | (0.7) | (0.7) | (0.8) | (0.8) | (0.8) |
| 28 yrs    | 64.8 | 54.1 | 51.8 | 52.1 | 50.4 | 42.8 | 44.5 | 36.8 | 35.6 | 37.0 | 38.3 | 36.3 |
| 47 yrs    | (2.2) | (1.4) | (1.4) | (1.7) | (1.9) | (1.9) | (1.2) | (0.8) | (0.8) | (0.9) | (1.0) | (1.1) |
| 59 yrs    | (2.1) | (2.3) | (2.3) | (2.6) | (2.9) | (3.0) | (2.0) | (1.3) | (1.3) | (1.5) | (1.8) | (2.0) |

2.2 Change in Conditional Means between 2002 and 2007

| Age (yrs) | | | | | | | | | | | | |
| 3 yrs     | (11.7) | (8.0) | (7.3) | (7.7) | (8.7) | (9.7) | (10.2) | (5.7) | (5.1) | (5.4) | (6.0) | (7.2) |
| 12 yrs    | (8.1) | (6.4) | (6.6) | (7.5) | (8.8) | (10.0) | (6.9) | (4.2) | (3.7) | (3.8) | (4.3) | (5.2) |
| 28 yrs    | (2.7) | (3.5) | (4.2) | (5.0) | (6.1) | (7.6) | (2.4) | (2.2) | (2.3) | (2.4) | (2.4) | (2.8) |
| 47 yrs    | (1.6) | (2.0) | (2.0) | (2.5) | (2.7) | (3.2) | (1.3) | (1.2) | (1.1) | (1.3) | (1.4) | (1.4) |
| 59 yrs    | (1.5) | (1.5) | (1.3) | (1.8) | (1.9) | (1.8) | (1.1) | (0.9) | (0.7) | (0.9) | (1.0) | (1.0) |

2.3 Change in Disease Severity Index

| Age (yrs) | | | | | | | | | | | | |
| 3 yrs     | (11.5) | (9.8) | (8.9) | (9.4) | (10.5) | (11.9) | (10.6) | (7.6) | (7.6) | (8.5) | (9.9) | (11.2) |
| 12 yrs    | (3.5) | (4.2) | (4.9) | (5.6) | (6.6) | (8.1) | (4.3) | (3.6) | (3.7) | (3.8) | (3.7) | (3.8) |
| 28 yrs    | (2.0) | (2.3) | (2.3) | (2.8) | (3.0) | (3.6) | (2.0) | (2.3) | (2.3) | (2.4) | (2.3) | (2.4) |
| 47 yrs    | (1.8) | (1.7) | (1.5) | (2.0) | (2.2) | (2.1) | (1.9) | (1.6) | (1.4) | (1.7) | (1.8) | (1.6) |
| 59 yrs    | (2.5) | (1.6) | (1.7) | (2.0) | (2.1) | (2.2) | (2.5) | (1.6) | (1.4) | (1.5) | (1.7) | (1.8) |

2.4 Change in Age

| Age (yrs) | | | | | | | | | | | | |
| 3 yrs     | (0.4) | (0.2) | (0.2) | (0.2) | (0.3) | (0.3) | (0.8) | (0.3) | (0.2) | (0.2) | (0.2) | (0.3) |
| 12 yrs    | (0.7) | (0.3) | (0.3) | (0.3) | (0.3) | (0.3) | (0.9) | (0.2) | (0.2) | (0.2) | (0.2) | (0.3) |
| 28 yrs    | 0.4 | 0.7 | 0.9 | 1.0 | 1.0 | 0.8 | 0.8 | 0.6 | 0.6 | 0.7 | 0.8 | 0.7 |
| 47 yrs    | (0.3) | (0.2) | (0.2) | (0.2) | (0.3) | (0.3) | (0.4) | (0.3) | (0.2) | (0.2) | (0.2) | (0.3) |
| 59 yrs    | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) |
| 69 yrs    | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) |
| 79 yrs    | (0.2) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) |
| 89 yrs    | (0.2) | (0.2) | (0.2) | (0.2) | (0.2) | (0.2) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) |
### Table 3: Diseases of the Digestive System

#### For details see Table 2

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<td>(0.8)</td>
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<td>(0.6)</td>
<td>(0.9)</td>
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<td>28 yrs</td>
<td>83.3</td>
<td>76.0</td>
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<td>47 yrs</td>
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<td>(0.8)</td>
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<td>69 yrs</td>
<td>81.9</td>
<td>64.4</td>
</tr>
<tr>
<td></td>
<td>(0.8)</td>
<td>(0.7)</td>
</tr>
<tr>
<td>79 yrs</td>
<td>73.5</td>
<td>57.1</td>
</tr>
<tr>
<td></td>
<td>(1.3)</td>
<td>(0.9)</td>
</tr>
<tr>
<td>89 yrs</td>
<td>68.1</td>
<td>53.2</td>
</tr>
<tr>
<td></td>
<td>(2.5)</td>
<td>(1.7)</td>
</tr>
</tbody>
</table>

#### 3.1 Conditional Means

3 yrs: 13.3, 5.7, 0.6 −1.7, 0.2, 5.6

12 yrs: 3.6, −3.6, −7.4, −7.7, −3.8, 3.4

28 yrs: −14.9, −16.5, −16.4, −14.0, −8.7, −1.2

47 yrs: −27.7, −22.9, −19.7, −16.6, −12.6, −7.4

59 yrs: −28.5, −20.9, −16.8, −14.3, −12.0, −9.4

69 yrs: −25.5, −16.8, −12.9, −11.4, −11.0, −10.9

79 yrs: −21.3, −12.5, −9.3, −9.1, −10.7, −13.4

89 yrs: −17.4, −9.1, −6.8, −7.9, −11.7, −17.3

#### 3.2 Change in Conditional Means between 2002 and 2007

3 yrs: 6.4, 7.4, 6.3, 2.0, −5.2, −1.2, 2.2, 4.1, 3.8, 0.2

12 yrs: 3.9, 5.5, 4.3, −0.5, −8.6, −3.3, 1.7, 4.0, 3.4, −1.4

28 yrs: −6.3, −0.4, 0.4, −3.4, −11.4, −8.0, −0.2, 2.9, 1.9, −3.9

47 yrs: −15.3, −5.2, −1.8, −3.5, −9.5, −10.6, −2.0, 1.3, 0.5, −4.4

59 yrs: −17.5, −6.1, −1.6, −1.9, −6.3, −9.9, −2.1, 0.9, 0.4, −3.6

69 yrs: −17.5, −5.7, −0.6, 0.2, −3.0, −8.8, −1.8, 0.9, 0.6, −2.9

79 yrs: −16.4, −4.5, 1.0, 2.8, 0.5, −7.7, −1.3, 1.3, 1.1, −2.1

89 yrs: −14.9, −3.0, 3.1, 5.8, 4.5, −6.7, −0.7, 2.0, 2.0, −1.2

#### 3.3 Change in Disease Severity Index

3 yrs: 6.4, 7.4, 6.3, 2.0, −5.2

12 yrs: 3.9, 5.5, 4.3, −0.5, −8.6

28 yrs: −6.3, −0.4, 0.4, −3.4, −11.4

47 yrs: −15.3, −5.2, −1.8, −3.5, −9.5

59 yrs: −17.5, −6.1, −1.6, −1.9, −6.3

69 yrs: −17.5, −5.7, −0.6, 0.2

79 yrs: −16.4, −4.5, 1.0

89 yrs: −14.9, −3.0, 3.1

#### 3.4 Change in Age

3 yrs: 2.4, 2.5, 2.5, 2.3, 2.1, 1.6, 1.7, 1.6, 1.5, 1.5, 1.4

12 yrs: 2.8, 2.3, 1.9, 1.6, 1.4, 1.1, 1.5, 1.3, 1.2, 1.1, 0.9

28 yrs: 1.6, 1.0, 0.6, 0.4, 0.3, 0.3, 0.6, 0.4, 0.2, 0.1, 0.0

47 yrs: −0.1, −0.4, −0.6, −0.6, −0.5, −0.3, −0.4, −0.4, −0.5, −0.5, −0.6

59 yrs: −0.8, −0.9, −0.8, −0.8, −0.6, −0.3, −0.6, −0.5, −0.5, −0.6, −0.5

69 yrs: −0.9, −0.8, −0.8, −0.6, −0.4, −0.0, −0.5, −0.4, −0.3, −0.3, −0.2

79 yrs: −0.7, −0.6, −0.4, −0.2, 0.1, 0.4, −0.3, −0.2, −0.1, 0.1, 0.0

89 yrs: −0.3, −0.2, 0.0, 0.2, 0.6, 1.0, 0.0, 0.1, 0.2, 0.3, 0.4, 0.6

(0.2) (0.1) (0.1) (0.2) (0.4) (0.1) (0.1) (0.1) (0.1) (0.1) (0.1) (0.1)
### 4.1 Conditional Means

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<td>71.4</td>
<td>64.0</td>
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<tr>
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<td>49.2</td>
<td>47.7</td>
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### 4.2 Change in Conditional Means between 2002 and 2007

<table>
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<th>2007</th>
</tr>
</thead>
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<td>3 yrs</td>
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<tr>
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<td>8.5</td>
<td>6.1</td>
</tr>
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<td>28 yrs</td>
<td>−1.3</td>
<td>−0.5</td>
</tr>
<tr>
<td>47 yrs</td>
<td>−5.4</td>
<td>−3.7</td>
</tr>
<tr>
<td>59 yrs</td>
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<td>−3.2</td>
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<td>69 yrs</td>
<td>−2.3</td>
<td>−1.4</td>
</tr>
<tr>
<td>79 yrs</td>
<td>0.8</td>
<td>−0.1</td>
</tr>
<tr>
<td>89 yrs</td>
<td>2.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

### 4.3 Change in Disease Severity Index

<table>
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<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 yrs</td>
<td>−4.5</td>
<td>−1.6</td>
</tr>
<tr>
<td>12 yrs</td>
<td>−4.7</td>
<td>−1.6</td>
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<td>28 yrs</td>
<td>−5.4</td>
<td>−2.0</td>
</tr>
<tr>
<td>47 yrs</td>
<td>−8.1</td>
<td>−2.9</td>
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<tr>
<td>59 yrs</td>
<td>−9.4</td>
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</tr>
<tr>
<td>69 yrs</td>
<td>−9.7</td>
<td>−5.3</td>
</tr>
<tr>
<td>79 yrs</td>
<td>−9.4</td>
<td>−5.4</td>
</tr>
<tr>
<td>89 yrs</td>
<td>−8.4</td>
<td>−5.4</td>
</tr>
</tbody>
</table>

### 4.4 Change in Age

<table>
<thead>
<tr>
<th>Age</th>
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<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 yrs</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>12 yrs</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
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<td>0.5</td>
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<tr>
<td>47 yrs</td>
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<td>0.1</td>
</tr>
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<td>59 yrs</td>
<td>−0.1</td>
<td>−0.3</td>
</tr>
<tr>
<td>69 yrs</td>
<td>−0.5</td>
<td>−0.5</td>
</tr>
<tr>
<td>79 yrs</td>
<td>−0.8</td>
<td>−0.7</td>
</tr>
<tr>
<td>89 yrs</td>
<td>−1.0</td>
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</table>

Table 4: Diseases of the nervous system

For details see Table 2.
Appendix A. Conditional Density Estimation

Appendix A.1. Estimation Method

As described by Gilleskie and Mroz (2004), the implementation of the CDE model involves several steps. First, we partition the range of the dependent variable $y$ into $K$ bins with corresponding boundaries: $y_0, y_1, \ldots, y_K$. Second, we estimate a survival model using polynomial approximation of a special kind. Third, we recover the conditional density function from a sequence of hazard functions approximated at the second step. Fourth, we compute a conditional expectation of interest. As we consider bins with unequal number of observations, we have to distinguish total number of bins $K$ from a partition associated with a particular choice of $y_0, y_1, \ldots, y_K$, $\Omega_K$, as there are $\binom{N-1}{K-1}$ partitions associated with $K$ bins. The probability that a random variable $Y$ falls in the first interval $p(y_0 \leq Y < y_1 | x)$ is the same as a time hazard for the same interval $\lambda(1, x|\Omega_K)$

$$
\lambda(1, x|\Omega_K) = p[y_0 \leq Y < y_1 | x, Y \geq y_0] = p[y_0 \leq Y < y_1 | x] = \int_{y_0}^{y_1} f(y|x)dy.
$$

(A.1)

They are not the same for the later intervals. However there is a one-to-one correspondence between the two. The probability that the random variable $Y$ falls in the $k$-th interval is given by

$$
p[y_{k-1} \leq Y < y_k | x] = \int_{y_{k-1}}^{y_k} f(y|x)dy
$$

And the probability that the random variable falls in the $k$ interval given that it did not fall in the first $k - 1$ intervals or the discrete time hazard $\lambda(k, x|\Omega_K)$ is

$$
\lambda(k, x|\Omega_K) = p[y_{k-1} \leq Y < y_k | x, Y \geq y_{k-1}] = \frac{\int_{y_{k-1}}^{y_k} f(y|x)dy}{1 - \int_{y_0}^{y_{k-1}} f(y|x)dy}
$$

(A.2)

$$
p[y_{k-1} \leq Y < y_k | x] = \lambda(k, x|\Omega_K) \prod_{j=1}^{k-1} [1 - \lambda(j, x|\Omega_K)]
$$

(A.3)

The function $p[y_{k-1} \leq Y < y_k | x]$ defines the probability that the random variable $Y$ falls in the $k$-th interval of a partitioning $\Omega_K$. The conditional expectation of a function $h(\cdot)$ of a random variable $Y$ given $x$ is

$$
E(h(Y)|x) = \int_{-\infty}^{\infty} h(y)f(y|x)dy
$$
\[ E(h(Y)|x) \text{ can be approximated using Eq. } (A.3) \]

\[
E(h(Y)|x) \approx \sum_{k=1}^{K} h^*(k|\Omega_K) \lambda(k,x|\Omega_K) \prod_{j=1}^{k-1} [1 - \lambda(j,x|\Omega_K)]
\] (A.4)

where

\[ h^*(k|\Omega_K) = E(h(Y)|Y \in [y_{k-1}, y_k)) \]

The adequacy of such an approximation depends on the variability of \( h(Y) \) with respect to \( x \) within an interval for a given partition \( \Omega_K \). Failure to acknowledge this fact creates a bias in marginal effects. An appropriate approximation of the marginal effect is:

\[
\frac{\partial E(h(y)|x)}{\partial x} \approx \sum_{k=1}^{K} \left[ h^*(k|\Omega_K) \frac{\partial p[y_{k-1} \leq Y < y_k|x]}{\partial x} + \frac{\partial h^*(k|\Omega_K)}{\partial x} p[y_{k-1} \leq Y < y_k|x] \right]
\] (A.5)

while, using Eq. (A.4)

\[
\frac{\partial E(h(y)|x)}{\partial x} \approx \sum_{k=1}^{K} \left[ h^*(k|\Omega_K) \frac{\partial p[y_{k-1} \leq Y < y_k|x]}{\partial x} \right]
\] (A.6)

as it is implicitly assumed that \( \frac{\partial h^*(k|\Omega_K)}{\partial x} = 0 \). For the purposes of estimation, \( h^*(k|\Omega_K) \) is approximated by

\[
\hat{h}^*(k|\Omega_K) = \frac{\sum_{y \in [y_{k-1}, y_k)} h(y)}{\sum_{y \in [y_{k-1}, y_k)} 1}
\] (A.7)

which is the sample average of the function of interest \( h(y) \) over the interval \([y_{k-1}, y_k)\). The sequence of \( \hat{\lambda}(k,x|\Omega_K) \) is estimated using logit regression. Once \( \hat{\lambda}(k,x|\Omega_K) \) and \( \hat{h}(k|\Omega_K) \) are estimated, an estimate of \( \hat{E}(h(y)|x) \) and the marginal effects are constructed.

The logit function is constructed in the following way. We build it around the simplest case of uniform distribution for waiting times \( y \) of patient \( i \) that does not depend on patients’ characteristics \( x_i \). In this case the probability of observing patient \( i \) in particular interval depends only on the width of this interval. Assume that the number of observations in \( k \)th interval \([y_{k-1}, y_k)\) of partitioning \( \Omega_K \) is \( N_k \). Under these conditions, a hazard function \( \lambda \) is solely a function of partitioning \( \Omega_K \)\(^{14}\):

\[
\lambda_k(\Omega_K) = P(Y \in [y_{k-1}, y_k)|Y \geq y_{k-1}) = \frac{N_k / \sum_{s=k}^{K} N_s}{\sum_{s=k}^{K} N_s / \sum_{l=1}^{K} N_l} = \frac{N_k}{\sum_{s=k}^{K} N_s}
\]

\(^{14}\) This is an extension of the formula in Gilleskie and Mroz (2004) which allows for different numbers of observations per bin.
which, for the case of logit, gives an intercept

\[ \alpha_k = \log \left( \frac{\lambda_k}{1 - \lambda_k} \right) \]

This intercept would fit the unconditional discrete distribution function if there are no other relevant covariates included. On the other side of the spectrum is the case when hazards are estimated independently for separate bins using all covariates available. The former approach is excessively restrictive, while the latter would require estimating an excessively large number of parameters. The CDE method, depending on specification, incorporates a wide range of models including the two extreme cases discussed above. It estimates a single logit model for all hazard rates simultaneously. The model depends on interactions of \( \alpha_k \) polynomial with polynomials of the covariates \( x \). The polynomial degree of \( \alpha_k \) determines the flexibility of the response function across bins. The polynomial degree of \( x \) determines the flexibility of the response function in a particular bin.

**Appendix A.2. Partition selection**

The proposed model estimation is partition-dependent. A natural question is how the likelihood function changes with partition. Gilleskie and Mroz (2004) answer this question for the case when the sample is divided into bins with an equal number of observations per bin. Under these conditions, the choice of partition is equivalent to the choice of number of bins. However, for the waiting times framework, their approach requires generalisation as the data demonstrate very strong temporal variation (the number of patients treated has a definite seven-day cycle), see Figure 2. We set the finest possible grid to be one week to eliminate this repetitive pattern.

Following the original paper, the likelihood function is defined over a strictly increasing partition with \( K \) bins \( \Omega_K = \{y_0, \ldots, y_K\} \) and can be represented as

\[
L(Y|\Omega_K) = \sum_{i=1}^{N} \ln [p(y_i|x_i; K)]
\]

where \( p(y_i|x_i; K) = \prod_{k=1}^{K} \{p(y_{k-1} \leq y(i) < y_k|x(i))\}^{I_i \in \{y_{k-1}, y_k\}} \). Please note that we allow for different numbers of observations in different bins. For ease of notation this formula is modified by indexing all sorted observations that fall into bin \( k \) as \( \{k, 1\}, \ldots, \{k, n\}, \ldots, \{k, N_k\} \), where \( N_k \) is the number of observations in bin \( k \) and \( \{N_k\} \) is the collection of all observations that belong to bin \( k \). In this case the likelihood function can be rewritten as

\[
L(Y|\Omega_K) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n})
\]  

(A.8)

with a natural relationship \( \sum_{k=1}^{K} N_k = N \). An additional division of bin \( k \) into \( i = 1, \ldots, I_k \) sub-bins with the corresponding number of observations in each sub-bin \( R_{k,i} \) creates a finer nested partition \( \Omega_{K,R} \), \( \Omega_K \subset \Omega_{K,R} \).
A modified probability for a particular observation \( \{k,n\} \) with a vector of covariates \( x_i \) falling in the sub-bin \( i \) is: \( \tilde{p}_{k,n} = P(\{k,n\} \in \{R_{k,i}\}|\{k,n\} \in \{N_k\}; x_i) \). Using Eq. (A.8) the resulting log-likelihood function is:

\[
L(Y|\Omega_{K,R}) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n,\tilde{p}_{k,n}}) = L(Y|\Omega_{K}) + \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (\tilde{p}_{k,n}) \tag{A.9}
\]

where the second term is non-positive by construction.

Assume that the model is correctly specified on a partition \( \Omega_{K0} \) with corresponding number of observations in bin \( k \) \( N_k^{(0)} \). Further assume that \( \Omega_{K0} \subset \Omega_{K,R1} \) and \( \Omega_{K0} \subset \Omega_{K,R2} \), i.e. each bin \( k \) of \( \Omega_{K0} \) is subdivided in \( I_{k}^{(1)} \) or \( I_{k}^{(2)} \) sub-bins with corresponding number of observations per sub-bin \( R_{k,i}^{(1)} \) and \( R_{k,i}^{(2)} \). Their maximum log-likelihoods can be represented as:

\[
L(Y|\Omega_{K0}) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}) \tag{A.10}
\]

\[
L(Y|\Omega_{K1}) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}\tilde{p}_{k,n}^{(1)}) = L(Y|\Omega_{K0}) + \sum_{k=1}^{K} \sum_{i=1}^{I_{k}^{(1)}} \ln (\tilde{p}_{k,i}^{(1)}) \tag{A.11}
\]

\[
L(Y|\Omega_{K2}) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}\tilde{p}_{k,n}^{(2)}) = L(Y|\Omega_{K0}) + \sum_{k=1}^{K} \sum_{i=1}^{I_{k}^{(2)}} \ln (\tilde{p}_{k,i}^{(2)}) \tag{A.12}
\]

and hence \( L(Y|\Omega_{K1}) \neq L(Y|\Omega_{K2}) \neq L(Y|\Omega_{K0}) \). As \( L(Y|\Omega_{K0}) \) reflects the true model, \( \tilde{p}_{k,i,r}^{(j)} = \frac{R_{k,i,r}^{(j)}}{N_k} \), \( j = 1,2 \).

Thus,

\[
L(Y|\Omega_{K1}) = L(Y|\Omega_{K0}) + \sum_{k=1}^{K} \sum_{i=1}^{I_{k}^{(1)}} \ln (R_{k,i}^{(1)}) - \sum_{k=1}^{K} N_k \ln(N_k) \tag{A.13}
\]

\[
L(Y|\Omega_{K2}) = L(Y|\Omega_{K0}) + \sum_{k=1}^{K} \sum_{i=1}^{I_{k}^{(2)}} \ln (R_{k,i}^{(2)}) - \sum_{k=1}^{K} N_k \ln(N_k) \tag{A.14}
\]

As \( \{R_{k,i}^{(j)}\} \) is known for \( \Omega_{Kj} \), subtracting \( \ln R_{k,i}^{(j)} \), \( j = 1,2 \) from each observation in Eq. (A.13) – (A.14) makes the two expressions equal. This is equivalent to introducing the finest possible partition with one observation per bin, \( \Omega_N \).

\[
L(Y|\Omega_{K0} \Rightarrow \Omega_N) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}\frac{1}{N_k}) = L(Y|\Omega_{K0}) - \sum_{k=1}^{K} N_k \ln N_k \tag{A.15}
\]

\[
L(Y|\Omega_{K1} \Rightarrow \Omega_N) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}\tilde{p}_{k,n}^{(1)}\frac{1}{R_{k,i}^{(1)}}) = L(Y|\Omega_{K0}) - \sum_{k=1}^{K} N_k \ln N_k \tag{A.15}
\]

\[
L(Y|\Omega_{K2} \Rightarrow \Omega_N) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln (p_{k,n}^{(0)}\tilde{p}_{k,n}^{(2)}\frac{1}{R_{k,i}^{(2)}}) = L(Y|\Omega_{K0}) - \sum_{k=1}^{K} N_k \ln N_k \tag{A.15}
\]

At the same time any partition \( \Omega_K \) can be reduced using Eq. (A.9) to a trivial partition with one bin \( \Omega_1 \). The
corresponding probabilities of falling in a particular bin are $\tilde{p}_{k,n}^{(0)} = N_k/N$, $\tilde{p}_{k,n}^{(1)} = R_{k,i}^{(1)}/N$, $\tilde{p}_{k,n}^{(2)} = R_{k,i}^{(2)}/N$ with likelihood functions:

$$L(Y|\Omega_K \Rightarrow \Omega_1) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln \left( \frac{p_0^{k,n} N}{N_{k}} \right) = L(Y|\Omega_K) + N \ln N - \sum_{k=1}^{K} N_k \ln N_k$$

$$L(Y|\Omega_K \Rightarrow \Omega_1) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln \left( \frac{\tilde{p}_{k,n}^{(1)} N}{R_{k,i}^{(1)}} \right) = L(Y|\Omega_K) + N \ln N - \sum_{k=1}^{K} N_k \ln N_k \quad (A.16)$$

$$L(Y|\Omega_K \Rightarrow \Omega_1) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \ln \left( \frac{\tilde{p}_{k,n}^{(2)} N}{R_{k,i}^{(2)}} \right) = L(Y|\Omega_K) + N \ln N - \sum_{k=1}^{K} N_k \ln N_k$$

As $N$ is fixed the following relationship between likelihood functions of arbitrary partitions $\Omega_K$ and $\Omega_{K'}$ holds:

$L(Y|\Omega_K \Rightarrow \Omega_N) < L(Y|\Omega_{K'} \Rightarrow \Omega_N) \iff L(Y|\Omega_K \Rightarrow \Omega_1) < L(Y|\Omega_{K'} \Rightarrow \Omega_1)$. If each bin contains the same number of observations, i.e. $N_k = N/K$, this becomes the adjusted likelihood in keeping with Gilleskie and Mroz

$$L(Y|\Omega_K \Rightarrow \Omega_1) = L(Y|\Omega_K) - \sum_{k=1}^{K} \frac{N}{K} \ln \left( \frac{N}{K} \right) = L(Y|\Omega_K) + N \ln (K)$$

**Appendix A.3. Method Implementation and Model Selection**

To implement the method we have to:

1. choose number of intervals and their width;
2. estimate a set of averages $\hat{h}^{*(1)}(1|\Omega_K), \hat{h}^{*(2)}(2|\Omega_K), \ldots, \hat{h}^{*(K)}(K|\Omega_K)$ as in Eq. A.4;
3. calculate changes in expectations for the function of the outcome of interest.

To eliminate the seasonality described in Subsection we first aggregate the data to weekly bins. Then we define several partitions. The first partition, A1, has the following structure: the first bin consists of people who are treated on the same day; the second to 27th bin are for patients who are treated, respectively, during the first, second, ..., 26th week; bins 28 – 57 are constructed on a bi-weekly basis; bins 58–62 are constructed on a four-week basis. Our choice of partitioning is summarized below:

15We use Norton (2010) to code the algorithm. All implementation errors are ours.
<table>
<thead>
<tr>
<th>Bin number</th>
<th>1</th>
<th>2–27</th>
<th>28–57</th>
<th>58–62</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days</td>
<td></td>
<td>same day</td>
<td>weekly</td>
<td>bi-weekly</td>
</tr>
<tr>
<td>Period covered (up to)</td>
<td>0</td>
<td>6 months</td>
<td>18 months</td>
<td>24 months</td>
</tr>
</tbody>
</table>

Two alternative partitions, A2 and A4, are generated by aggregating partition A1 by further uniting, respectively, every two or every four adjacent bins. The other set of models is based on approximately equal number of observations after weekly aggregation. In particular, we consider four partitions with 5, 10, 15 and 20 bins with approximately the same number of observations in each bin with corresponding names B5, B10, B15, and B20.

Model A1 has the largest number of bins and thus has the best chance to be sufficient across the above specified alternatives. If the partitioning of A1 is finer than necessary, there will be no gain in maximum likelihood compared to alternative specifications. We compare these partitions using different disease categories and years (36 samples). We use Eq. A.15 and report the fraction of times model A1 is better than the alternative models. Results in Table A.5 show that model A1 is better in at least 25% of the cases, and thus none of the alternatives can be considered sufficient.

<table>
<thead>
<tr>
<th>A2</th>
<th>A4</th>
<th>B5</th>
<th>B10</th>
<th>B15</th>
<th>B20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>0.389</td>
<td>0.528</td>
<td>0.361</td>
<td>0.25</td>
<td>0.361</td>
</tr>
</tbody>
</table>

In our estimation of the “hazard” function which conditions on covariates we include a polynomial of the third order in $\alpha_k$ in addition to the polynomials in the observed covariates. Our set of covariates $X$ consists of all cross-products up to the fourth power of age and disease severity index. These are interacted with polynomials of $\alpha_k$. The analysis is performed separately for 18 disease chapters.

We next calculate an approximation of the conditional expectation for a particular set of covariates as in Equation A.4. We calculate approximations of the derivatives which are our main effects of interest. For the purposes of statistical inference we bootstrap the standard errors.