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A Novel Spherical Actuator: Design and Control

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Abstract — The paper describes the design and control of a novel spherical permanent magnet actuator which is capable of two-degrees-freedom and a high specific torque. Based on an analytical actuator model, an optimal design procedure is developed to yield maximum output torque or maximum system acceleration for a given payload. The control of the actuator, whose dynamics are similar to those of robotic manipulators, is facilitated by the establishment of a complete actuation system model. A robust control law is applied, and its effectiveness is demonstrated by computer simulation.

I. INTRODUCTION

Recent advances in robotics, office automation, and intelligent flexible manufacturing and assembly systems have necessitated the development of precision multiple degree-of-freedom actuation systems. In general, however, motion with several degrees of freedom is realized almost exclusively by the use of a separate motor/actuator for each axis, which results in complicated transmission systems and relatively heavy structures. Thus, it is difficult to achieve a high dynamic performance, due to the effects of inertia, backlash, non-linear friction, and elastic deformation of gears. Actuators which are capable of controlled motion in two or more degrees-of-freedom can alleviate the problem, whilst being lighter and more efficient. However, although such actuators have been the subject of some research [1]-[3], they have rarely been commercialized, due to their complexity, and related difficulties in modelling their electromagnetic behaviour and optimizing their design.

II. DESIGN OPTIMIZATION

Due to the simplicity of the actuator topology, the magnetic field distribution, and torque vector and back-emf constant can be derived analytically[4]. This allows for the design of the actuator to be optimized with respect to a given criterion. The prime considerations in this paper are either maximum torque capability or maximum achievable acceleration with a given payload, although other criteria, e.g. minimum cost, may similarly be addressed.

A. Maximum Output Torque Design

Without loss of generality, an air-cored spherical actuator is assumed. Its electromagnetic torque is given by [4]:

$$T_{elm} = T_{m0} R_s^4 x_i^2 (1 - x_r - G_p / R_s)$$

(1)

where $x_r$ is equivalent to the split ratio of conventional permanent magnet machines, and is the ratio of the rotor radius $R_m$ to the outer radius of the stator $R_s$, and $G_p$ is the airgap length. $T_{m0}$ is a constant related to the remanence of the magnet, $B_{rem}$, the winding current density, $J$, the packing factor, $P_f$ and the winding geometrical angles, $\delta_1$ and $\delta_0$, and is given by:

$$T_{m0} = 2\pi P_f B_{rem} J (\delta_1 - \delta_0 + 0.5 (\sin 2\delta_0 - \sin 2\delta_1)) / 3$$

(2)

As has been shown [4], for a given $R_s$, there is an optimal split ratio, viz. $x_r = [1 - G_p / R_s] / 4$, which yields maximum torque. This result is obtained when friction-free conditions are assumed. In the present actuator, the rotor magnet is in direct contact with the stator housing, and the friction torque is, therefore, not negligible, although it can be minimized by using a low friction coating or a lubricant. Over the range of
rotor operating speeds, Coulomb friction, which is proportional to the rotor weight, is a dominant factor, and is given by:

\[ T_f = 4\pi \rho R_0^4 g f_c / 3 \]  

(3)

where \( f_c \) is the Coulomb friction coefficient, \( \rho \) is the mass density of the magnet and \( g \) is the gravitational acceleration.

The effective output torque of the actuator can, therefore, be written as:

\[ T_{\text{eff}} = T_{\text{em}} - T_f = R_0^4 \frac{x_f^4 (b - x_f) - cx_f}{x_f^3} \]  

(4)

where \( b = (1 - G/R_s) \) and \( c = 4\pi \rho g f_c / 3 \). It is evident that the optimal value of the ratio \( x_f \) is:

\[ x_f = 3T_{\text{m0}} b / 4(T_{\text{m0}} + c) \]  

(5)

Fig. 2 shows \( T_{\text{em}} \) and \( T_{\text{eff}} \) as functions of \( x_f \), assuming \( R_s = 0.036 [m] \), \( B_{\text{em}} = 1.2 \) [T], \( J = 4.0 \) [A/mm²], \( P_f = 0.5 \), \( \delta_1 = 0.6298 \) [rad], \( \delta_0 = 0.1222 \) [rad], \( g = 9.8 \) [m/s²], \( \rho = 7.5 \times 10^3 \) [kg/m³] and \( f_c = 0.12 \). As is seen, the effective output torque is reduced by an amount corresponding to the friction torque, which is proportional to \( x_f \). Consequently, the optimal split ratio \( x_f \) is reduced, compared with the optimal friction-free value. Since the optimal value of \( x_f \) is proportional to (1 - \( G/R_s \)), it decreases as \( R_s \) decreases and approaches 3\( T_{\text{m0}} / 4(T_{\text{m0}} + c) \) as \( R_s \) increases, provided that \( G/R_s < 1 \).

**B. Maximum Acceleration Design**

A common requirement is for maximum acceleration from an actuator so as to achieve the fastest dynamic response for a given payload. Assuming that the payload can be approximated by a point mass, \( m_s \), with its center of gravity at \( r_s = R_s + l_c \), then the additional inertia \( I_c \), friction torque \( T_f \) and gravitational torque \( T_g \), due to the payload, are given, respectively, by:

\[ I_c = m_s l_c^2 \] ; \( T_f = m_s g R_m f_c \) ; \( T_g = m_s g r_c \)  

(6)

The maximum attainable acceleration, when a pair of diametrically opposite windings is excited, is:

\[ A_{\text{eff}} = \left( T_{\text{em}} - (T_f + T_f + T_g) \right) / (I_c + I_c) \]  

\[ = \frac{R_0^4 x_f^4 (b - x_f) - cx_f}{8\pi \rho R_0^5 x_f^3 / 15 + m_c (r_c)^2} \]  

(7)

For a given \( R_s \), the optimal value of \( x_f \) is obtained from the solution of the following equations:

\[ \begin{align*}  
\alpha_1 x_f^4 + \alpha_2 x_f^3 + \alpha_3 x_f^2 + \alpha_4 x_f + \alpha_5 + \alpha_6 &= 0 \\
0 &< x_f < 1 
\end{align*} \]  

(8)

where

\[ \begin{align*}  
\alpha_1 &= c_1 (T_{\text{m0}} + c) R_s^8 \\
\alpha_2 &= -2 c_1 T_{\text{m0}} b R_s^8 \\
\alpha_3 &= 4 c_1 m_s g f_c R_s^5 \\
\alpha_4 &= 5 c_1 m_c g r_c R_s^4 \\
\alpha_5 &= -4 (T_{\text{m0}} + c) I_c R_s^3 \\
\alpha_6 &= 3 T_{\text{m0}} b I_c R_s^3 \\
a_0 &= -m_c g f_c I_c \\
c_1 &= 8\pi \rho / 15 
\end{align*} \]

Equation (8) may be solved numerically, e.g., using the Matlab routine *Roots*. Fig. 3 shows the maximum acceleration, as a function of \( R_s \) and \( x_f \), assuming \( m_s = 0.05 \) [kg], \( l_c = 0.017 \) [m], which correspond to a payload such as a miniature high resolution electronic camera, the other parameters being the same as those specified earlier. At \( R_s = 0.036 \) [m], the corresponding optimal split ratio \( x_f \) is 0.583, which is lower than the optimal value for maximum output torque. As can be seen from Fig. 3, the optimal ratio decreases slightly as \( R_s \) increases. This is due to the fact that the electromagnetic torque increases with \( R_s^4 \) whilst the moment of inertia of the rotor increases with \( R_s^5 \). Thus, in order to maintain maximum acceleration, any increase in the value of \( R_s \) should be proportionally less than any increase of \( R_s \).

Based on the above results, an integrated design procedure can be formulated to yield optimal designs in terms of a chosen criterion for a given specification.

**III. CONTROL OF SPHERICAL ACTUATOR**

A complete dynamic model for the actuator is given by:[5]

\[ \begin{align*}  
M \dot{Q_{\text{E}}} + C Q_{\text{E}} + G + \tau_{\text{ef}} &= K_{\text{EF}} I_w \\
L I_w + R I_w - K_{\text{EF}} Q_{\text{E}} &= u_E 
\end{align*} \]  

(9)
where $Q_e = [\beta \alpha]^T$ are the Euler angles representing the rotor orientation, the inertia matrix $M$, the Coriolis and centripetal force matrix $C$, and the gravitational torque vector $G$ being given by:

$$M = J_s \begin{bmatrix} (c\alpha)^2 & 0 \\ 0 & 1 \end{bmatrix}; C = I_s c\alpha \alpha \begin{bmatrix} -\alpha & -\beta \\ -\beta & 0 \end{bmatrix}; G = r_m e \begin{bmatrix} -c\beta \alpha \\ c\beta \alpha \end{bmatrix}$$

where a shorthand notation for sine and cosine functions is used for clarity, e.g., $c\alpha$ represents for $\sin\alpha$. $I_s$ is the combined moment of inertia of the rotor and payload referred in the rotor co-ordinate system, $u_e = [u_A u_B u_C]^T$ is the winding terminal voltage vector, $i_w = [i_A i_B i_C]^T$ is the winding current vector, $L = \text{diag}[L_A L_B L_C]^T$ is the winding self-inductance matrix, $R = \text{diag}[R_A R_B R_C]^T$ is the diagonal winding resistance matrix, $F = \text{diag}[F_A F_B F_C]^T$ is the diagonal force matrix, and $K_{ET}$, defined as the actuator torque matrix, is related to the actuator torque constant $K_T$ [4] by:

$$K_{ET} = K_T \begin{bmatrix} -s\beta \alpha & 0 & -c\beta \alpha \\ -c\beta \alpha & c\alpha & s\beta \alpha \end{bmatrix}$$

Note that (9) has a singularity at $\alpha = 90^\circ$. However, with the present actuator design the angular excursion of $\alpha$ is within $\pm 45^\circ$, and this singular point will never be encountered. Also, it will be noted that in non-singular regions, (9) constitutes a Hamiltonian system, and, therefore, possesses an understood structure and similar important properties as the dynamic equations for robotic manipulators.

A design methodology for a spherical permanent magnet actuator to achieve maximum output torque or maximum acceleration has been developed, and a control strategy for the closed-loop actuation system has been described. The stability and performance of this control strategy is guaranteed through the properties of its dynamic equations, and has been further verified by realistic computer simulation.

**REFERENCES**


