Please cite this article in press as:

Wu, W., Shen, L., Jin, W. and Liu, R. (2015) Density-based mixed platoon dispersion modeling with truncated mixed Gaussian distribution of speed. *Transportmetric B: Transport Dynamics.* Vol 3(2), 114-130. DOI: 10.1080/21680566.2014.960502.

Density-based mixed platoon dispersion modeling with truncated mixed Gaussian distribution of speed

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Abstract

On China urban arterials traffic presents a mixed flow feature because the percentage of bus flow is relatively high. This affects the applicability of traditional platoon dispersion models which generally only suitable for homogeneous traffic flow. Based on field observations, this paper proposes a mixed platoon dispersion model (MPDM) to macroscopically simulate the mixed platoon dispersion process along the road segment between two successive signalized intersections from the density view. In order to capture the heterogeneity in mixed platoon speeds, the truncated mixed Gaussian distribution (TMGD) is adopted here to fit the speed data collected in the field, and expectation maximization (EM) algorithm is employed to estimate the distribution parameters. Later, the piecewise platoon density function is developed to examine the platoon dispersion characteristics. By applying this density function, the formulation of the expected number of vehicles in the front of the platoon that have passed and the expected number of vehicles at the rear of the platoon that have not passed a downstream intersection, as well as the downstream arriving flow function are derived. Furthermore, numerical calculation for signal coordination verifies the effectiveness of the proposed MPDM.

Keywords: mixed traffic flow, platoon dispersion model, truncated mixed Gaussian distribution, EM algorithm, signal coordination

1. Introduction

Due to signals' compression and splitting along urban arterials, traffic flow is separated into series and moves downstream in platoons which present typical feature of interrupted flow. Vehicles in platoons are traveling at different velocities due to the differences in behaviors of drivers and maneuverability characteristics of vehicles. This dispersion of traffic platoons occurs when the platoon starts diffusing along the arterial link toward the downstream. By simulating the dispersion process, platoon dispersion models are developed to estimate vehicle arrivals at downstream intersections, which provide theoretical basis for signal coordination control.

Platoon dispersion has been studied by many researchers using different assumptions and methodologies. Pacey(1956) first proposed the traffic diffusion model assuming normal distribution of speed. Grace and Potts(1964) further investigated Pacey's model from the density aspect. Later, Robertson(1969) developed a recurrent dispersion model with field data gathered by Hillier and Rothery(1967), which is widely implemented in various signal plan design software and signal control systems, including TRANSYT(2006), SCOOT(1981), SATURN(1980), and TRAFLO(1980). Seddon(1972)reported that Robertson's model was intrinsically based on shifted geometric distribution of travel time. Tracz(1975) and Polus(1979) reported that distribution of vehicle's travel time is not always a shifted geometric distribution as in Robertson's model, but more consistent with a normal, lognormal, or a gamma distribution. Liu et al. (1996, 2000, 2001), conducted a field observation in Shanghai, China, and developed a methodology to fix the vehicle start-up time loss in Grace's model along with the front and rear of the platoon problem analysis. Wang et al.(2009) established a model based on the non-transformation normal distribution of travel time using data in Changchun, China, and proved to provide a better traffic flow predicting precision comparing with Pacey's model. Wei at al. (2012) proposed a platoon dispersion model for cars from the aspect of density with truncated normal distribution of speed assumption.

Plenty of works have been conducted for parameter calibration since the development of Robertson's model. A best-fit approach is used in most of these studies to explore the rational parameters as summarized by McCoy et al.(1983). From a different view, Manar(1994)measured the impact of unsuitable settings of the model's parameters when applying the TRANSYT-7F software for coordination of three intersections in Montreal, Canada, and found that the use of the recommended platoon dispersion factor of 0.25incurred 65,250 CND per year in additional user costs when applying inefficient signal timings. Other than using a goodness-of-fit approach, Yu(2000) developed calibrating technique for the parameters of Robertson's model directly from the mean and standard deviation of the link travel time. Rakha and Farzaneh(2006) improved on Yu's calibration method through explicitly accounting for the time step duration on platoon dispersion.

Three external friction levels is implied in Robertson's model in TRANSYT for the platoon dispersion factor. Manar and Baass(1996) pointed out that platoon dispersion relies on not only the external friction but also the internal friction measured with density and volume, and established mathematical models to reflect the dynamic changes in traffic conditions. A multiclass traffic flow model is developed by Wong and Wong(2002) to extend the LWR model considering heterogeneous drivers. Recently, a procedure is proposed by Bonneson et al.(2010) to predict the profile of arrival flow for an intersection by considering platoon decay due to mid-segment driveway access and egress.

Some literatures have doubted about the distribution assumptions of both Pacey's and Robertson's models. Moreover, researchers recently start looking into the heterogeneity and impact of internal frictions on platoon dispersion.

Both Pacey's and Robertson's models assume homogeneous traffic flow. While, in urban areas of China there is a large amount of bus traffic besides car flow. Typically, buses run on three types of facilities: normal lanes with mixed traffic, dedicated bus lanes, and bus rapid transit (BRT) lanes as shown in Figure 1. Dedicated bus lanes and BRT lanes are facilities which are isolated from the general traffic by road markings or physical barriers, and present special operational characteristics. However, most of urban arterials in China are classified as the first type, which present mixed traffic flow. Because the maneuverability of buses is lesser than cars, and buses need to stop at bus stops, special characteristics present for bus platoon comparing to car platoon. These differences in the microscopic performance are likely to have an impact on the macroscopic platoon dispersion behavior, and require modifications when applying traditional models. A study by Chen et al. (2012) confirmed that the bus traffic has a great effect on mixed platoon speed distribution.

(A) Normal Lane (B) Bus Lane (C) BRT Lane

Fig.1. Three typical road facilities for bus traffic

Performance of platoon dispersion models depends on the distribution assumption of platoon speed. As reported in some recent studies, the assumption of a simple form speed distribution could not capture the characteristics of mixed traffic flow. Therefore, it will be beneficial to investigate the new feature of mix traffic platoon dispersion based on field collected data. The motivation of this work is to develop a mixed platoon dispersion model based on surveyed data and investigate the impact of bus traffic on the mixed platoon dispersion, which will provide the theoretical support for signal coordination and bus priority control.

In the remainder of this paper, data acquisition and analysis are presented firstly, and the impact of bus traffic on mixed platoon dispersion is investigated by calibration results; following that, a density based mixed platoon dispersion model is developed; then, numerical computations for the model application in signal coordination are presented; at the end, conclusions and future work are provided.

2. Data acquisition and analysis

Wushan Road, Guangzhou is surveyed for model development and validation. This road segment is a typical arterial street of bidirectional four roadways, which is generally in unsaturated traffic state. There are totally 14 bus routes along this road, and the posted speed limit is 50 km/h (13.89 m/s). License numbers were recorded by video detection at two points, one is right after the signal at Yuehan Road, and the other is situated downstream with 650 m distance, shown in Figure 2. The downstream location is considered as the hypothetic downstream signal. The queuing in front of signal can be modeled typically using shock wave theory, but this is out of the scope of this work. Travel times were directly calculated from video records, and the original speeds (journey speed for buses, running speed for cars) were derived from the travel time and distance. The data was gathered from 7:45 am to 10:40 am. The whole survey period were divided into three sub-time sections according to different traffic volume levels. Table1 gives the statistical results of the original speed data for each period. The plots of the mixed platoon speed histogram are shown in Figure 3.

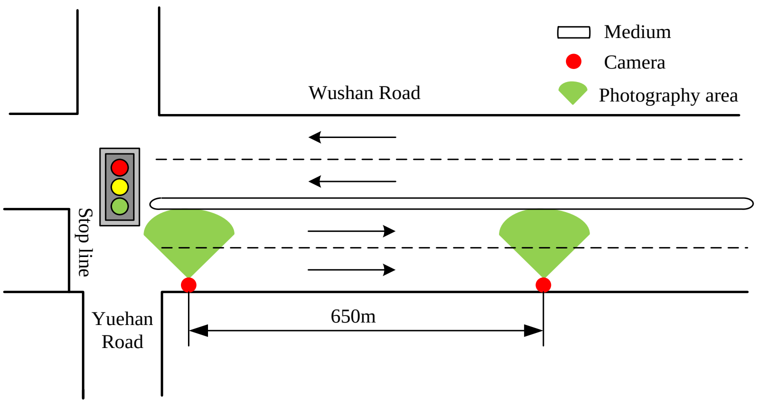
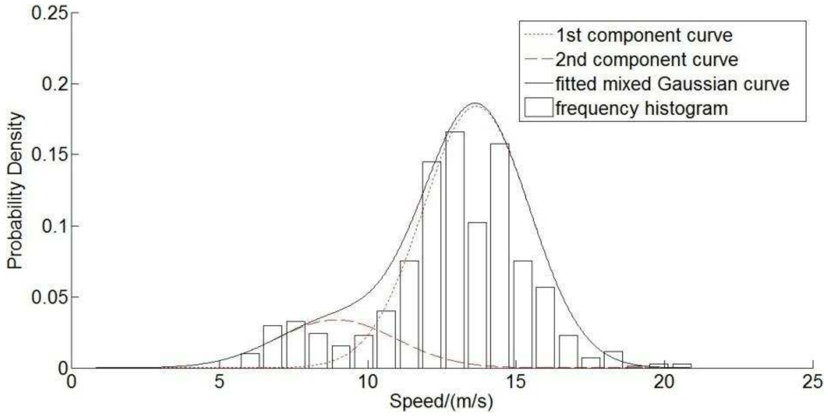


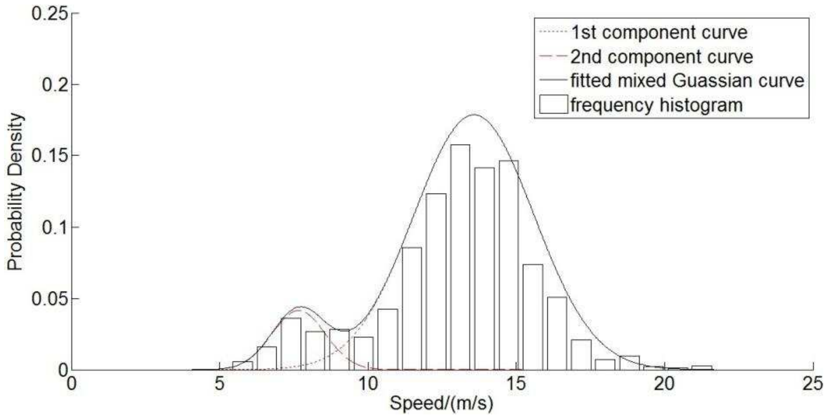
Fig.2. Diagram of the survey road section

Table 1 Original speed data

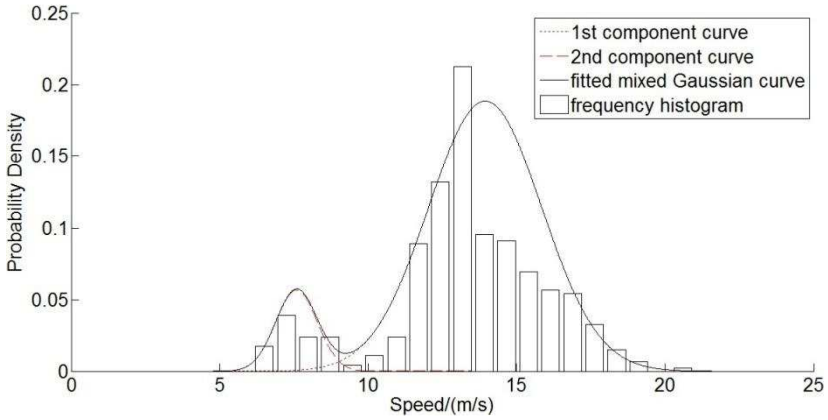
	Period 1 (7:45-8:25 am)			Period 2 (8:25-10:00 am)			Period 3 (10:00-10:40 am)		
	Car	Bus	Mixed	Car	Bus	Mixed	Car	Bus	Mixed
Distance (m)					650				
Mixed flow rate (veh/h)		1007			881			648	
Bus percentage (%)		13.1			10.8			11.8	
Sample size	617	88	705	1244	151	1395	410	51	461
Minimum speed (m/s)	8.67	5.65	5.65	7.74	5.37	5.37	7.22	6.13	6.13
Maximum speed (m/s)	20.97	14.44	20.97	21.67	12.50	21.67	20.96	13.27	20.96
Mean speed (m/s)	13.52	7.80	12.85	13.62	8.14	13.23	13.97	8.10	13.31
Standard deviation	1.99	1.79	2.64	1.90	1.34	2.56	1.85	1.61	2.44



a) Time period 1



b) Time period 2



c) Time period 3

Fig.3. Speed distribution histograms and fitted mixed Gaussian distribution curves of the study segment

From the three plots in Figure 3, two humps can be apparently identified from the speed data histogram of the mixed platoon, which represent bus and car categories, respectively. This confirmed the conclusions by Cheng(2012), Chen(2012), Park, et.al,(2010) and Wu, et.al,(2013). Therefore, the speed distribution of mixed platoons is difficult to be fitted with a single statistic distribution like: Normal or Lognormal. To effectively capture the speed heterogeneity, mixture distribution is needed, which is an efficient tool for fitting any complex continuous random variable. In the literatures, May(1990) and Johnson(1970) once proposed the employment of mixture distributions as a mix of a number of simple form distributions.

Among those mixture distributions, mixed Gaussian distribution is the most widely used, and has a simple mathematics form. The density function of mixed Gaussian distribution is as follow.

$$f(v)=\sum_{i=1}^{M}\beta_i\frac{1}{\sqrt{2\pi}\sigma_i}e^{-0.5(\frac{v-\mu_i}{\sigma_i})^2}\ ,\quad \sum_{i=1}^{M}\beta_i=1\ (1)$$

Where, β_i , μ_i , σ_i M are parameters.

In this study, four traditional distributions: normal, lognormal, Weibull, and Gamma are firstly fitted for the mixed platoon speed data. The fitting results show p-values < 0.01 with confidence levels of 95% in K-S evaluation[1]. It can be concluded that those four distributions are considered not suitable for the data, which is not a surprise since the speed distributions of buses and cars present different characteristics in the mixed platoon. Later mixed Gaussian distributions are explored for the speed data using EM algorithm through the software of Matlab, and the fitted curves are also presented in Figure 2. Since EM algorithm is widely known, detailed process is not discussed here for conciseness[2].

The parameter M of the mixed Gaussian distribution is generally set as the number of curve peaks F of the histogram plots, two curve peaks are easily identified for all three time periods. With $M=2$, the fitted parameters for all time periods are list in Table 2.

Table 2 Estimated Parameter variables of mixed Gaussian Distributions using EM Algorithm

	1st component			2nd component			Iterations
	β_1	μ_1	σ_1	β_2	μ_2	σ_2	
Period 1	0.829	13.664	3.234	0.171	8.930	4.087	3
Period 2	0.907	13.576	4.102	0.094	7.666	0.809	22
Period 3	0.900	13.945	3.632	0.099	7.592	0.491	7

Results in table1and table2 show that parameter μ_1 for all the three time periods are close to the observed car mean speeds, and parameter β_2 varies proportional to the bus percentage. This means that the impact of buses on the platoon dispersion is mainly reflected by β_2 .

As expected, the mixed Gaussian distribution has p-values > 0.15 in K-S evaluation for each time period. This consolidates the selection of mixed Gaussian distribution for the mixed platoon dispersion modeling because it statistically passes the K-S goodness-of-fit. Moreover, as indicated from the original speed data in Table 1, the speeds are not ranging from negative infinity to positive infinity, but vary between a minimum speed and a maximum speed (e.g. for period 1, the minimum speed and maximum speeds are 5.65m/s and 20.97m/s, respectively). This is reasonable because vehicles in platoons with speed $v<v_{min}$ and $v>v_{max}$ (v_{min} and v_{max} denote the minimum and maximum speed, respectively) seldom exist in the real world. Therefore, it may be more rational to assume truncated distribution for the platoon speed (May, 1990, Johnson, 1970, Wang et.al, 2012, Wei et.al, 2012), which will be discussed in the following model development section.

Performance of platoon dispersion models depend on the distribution assumption of platoon speed. However, the speed distribution is only descriptive parameter which doesn't study the mechanism inside the traffic flow since detailed

[1] K-S evaluation is a nonparametric test method for inferring the distribution pattern using sample data within significant levels.

[2] The expectation-maximization (EM) algorithm is one of maximum likelihood methods for estimating a mixture model and has been most commonly applied based on the work of Dempster et al.(1977)

interactions between vehicles is not the scope of macroscopic flow modeling other than microscopic flow modeling. But, speed distributions still sufficiently describe the results of microscopic interrelations between bus and car platoons. Here, as concluded from the field data, mixed Gaussian distribution is found better fit the data other than those simple form distributions. This does describe the phenomena of mixed platoon dispersion. However, further studies such as does the mixed Gaussian distribution will always fit the data with varied percentage of buses in the mixed flow, how to define the mixed flow, what is the thresh hold of bus percentage? Due to limited resources, this will be an opening research area and needs more future efforts.

In the practice of developing timing plans for coordinated signals, the speed data is collected in the field. While the speed distributions vary section by section, engineering experience shows that the characteristics of the speed distribution can be classified into several categories with different parameter sets. However, details of the application are not presented here, which can be explored in future works.

3. Model Development

3.1 Assumption for speed function of mixed platoon

As discussed previously that a simple distribution like normal and lognormal can't capture the speed heterogeneity of the mixed flow. Altogether, study like Pacey's platoon dispersion model does not appropriately reflect the practical situation, with the assumption that the speed follows normal distribution spreading from negative to positive infinity. Hence, the truncated distribution with range limit of speed is more suitable. In this study, the truncated mixed Gaussian distribution (TMGD) is selected to examine the mixed platoon dispersion model development.

By remedying equation (1), the proposed TMGD is shown in the following formula:

$$f(v)=\begin{cases}\sum_{i=1}^{M}\beta_i p_i(v|\mu_i,\sigma_i)=c\sum_{i=1}^{M}\beta_i\frac{1}{\sqrt{2\pi}\sigma_i}e^{-0.5(\frac{v-\mu_i}{\sigma_i})^2}, v_{min}\le v\le v_{max} \\ 0 \qquad ,\text{others}\end{cases},\quad \sum_{i=1}^{M}\beta_i=1\,(2)$$

where, β_i, μ_i and $\sigma_i, i=1,2\cdots,M$ are the parameters of mixed Gaussian distribution，which can be obtained by EM algorithm(Yu, 2009; Wu, 2013); M is the number of components; β_i is the proportion of component i; c is the normalizing constant that insures accumulated probability of $f(v)$ within range $[v_{min}, v_{max}]$ integrates to 1, which is the parameter different to non-truncated distribution. As for $v_{min}\le v\le v_{max}$, because $\int_{v_{min}}^{v_{max}} f(v)dv=1$, then:

$1/c=\sum_{i=1}^{M}\beta_i[\Phi(v_{max}/\sigma_i-\mu_i/\sigma_i)-\Phi(v_{min}/\sigma_i-\mu_i/\sigma_i)]$, where, Φ denotes the cumulative standard normal distribution function.

3.2 Mixed Platoon Dispersion Model (MPDM) Development

It is assumed that the green phase start time of the upstream signal $t=0$, and the stop line $x=0$, the platoon density distribution function $k(x,t=0)$ of the queuing vehicles at time $t=0$ is:

$$k(x,t=0)=\begin{cases}0, x\ge 0\\ k_j, -a\le x\le 0\ (3)\\ 0, x<-a\end{cases}$$

where, a is the queue length and k_j is the jam vehicle density.

This paper investigates the process whereby the queuing vehicles were discharged from upstream intersection and travel at constant speed v from their stop position $x-vt\in[-a,0]$ after time t. $A(x,t)$ denotes the number of vehicles having passed the downstream intersection location x (can be real or virtual) and $B(x,t)$ denotes the number of vehicles not having passed the downstream intersection. $B(x,t)$ is used to estimate the number of vehicles in the rear of the platoon might be stopped at the signal light and how much delay is caused. The formulas for $A(x,t)$ and $B(x,t)$ are presented based on the above physical definition and expressed as follows:

$$A(x,t)=\int_{x}^{t\cdot v_{max}} k(y,t)dy \quad (4)$$

$$B(x,t)=k_j a - A(x,t)=\int_{tv_{min}-a}^{x} k(y,t)dy \quad (5)$$

where, $k(y,t)$ is the mixed density function at location $x=y$ and time t, which can be computed with the following piecewise functions:

$$k(x,t)=\int_{v_{min}}^{v_{max}} f(v)k(x-vt,0)dv=\begin{cases}0, x>tv_{max} \cup x<tv_{min}-a \\ k_j\int_{v_{min}}^{\frac{x+a}{t}} f(v)dv, tv_{min}-a\le x<tv_{min} \\ k_j\int_{x/t}^{\frac{x+a}{t}} f(v)dv, tv_{min}\le x\le tv_{max}-a \\ k_j\int_{\frac{x}{t}}^{v_{max}} f(v)dv, tv_{max}-a<x\le tv_{max}\end{cases} \quad (6)$$

Let $u=(v-\mu)/\sigma$ and the dispersion rate $\alpha=\sigma/\mu$, because:

$$\int_{v_1}^{v_2} f(v)dv = c\sum_{i=1}^{M}\beta_i\int_{v_1}^{v_2}\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{1}{2}(\frac{v-\mu_i}{\sigma_i})^2}dv$$

$$=c\sum_{i=1}^{M}\beta_i\int_{\frac{v_1-\mu_i}{\sigma_i}}^{\frac{v_2-\mu_i}{\sigma_i}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}u^2}du = c\sum_{i=1}^{M}\beta_i\int_{\frac{tv_1/\mu_i-t}{\alpha_i t}}^{\frac{tv_2/\mu_i-t}{\alpha_i t}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}u^2}du \quad (7)$$

$$=\frac{c}{2}\sum_{i=1}^{M}\beta_i[\Phi(z)]_{z_{i,1}}^{z_{i,2}}$$

Where, $z_{i,2}=\frac{tv_2/\mu_i-t}{\sqrt{2}\alpha_i t}$, $z_{i,1}=\frac{tv_1/\mu_i-t}{\sqrt{2}\alpha_i t}$, v_1 and v_2 are constants, because: $\Phi(z)=2\int_0^{\sqrt{2}z}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}u^2}du=(2/\sqrt{\pi})\int_0^z e^{-u^2}du$ is the accumulated probability function of standard normal distribution. Following equations (6) and (7), the formulation of $k(x,t)$ can be rewritten as follows:

$$k(x,t)=\begin{cases}0, x>tv_{max} \cup x<tv_{min}-a \\ \frac{ck_j}{2}\sum_{i=1}^{M}\beta_i[\Phi(z)]_{\frac{tv_{min}/\mu_i-t}{\sqrt{2}\alpha_i t}}^{\frac{x/\mu_i+a/\mu_i-t}{\sqrt{2}\alpha_i t}}, tv_{min}-a\le x<tv_{min} \\ \frac{ck_j}{2}\sum_{i=1}^{M}\beta_i[\Phi(z)]_{\frac{x/\mu_i-t}{\sqrt{2}\alpha_i t}}^{\frac{x/\mu_i+a/\mu_i-t}{\sqrt{2}\alpha_i t}}, tv_{min}\le x\le tv_{max}-a \\ \frac{ck_j}{2}\sum_{i=1}^{M}\beta_i[\Phi(z)]_{\frac{x/\mu_i-t}{\sqrt{2}\alpha_i t}}^{\frac{tv_{max}/\mu_i-t}{\sqrt{2}\alpha_i t}}, tv_{max}-a<x\le tv_{max}\end{cases} \quad (8)$$

Let $G(z)=\int\Phi(z)dz=z\Phi(z)+(1/\sqrt{\pi})\exp(-z^2)$, based on equation (8), the number of vehicles $\int_{x_1}^{x_2}k(y,t)dy$ scattering along the road section $[x_1,x_2]$ can be calculated using the following equation:

$$\int_{x_1}^{x_2}k(y,t)dy=\frac{ck_j}{2}\sum_{i=1}^{M}\beta_i\int_{x_1}^{x_2}[\Phi(z_{i,2}(y))-\Phi(z_{i,1}(y))]dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i[\int_{z_{i,2}(x_1)}^{z_{i,2}(x_2)}\Phi(y)dy-\int_{z_{i,1}(x_1)}^{z_{i,1}(x_2)}\Phi(y)dy]=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{z_{i,2}(x_1)}^{z_{i,2}(x_2)}-[G(z)]_{z_{i,1}(x_1)}^{z_{i,1}(x_2)})\ (9)$$

With equations (8) and (9), $A(x,t)$ and $B(x,t)$ can be computed by the following five cases:

1) where $x > tv_{max}$:

$$A(x,t)=0$$

$$B(x,t)=\int_{tv_{min}-a}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{max}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t})+\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i(\Phi((tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)-\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t))$$

2) where $tv_{max}-a < x \le tv_{max}$:

$$A(x,t)=\int_{x}^{t\cdot v_{max}}k(y,t)dy=\frac{ck_j}{2}(tv_{max}-x)\sum_{i=1}^{M}\beta_i\Phi(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)-\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i[G(z)]_{(x/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$B(x,t)=\int_{tv_{min}-a}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{x}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\alpha_i\sigma_i([G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(x/\mu_i-t)/\sqrt{2}\alpha_i t})-\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t)+\frac{ck_j}{2}(x-tv_{max}+a)\sum_{i=1}^{M}\beta_i\Phi(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)$$

3) where $tv_{min} \le x \le tv_{max}-a$:

$$A(x,t)=\int_{x}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(x/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t})$$

$$+\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i\Phi((tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)$$

$$B(x,t)=\int_{tv_{min}-a}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{x}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(x/\mu_i-t)/\sqrt{2}\alpha_i t})$$

$$-\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t)$$

4) where $tv_{min}-a \le x < tv_{min}$:

$$A(x,t)=\int_{x}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(t\cdot v_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t})-\frac{ck_j}{2}(tv_{min}-x)\sum_{i=1}^{M}\beta_i\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t)+\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i\Phi((tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)$$

$$B(x,t)=\int_{t\cdot v_{min}-a}^{x}k(y,t)dy=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}-\frac{ck_j}{2}(x-tv_{min}+a)\sum_{i=1}^{M}\beta_i\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t)$$

5) where $x<tv_{min}-a$:

$$A(x,t)=\int_{tv_{min}-a}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy$$

$$=\frac{ck_j\sqrt{2}t}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t})+\frac{ck_j}{2}a\sum_{i=1}^{M}\beta_i(\Phi((tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)-\Phi((tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t))$$

$$B(x,t)=0$$

Typically, shock wave theory is applied to calculate delay, stops, and queue length when knowing the distribution of the arriving flow from upstream.

The vehicle flow function of downstream location x at time t can be calculated as $q(x,t)=\partial A(x,t)/\partial t=-\partial B(x,t)/\partial t$, which is derived from equation (4) and (5), and can be computed by the following five cases:

1) where $x>tv_{max}$:

$$q(x,t)=0$$

2) where $tv_{max}-a<x\le tv_{max}$:

$$q(x,t)=\frac{\partial(\int_{x}^{tv_{max}}k(y,t)dy)}{\partial t}$$

$$=\frac{ck_j v_{max}}{2}\sum_{i=1}^{M}\beta_i\Phi(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t)-\frac{ck_j\sqrt{2}}{2}\sum_{i=1}^{M}\beta_i\sigma_i[G(z)]_{(x/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-\frac{cxk_j t^{-1}}{2}\sum_{i=1}^{M}\beta_i\Phi((x/\mu_i-t)/\sqrt{2}\alpha_i t)$$

3) where $tv_{min}\le x\le tv_{max}-a$:

$$q(x,t)=\frac{\partial(\int_{x}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy)}{\partial t}$$

$$=\frac{ck_j\sqrt{2}}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(x/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t})$$

$$+\frac{ck_j t^{-1}}{2}\sum_{i=1}^{M}\beta_i((x+a)\Phi((x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t)-x\Phi((x/\mu_i-t)/\sqrt{2}\alpha_i t))$$

4) where $tv_{min}-a\le x<tv_{min}$:

$$q(x,t)=\frac{\partial(\int_{x}^{tv_{min}}k(y,t)dy+\int_{tv_{min}}^{tv_{max}-a}k(y,t)dy+\int_{tv_{max}-a}^{tv_{max}}k(y,t)dy)}{\partial t}$$

$$=\frac{ck_j\sqrt{2}}{2}\sum_{i=1}^{M}\beta_i\sigma_i([G(z)]_{(x/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}+[G(z)]_{(tv_{min}/\mu_i+a/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-t)/\sqrt{2}\alpha_i t}-[G(z)]_{(tv_{min}/\mu_i-t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i-a/\mu_i-t)/\sqrt{2}\alpha_i t}$$

$$-[G(z)]_{(tv_{max}/\mu_i - a/\mu_i - t)/\sqrt{2}\alpha_i t}^{(tv_{max}/\mu_i - t)/\sqrt{2}\alpha_i t}) + \frac{c(x+a)k_j t^{-1}}{2}\sum_{i=1}^{M}\beta_i(\Phi((x/\mu_i + a/\mu_i - t)/\sqrt{2}\alpha_i t)) - \frac{ck_j v_{min}}{2}\sum_{i=1}^{M}\beta_i\Phi((tv_{min}/\mu_i - t)/\sqrt{2}\alpha_i t)$$

5) where $x < tv_{min} - a$:

$q(x,t) = 0$

Assuming that the downstream signal locations x is given, the mixed platoon dispersion dynamic, the front of mixed platoon and the rear of mixed platoon behaviors, as well as the impact on the start time of the green phase at the downstream intersection can be quantitatively analyzed. The outcomes can be used for the calculation of timing plan evaluation parameters including delay, stop, and queue length.

4. Mixed platoon dispersion analysis

Because the proposed MPDM is based on speed TMGD assumption, the parameters of the model should be transferred from the mixed Gaussian distribution. In this section, the MPDM application in Period 1 is performed, and the statistical summary of the field data and the parameters are list in Table 3.

Table 3 Statistics of TMGD of Time Period 1

	Notation	Variable
TMGD coefficient	c	1.055
Jam Density (veh/m/l)	k_j	1/5.5
Queue Length (m)	a	70
Minimum speed(m/s)	v_{min}	5.65
Maximum speed(m/s)	v_{max}	20.97
Mean speed(m/s)	μ	12.85
	β_1	0.829
	β_2	0.171
Parameters of Gaussian	μ_1	13.664
mixture distribution	μ_2	8.930
	σ_1	3.234
	σ_2	4.087

The features of different modeling methods are summarized in Table 4. Since Robertson's and Pacey's model are widely known, details are not presented.

Table 4 Definitions of Different Models

Modeling Framework	Robertson Model	Pacey Model
✧ Model mixed platoon with truncated mixed Gaussian distribution of speed;	✧ Model mixed platoon directly, regardless of the influence of bus traffic;	✧ Model platoon with normal distribution of speed(cannot model mixed distribution);
✧ Density based;	✧ Flow based;	✧ Density based;
✧ Using original mixed platoon speed data.	✧ Using original mixed platoon travel time and experiential data.	✧ Using original homogeneous platoon speed data.

4.1 Mixed Platoon density distribution function

The density function $k(x,t)$ is a key modeling issue in platoon dispersion, which is of great importance to the offset setting in signal coordination design.

Generally, the offset of downstream intersection is set as $t_0 = x/\mu$, which is the time traveled by vehicles from upstream intersection to downstream intersection at the average speed. In this case study the downstream intersection location is $x = x_d = 300(\mathrm{m})$, the platoon density distributions along the downstream section of the proposed model at three time points: $t = x/v_{max}$, x/μ and $(x+a)/v_{min}$ are shown in Figure 4.

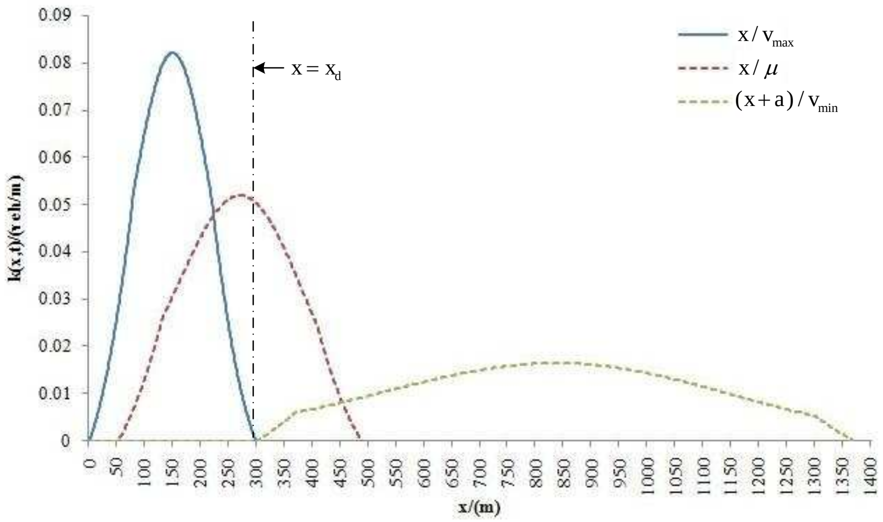


Fig.4. Mixed platoon density distributions of different times.

The following can be concluded from Figure 4:

a) During time period $t \in [0, x/v_{max})$, the platoon spreads along segment $[tv_{min} - a, tv_{max}]$, and the front of the platoon has not arrived at location $x = x_d$, and $k(x_d, t) = 0$ for location $tv_{max} < x_d$; during time period $t \in [x/v_{max}, (x+a)/v_{min}]$, there are some vehicles that have passed location $x = x_d$, $k(x_d, t) \neq 0$ for location $tv_{min} - a \le x_d \le tv_{max}$; during time period $t \in [(x+a)/v_{min}, +\infty)$, the rear of platoon has passed location $x = x_d$, $k(x_d, t) = 0$ for location $tv_{min} - a > x_d$.

b) At time t, the vehicles spread along the segment $x \in [tv_{min} - a, tv_{max}]$. Because speed density function follows TMGD, the density $k(x,t)$ in the middle of the platoon is higher compared to those of in the front and rear of the platoon. With passing time the platoon becomes more dispersed along the road section $[tv_{min} - a, tv_{max}]$, and the hump of the platoon density distribution function $k(x,t)$ becomes flatter.

4.2 Number of vehicles forced to stop in the front of the mixed platoon

If the downstream intersection green phase start time is normally set as $t_0 = x_d / \mu$, which assumes that the vehicles are traveling at average speed μ, the downstream light turns green when the upstream vehicles arrive. Because vehicles in the front of the platoon travel at a speed higher than μ, the downstream signal should turn green t_h ahead of t_0 to allow more vehicles to pass during the green phase. As for $t_0 = 45, 60, 90, 120(\mathrm{s})$, and $t_h = 0, 1, 2, \cdots, 10(\mathrm{s})$, the number of vehicles having passed downstream intersection $A(x_d, t_0 - t_h)$ are presented in Table 4.

Table4 Number of vehicles stopped at the front of mixed platoon

Offset	**Preset t_h or extension (s)**										
t_0 (s)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
45	3.18	2.76	2.40	2.09	1.86	1.71	0	0	0	0	0
60	3.40	3.15	2.86	2.51	2.22	1.97	1.79	1.71	0	0	0
90	3.61	3.45	3.28	3.11	2.93	2.75	2.47	2.22	2.01	1.84	1.71
120	3.72	3.60	3.47	3.34	3.22	3.09	2.96	2.82	2.68	2.47	2.25

As seen in Table 4, with the increment of the distance between the adjacent intersections, the speeds of the vehicles at the front of platoon vary in a large range $[\mu + a/t_0, v_{max}]$, and the $A(x_d, t_0 - t_h)$ gets larger. Therefore, more preset t_h is required to allow more vehicles in the front of the platoon to pass the downstream intersection.

4.3 Number of vehicles not having passed downstream intersection at the rear of mixed platoon

Normally, assuming that the green phase end time of the downstream signal is $t_0 = (x_d + a)/\mu$, and the speed of the vehicles at the rear of the platoon is lower than μ. Hence, the green phase should be postponed for time t_t to allow more vehicles at the rear of platoon to pass, which signifies the number of vehicles not having passed downstream intersection at time $t = t_t + t_0$ is $B(x_d, t_t + t_0)$. As for $t_0 = 45, 60, 90, 120(s)$, and $t_t = 0, 1, 2, \cdots, 10(s)$, according to equation (5), the number of vehicles not having passed downstream intersection $B(x_d, t_t + t_0)$ of MPDM is presented in Table 5.

Table5 Number of vehicles not having passed downstream intersection at the rear of mixed platoon

Offset	**Preset t_t or extension (s)**										
t_0 (s)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
45	9.54	9.21	8.88	8.55	8.21	7.86	7.51	7.14	6.76	6.38	5.99
60	9.33	9.08	8.83	8.58	8.32	8.07	7.80	7.53	7.25	6.97	6.68
90	9.12	8.95	8.78	8.61	8.44	8.27	8.10	7.92	7.74	7.55	7.36
120	9.01	8.88	8.76	8.64	8.50	8.37	8.24	8.11	7.97	7.84	7.70

It is shown in Table 5 that as the distance between the two consecutive intersections increases; more preset t_t is in need to allow more vehicles at the rear of the platoon to pass the downstream intersection.

4.4 Mixed platoon flow distribution at downstream intersection

In general, the downstream intersection green phase start time is set as $t_0 = x_d / \mu$. As for $x = 0, 300(m), 500(m)$ the flow function $q(x,t)$ at downstream location $x = x_d$ for any time t is computed for MPDM and is shown in Figure 5.

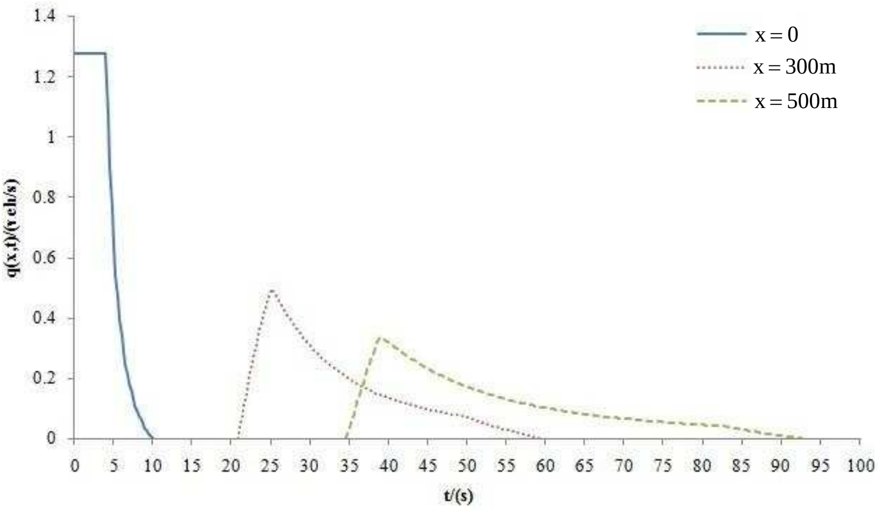


Fig.5. Flow functions of MPDM at different locations.

The following can be concluded from Figure 4.

a) When offset $t_0 = 0(s)$, at location $x_d = 0(m)$, the flow function $q(0,t)$ is the departing flow function from the upstream intersection. During time period $0 < t \le 5(s)$, the mixed platoon is discharging at the saturation flow rate; as t increases, $q(0,t)$ decreases, and after10 s, the departing flow decreases to zero, which means all vehicles have passed the stop bar.

b) As for offset $t_0 = 21, 34(s)$, locations $x_d = 300(m), 500(m)$ represents two virtual downstream intersections, $q(x_d, t)$ is the flow function arriving at the downstream intersection at location $x = x_d$ at time t. According to the menbership between $q(x,t)$ and $k(x,t)$, during time period $\forall t \notin [t_0\mu / v_{max}, (t_0\mu + a)/v_{min}]$, the flow rate $q(x_d, t) = 0$; during time period $\forall t \in [t_0\mu / v_{max}, t_0]$, the flow rate $q(x_d, t)$ increases as t increases; in time period $\forall t \in [t_0, (t_0\mu + a)/v_{min}]$, the flow rate $q(x_d, t)$ becomes smaller. Furthermore, as t_0 increases, the peak flow rate decreases, so it will take a longer time $(t_0\mu + a)/v_{min} - t_0\mu / v_{max}$ for all vehicles to pass the downstream intersection.

c) Pacey's normal distribution assumption and Robertson's shifted geometric distribution present vehicles traveling to downstream/upstream infinity distance, which is not realistic, and leads to flow lost between successive intersections. While MPDM presents the exact time when the first vehicle reaches and the last vehicle clears the downstream intersection, which ensure the flow conservation. This reflects the field situation properly.

5. Conclusion section

5.1 Conclusion

Large percentage of buses in mixed traffic flow affects the accuracy of platoon dispersion modeling in Pacey's and Robertson's model. A mixed platoon dispersion model is developed to capture the characteristics of the mixed platoon dispersion from the density view based on a TMGD of speed. Field data is collected to fit the parameters of the speed distribution. The application on adjacent signal coordination is presented, which indicates that TMGD can effectively capture the heterogeneity of the speed distribution of mixed traffic caused by bus traffic.

By imposing a minimum and maximum speed to the speed distribution, the truncated distribution fits field observations more realistically. The truncated assumption fixes the defects of Pacey's normal distribution and Robertson shifted geometric distribution both presenting vehicles with infinite speeds.

When fitting parameters from field data, the model can be applied in practice, which will improve the signal timing plan development, and lead to delay reduction and congestion relieving.

5.2 Future work

This paper proposed a theoretical model, and some on-going issues still need to be explored in future work, like model calibration and validation using extensive field data for different traffic conditions, number of lanes, geometry, roadway function classes. Besides, departing flow data at upstream and arriving flow data at downstream are needed to test the prediction performance of the proposed data.

Furthermore, in order to conveniently applying the proposed model in signal coordination timing software, the development of a macroscopic simulation program is the direction of future works.

Acknowledgment

This work is funded by the Project (No.61174188) supported by National Science Foundation of China. And the authors wish to thank the anonymous referees for their helpful comments and useful suggestions which considerably improved this paper.

References

[1] Pacey, G. M., The progress of a bunch of vehicles released from a traffic signal. Research Note No. RN/2665/GMP, Road Research Laboraroty, Crowthorne, Berkshire, England, 1956.

[2] Grace, M. J., and R. B.Potts, A theory of the diffusion of traffic platoons. Operation Research, 12, 255-275, 1964.

[3] Robertson, D. I., TRANSYT-A Network Study Tool. RRL Report LR 253, Road Research Laboratory, Crowthorne, Berkshire, U.K, 1969.

[4] Hillier, J. A., and R. W.Rothery, The synchronization of traffic signals for minimum delay. Transportation Science, 1, 81-93, 1967.

[5] TRANSYT-7F User's Manual Release 10 (TM), 2006.

[6] Hunt, P.B., F.I.Robertson, R.D.Bretherton, and R.I. Winton, SCOOT – A Traffic Responsive Method of Coordinating Signals, RRL Tool. RRL Report LR 1041, Road Research Laboratory, Crowthorne, Berkshire, U.K, 1981.

[7] Hall, M.D., D.VanVliet, and L.G. Willumsen, SATURN – A Simulation/Assignment Model for the Evaluation of Traffic Management Schemes. Traffic Engineering and Control, Vol. 21, No. 4, 168-176, 1980.

[8] Lieberman, E.B. and B.J. Andrews, TRAFLO – A New Tool to Evaluate Transportation Management Strategies. Transportation Research Record 772, TRB, National Research Council, Washington D.C., 9-15, 1980.

[9] Seddon, P. A.,Another Look at Platoon Dispersion: 3. The Recurrence Relationship. Traffic Engineering and Control, 13(10), 442-444, 1972.

[10] Tracz, M., The Prediction of Platoon Dispersion Based on Rectangular Distribution ofJourney Time. Traffic Engineering and Control, Vol. 16, 490-492, 1975.

[11] Polus, A., A Study of Travel Time and Reliability on Arterial Routs. Transportation, vol.8, No. 2, 141-151, 1979.

[12] Liu C., and P.Yang, Diffusion Models of Traffic Platoon on Signal-intersection and Control of Coordinated Signals. Journal of TongjiUniversity(Nature Science Edition), Vol. 24, No. 6, 636-641, 1996.

[13] Liu C., and P.Yang, Diffusion Models of Density of Traffic Platoon and Signal Coordinated Control. China Journal of Highway and Transport, Vol. 14 No. 1, 89-91, 2001a.

[14] Liu C., and X.Yang, Modification of Grace's Density Diffusion Model and Its Application. Journal of Highway and Transportation Research and Development, Vol.18, No.1, 62-64, 2001b.

[15] Wang D., F.Li, and X. Song,A New Platoon Dispersion Model and Its Application. Journal of Jilin University (Engineering and Technology Edition), Vol.39, No.4, 891-896, 2009.

[16] Wang, D., Y.ZhangandZ.Wang, Study of Platoon dispersion Models. TRB Annual Meeting, Washington, 2003.

[17] Wei M., W.Jin and L.Shen, A Platoon Dispersion Model Based on a Truncated Normal Distribution of Speed. Journal of Applied Mathematics, Volume 2012, Article ID 727839, 13 pages, 2012.

[18] B.J. Park, Y. Zhang, D. Lord, Bayesian mixture modeling approach to account for heterogeneity in speed data. Transportation Research Part B, Vol. 44, 662-673, 2010.

[19] Wu W., W. Jin and L. Shen, Mixed Platoon Flow Dispersion Model Based on Speed-Truncated Gaussian Mixture Distribution. Journal of Applied Mathematics, Volume 2013, Article ID 480965, 9 pages, 2013.

[20] McCoy, P.T., E.A.Balderson, R.T. Hsueh, and A.K.Mohaddes, Calibration ofTRANSYT Platoon Dispersion Model for Passenger Cars under Low-Friction TrafficFlow Conditions. Transportation Research Record 905, TRB, National ResearchCouncil, Washington D.C., 48-52, 1983.

[21] Manar, A.,Modelisation de la Dispersion du Trafic Entre les Carrefours. Ph.DDissertation at the Civil Engineering Department, Universite de Montréal, Montréal,Canada, 1994.

[22] Yu, L., Calibration of Platoon Dispersion Parameters on the Basis of Link Travel TimeStatistics. Transportation Research Record 1727, TRB, National Research Council,Washington D.C., 89-94, 2000.

[23] Rakha H. and M. Farzaneh, Issues and Solutions to Macroscopic TrafficDispersion Modeling, Journal of Transportation Engineering, Vol. 132,No. 7, 555-564, 2006.

[24] Manar, A., and K. G. Baass, Traffic Platoon Dispersion Modeling on Arterial Streets. Transportation Research Record 1566, TRB, National Research Council,Washington D.C., 49-53, 1996.

[25] Wong G., and S.Wong, A multi-class traffic flow model - an extension of LWRmodel with heterogeneous drivers, Transportation Research Part A, 36, 827-841, 2002.

[26] Bonneson, J. A., M. P. Pratt, and M. A. Vandehey, Predicting Arrival Flow Profiles and Platoon Dispersion for Urban Street Segments. Transportation Research Record 2173, TRB, National Research Council,Washington D.C., 28-35, 2010.

[27] Cheng, X., Distribution of Vehicle Free Flow Speeds Based on Gaussian Mixture Model, Journal of Highway and Transportation Research and Development, Vol. 29, No. 8,132-135, 2012.

[28] Chen J., T. Wang, C. Li, and C. Yuan, Speed models of mixed traffic flow on bus-car and vehicle and analysis of traffic running state, China Journal of Highway and Transport, Vol. 25, No. 1, 2012.

[29] Yu L., Finite Weibull mixture distribution model of VOT distribution forecasting. China Journal of Highway and Transport, 2009, 22(4): 96-101.

[30] Tao P, D. Wang and S. Jin, Mixed Distribution Model of Vehicle Headway. Journal of Southwest JiaoTong University, 2011,46(4)：633-637.

[31] Transit Capacity and Quality of Service Manual - 2nd Edition, Transportation Research Board, Washington D.C., 2003.

[32] May, A.D. Traffic flow fundamentals. Prentice Hall, 1990.

[33] Johnson, N.H., Kotz, S. Continuous univariate distributions. John Wiley&Sons, 1970.

[34] Yibing Wang, Wei Dong, Liangqi Zhang, etc. A Note on Speed Modeling and Travel time Estimation based on Truncated Normal and log-normal Distributions. 91st Annual Meeting for the Transportation Research Board, January 22-26, 2012.

[35] Dempster, A.P., Laird, N.M., Rubin, D.B., 1977. Maximum likelihood from incomplete data via the EM algorithm. Journal of the Royal Statistical Society Series B, 1977, 39(1), 1-37.