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Cancellieri, E. orcid.org/0000-0002-4099-2716, Boulier, T., Hivet, R. et al. (6 more authors) (2014) Merging of vortices and antivortices in polariton superfluids. *Physical review B*, 90 (21). ARTN 214518. ISSN 1098-0121

<https://doi.org/10.1103/PhysRevB.90.214518>

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Merging of vortices and antivortices in polariton superfluids

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Quantised vortices are remarkable manifestations on a macroscopic scale of the coherent nature of quantum fluids, and the study of their properties is of fundamental importance for the understanding of this peculiar state of matter. Cavity-polaritons, due to their double light-matter nature, offer a unique controllable environment to investigate these properties. In this work we theoretically investigate the possibility to deterministically achieve the annihilation of a vortex with an antivortex through the increase of the polariton density in the region surrounding the vortices. Moreover we demonstrate that by means of this mechanism an array of vortex-antivortex pairs can be completely washed out.

PACS numbers: 03.75.Lm, 42.65.Hw, 71.36.+c

I. INTRODUCTION

Topological excitations such as quantised vortices, characterised by a phase winding from 0 to $2\pi m$ (with m an integer number) around a vortex core, have been extensively studied in several systems such as non-linear optical systems¹, superconductors², superfluid ^4He ³, vertical-cavity surface-emitting lasers⁴, and more recently in cold atoms^{5,6,7}. Finally, in recent years, the study of vortices and vortex lattices has attracted much attention also in the field of coherent cavity-polariton fluids. First because, being intrinsically out-of-equilibrium, they constitute a novel system to study Bose-Einstein condensation phenomena, and secondly because, due to their light-matter nature, polaritonic systems are fully controllable by optical techniques and therefore allow very detailed studies of quantum turbulence.

In particular, in the context of cavity-polariton systems, it has been shown that stable vortices and half-vortices^{8,9,10}, as well as single vortex-antivortex (V-AV) pairs^{11,12,13,14} can be generated. The formation of lattices of vortices and of vortex-antivortex pairs has also been theoretically and experimentally studied in several different configurations: in the optical parametrical oscillator configuration¹⁵, in non-resonantly generated condensates^{16,17,18}, and in the case of patterns induced by metallic deposition on the surface of the cavity¹⁹. However, the mechanisms lying beneath vortex-antivortex annihilation and vortex-vortex interaction are still not fully understood. This is related to the high degree of control needed to study such interactions.

For example, in the first experimental observations of vortex-antivortex lattices^{17,18,19} the formation and the properties of the array were only partially controllable due to either the presence of a strong exciton reservoir,

that influences the position of the formed vortex array and its disappearance, or due to the fact that the formed array depends on the structure of the metallic depositions over the cavity surface. To achieve a higher degree of control resonant pumping schemes have been proposed²⁰. By using masks in the pumping beam, the formation of vortex-antivortex arrays with controllable shape and vortex distribution was achieved²¹ and the evolution of the arrays was studied in correlation with the local onset of the superfluid regime. However, in this study the case of high polariton densities was not experimentally achievable since the masks used to generate the vortex array were blocking most of the laser power, and therefore was not theoretically investigated.

In this work we theoretically study the annihilation of a vortex with an antivortex when injecting polaritons resonantly, in a broad range of polariton densities. In our model we assume an excitation with four coherent laser beams resonant with the lower polariton branch similarly to¹⁷ and¹⁸ where, however, polaritons were injected non-resonantly. The four pump spots are supposed to have the same energy and k-vector modulus and to generate polaritons propagating toward the centre of a common area. The advantage of our model is that in the resonant configuration it is well known that there is no exciton reservoir and that, since the entire pump intensity is used to inject the coherent fluid, high polariton densities can be achieved. Moreover, since the pumps set the momentum and the density of the injected fluid we can directly correlate the vortex-antivortex annihilation and the washing out of an array of vortices with the increase of the regions where the fluid is subsonic and with the change in the polariton flow that comes with it.

The manuscript is structured as follows: In Sec. II we introduce the theoretical model used to simulate the sys-

tem, describe more in details the setup we have in mind and introduce the *generalized local speed of sound* that will be needed for the analysis of the results. In Sec. III we demonstrate the annihilation of V-AV pairs and correlate this annihilation with the change in the polariton flow caused by the widening of the regions where the fluid is subsonic. Finally in In Sec. IV we draw some conclusions and give prospects for future developments of the work.

II. MODEL

A standard way to model the dynamics of resonantly-driven polaritons in a planar microcavity is to use a Gross-Pitaevskii (GP) equation²² for coupled cavity and exciton fields (Ψ_C and Ψ_X) generalized to include the effects of the resonant pumping and decay ($\hbar = 1$):

$$\partial_t \begin{pmatrix} \Psi_X \\ \Psi_C \end{pmatrix} = \begin{pmatrix} 0 \\ F \end{pmatrix} + \left[H_0 + \begin{pmatrix} g_X |\Psi_X|^2 & 0 \\ 0 & V_C \end{pmatrix} \right] \begin{pmatrix} \Psi_X \\ \Psi_C \end{pmatrix},$$

where the single particle polariton Hamiltonian H_0 is given by

$$H_0 = \begin{pmatrix} \omega_X - i\kappa_X/2 & \Omega_R/2 \\ \Omega_R/2 & \omega_C(-i\nabla) - i\kappa_C/2 \end{pmatrix},$$

and

$$\omega_C(-i\nabla) = \omega_C(0) - \frac{\nabla^2}{2m_c}$$

is the cavity dispersion, with the photon mass $m_C = 5 \times 10^{-5} m_0$ and m_0 the bare electron mass. For our simulations we assumed a flat exciton dispersion relation $\omega_X(\mathbf{k}) = \omega_X(0)$, set the exciton-photon detuning to zero $\delta_{ex-ph} = \omega_X(0) - \omega_C(0) = 0$ and set this energy value as the reference of the zero energy. The parameters Ω_R , κ_X and κ_C are the Rabi frequency and the excitonic and photonic decay rates respectively and have been given values close to the usual experimental ones: $\Omega_R = 5.1$ meV, $\kappa_X = 0.05$ meV, and $\kappa_C = 0.08$ meV²¹. In this model polaritons are injected into the cavity by four coherent and monochromatic laser fields with pump intensity f_p and Gaussian spatial profiles with σ_p of 20 μm : $F(\mathbf{x}) = \sum_{i=1}^4 f_p e^{i\mathbf{k}_{p_i} \cdot \mathbf{x}} e^{-(\mathbf{x}-\mathbf{x}_i)^2/2\sigma_p^2}$. Where \mathbf{k}_{p_i} are the four wave vectors of the four pumps that we fix to have the same modulus $|\mathbf{k}_{p_i}| = |\mathbf{k}|$. In order to ensure that the phase of the polariton fluid is not imposed by the laser pumps in the central region of the system we set to zero the pump intensity outside of a $\sigma_{pin} = 9\mu\text{m}$ radius circle. The exciton-exciton interaction strength g_X is set to one by rescaling both the cavity and excitonic fields and the pump intensities. The numerical solution of the GP equation is obtained over a two-dimensional grid (of 512×512 points) in a box with sides of $150 \times 150 \mu\text{m}^2$ using a fifth-order adaptive-step Runge-Kutta algorithm.

All the analysed quantities are taken when the system has reached a steady state condition after a transient period of 200 ps.

To understand the role of the subsonic character of the fluid in the annihilation of a vortex-antivortex pair we define the local fluid velocity $v_f(\mathbf{x}) = \hbar|\mathbf{k}(\mathbf{x})|/m_{LP}$, where m_{LP} is the lower polariton mass and $\mathbf{k}(\mathbf{x})$ is the locally evaluated derivative of the phase at the point \mathbf{x} . Moreover we define the quantity $c_s(\mathbf{x}) = \sqrt{\hbar g_{LP} |\Psi_{LP}(\mathbf{x})|^2 / m_{LP}}$, where $|\Psi_{LP}(\mathbf{x})|^2$ and g_{LP} are the local polariton density and the coupling constant. Since in the local density approximation $c_s(\mathbf{x})$ corresponds to the speed of sound defined in the case of high densities^{23,24}, we can take it as definition of a *generalized local speed of sound* valid also for low polariton densities and we define a generalized Mach number:

$$M(\mathbf{x}) = \frac{v_f(\mathbf{x})}{c_s(\mathbf{x})} = \frac{\hbar|\mathbf{k}(\mathbf{x})|/m_{LP}}{\sqrt{\hbar g_{LP} |\Psi_{LP}(\mathbf{x})|^2 / m_{LP}}}. \quad (1)$$

This will allow to establish a direct correlation between the subsonic or supersonic character of the fluid and the annihilation of a V-AV pair. Since the subsonic character of the fluid is induced by polariton-polariton interaction, this corresponds to studying the role of polariton-polariton interactions in the annihilation of the pairs and of the vortex-array. Although other techniques, like the study of the vortex-antivortex correlation function²⁵ or Reynolds-averaged Navier-Stokes equations²⁶, can be used to address this problem, we chose to focus on the study of the Mach number since it since it allows a simple and clear physical understanding of the V-AV merging process.

III. RESULTS

The mechanism lying beneath the annihilation of V-AV pairs and the role of polariton-polariton interactions in this annihilation can be better highlighted by studying the system behaviour as a function of the pump intensity and therefore of the polariton density. We start by studying the system in the two limiting cases of very low and very high pump intensities.

In the low intensity case the polariton density lies on the lower branch of the bistability curve everywhere in space and the system behaviour is purely linear. In this regime (fig. 1A-C), the formation of an array of vortices and antivortices is observed as in^{17,18,21}. Here, like in²¹, the shape and size of the unit cell only depend on the geometry of the pumping configuration and on the angle of incidence of the laser beams. Since we use four pumps with $|\mathbf{k}| = 0.7\mu\text{m}^{-1}$ the formed array has square unit cells with unit cell size of approximately $9\mu\text{m}$ (fig. 1A-C). This interference pattern generates an array of vortices (with a clockwise phase winding from $-\pi$ to π) and antivortices (with an anti-clockwise phase winding from $-\pi$ to π) that

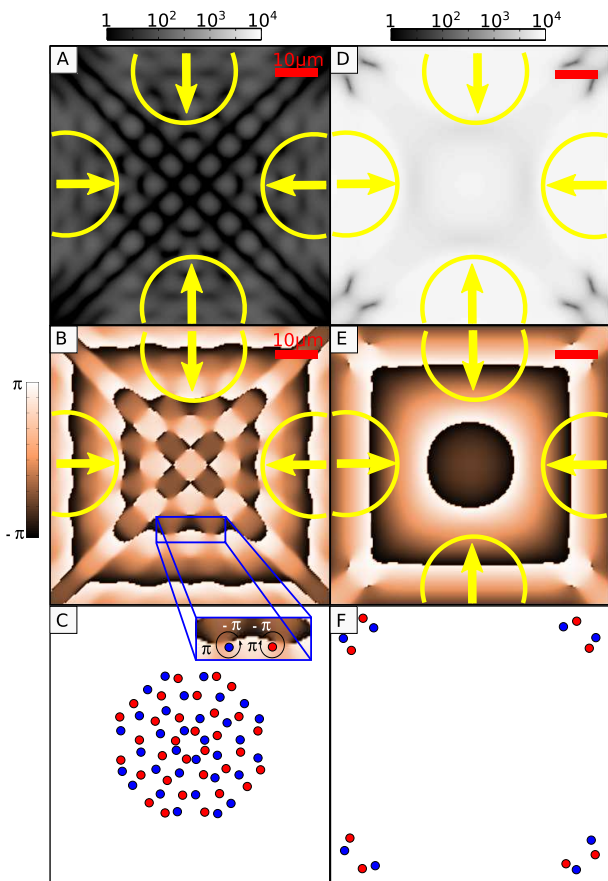


FIG. 1: Numerical Real space emission intensity, phase, and vortex distribution in the low and high density regimes. The four polariton ensembles are generated by pumps at $k_{pump} = 0.7 \mu\text{m}^{-1}$ in different directions (yellow arrows) with $\omega_{pump} = -2.25$ meV. A: Real space image in the low density regime with pump intensity $f_p = 0.07f_{th}$, a square interference pattern with a unit cell of about $9\mu\text{m}$ is formed. D: real space image in the high density regime $f_p = 3.33f_{th}$. B and E: Phases corresponding to A and D showing an array of vortices and antivortices (B) and no phase modulations (E). C and F: Vortices (red) and anti-vortices (blue) distribution in real space corresponding to phase diagrams B to E.

is therefore due to purely linear mechanism (fig. 1C and inset for the definition of vortices and antivortices).

In the opposite limit the pump intensity is strong and polariton-polariton interactions play a dominant role. This regime is characterised by a polariton density everywhere lying on the upper branch of the bistability curve. As it can be seen in fig. 1D-F, in this regime the array of vortices and antivortices has completely disappeared due to the renormalisation of the lower-polariton branch.

From this, we can deduce that in the transition region between the low and the high density regimes vortices and antivortices either are expelled from the fluid or annihilate each-other. In order to investigate this transition we vary the pump intensity around the threshold value (f_{th}) at which V-AV pairs disappear. Note that in this non-homogeneous system the density is different in the

regions within and outside the pumping spot, resulting in four threshold intensities: two for increasing and two for decreasing pump intensities. When the intensity of the four pumps increases the regions directly pumped by the lasers jump from the lower to the upper branch of the bistability curve (first threshold). At this point, since polaritons have a finite lifetime, the central region between the four laser spots is still in the lower part of the bistability curve. As the pumps intensity is further increased also the central region eventually jumps from the lower to the upper branch (second threshold). This threshold corresponds to the intensity f_{th} at which V-AV pairs disappear. Similarly one can observe two threshold for decreasing pump intensity.

Figures 2 A-C(D-F) represent the polariton distribution (phase) for increasing pump intensity from just below to just above f_{th} . In fig. 2A ($f_p = 0.66f_{th}$) some remaining of the interference pattern of fig. 1A is still visible together with four dark segments surrounding the centre of the image that correspond to four V-AV pairs. In this plot of the intensity distribution each V-AV pair looks like a straight dark segment rather than like two separated vortices because the core of the vortex is extremely close to the core of the antivortex. The fact that these four dark segments correspond to V-AV pairs is confirmed by the phase distribution of fig. 2D where the two phase rotations of the vortex and of the antivortex are visible in correspondence of each dark segment of fig. 2A. For the sake of clarity the phase distribution of the V-AV pair delimited by the red square in fig. 2D is also reported enlarged in fig. 2G. When the pump intensity is increased (fig. 2B ($f_p = 0.8f_{th}$) and fig. 2C ($f_p = 1.0f_{th}$)) the vortex and the antivortex cores get closer (i.e. the dark segments become shorter) until the four V-AV pairs disappear. For even higher pump intensities the density distribution becomes homogeneous as in fig. 1D. Again this moving closer and merging of the V-AV is confirmed by the corresponding phase distributions (fig. 2D-F), and by the corresponding zoom of the regions delimited by the red squares (fig. 2G-I).

This detailed analysis of the density and phase distributions and of the "on site" annihilation of V-AV pairs shows that the disappearance of the vortex array is not due to the expulsion of the vortices from the fluid. Therefore, the mechanism lying beneath the V-AV annihilation cannot be ascribed to a simple renormalisation of the lower polariton branch. First, the effect of the renormalisation is to decrease the wave vector of the injected polaritons and correspondingly to increase the size of the interference pattern of the array therefore leading to the expulsion of vortices from the fluid rather than the observed merging. Second, it can be seen in fig. 2C that when V-AV annihilation takes place, part of the interference pattern is still visible. Third, vortices and antivortices can exist in a fluid at rest, so the renormalization of the lower polariton branch can not justify *per se* the disappearance of the array.

Moreover, since all the plots show the system steady

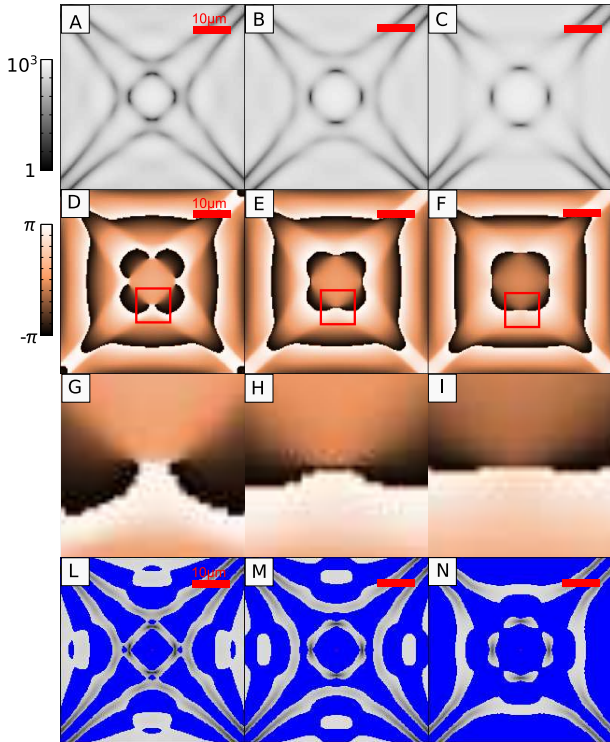


FIG. 2: A-C: Numerical Real-space emission intensity for three different pump intensities. D-F: real space phase diagrams corresponding to the pumping condition of A-C. G-I: Enlargement of the red squared area in D-F. L-N: Mach charts corresponding to panels A-C, where the fluid is supersonic the Mach chart show the real-space emission intensity at that point, when the fluid is subsonic the Mach chart is blue. A V-AV pair is clearly visible in G, V and AV have almost merged in H, and no vortex pair is present any more in I. The pump parameters are the same as in fig. 1 and the pump intensities (from left to right) are $f_p = 0.66f_{th}$, $0.8f_{th}$, $1.0f_{th}$.

state for a given pump intensity, the fact that vortices coexist with antivortices means that their position is the result of a tradeoff between vortex-vortex and vortex-antivortex interactions, direction of the polariton flow, and relative phase of the four pumps. In other words, this means that vortices and antivortices are not completely free to move in the system and therefore V-AV annihilation cannot be simply ascribed to V-AV attraction. In fact, if vortices and antivortices are completely free to move, due to their mutual attraction they must always annihilate, and the only possible steady state must be completely free of vortex dislocations independently from the intensity of the laser pumps. Finally, it is worth noting that due to the choice of the continuous-wave resonant pumping setup no exciton-reservoir is present in the system and all polaritons have the same energy therefore no trapping mechanism can be advocated to explain this annihilation as in¹⁸.

To have a physical insight of the mechanism lying beneath V-AV annihilation it is useful to study the Mach-

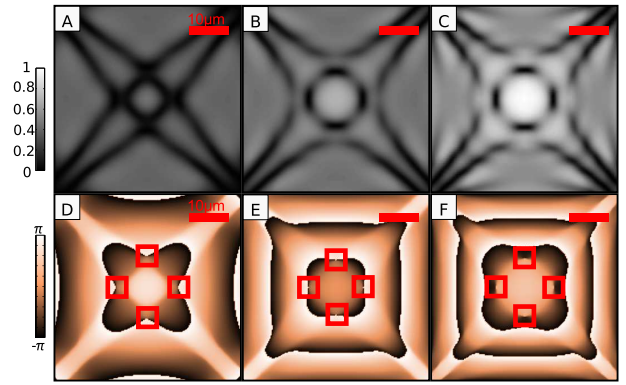


FIG. 3: A-C: Numerical real-space emission intensity (in linear greyscale) . D-F: Real space phase diagram (color scale) corresponding to A-C. The red squares indicate the regions where vortex and anti-vortex pairs have been annihilated. The plots correspond to pumps with increasing in plane momenta from left to right: $k_{pump} = 0.5, 0.7$ and $0.9 \mu\text{m}^{-1}$. In each case the pump intensity is chosen in order to drive the system at the threshold at which V-AV annihilation occurs ($f_p(B) = 4.33f_p(A)$, $f_p(C) = 8.66f_p(A)$).

number charts in fig. 2L-N where blue regions correspond to a subsonic character of the fluid and grey regions correspond to a supersonic character. Clearly as the pump intensity is increased (from left to right) the polariton density increases and the regions where the fluid is subsonic becomes wider and wider. This spreading of the subsonic regions rearranges the polariton flow therefore changing the steady state position of vortices and antivortices until, when the pump threshold f_{th} is reached, vortices and antivortices annihilate. This mechanism can also be understood observing that vortices cannot enter subsonic regions since they are intrinsically related to regions where the fluid is supersonic, because at the centre of their core the fluid density vanishes while the fluid velocity does not and because a subsonic fluid tends to avoid strong phase modulations²⁷. Since in our system the subsonic regions surround the V-AV distribution, when the pump intensity changes the polariton flows rearrange letting vortices overlap and annihilate with antivortices. This is different from the case of atomic Bose-Einstein condensates where a single vortex cannot be destroyed by the superfluid character of a quantum fluid since the total angular momentum of the system must be conserved. Here, polariton-polariton interactions can affect the fluid distribution and make a vortex overlap with an antivortex, so that V-AV pairs can be completely suppressed in the fluid.

An additional proof of the correlation between the subsonic character of the fluid and the disappearance of V-AV pairs is given by the fact that higher sound velocities (i.e. higher polariton densities) are needed to destroy the array when polaritons have higher velocities. Figures 3 A-C show the real space distribution of the photonic field for three different increasing velocities of the injected polaritons ($k_{pump} = 0.5, 0.7$ and $0.9 \mu\text{m}^{-1}$) and a

pump intensity corresponding to the threshold ($f_p = f_{th}$) where V-AV pairs disappear. We find that the polariton density, and therefore the sound velocity, at which V-AV pairs annihilate is higher when the velocity of the injected polaritons is higher (see fig. 3 D-F where no phase cut corresponding to a V-AV pair can be observed in the four regions delimited by the red squares). This confirms that polariton-polariton interactions, causing the widening of the subsonic regions and the consequent rearrangement of the polariton flows, induce the disappearance of V-AV pairs.

IV. CONCLUSIONS

We have investigated the washing out of vortex-antivortex lattices in exciton polariton systems as a function of the polariton density. Our detailed analysis shows that V-AV annihilation and the washing out of the vortex lattice is due to polariton-polariton interactions that, through the renormalization of the lower polariton branch, induce the widening of the regions where the fluid is subsonic as the pump intensity is increased.

This widening induce a modification in the polariton flows therefore changing the steady state position of vortices and antivortices until a threshold pump intensity is reached at which V-AV merging takes place. The complete washing out of the V-AV array can take place because in our system the number of vortices is equal to the number of antivortices, i.e. the system has zero angular momentum. Our analysis applied to a system with net angular momentum could open the way to the study of vortex-vortex interactions.

Acknowledgments

We would like to thank C. Tejedor for the use of the computational facilities of the Universidad Autonoma de Madrid and I. Carusotto and F. M. Marchetti for useful discussions. This work has been partially funded by the Quandt project of the ANR France, by the POLATOM ESF Research Network Program and by the CLERMONT4 Network Program. MHS from the EPSRC (grants EP/I028900/2 and EP/K003623/2). A. B. is member of Institut Universitaire de France (IUF).

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