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On the Network Connectivity of Platoon-based Vehicular Cyber-Physical Systems

Dongyao Jia\textsuperscript{a}, Kejie Lu\textsuperscript{b}, Jianping Wang\textsuperscript{a}

\textsuperscript{a}Department of Computer Science, City University of Hong Kong, China.
\textsuperscript{b}Department of Electrical and Computer Engineering, University of Puerto Rico at Mayagüez, PR, USA.

Abstract

In the past few years, vehicular ad-hoc network (VANET) has attracted significant attention and many fundamental issues have been investigated, such as network connectivity, medium access control (MAC) mechanism, routing protocol, quality of service (QoS), etc. Nevertheless, most related work has been based on simplified assumptions on the underlying vehicle traffic dynamics, which has a tight interaction with VANET in practice. In this paper, we try to investigate VANET performance from the vehicular cyber-physical system (VCPS) perspective. Specifically, we consider VANET connectivity of platoon-based VCPSs where all vehicles drive in platoon-based patterns, which facilitate better traffic performance as well as information services. We first propose a novel architecture for platoon-based VCPSs, then we derive the vehicle distribution under platoon-based driving patterns on a highway. Based on the results, we further investigate inter-platoon connectivity in a bi-directional highway scenario and evaluate the expected time of safety message delivery among platoons, taking into account the effects of system parameters, such as traffic flow, velocity, platoon size and transmission range. Extensive simulations are conducted which validate the accuracy of our analysis. This study will be helpful to understand the behavior of VCPSs, and will be helpful to improve vehicle platoon design and deployment.
keyword:
vehicular ad-hoc networking (VANET) platoon-based vehicular cyber-physical systems inter-platoon connectivity expected transmission delay
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1. Introduction

Vehicular ad-hoc networking (VANET) is a promising technique for future inter-vehicle communications. Vehicles driving under VANET environment can be regarded as a typical vehicular cyber-physical system (VCPS), which is characterized by the tight coupling between a vehicle’s physical dynamics (mobility) and the computing and communications aspects of the vehicle (Fallah et al., 2010). In a typical VCPS, VANET communication plays a critical role in both vehicle safety applications and infotainment services. Therefore, a comprehensive study on VANET performance in various traffic\(^1\) conditions is an essential topic for VCPS, including the performance of network connectivity, medium access control (MAC) mechanism, routing protocol, quality of service (QoS), etc.

In the past few years, a lot of studies have been conducted on aforementioned issues. For example, VANET connectivity has been extensively investigated in different highway scenarios (Yousefi et al., 2008; Sou and Tonguz, 2011; Wu, 2009; Neelakantan and Babu, 2012; Ng et al., 2011), among which some important probability distributions have been obtained. Optimization of the IEEE 802.11p MAC mechanism is another hot topic which has been discussed in terms of both contention-free based and contention-based approaches (Katrin and Elisabeth, 2009; Almalag et al., 2012; Han and Dianati, 2012; Song et al., 2011; Park et al., 2012). To tackle the interaction between VANET performance and vehicle dynamics, the relationship of three fundamental issues: traffic flow, safety, and communications capacity, within a simple transportation system has been investigated in (Nekoui, 2010), which initiated a more comprehensive study combining transportation with communication fields and sought to address their mutual dependencies. A case study is illustrated in (Fallah and Sengupta, 2012), where a cooperative vehicle safety system is designed by a systematic CPS approach.

\(^1\)In this paper, “traffic” is limited to the context of vehicle transportation.
Nevertheless, from the perspective of physical process in the VCPSs, most previous work takes individual vehicles as the objects, seldom considering the behavior of a group of vehicles on the VCPS performance. In practice, some consecutive vehicles with close space on the same direction can naturally be grouped into a platoon, in which a non-leading vehicle maintains a small distance with the preceding one, as shown in Fig. 1. Platoon-based driving pattern in highway is regarded as a promising driving manner and has been verified to bring benefits in many ways (van Arem et al., 2006). First, since vehicles in the same platoon are much closer to each other, the road capacity can be increased and the traffic congestion may be decreased accordingly. Second, the platoon pattern can reduce the energy consumption and exhaust emissions considerably because the streamlining of vehicles in a platoon can minimize air drag. Third, steady platoon formation facilitates more efficient information dissemination and sharing among vehicles in the same platoon.

Although many platoon related issues have been studied in the past several decades, such as traffic performance optimization by managing and controlling platoon (Hall and Chin, 2005; Chen et al., 2006; Uchikawa et al., 2010; Puebboobpaphan, 2010), platoon control method called cooperative adaptive cruise control (CACC) (Puebboobpaphan, 2010) with the help of VANET, etc., there is still a lack of the analysis and evaluation of the impact of platoon-based driving pattern on the performance of VCPS. For instance, with a given traffic flow rate, excessive vehicles in a single platoon could lead to transmission delay and packet loss due to contention-based CSMA/CA access mechanism of IEEE 802.11p, which cannot guarantee stringent real-time delivery for some critical safety applications such as collision avoidance and platoon control. Moreover, inter-platoon spacing is enlarged in this case and would impair the VANET connectivity among consecutive platoons. On the other side, few vehicles in a single platoon would discount the benefits obtained by vehicle platooning. Finally, intra-platoon communication could be interfered by adjacent platoons because of the smaller inter-platoon spacing in practice.
To summarize, platoon-based driving pattern can reshape the whole traffic flow distribution, compared to the original individual driving pattern, which could significantly affect the VANET communication in the VCPSs. Therefore, it is critical to re-evaluate the communication performance of VANET under platoon-based driving pattern. To this end, in this paper, we first propose a novel architecture for platoon-based VCPSs, taking into consideration the tight interaction between platoon dynamics and VANET. Then we analyze the probability distribution of platoon-based traffic flow. Based on the result, we further investigate inter-platoon connectivity and calculate the expected transmission delay between adjacent platoons. Finally, we conduct extensive simulation studies to validate the analytical results, taking into account the effect of various system parameters, such as traffic flow, velocity, platoon size and transmission range. To the best of our knowledge, this is the first time to address the VANET connectivity of platoon-based VCPSs.

The organization of this paper is described as follows. In Section 2, we first review the related work, especially on VANET connectivity analysis and platoon dynamics in various traffic conditions. In Section 3, we propose a general platoon-based VCPS architecture, illustrate all modules of the architecture, then specify a particular one to be investigated. In Section 4, we derive analytical expression of inter-platoon spacing distribution and calculate the expected message transmission delay of inter-platoon. In Section 5, we conduct extensive simulation experiments to validate the theoretical analysis, before concluding the paper in Section 6.

2. Related Work

In this section, we will first review related work on VANET connectivity. Next, we give a short overview of the platoon dynamics because it essentially reflects the physical process in VCPSs and plays a critical role on VANET performance. Finally, we highlight our contributions by comparing our work with existing ones.
2.1. VANET Connectivity

VANET connectivity is the fundamental issue regarding VANET performance and some important probability distributions have been obtained in the literature. In (Yousefi et al., 2008), the authors investigated connectivity between vehicles in a sparse traffic condition where the number of vehicles passing the observer point is assumed to follow a Poisson process and vehicle speeds are independent and identically distributed. It has been shown that increasing the traffic flow and the vehicle transmission range facilitate inter-vehicle connectivity. Moreover, if the variance of the speed distribution is increased, then, in case of normally distributed speeds with fixed average value, the connectivity is improved in the free-flow traffic state. Different from conventional graph-theoretic approach, network connectivity is investigated in terms of a physical layer-based QoS constraint in (Neelakantan and Babu, 2012), i.e., the average route bit error rate (BER) meeting a target requirement. Lifetime of individual links in a VANET is investigated in (Yan and Olariu, 2011), analytical results show that link duration is subject to log-normal distribution.

To effectively transmit safety message in such intermittently connected networks, an innovative Store-Carry-Forward scheme has been proposed which exploits opportunistic connectivity between vehicles moving on opposing directions to achieve greedy data forwarding (Kesting et al., 2010a; Agarwal, 2012; Baccelli et al., 2012; Sou and Tonguz, 2011). In (Kesting et al., 2010a), the authors proposed transversal message hopping strategy to transfer message between consecutive vehicles. They derived analytical probability distributions for message transmission times for a Poissonian distance distribution between equipped vehicles. Agarwal et al. studied message propagation (Agarwal, 2012) in a 1-D VANET where vehicles are Poisson distributed and move at the same speed but on either direction on a bi-directional roadway. They derived the upper and lower bounds for the average message propagation speed, which provided a hint on the impact of vehicle density on the message propagation. In (Baccelli et al., 2012), the authors analyzed the information propagation speed in bi-directional highways. The conclusion shows that under a certain threshold
of vehicle density, information propagates on average at the vehicle speed, while above this threshold, information propagates increases quasi-exponentially with respect to vehicle density. In addition, to enhance the connectivity in VANET, roadside units (RSUs) can also be deployed to forward information between disconnected vehicles (Sou and Tonguz, 2011).

In summary, most previous work assumes that vehicles drive in free traffic state with sparse density, i.e., each vehicle runs randomly and independently, and thus, the interaction between vehicles, for example the car-following model, is seldom taken into account.

2.2. Platoon Dynamic

Platoon dynamics normally describes the transient and steady responses of platoon, such as intra/inter-platoon spacing and velocity trajectory of each vehicle, etc., under a certain spacing policy and control strategy. As already noted, platoon dynamics can dramatically affect VANET communication. Therefore, it is important to understand the characteristics of platoon dynamics under different spacing policies and control strategies.

In (Seiler et al., 2004), the authors analyzed disturbance propagation in a platoon and showed error amplification of intra-platoon spacing under a predecessor-following control strategy, in which each vehicle only has the relative position to its preceding vehicle. To maintain constant intra-platoon spacing, predecessor-leader control strategy (Rajamani et al., 2000) has been proposed wherein each vehicle should get information from both its preceding vehicle and the platoon leader. To realize this strategy, the CACC has been proposed to maintain the stability of a given platoon (Fernandes, 2012).

2.3. Our Contributions

Compared to existing studies, the contributions in this paper can be summarized as follows. First, we investigate the impact of vehicle platooning on the traffic flow by a novel architecture for platoon-based VCPSSs. Based on the analysis, we try to explore the characteristics of VANET performance from the
VCPS’ perspective, especially focusing on inter-platoon connectivity in intermediate traffic flows. The obtained results can comprehensively demonstrate the fundamental relationships among traffic flow, platoon parameters and VANET connectivity, which could be utilized as the reference for VCPS performance optimization.

3. A Novel Architecture for Platoon-based Vehicular Cyber-physical Systems

In this section, we first propose a general architecture for platoon-based VCPSs. We then specify one particular instance which will be investigated in this paper.

3.1. Architecture for Platoon-based VCPSs

Platoon-based VCPSs describe vehicular applications, such as safety applications or infotainment services, in a VANET environment from the CPS’ perspective where each vehicle drives in a platoon-based pattern. The two main processes of the system are the networking/communication process which implements information dissemination upon the request of VANET applications, and the platoon mobility process that is determined by the control strategy of the platoon and the received state information of neighborhood.

To demonstrate the two different processes as well as their relationship, we propose an architecture for platoon-based VCPSs, where we jointly consider VANET operation and traffic dynamics.

place Fig. 2 about here

Fig. 2 illustrates a general architecture for platoon-based VCPSs. In the envisioned architecture, the unity of vehicle is composed of two parts: platoon-based mobility/control model which regulates the vehicle dynamics under a platoon-based driving pattern, and networking/communication model that generalizes the networking request of VANET applications of a vehicle, such as the rate of message generation, transmission range, networking topology, etc.
Platoon mobility process can be presented such that the original individually driven vehicles are formed into series of platoons under the regulation of a certain platoon mobility control model. *Platoon parameters* as the reference input of the control model describe the expected platoon profile, such as platoon size, intra-platoon spacing and inter-platoon spacing. As a result, platoon mobility process reshapes the whole traffic flow into *platoon-based distribution*. On the other hand, networking process mainly demonstrates data dissemination on the request of platoon-based VCPS application, which may exhibit different VANET performance under various platoon-based traffic flow scenarios, namely *platoon-based VANET*. For a typical platoon-based VCPS, there exists a tight couple between the platoon mobility process and the networking process, which can be explored from two sides:

1. To illustrate how the two different processes affect the platoon-based VANET performance, we take the example of collision risk warning application, where each vehicle periodically broadcasts its current kinematic status (e.g., location, speed, acceleration/braking information) to neighborhood in the same platoon. In case of small platoon size and large inter-platoon spacing, packet delay and loss seldom happen within a single platoon even at high rate of message generation; while under the condition of large platoon size and small inter-platoon spacing, packet delay and loss would rise sharply at the same message generation frequency. If the message generation frequency is decreased, the ratio of packet loss would become lower accordingly. Therefore, the performance of platoon-based VANET is determined by both the platoon mobility process and the networking process jointly.

2. To illustrate how the two processes affect the performance of the platoon-based mobility, we address the CACC system with the help of VANET. The main control objective of CACC is to maintain a desired distance among inter-vehicles or inter-platoons. To this end, the control strategy normally needs the status of neighboring vehicle which are acquired
by means of inter-vehicle communication. Thus CACC system can be modeled as a networked control system wherein feedback loop design couples both VANET and platoon mobility. Some uncertainties of practical VANET could have negative impact on the control performance, such as packet loss and probabilistic transmission delay. In (Lei et al., 2011), C. Lei et al. investigated platoon stability of a CACC controller in the presence of imperfect communication. Experimental results indicate that beacon sending frequency and packet loss ratio have significant impact on the performance of the evaluated CACC controller. Specifically, lower beacon sending frequency and higher packet loss ratios of vehicle-to-vehicle (V2V) communication will impair the CACC controller performance on platoon stability, which might lead to collisions.

To summarize, the platoon-based VCPSs heavily depend on both networking process and control process, which closely integrate communication, computation and physical processes together.

3.2. Mobility Specification for Platoon-based VCPSs

3.2.1. Log-normal Distribution of Traffic Flow

In this paper, we consider a traffic scenario of straight two-lane highway that goes on opposite directions (which means overtaking is not allowed for the vehicle). In Fig. 2, platoon-based traffic flow is formed from individual random traffic flow. To model the original traffic flow, we adopt the statistics of time headway as the fundamental parameter to describe the traffic flow distribution.

Time headway is defined as the time (or, equivalently, distance) between two consecutive vehicles passing the same point and traveling on the same direction. Normally it is assumed that time headways are independent and identically distributed random variables. Since the 1960s, many time headway models have been developed. The typical representatives of such distribution models include exponential distribution, normal distribution, gamma distribution, and log-normal distribution. It is confirmed in (Ha et al., 2012) that log-normal
distribution fits well the intermediate traffic demand level, between about 700 and 1700 vehicles per hour (vph). According to (Chen et al., 2010), Chen et al. employed a unified car-following model integrated with Markov process description to simulate different driving scenarios. Headway time is verified to be log-normally distributed by NGSIM Trajectory Data. Therefore, we assume that the original distribution of individual vehicle is log-normal in this paper, which is expressed as:

\[ f(t_h; \mu, \sigma, \tau) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(\log(t_h - \tau) - \mu)^2}{2\sigma^2} \right), t_h > \tau \]  

(1)

where \( t_h \) represents the possible value of the time headway, \( \tau \) is the location parameter, representing the minimum value of the time headway, \( \mu \) is the scale parameter and \( \sigma \) is the shape parameter. Accordingly, we can calculate the mean and variance of the time headway:

\[ \mu(T_h) = \tau + e^{\mu + \frac{1}{2} \sigma^2} \]  

(2)

\[ \sigma^2(T_h) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) \]  

(3)

In the steady state of a traffic flow, we assume that vehicles run at about the same velocity \( v_{stb} \), which is a constant. Therefore, we can get the corresponding distance headway for individual driving patterns:

\[ s_h \approx v_{stb} t_h. \]  

(4)

Obviously, \( s_h \) is subject to log-normal distribution.

In addition, we assume that all vehicles run at the same velocity \( v_{stb} \) after forming the platoons and driving in the steady state.

3.2.2. The Platoon Driving Strategy and Platoon Parameters

To describe the distribution of the formed series of platoons, we let intra-platoon spacing be the distance between adjacent vehicles in the same platoon,
and inter-platoon spacing be the gap between the tail of the preceding platoon and the leader of the next platoon. Platoon parameters are illustrated in Fig. 3, where $P^i$ means the $i$-th platoon, $C^i_j$ denotes the $j$-th vehicle in $P^i$, $s^i_j$ denotes the intra-platoon spacing between $C^i_j$ and $C^i_{j-1}$, and $S^i$ is the inter-platoon spacing between $P^{i-1}$ and $P^i$. Note that, for convenience, the platoon index is skipped when we discuss a single platoon.

To facilitate further discussions, we summarize important notations in Table 1, where variables have been sorted according to the alphabetic order.

**place Table 1 about here**

Due to the strong interaction among adjacent vehicles within the same platoon, the most common vehicle mobility model is the car-following model, which can effectively describe ACC-equipped platoon dynamics (Kesting et al., 2010b). In this paper, we consider that all vehicles, except the leaders, move according to a car-following model. Specifically, we apply a typical car-following model for ACC-equipped vehicles, known as the Intelligent Driver Model (IDM) (Treiber and Hennecke, 2000), which is based on the stimulus-response approach and can be expressed as follows:

\[
s^*_j(t) = s_0 + v_j(t)T_0 + \frac{v_j(t)\Delta v_j(t)}{2\sqrt{ab}} \tag{5}\]

\[
a_j(t) = a\left[1 - \left(\frac{v_j(t)}{v_0}\right)^4 - \left(\frac{s^*_j(t)}{s_j(t)}\right)^2\right] \tag{6}\]

where $s^*_j(t)$ is the desired gap to the preceding vehicle and the other parameters can be found in Table 1. In the IDM, the instantaneous acceleration consists of a free acceleration on the road where no other vehicles are ahead $a[1 - (v_j(t)/v_0)^4]$, and an interaction deceleration with respect to its preceding vehicle $-a(s^*_j(t)/s_j(t))^2$.

Accordingly, we can derive the intra-platoon spacing in the steady state:

\[
s_{stb} = \frac{s_{stb}^2}{\sqrt{1 - \left(\frac{v_{stb}}{v_0}\right)^4}} = \frac{s_0 + v_{stb}T_0}{\sqrt{1 - \left(\frac{v_{stb}}{v_0}\right)^4}} \tag{7}\]
For all platoon leaders, on the other hand, we assume that they all run at the equal velocity $v_{stb}$ in the steady state since it has relatively long distance to the preceding platoon.

### 3.3. Networking Specification for Platoon-based VCPSs

In this paper, our objective is to evaluate VANET connectivity of platoon-based VCPSs specifically for VANET safety applications. Cooperative vehicular safety application is one critical issue of VCPSs which primarily provides vehicular status through inter-vehicle communication to neighbors to avoid dangerous situations beforehand. To this end, two types of message transmissions need to be handled:

1. Beacon message dissemination: A vehicle is required to periodically disseminate its current kinematic status to its neighboring vehicles.
2. Safety message delivery: The critical safety message should be timely disseminated to the following vehicles.

For the former, the major problem is to deal with packet loss due to the MAC contention for dense vehicles within the same platoon, while the latter mainly tackles packet delay because of possible disrupted inter-platoon connectivity. The main objective in this paper is to evaluate the performance of safety message transmission for platoon-based VCPSs.

Upon the requirement for the real-time safety application, we define the beacon frequency $f_{bsm}$, normally in range of 3-10Hz. Thus each vehicle can timely collect all needed local information from neighbors, such as acceleration, velocity, location, direction, etc., and can maintain its local topology accordingly, which contains the list of one-hop neighbors who are leading, following or moving on the opposite direction, respectively.

In the following parts, we identify some assumptions and models applied in platoon-based VCPSs, including the VANET protocol layers, the platoon topology and message dissemination scheme.
3.3.1. Protocol Layers

For the physical layer, we only consider the transmission range as the major impact on VANET connectivity. Each vehicle is assumed to have the same fixed minimum transmission range \( R \) within which reliable V2V communication is guaranteed. In addition, we do not take into account the impact of a highway’s lane width on the communication distance as its value is negligible compared to \( R \).

For the MAC layer, we consider the standard IEEE 802.11p implemented on each vehicle, where an Enhanced Distributed Channel Access (EDCA) MAC protocol is designed based on that of IEEE 802.11e with some modifications to the transmission parameters (Han and Dianati, 2012).

Note that different MAC mechanisms are applied on different transmission manners. For unicast transmission, when a vehicle tries to access the medium and finds the channel busy, it delays the medium access for the duration of backoff upon the defined value of the contention window (CW). If no acknowledgement is received (e.g. a collision occurs) the CW size is increased and the process starts over. While for broadcast transmission, message is not retransmitted and the CW size maintains unchanged due to the lack of acknowledgement mechanisms in the MAC layer.

3.3.2. Platoon Topology and Safety Message Dissemination Scheme

We assume that all vehicles in the same platoon can directly communicate with each other, which means that the platoon length does not exceed one hop transmission range, as shown in Fig. 4. The leader is the leading vehicle in the platoon, which is responsible for creating and managing the platoon. The leader also acts as the receiver obtaining the information originally disseminated from the preceding platoon and then broadcasts it to other vehicles within the same platoon. Moreover, a platoon leader can be selected as a message carrier to forward the message from vehicles on the opposite direction. The tail vehicle locates at the end of a platoon and is responsible for communicating with the following platoon leader.
With the defined topology, the safety message dissemination among platoon-based traffic flow alternately undergoes two stages: intra-platoon message dissemination and inter-platoon message delivery. The corresponding scheme is proposed as follows:

1. Intra-platoon message dissemination: Since all vehicles within the same platoon can communicate with each other, safety message from the preceding platoon can be instantly broadcasted to all vehicles by the platoon leader. Thus the message transmission delay in this stage is negligible.

2. Inter-platoon message delivery: In case of connected platoons, the tail vehicle can directly forward the information to the following platoon leader. In case of disrupted inter-platoon connectivity, the tail vehicle (as sender) would first forward the information to the vehicles on the opposite direction and choose the one closest to the following platoon leader (as receiver) as the forwarder. If the forwarder cannot directly communicate to the receiver, it would try to retransmit the message to the connected preceding vehicle and set it as new forwarder until the message cannot be retransmitted. Then the final forwarder would store the information and continuously broadcast it. Eventually, the information might be received by the following platoon on the original direction. The process of message delivery is illustrated in Fig. 5. This greedy forwarder-selecting scheme delivers inter-platoon messages as soon as possible and can significantly reduce the transmission delay among platoons.

4. Connectivity Analysis of Platoon-based VCPSs

In this section, we investigate inter-platoon connectivity in VCPSs. We first derive analytical expression of inter-platoon spacing distribution. Then we calculate the expected message transmission delay of inter-platoon upon the distribution of platoon-based traffic flow.
4.1. Distribution of Inter-platoon spacing

To simplify the analysis, we assume that all platoons are formed uniformly, i.e., they have the same platoon size \( n \) and the same IDM parameters. In this case, the platoon index is skipped for platoon-based parameters. Thus, in Fig. 3, the inter-platoon spacing can be expressed as follows:

\[
S = S_L - L
\]  

(8)

Accordingly, we have the following Lemma.

**Lemma 1.** Assume all platoons are formed uniformly and controlled by IDM, inter-platoon spacing is lognormal distributed in the traffic steady state with all platoon leaders driving at the same velocity \( v_{stb} \).

**Proof.** In the steady state, inter-platoon spacing can be given by

\[
S = S_L - L = v_{stb} \sum_{j=1}^{n} t_{h,j} - (n-1)s_{stb} = v_{stb} \sum_{j=1}^{n} t_{h,j} - n v_{stb} \theta T_0 = v_{stb} (\sum_{j=1}^{n} t_{h,j} - n \theta T_0)
\]

where

\[
\theta = \left( \frac{n-1}{n} \right) \frac{s_0}{v_{stb}} + \frac{1}{\sqrt{1 - \left( \frac{v_{stb}}{v_0} \right)^4}}
\]

(9)

We define \( t_{ph} = \sum_{j=1}^{n} t_{h,j} \), which represents the convolution of \( n \) independent lognormal random time headways. As shown in (Beaulieu and Xie, 2004), \( t_{ph} \) is approximately lognormal, where \( \mu_P \) and \( \sigma^2_P \) can be obtained by (Fenton, 1960):

\[
\sigma^2_P = \log \left[ \frac{\sum_{j=1}^{n} \sigma^2(T_h)_j}{\left( \sum_{j=1}^{n} \left( \mu(T_h)_j - \tau \right) \right)^2 + 1} \right]
\]

As all \( t_{h,j} \) are subject to the same distribution with the parameters \((\tau, \mu, \sigma)\), \( \sigma^2_P \) can be calculated by:

\[
\sigma^2_P = \log \left[ \frac{\sigma^2(T_h)}{n \left( \mu(T_h) - \tau \right)^2 + 1} \right]
\]

(10)
Accordingly, $\mu_P$ is calculated:

$$\mu_P = \log \left( \sum_{j=1}^{n} (\mu(T_h)_j - \tau) \right) - \frac{\sigma^2_P}{2} = \log \left( n(\mu(T_h) - \tau) \right) - \frac{\sigma^2_P}{2} \quad (11)$$

As a result, $t_{ph} \in \log N(n\tau, \mu_P, \sigma_P)$. Therefore, we can derive the PDFs of $S_L$ and $S$, respectively:

$$f_{S_L}(x) = \frac{1}{\sqrt{2\pi} \sigma_P(x - n\tau v_{stb})} \exp \left( -\frac{(\log(x - n\tau v_{stb}) - (\mu_P + \log v_{stb}))^2}{2\sigma^2_P} \right), \quad x > n\tau v_{stb}$$

$$f_S(x) = \frac{1}{\sqrt{2\pi} \sigma_P(x - n(\tau - \theta T_0) v_{stb})} \exp \left( -\frac{(\log(x - n(\tau - \theta T_0) v_{stb}) - (\mu_P + \log v_{stb}))^2}{2\sigma^2_P} \right), \quad x > n(\tau - \theta T_0) v_{stb}$$

Obviously, to guarantee realistic value of inter-platoon spacing after vehicle platooning, the following constraint can be obtained: $n(\tau - \theta T_0) v_{stb} \geq 0$, i.e.,

$$T_0 \leq \frac{\tau}{\theta} \quad (12)$$

For convenience, we choose the equal case of the Eq. (12) and in practice we have $v_{stb} < < v_0$, thus $\theta \approx 1$, then we get the appropriate time headway for a platoon: $T_0 \approx \tau$. Accordingly, the platoon length is calculated by:

$$L \approx n\tau v_{stb} \quad (13)$$

For convenience, we denote $\mu_D$ and $\sigma^2_D$ as follows.

$$\begin{cases}
\sigma^2_D = \sigma^2_P = \log \left[ \frac{\sigma^2(T_h)}{n(\mu(T_h) - \tau)^2} + 1 \right] \\
\mu_D = \mu_P + \log v_{stb} = \log \left( n v_{stb} (\mu(T_h) - \tau) \right) - \frac{\sigma^2_D}{2}
\end{cases} \quad (14)$$

Accordingly, $f_{S_L}(x)$ and $f_S(x)$ can be rewritten as follows:

$$f_{S_L}(x) = \frac{1}{\sqrt{2\pi} \sigma_D(x - L)} \exp \left( -\frac{(\log(x - L) - \mu_D)^2}{2\sigma_D^2} \right), \quad x > L \quad (15)$$

$$f_S(x) = \frac{1}{\sqrt{2\pi} \sigma_D x} \exp \left( -\frac{(\log(x) - \mu_D)^2}{2\sigma_D^2} \right), \quad x > 0 \quad (16)$$
4.2. Expected Message transmission delay Among Inter-platoons

In this section, we study the expected time of safety message transmission between two adjacent platoons (called expected transmission delay), which is regarded as a critical indicator for safety VCPSs applications. To simplify the analysis, we assume that vehicles run at the same velocity $v_{stb}$ on both directions of the highway and the safety message is generated by $C_s$ (platoon tail) at time $t_0$. Also we assume $C_s$ is located at the position point 0, $C_r$ (the following platoon leader) is at $-S_e$, and $C_{f0}$ (possible forwarder) is at $S_{w0} - (L + R)$ at time of $t_0$ in the same coordinate system, as illustrated in Fig. 5. Furthermore, we assume that the radio propagation speed is infinite and message store and forward processing time is negligible, which means as long as the continuous connectivity between $C_s$ and $C_r$ is built up, the safety message can immediately be transmitted to $C_r$. Based on the proposed scheme of inter-platoon message delivery, safety message can be delivered via two kinds of routing paths according to the platoon-based traffic flow spatial distribution at time $t_0$.

(1) when the inter-platoon spacing between $C_s$ and $C_r$ is less than $R$, i.e., $0 < S_e \leq R$:

In this case, the message would directly be transmitted from $C_s$ to $C_r$ and transmission delay $T_{d0} = 0$. This happens with probability

$$P_r(0 < S_e < R) = \int_{0}^{R} f_S(s_e) ds_e = \Phi \left( \frac{\log R - \mu_D}{\sigma_D} \right)$$

(17)

Thus the corresponding expected transmission delay $E[T_{d0}] = P_r(0 < S_e \leq R) \times T_{d0} = 0$.

(2) when the inter-platoon spacing between $C_s$ and $C_r$ is greater than $R$, i.e., $S_e > R$:

Obviously, the probability of this case is calculated by:

$$P_r(S_e > R) = \int_{R}^{\infty} f_S(s_e) ds_e = \Phi \left( \frac{\mu_D - \log R}{\sigma_D} \right)$$

(18)

In this case, the message would be first transmitted to the opposite vehicle, then conveyed by the selected forwarder, finally sent to $C_r$. According to the
greedy scheme for selecting forwarder, the first and final forwarder must be some platoon leader on the opposite direction since one hop communication is guaranteed for a single platoon. In Fig. 5, in case of \( S_{w0} - (L + R) = -L - R \), i.e., \( S_{w0} = 0 \), which means there exists one platoon leader at the limit of the transmission range on the desired direction. When \( S_{w0} \geq 0 \), the platoon leader can be selected as the first forwarder \( C_f_0 \).

Moreover, since platoons run at steady velocity \( v_{stb} \), the probability that a platoon leader occur at any position is approximately uniformly distributed in enough long observation time. Therefore, a rough estimate for the probability of \( C_f_0 \) in road segment of \([-L - R, s_{w0} - L - R)\), i.e., \( P_r(-L - R \leq S_{w0} - L - R < s_{w0} - L - R) = P_r(0 \leq S_{w0} < s_{w0}) \) is expressed:

\[
P_r(0 \leq S_{w0} < s_{w0}) = \int_{0}^{s_{w0}} \frac{1}{c} dx = \frac{s_{w0}}{c}, 0 \leq s_{w0} < c
\]

where \( 1/c \) is the probability density function of \( S_{w0} \).

On the other hand, in the aforementioned analysis, we can also regard the position \( S_{w0} = 0 \) as the median probability point between any two adjacent platoon leaders, therefore we can derive the value of \( c \) by the equation \( \int_{0}^{c} f_{S_L}(s_{w0}) ds_{w0} = \frac{1}{2} \), that is, \( c = e^{\mu_D} + L \). Accordingly, Eq. (19) is replaced by:

\[
P_r(0 \leq S_{w0} < s_{w0}) = \frac{s_{w0}}{e^{\mu_D} + L}, s_{w0} \leq e^{\mu_D} + L
\]

Next, we evaluate the expected transmission delay, which can be classified into two cases with respect to the position of \( C_s \).

(a) When \( C_f_0 \in (0, +\infty) \), i.e., \( S_{w0} > L + R \):

In this case, the first forwarder candidate \( C_f_0 \) is located east to the \( C_s \) and must be the only forwarder to transmit the message to \( C_r \), because inter-platoon spacing between \( C_f_0 \) and the preceding platoon tail \( C_r \) exceeds one hop transmission range. The probability of this case is calculated by:

\[
P_r(S_{w0} > L + R) = \begin{cases} 
\frac{e^{\mu_D} - R}{e^{\mu_D} + L}, & e^{\mu_D} \geq R \\
0, & \text{others}
\end{cases}
\]
The transmission delay $T_{d1}$ is related to the distance between $C_{f0}$ and $C_r$, which can be calculated by $T_{d1} = ((S_e - R) + (S_{w0} - (L + R)))/2v_{stb}$. We define the joint probability $P_r(S_{w0} > s_{w0}, S_e > s_e)$, and due to independence of $S_{w0}$ and $S_e$ in this case, $P_r(S_{w0} > s_{w0}, S_e > s_e) = P_r(S_{w0} > s_{w0}) \times P_r(S_e > s_e)$.

The corresponding expected transmission delay $E(T_{d1})$ is derived by:

$$E(T_{d1}) = E\left[\frac{(S_{w0} - (L + R)) + (S_e - R)}{2v_{stb}} \bigg| S_{w0} > L + R, S_e > R\right] \times P_r\left(\frac{S_{w0} > L + R, S_e > R}{\frac{S_{w0} - (L + R)}{2v_{stb}}}ight)$$

$$= \left(E\left[\frac{S_{w0} - (L + R)}{2v_{stb}}|S_{w0} > L + R\right] + E\left[\frac{S_e - R}{2v_{stb}}|S_e > R\right]\right) \times P_r\left(\frac{S_{w0} > L + R, S_e > R}{\frac{S_{w0} - (L + R)}{2v_{stb}}}\right)$$

$$= \left(\frac{1}{2v_{stb}} \int_{L+R}^{\infty} \frac{s_{w0}}{e^{\mu_D} + L} ds_{w0} + \frac{1}{2v_{stb}} \int_{R}^{\infty} \frac{s_e f_S(s_e) ds_e}{P_r(S_e > R)} \right) \times P_r(S_{w0} > L + R) \times P_r(S_e > R)$$

$$= \frac{(e^{\mu_D} + R + 2L)(e^{\mu_D} - R)}{4v_{stb}(e^{\mu_D} + L)} \Phi\left(\frac{\mu_D - \log R}{\sigma_D}\right)$$

$$+ \frac{e^{\mu_D} - R}{2v_{stb}(e^{\mu_D} + L)} e^{\mu_D + \frac{\sigma_D^2}{2}} \Phi\left(\frac{\mu_D + \sigma_D^2 - \log R}{\sigma_D}\right)$$

$$- \frac{(e^{\mu_D} - R)(L + 2R)}{2v_{stb}(e^{\mu_D} + L)} \Phi\left(\frac{\mu_D - \log R}{\sigma_D}\right)$$

(22)

(b) When $C_{f0} \in [-R - L, 0]$, i.e., $0 \leq S_{w0} \leq L + R$:

In this case, the safety message can be directly transmitted to the first forwarder $C_{f0}$, as illustrated in Fig. 6. The corresponding probability of $C_{f0}$ in this case is calculated by:

$$P_r(0 \leq S_{w0} \leq L + R) = \begin{cases} 
\frac{R + L}{e^{\mu_D} + L} e^{\mu_D} \geq R \\
1, \text{others} 
\end{cases}$$

(23)

Nevertheless, we cannot identify $C_{f0}$ as the only forwarder before it delivers the message to $C_r$ because another possible forwarder on the westbound may be out of the transmission range of $C_r$. We assume that there are $m$ consecutive platoons on the westbound and each inter-platoon spacing is less than $R$, i.e., each inter-platoon leader spacing (denoted as $S_{L,i}$) is less than $R + L$. Then the
safety message should be immediately forwarded to $C_{fm}$ and carried for some time before being delivered to $C_r$. Thus the possible distance traversed in this process is given by $S_e - R - (L + R - S_{w0}) - \sum_{i=1}^{m} S_{L,i}$, as shown in Fig. 6.

**place Fig. 6 about here**

The expected transmission delay $E(T_{d2})$ in this case is calculated by:

$$E[T_{d2}] = \sum_{m=0}^{\infty} E \left[ \frac{S_e + S_{w0} - (L + 2R) - \sum_{i=1}^{m} S_{L,i}}{2v_{stb}} \right] P_r \left( \frac{S_{w0} \leq L + R}{0 \leq S_{w0} \leq R + L} \right) \times P_r \left( \frac{S_e \leq R + \sum_{i=1}^{m} S_{L,i}}{0 \leq S_{w0} \leq R + L} \right)$$

(24)

where the case of $m = 0$ indicates that there is no available platoon leader as valid forwarder between $C_{f0}$ and $C_r$. Obviously, it is complicated to precisely calculate the expected value $E[T_{d2}]$. Here we adopt the method in (Agarwal, 2012) to estimate $E[T_{d2}]$. First, we discretize the westbound roadway segment $S_e - R$ into multiple cells with length $R + L$, as illustrated in Fig. 6. According to the analysis of (Agarwal, 2012), the optimal necessary condition for safety message continuous forwarding is: if each adjacent cell between $C_r$ and $C_s$ is occupied by at least one platoon leader, the safety message can be continuously forwarded by each platoon leader and eventually received by $C_r$.

The probability of each cell being occupied by at least one platoon leader $P_w$ is calculated by

$$P_w = \int_0^{R + L} f_{S_e}(s_{w}) ds_w = \Phi \left( \frac{\log R - \mu_D}{\sigma_D} \right)$$

(25)

Next, we calculate the expected transmission delay $E[T_{d2}|M = m]$ for a separation distance between $C_s$ and $C_r$ with given value of $(m + 1)(L + R) + R$.

(i) When $m = 0$, i.e., $R < S_e \leq R + (R + L)$,

In this case, since $C_{f0}$ is uniformly distributed in $[0, -R - L,)$, we can assume $C_{f0}$ is fixed at position $S_{w0} - (R + L) = -(R + L)/2$, that is $S_{w0} = 2/(R + L)$. The traversed distance between $C_{f0}$ and $C_r$ is $S_e - R + S_{w0} - (R + L) = S_e - R - (R + L)/2$, therefore only $R + (R + L)/2 < S_e \leq 2R + L$, the value of expected transmission delay is positive. Accordingly, $E[T_{d2}|M = 0]$ is calculated
by

\[ E[T_{d2}|M = 0] = E\left[ \frac{S_e - R - (R + L)/2}{2v_{stb}} | M = 0 \right] \]
\[ = \frac{1}{2v_{stb}} \int_{2R+L}^{2R+2L} \frac{s_c f_S(s_c) ds_c}{\left( \frac{3R + L}{2} < S_e \leq 2R + L \right)} - \frac{R + (R + L)/2}{2v_{stb}} \]

(ii) When \( m = 1 \),

As aforementioned analysis, there is one cell with length of \( R + L \) between \( C_{f0} \) and \( C_r \) in this case, so we have:

\[ E[T_{d2}|M = 1] = \frac{R + L}{2v_{stb}} \left[ 1 - p_w \right] \tag{27} \]

(iii) When \( m \geq 2 \),

In this case, we can calculate \( E[T_{d2}|M = m] \) by the following equation:

\[ E[T_{d2}|M = m] = \frac{R + L}{2v_{stb}} \left[ m - \sum_{i=1}^{m-1} ip_i(1 - p_w) + mp_m \right] \tag{28} \]

Thus the expected transmission delay \( E(T_{d2}) \) can be evaluated by:

\[ E[T_{d2}] = \sum_{m=0}^{\infty} E[T_{d2}|M = m] P_r \left( M = m, 0 \leq S_{w0} \leq R + L \right) \tag{29} \]

where

\[ P_r \left( M = m, 0 \leq S_{w0} \leq R + L \right) = P_r \left( M = m \right) \times P_r \left( 0 \leq S_{w0} \leq R + L \right) \tag{30} \]

Combined with above equations Eq. (26)-Eq. (30), we can calculate the expected transmission delay \( E[T_{d2}] \).

Consequently, the total expected time for inter-platoon message transmission is:

\[ E(T_d) = E(T_{d0}) + E(T_{d1}) + E(T_{d2}) \tag{31} \]
Furthermore, to evaluate the transmission delay for individual vehicles, we define the metric of average expected transmission delay, \( E[T_d]_{\text{avg}} \), where we assume that platoon size is set as the maximum available value \( n_{\text{max}} \) for all platoons. In Eq. (13), \( n_{\text{max}} = L/\tau v_{sth} = R/\tau v_{sth} \), thus \( E[T_d]_{\text{avg}} \) can be expressed as:

\[
E[T_d]_{\text{avg}} = \frac{E[T_d]}{n_{\text{max}}} = \frac{E[T_d] \tau v_{sth}}{R}
\]  

(32)

5. Simulation

In this section, we conduct extensive simulation experiments to validate theoretical analysis in the previous sections and to explore how the platoon-based driving pattern affects safety message transmission in VANET environments. In the rest of this section, we first explain the simulation settings, then verify our analysis on platoon spatial distribution, and finally we extensively discuss the impact of platoon-based driving parameters (such as platoon size, velocity, transmission range) on safety transmission delay.

5.1. Simulation Settings

In this paper, we use a software tool, Veins (Sommer et al., 2010), to implement our experiments. Veins is an open source inter-vehicular communication simulation framework composed of network simulator OMNeT++/MiXiM and SUMO. OMNeT++/MiXiM is used to simulate V2V communication based on IEEE 802.11p standard, while SUMO can simulate the vehicle dynamics with the IDM. Both components are coupled with each other through standard traffic control interface (TraCI) by exchanging TCP messages, while OMNeT++/MiXiM is acting as the TraCI client and SUMO is acting as the TraCI server.

5.2. Verification for the distribution of inter-platoon spacing

Inter-platoon spacing determines the spatial distribution of traffic flow for platoon-based driving pattern in a highway, which also has critical impact on the performance of inter-platoon communication. In this part, we conduct the
experiments for vehicle platooning in a highway to explore the spatial distribution of inter-platoon spacing under different parameter settings such as platoon size, traffic flow rate, and vehicle velocity.

As aforementioned, the original individual vehicle time headway before vehicle platooning is log-normal distributed in intermediate traffic demand level (normally between 600 and 1800 vph). The value of $\sigma$ obtained from freeway traffic is about 0.4, which normally does not vary very much over different traffic flow levels (Baras et al., 1979). The realistic location parameter $\tau = 1s$. Thus by setting different value of $\mu$, we can simulate the traffic scenarios with various traffic flow rates. To calculate intra-platoon spacing as well as platoon length in the steady state, we set IDM parameters as follows: $v_0 = 40m/s$, and $T_0 = \tau = 1s$, while $s_0$ and $l_0$ are chosen with tiny value so as to simplify the calculation of steady intra-platoon spacing in Eq. (7).

Fig. 7 illustrates distribution of inter-platoon spacing with different parameter settings including traffic flow rate, velocity, and platoon size. Simulation results show that inter-platoon spacing is log-normal distributed after vehicle platooning under the regulation of IDM model, which closely matched up with the analytical results. In addition, we can see inter-platoon spacing interval distribution is enlarged by reducing the traffic flow rate in case (a), approximately from $[140m, 300m]$ at flow rate 1800vph to $[550m, 1100m]$ at 720vph. Similar tendency occurs in both case (b) and case (c) when platoon size and velocity increase, respectively.

place Fig. 7 about here

5.3. Expected Transmission delay for inter-platoon

In this part, we investigate expected message transmission delay $E[T_d]$ between two adjacent platoons. Towards this, we assume that the safety message is generated by one platoon tail at a certain time and then transmitted to the following platoon leader through the forwarder-selecting scheme. In addition, to simplify the analysis, we do not take into account the impact of packet loss on message transmission. Extensive experiments have been conducted under
different traffic and communication conditions. For each experiment with specific parameter settings, we ran the simulation 1000 times with different random seeds and measured $E[T_d]$ in each run using OMNET simulator. To timely collect neighbor information, the value of beacon message frequency $f_{bsm}$ in the simulation is set to 5Hz.

5.3.1. Effect of the Transmission Range

Transmission range is a critical parameter which significantly affects inter-platoon communication in VANET. Given a constant platoon size $n = 10$ and platoon velocity $v_{stb} = 20m/s$, we explore the relationship between expected transmission delay $E[T_d]$ and transmission range $R$ under different traffic flow rate conditions.

Simulation results and analytical results are both illustrated in Fig. 8, where we can observe that simulation results match very well with analytical results for all cases of traffic flow rates. Obviously, $E[T_d]$ decreases as $R$ increases in various traffic flow rate conditions, because the performance of connectivity of inter-platoon is improved as transmission range increases. For a given value of $R$, on the other hand, $E[T_d]$ in dense traffic condition is smaller than that in sparse traffic, which is due to smaller inter-platoon spacing in dense traffic condition. Specifically, when the traffic flow rate is greater than 1200vph, $E[T_d]$ approximately equals to zero, which indicates that safety message can be directly transmitted from the preceding platoon tail to the following platoon leader, or be continuously forwarded by vehicles on the opposite direction and eventually received by the following platoon leader without disruption in most cases. The profile of relationship between $E[T_d]$ and traffic flow rate is illustrated in Fig. 9.

5.3.2. Effect of the Platoon Size

In this part, we investigate how platoon size affects the performance of inter-platoon safety message transmission. Similar to the previous experiments, we
conduct the simulations under different traffic flow rate conditions, where the value of transmission range $R$ is set to $400m$ and platoon velocity $v_{setb}$ is set to $20m/s$. In Eq. (13), the maximum of platoon size is $n_{max} = 20$ in this case.

Simulation results and analytical results are both illustrated in Fig. 10. We can observe that analytical results agree with the simulation results for all cases of traffic flow rates. It is easy to see that $E[T_d]$ increases as $n$ increases in various traffic flow rate conditions. This is because inter-platoon spacing will be enlarged as $n$ increases, and as a result, the possible distance traversed during message transmission will be maximized. In addition, $E[T_d]$ rises significantly faster in case of spare traffic flow than in dense traffic flow condition. Basically, the platoon size is supposed to be no more than 10 for time-critical applications, under which $E[T_d]$ is about one second in most traffic conditions.

5.3.3. Effect of the Platoon velocity

Speed is one of the important mobility characteristics of VCPSs. In this part, we discuss how platoon velocity affects the performance of inter-platoon connectivity. Following the same aforementioned scheme, we conduct the simulation under different traffic flow rate conditions, where transmission range $R$ is set to $400m$ and platoon size $n$ is set to 10. From the simulation results and analytical results illustrated in Fig. 11, we can clearly observe that platoon velocity has the similar impact on $E[T_d]$, i.e., $E[T_d]$ increases with the increase of platoon velocity. This is because increasing platoon velocity enlarges inter-platoon spacing for a given traffic flow rate, according to the fundamental relationship between the three traffic flow parameters, traffic density, velocity and flow rate.

5.3.4. Average transmission delay for individual vehicles

To evaluate the average transmission delay for individual vehicles, we conduct the experiments in which platoon size is configured with the maximum value for each platoon. Then we observed the metric of $E[T_d]_{avg}$ under different
traffic conditions. From the simulation results and analytical results illustrated in Fig. 12, we can observe that, in case of fixed traffic flow rate, \( E[T_d]_{avg} \) maintains a constant value as the transmission range increases. Moreover, we also verify the variation law of \( E[T_d]_{avg} \) with respect to \( v_{stb} \) and we can observe the same results. In addition, \( E[T_d]_{avg} \) decreases as traffic flow rate increases. Consequently, we can conclude that \( E[T_d]_{avg} \) is only related to traffic flow rate, regardless of the transmission range.

**place Fig. 12 about here**

6. Conclusion

In this paper, we have investigated VANET connectivity of a platoon-based VCPS. Towards this end, we first proposed a general architecture for platoon-based VCPS, taking into consideration the tight interaction between platoon dynamics and VANET. Then we derived the probability distribution of platoon-based traffic flow, including inter-platoon spacing and inter-platoon leader spacing. Based on the results, we further investigated inter-platoon connectivity in a practical bi-directional highway scenario and evaluated the expected time of safety message delivery among platoons, taking into account the effects of system parameters, such as traffic flow, velocity, platoon size and transmission range. Extensive simulations have been conducted, which demonstrate that our analysis results are very accurate.


Figure 1: Platoon-based driving pattern on a highway.
Figure 2: Architecture for platoon-based VCPSs.
Figure 3: Platoon parameters.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>the maximum acceleration</td>
</tr>
<tr>
<td>$a_{ji}^i$</td>
<td>the acceleration of $C_{ji}^i$</td>
</tr>
<tr>
<td>$b$</td>
<td>the comfortable deceleration</td>
</tr>
<tr>
<td>$f_{bam}$</td>
<td>beacon message frequency</td>
</tr>
<tr>
<td>$l_0$</td>
<td>the length of a vehicle</td>
</tr>
<tr>
<td>$L^i$</td>
<td>the length of platoon $P^i$</td>
</tr>
<tr>
<td>$n^i$</td>
<td>the number of vehicles in platoon $P^i$</td>
</tr>
<tr>
<td>$R$</td>
<td>a fixed minimum transmission range</td>
</tr>
<tr>
<td>$s_{ji}^i$</td>
<td>intra-platoon spacing between $C_{ji-1}^i$ and $C_{ji}^i$</td>
</tr>
<tr>
<td>$s_0$</td>
<td>minimum intra-platoon spacing (at standstill)</td>
</tr>
<tr>
<td>$S^i$</td>
<td>the inter-platoon spacing between $P^{i-1}$ and $P^i$</td>
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<tr>
<td>$S_{L_i}$</td>
<td>the distance between $C_{1}^{i-1}$ and $C_{1}^i$</td>
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<tr>
<td>$T_0$</td>
<td>the desired time gap</td>
</tr>
<tr>
<td>$v_0$</td>
<td>the maximum speed</td>
</tr>
<tr>
<td>$v_{ji}^i$</td>
<td>the velocity of $C_{ji}^i$</td>
</tr>
</tbody>
</table>
Figure 4: A example of platoon topology.
Figure 5: The scheme for inter-platoon message delivery.
Figure 6: Multiple hops for inter-platoon message delivery.
Figure 7: The probability density function for inter-platoon spacing.
Figure 8: $E[T_d]$ vs. transmission range.
Figure 9: $E[T_d]$ vs. traffic flow rate.

- Simulation results
- Analytical results

$R = 400$ m
$n = 10$
$v_{stb} = 20$ m/s
flow rate = 600vph, simulation results
flow rate = 720vph, simulation results
flow rate = 900vph, simulation results
flow rate = 1200vph, simulation results
flow rate = 600vph, analytical results
flow rate = 720vph, analytical results
flow rate = 900vph, analytical results
flow rate = 1200vph, analytical results

Figure 10: $ET_d$ vs. platoon size.
flow rate = 600vph, simulation results
flow rate = 720vph, simulation results
flow rate = 900vph, simulation results
flow rate = 1200vph, simulation results
flow rate = 600vph, analytical results
flow rate = 720vph, analytical results
flow rate = 900vph, analytical results
flow rate = 1200vph, analytical results

Figure 11: $ET_d$ vs. platoon velocity.
Figure 12: \( E[T_d]_{\text{avg}} \) vs. transmission range.