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Optimal Torque Control of Fault-Tolerant Permanent Magnet Brushless Machines

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Abstract—This paper describes a novel optimal torque control strategy for fault-tolerant permanent magnet brushless ac drives operating in both constant torque and constant power modes. The proposed control strategy enables ripple-free torque operation to be achieved while minimizing the copper loss under voltage and current constraints. The utility of the proposed strategy is demonstrated by computer simulations on a five-phase fault-tolerant drive system.

Index Terms—Fault tolerant, optimal control, permanent magnet machines.

I. INTRODUCTION

Due to their high-power density relative to other competing machine technologies [1], [2], fault-tolerant permanent magnet brushless drives are emerging as a key enabling technology for safety-critical applications, in the aerospace and automotive sectors, for example. A typical fault-tolerant brushless drive system comprises a permanent magnet machine with \( n \) magnetically and physically isolated phases, each having one per unit inductance and being supplied from an electrically isolated single-phase converter. Thus, in the event of a short circuit, the fault current is limited to the rated full-load current [3], [4]. Generally, it is assumed that such fault-tolerant machines have a sinusoidal back-emf waveform, so that the current controller is required to track a set of sinusoidal current commands proportional to a given torque demand. However, this simple control strategy inevitably results in an undesirable torque ripple during normal operation, if the back-emf waveform is nonsinusoidal, and a large torque pulsation under a fault condition, such as an open-circuited or a short-circuited phase. While this problem may be partly overcome by adopting an optimal torque control strategy which is aimed at minimizing the copper loss when delivering the required torque demand [5]–[7], the effectiveness of existing strategies is limited by the available converter voltage, since, by minimizing the copper loss, the resulting current command tends to be in-phase with the back-emf. Consequently, at high speed the controller cannot track the commanded currents due to the limited converter voltage.

Many safety-critical applications require a fault-tolerant drive to operate over a wide speed range, encompassing both constant torque and constant power operating modes. Although, theoretically, a fault-tolerant brushless machine with one per unit inductance should be able to operate over an infinitely wide constant power region [8], existing control methods do not provide an effective means of facilitating such operation under both normal and fault conditions. As a result, the potential power capability of fault-tolerant machines is significantly compromised.

This paper describes a new optimal torque control strategy for fault-tolerant permanent magnet brushless drives which is appropriate for both the constant torque and constant power operating regions.

II. OPTIMAL TORQUE CONTROL STRATEGY

For an \( n \)-phase permanent magnet machine equipped with a surface-mounted magnet rotor, the electromagnetic torque \( T_e \), which results when a fault occurs on phase \( k \), is given by

\[
T_e = \sum_{j \neq k}^{n} a_j(\theta) i_j + \begin{cases} 0, & \text{for an open-circuit fault} \\ a_k(\theta)i_k, & \text{for a short-circuit fault} \end{cases}
\]

(1)

where \( i_j \) is the instantaneous current in phase \( j \), and \( a_j(\theta) i_j = \frac{p}{L} \frac{d\psi_j}{d\theta} \) is the instantaneous torque of phase \( j \) at a given rotor angular position \( \theta \), where \( \psi_j \) is the magnet flux-linkage with phase \( j \) and \( p \) is the number of poles-pairs, and similarly for the faulted phase \( k \). For a given torque demand \( T_d \), the optimal instantaneous currents in the healthy phases can be determined in closed-form by minimizing a cost function \( F \), defined as

\[
F = \sum_{j \neq k}^{n} (L_i j + w \psi_j)^2
\]

(2)

subject to the following equation:

\[
T_d = \sum_{j \neq k}^{n} a_j(\theta) i_j + \begin{cases} 0, & \text{for an open-circuit fault} \\ a_k(\theta)i_k, & \text{for a short-circuit fault} + T_{cp}(\theta) \end{cases}
\]

(3)

under current and voltage constraints using the quadratic programming technique, where \( L \) is the self-inductance of each phase and \( T_{cp}(\theta) \) is the cogging torque which may still exist even after implementing design features for its minimization [5]. \( w \) is a weighting factor which is dependent on the speed and torque demand. At speeds below the base-speed, \( \omega_b \), which is defined as the maximum speed at which the drive system can produce rated smooth torque without flux-weakening, the weighting factor \( w \) is set to zero. Therefore, the objective function becomes \( L^2 \sum_j \psi_j^2 \), which is equivalent to minimizing the copper loss \( R \sum_j i_j^2 \). At speeds above the base speed, \( w \) is a nonzero positive number, which dictates the magnitude of the...
flux-weakening and varies as a function of the speed $\omega$ and the torque demand $T_d$ in a manner given by

$$w = \left[ (\omega - \omega_b) / \omega \right] \times T_d / T_e$$  \hspace{1cm} (4)

where $T_e$ is the rated torque at the base-speed $\omega_b$. The optimization is, therefore, weighted to minimize the copper loss and the total flux-linkage. As a result, the back-emf is suppressed by the effect of flux-weakening, and the machine can deliver smooth torque throughout the constant power operating region.

### III. SIMULATION RESULTS

The proposed optimal torque control strategy has been implemented by computer simulation on a five-phase, ten-slot, 12-pole fault-tolerant machine, a schematic of which is shown in Fig. 1.

Fig. 2 compares the measured and predicted emf and cogging torque waveforms, while the fundamental emf, the phase inductance, and the resistance of the machine are given in Table I.

Each phase is controlled separately from an H-bridge with a dc supply of 45 V. Fig. 3 shows the torque waveforms which result when a conventional control strategy is employed and the controller is required to track a set of sinusoidal current commands to produce the rated torque of 23.2 (Nm) at a rotor speed of 100 (rad/s) when a phase is open-circuited or short-circuited. As will be seen, due to the emf harmonics and fault condition, a torque ripple of 43.4% and 73.8% exists under such open-circuit or short-circuit fault conditions, respectively. However, the torque ripple can be eliminated completely by using the proposed optimal torque control strategy. By way of example, Fig. 4 shows the current, voltage, emf, and torque waveforms which result with the proposed control strategy when the machine is operating in the constant torque region with a short-circuit fault.
creased current harmonics and will be evident, smooth torque is produced at the expense of increased copper loss. Therefore, in order to achieve a smooth torque output, it is necessary to introduce a degree of flux-weakening proportional to the speed and torque demand. Again, by way of example, Fig. 5 shows the current, voltage, emf, and torque waveforms which result with the proposed control strategy when the machine is operating in the constant power region at a rotor speed of 600 (rad/s) with a short-circuit fault. As can be seen, even under the short-circuit condition, a ripple-free torque of 11.6 (Nm) is achieved in the flux-weakening mode. It is also evident from the current and emf waveforms in Fig. 5 for a healthy phase two that a significant phase advance in the current with respect to its emf has been produced in order to achieve the required flux-weakening.

IV. CONCLUSION

A generalized optimal torque control strategy for fault-tolerant permanent magnet brushless drives has been proposed. It has been shown that a ripple-free torque can be produced under both healthy and faulted conditions in both the constant torque and constant power operating regions. The utility of the proposed strategy has been demonstrated by computer simulation. Its experimental verification will be reported in due course.

REFERENCES