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Dynamic snap-through for morphing of bi-stable composite plates

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Abstract

Composite laminate plates designed to have two statically stable configurations have been the focus of recent research, with a particular emphasis on morphing applications. In this paper we consider how external vibration energy can be used to assist with the actuation between stable states. This is of interest in the case when surface mounted macro-fibre composites (MFC) actuators are employed as the actuation system. Typically, these type of actuators have been found to require very high voltage inputs to achieve significant levels of actuation authority. Therefore, assisting the actuation process will allow lower voltages and/or stiffer plates to be actuated. Two bi-stable plates with different thickness, $[0_4 - 90_4]_T$ and $[0_2 - 90_2]_T$, were tested. The results show a significant reduction in the force required to change state for the case where dynamic excitation provided by an MFC actuator is used to assist the process. This strategy demonstrates the potential of dynamically assisting actuation as a mechanism for morphing of bi-stable composites.

Keywords: Bi-stable composites, morphing structures, dynamic snap-through, macro-fibre composites.

1 INTRODUCTION

Composite laminates are becoming increasingly important in a wide variety of applications, particularly in aerospace engineering. A novel class of composite laminates exhibiting multiple stable configurations, or states, have been the focus of considerable recent research — see Dano and Hyer (2002); Hyer (1981) and references therein. The property of multi-stability appears as a result of asymmetric residual thermal stresses induced during the curing process (Dano and Hyer, 1998). The change between stable configurations is achieved by a strongly nonlinear mechanism known as snap-through (Sokorin and Terentiev, 1998). Recent research has focused on the application of multi-stable composites to aerospace morphing structures, such as morphing aerofoils — see for example Diaconu et al. (2008); Mattioni et al. (2005). This is possible, as the production of predetermined stable-shapes has now become feasible (Potter and Weaver, 2004).

By coupling bi-stable composite specimens with actuators to induce changes of shape (snap-through), morphing structural elements are created which have the potential to be incorporated into more complex structures.

In the past, different types of actuators have been employed to achieve this goal. The concept of using shape memory alloy (SMA) wires to induce snap-through has been investigated, however its integration proved difficult and the implementation with bi-stable composites remains challenging (Dano and Hyer, 2003). More suitable for integration with bi-stable composites are flexible piezo-electric actuators such as macro-fibre composite (MFC) actuators (Wilkie et al., 2000). In previous studies MFC actuators driven by a quasi-static voltage have been used to induce snap-through by statically loading bi-stable composites (Bowen et al., 2007; Portela et al., 2008; Schultz et al., 2006). A key limitation of the applicability of this concept is the low actuation authority achieved with static actuation from MFC actuators.

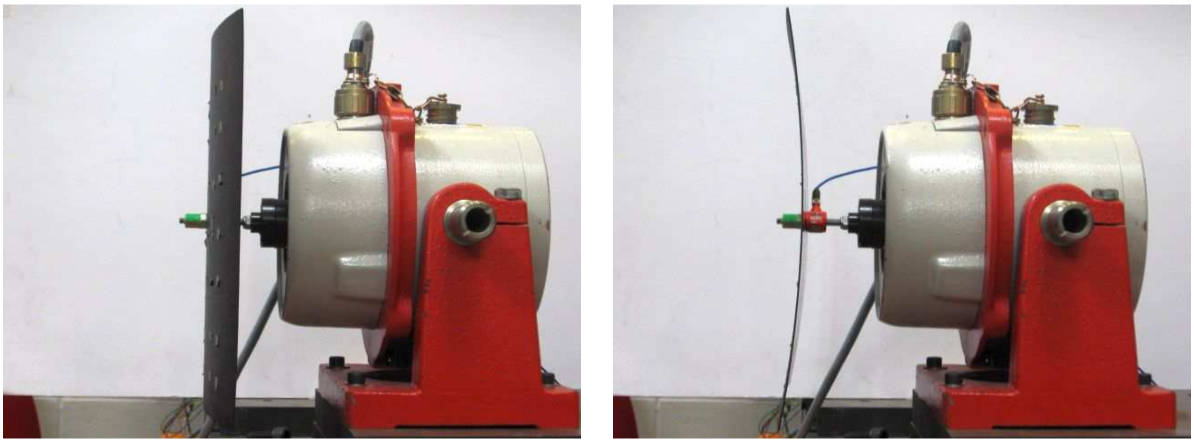
To address this problem, the idea of dynamically inducing snap-through in multi-stable composite plate is investigated in this paper. This actuation strategy exploits the dynamics of the bi-stable structure in order to achieve the change between stable shapes more efficiently — effectively increasing the actuator's authority to induce snap-through. The justification for aerospace applications is that all these structures will operate in a dynamic environment, and so exploiting the dynamic excitation can be thought of as a form of energy harvesting.

In this paper, two bi-stable plate specimens, $[0_4 - 90_4]_T$ and $[0_2 - 90_2]_T$, with surface mounted MFCs as actuators, are employed to experimentally demonstrate the morphing concept of dynamically induced snap-through. The main characteristics of the dynamic response of the plates are presented, based on previous works by the authors for similar bi-stable composites (Arrieta et al., 2007, 2009a). Taking advantage of key features of the response, particular excitation frequencies are selected such that large amplitude oscillations of the plate can be induced. This mechanism generates a mechanical advantage increasing the authority of the actuator to trigger snap-through.

Two types of tests are conducted. First, passive testing in which dynamically induced snap-through is triggered using a shaker as external source of excitation is carried out. The snap-through force is measured as a function of the forcing frequency for the experimental specimen. Second, active tests are conducted in which the force required to dynamically trigger snap-through under the combined action of the MFC actuator and the external shaker is measured. The resulting actuation strategy is shown to offer potential as a configuration control strategy for morphing applications involving bi-stable composites elements.

2 DYNAMIC RESPONSE

The concept of dynamically inducing snap-through as a morphing strategy is demonstrated by testing two $[0_4 - 90_4]_T$ 300x300mm and $[0_2 - 90_2]_T$ 200x200mm bi-stable plates with an MFC 8557-S1 bonded to the surface. The two stable configurations of the 8-ply $[0_4 - 90_4]_T$ bi-stable specimen are shown in Figure 1. The material properties of the 8-ply bi-stable composite plate specimen are given in Table 1. The plate is mounted on a Ling shaker which is used as the source of external excitation. The experimental assembly used for testing both plates is shown in Figure 2. The MFC actuator was driven by a voltage amplifier providing a maximum voltage of (550,-550) V. The mea-



(a) Stable state 1 (b) Stable state 2

Figure 1: Stable states of a bi-stable plate.

Property	*Fibrevol.* [%]	*Ply thickness* [mm]	*Density* ρ [kg/m^3]	E_{xx} [GPa]	E_{yy} [GPa]	G^a_{xy} [GPa]	$\nu^a_{xy} = \nu^a_{yx}$
Value	57.7	0.131	0.128	164	12	4.6	0.3

Table 1: Material properties for a ply of HexPly 8557 IM7 used to manufacture the bi-stable experimental specimens.

sured signals are pass through a *Kistler* signal conditioner. The experimental assembly is shown schematically in Figure 3.

The dynamic behaviour of similar bi-stable plates without a surface mounted MFC actuator were studied in previous papers by the authors (Arrieta et al., 2009a,b). In this paper, the same testing procedure is used to study the dynamics of the bi-stable plate specimens with MFC actuator attached to their surface. For further details of the dynamic testing methodology can be found in (Arrieta et al., 2009a,b).

The dynamic response of a grid of points on the bi-stable specimens is measured with a laser vibrometer to obtain a series of frequency response diagrams. The results allowed both the linear and nonlinear dynamic response of the bi-stable plate specimens to be investigated. The measured points P_x, P_y, and P_s for both bi-stable plate specimens are schematically shown in Figure 4. The position of the measured points are $(225, 150)_{P_x}$, $(150, 225)_{P_y}$, and $(180, 150)_{P_s}$, for the 8-ply plate, and $(150, 100)_{P_x}$, $(100, 150)_{P_y}$, and $(130, 100)_{P_s}$, for the 4-ply plate with respect to the frame of reference shown in Figure 4.

The frequency response diagrams for points P_x and P_y obtained for a constant forcing amplitude of 1 N for the 8-ply specimen in stable state 2 are shown in Figure 5. Three modes of vibration can be observed, $w_1^{(8p)}$ at 17.2 Hz, $w_2^{(8p)}$ at 18.2 Hz, and $w_3^{(8p)}$ at 47.1 Hz, dominating the response of the 8-ply plate for the frequency range of interest in this study. Nonlinear oscillations are identified in the response with the aid of the frequency response diagrams. In these diagrams, each dot represents the amplitude of response for a given forcing frequency of the plate. For linear oscillations, the response to a single frequency sinusoidal forcing results on only one dot per forcing frequency. Conversely, multiple dots resulting from a single frequency sinusoidal forcing are an indication of

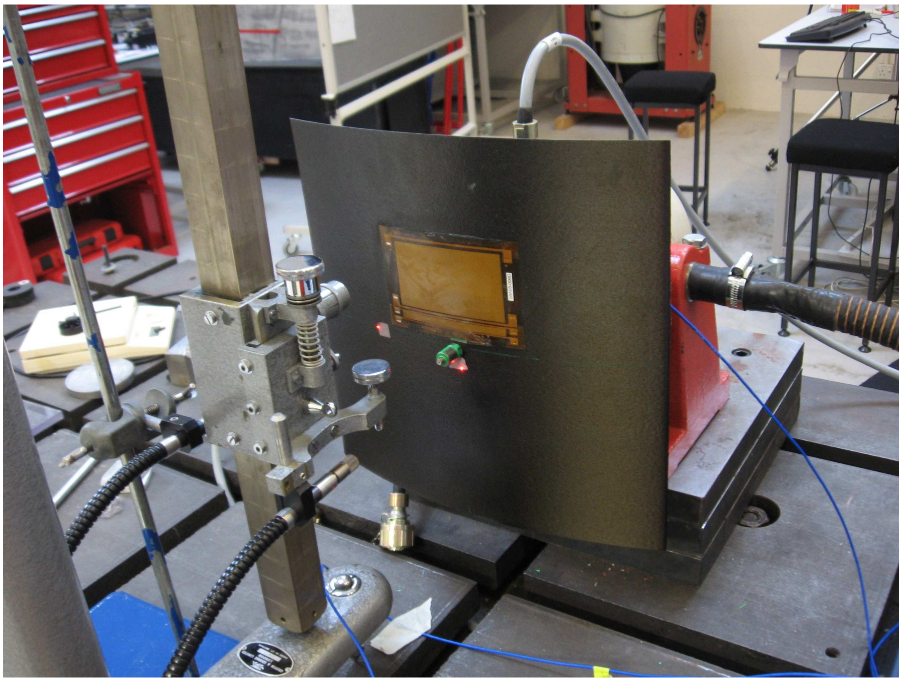


Figure 2: Bi-stable plate $[0_4 - 90_4]_T$ 300x300 [mm] made of carbon fibre epoxy HexPly 8557 IM7 with an MFC 8557-S1 surface mounted directly above the central attachment point used to connect the plate to the shaker.

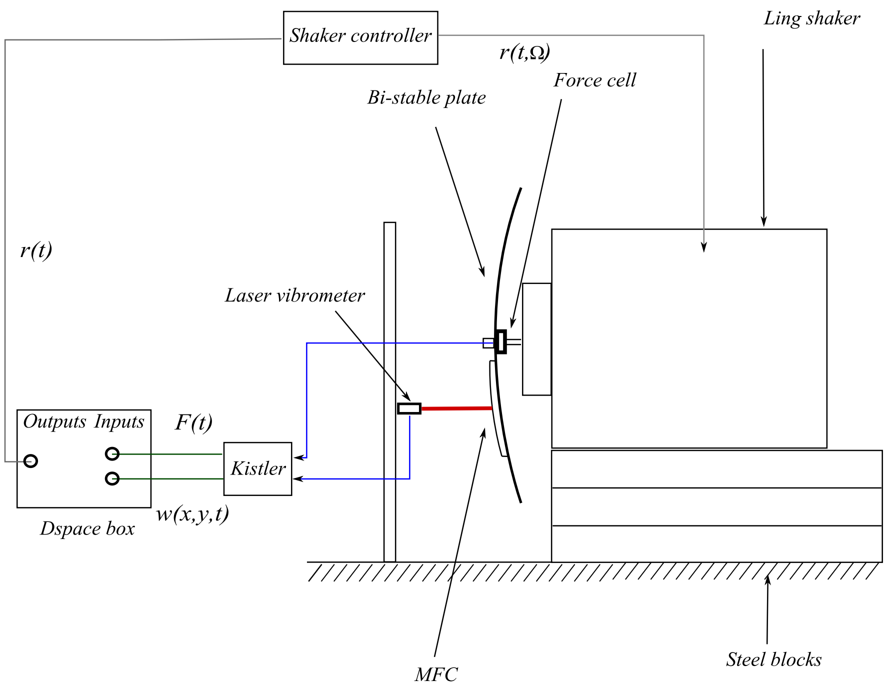


Figure 3: Schematic diagram of the experimental assembly and test set up.

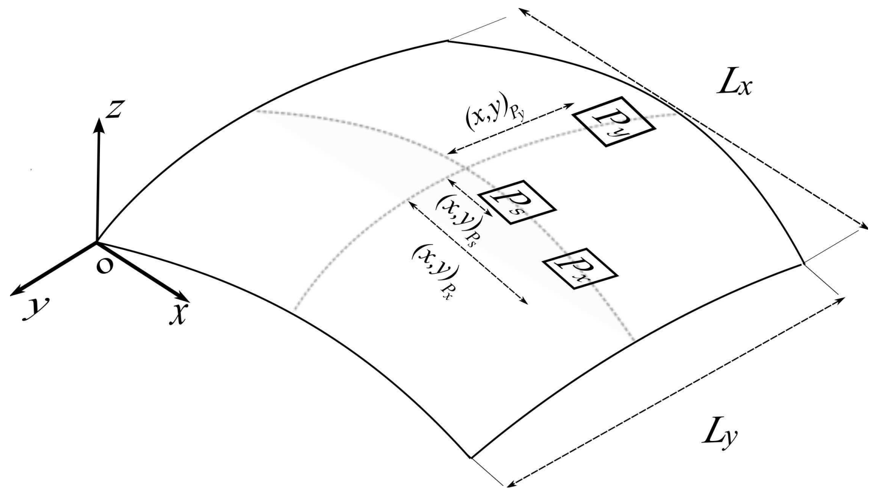


Figure 4: Grid of points measured on the tested specimens. Point P_x describes the out-of-plane deflection in low curvature-direction, coinciding with the x-coordinate, and point P_y describes the out-of-plane deflection in the high curvature-direction for state 2. Point P_s is used to measure large amplitude oscillations for the plate. This is because for some parameter values, points P_x and P_y can go beyond the range of the laser vibrometer.

the presence of nonlinear behaviour, as seen in Figure 6 for regions around twice the frequency of modes $w_1^{(8p)}$ and $w_2^{(8p)}$. The nature of the nonlinear oscillations indicate the characteristics of 1/2 subharmonic resonances of modes $w_1^{(8p)}$ and $w_2^{(8p)}$, as expected from well known literature results (Emam and Nayfeh, 2004; Nayfeh and Mook, 1979).

To test the actuation authority of the MFC actuator for the 8-ply plate, frequency response diagrams for both stable states of the specimen are acquired. These diagrams were obtained while exciting the 8-ply specimen with the MFC actuator driven with a constant 500 V voltage signal. The frequency response diagram for point P_x in state 1 and point P_y in stable state 2, are shown in Figures 7(a) and 7(b). These figures give an indication of the MFC actuator's authority showing the achievable displacements obtained for the 8-ply specimen. The observed dominant modes coincide with those shown in the frequency response functions obtained with the external shaker as an excitation source. The large modal responses for modes w_1^{8p} and w_2^{8p} are, in part, a result of the chosen position of the MFC actuator. In this case, mode w_3^{8p} is not excited as the location of the moments applied by the MFC lie over nodal lines of the associated mode shape. The positioning of the actuator was chosen so that its edges perpendicular to the fibre direction did not coincide with the nodal lines of the deflection shapes for state 2. This is done as it was found that the state 2 was the more compliant configuration of the tested bi-stable plates. Due to the bi-stability and the orthogonal symmetry of the curvatures (Arrieta et al., 2009b) of bi-stable plates this optimized positioning could only be achieved for one stable configuration. As a result we focused first on achieving snap-through from state 2 to state 1.

The broad resonant peak centred around 30 Hz in Figures 7(a) and 7(b) is not a characteristic of the bi-stable plate, but occurs as a result of the dynamics of the shaker. To address this, a controller is implemented to counter the shaker dynamics for the combined shaker-MFC tests shown in the next section.

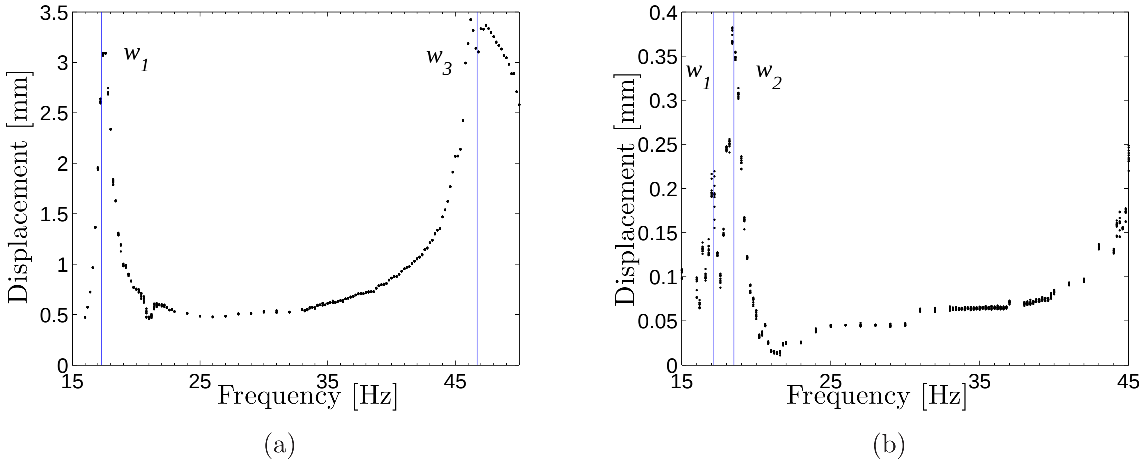


Figure 5: Experimental frequency response diagrams of $[0_4 - 90_4/MFC]_T$ 8-ply plate with surface mounted MFC for state 2, forcing amplitude $F_o = 1\ N$. (a) Point P_y. (b) Point P_x. Vertical lines show linear natural frequencies.

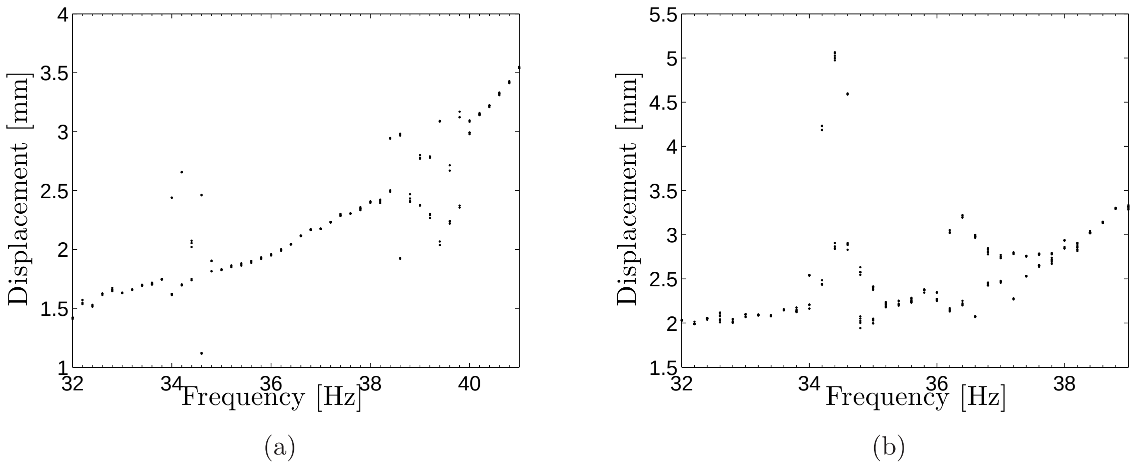


Figure 6: Experimental frequency response diagrams showing subharmonic oscillations for the 8-ply plate with surface mounted MFC for a forcing amplitude F_o= 7 N, for (a) point P_x of state 1 and (b) point P_y of state 2.

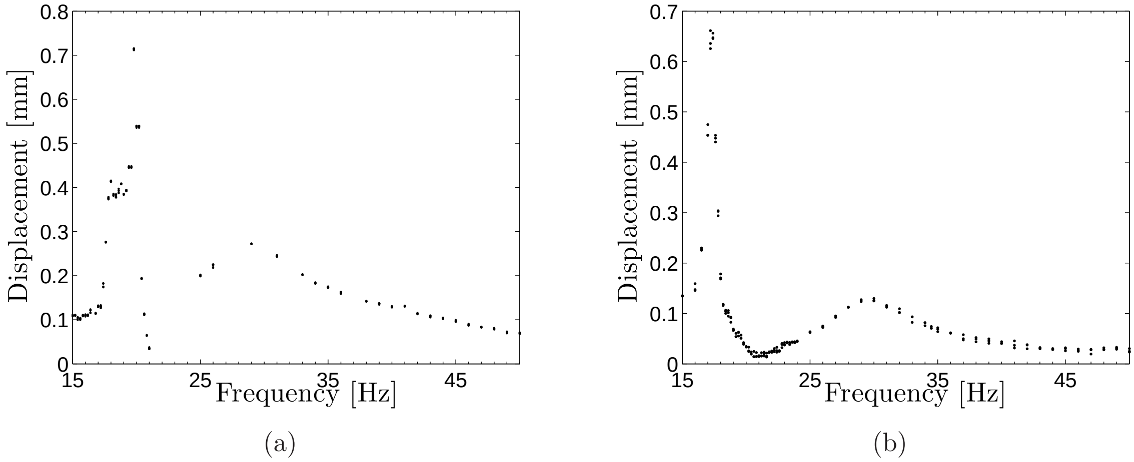


Figure 7: Experimental frequency response of 8-ply plate with MFC as actuator for a peak to peak voltage of 500V. (a) Response for point P_x of state 1. (b) Response for point P_y of state 2.

3 DYNAMICALLY INDUCED SNAP-THROUGH AS A MORPHING STRATEGY

The idea of dynamically inducing snap-through is achieved by exploiting the dynamics of the snap-through mechanism. In terms of a potential energy function, the two statically stable configurations of a bi-stable composite plate correspond to stable equilibria points separated by a unstable equilibrium. The potential energy function is shown schematically by the bold line in Figure 8. To trigger snap-through from one stable state to the other, energy needs to be put into the system such that it is pushed up and over the energy hilltop separating the two potential wells. This is shown schematically as the ball representing system's state, as shown in Figure 8. In other words, the dynamic response of the bi-stable plates is restricted to oscillations confined to a stable state unless enough energy is put into the system to deflect it beyond the hilltop. In physical terms this hilltop is defined as a deflection threshold or *critical displacement*. The critical displacement for an arbitrary point of a bi-stable plate is related to the static deflection required to change from one stable state to the other as seen in Figure 9. The asymmetry in the potential energy function is explained below.

We assume that the deflection shape of the bi-stable plate may be modelled with linear mode shapes, such that the relative deflection at any instant of the forcing period between points remains linearly proportional. Under this assumption, once a point is deflected past its critical displacement a snap-through is triggered as all the points in the plate must have also reached their particular threshold due to the proportionality relation of the mode shapes. For the 8-ply specimen the critical displacement for point P_x in stable state 1 was 5 mm, and for point P_y and P_s in stable state 2 is 1.5 mm and 0.2 mm respectively. In principle, the stable shapes of a bi-stable plate are orthogonally symmetric, i.e. the radius of curvature in the x-direction in state 1 would be equal to the radius of curvature in the y-direction in state 2. However, asymmetries caused by the manufacturing process arise in real plates explaining the difference in the critical displacements between points P_x in state 1 and P_y in state 2 this also explains difference in energy E_1 and E_2 in Figure 8.

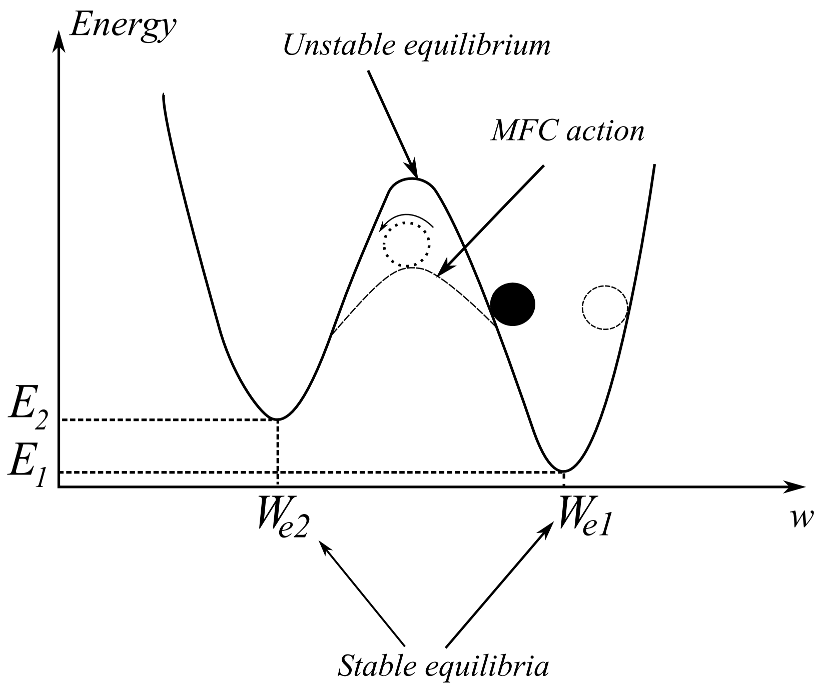


Figure 8: Potential well schematic diagram for a bi-stable plate under the action of an MFC actuator.

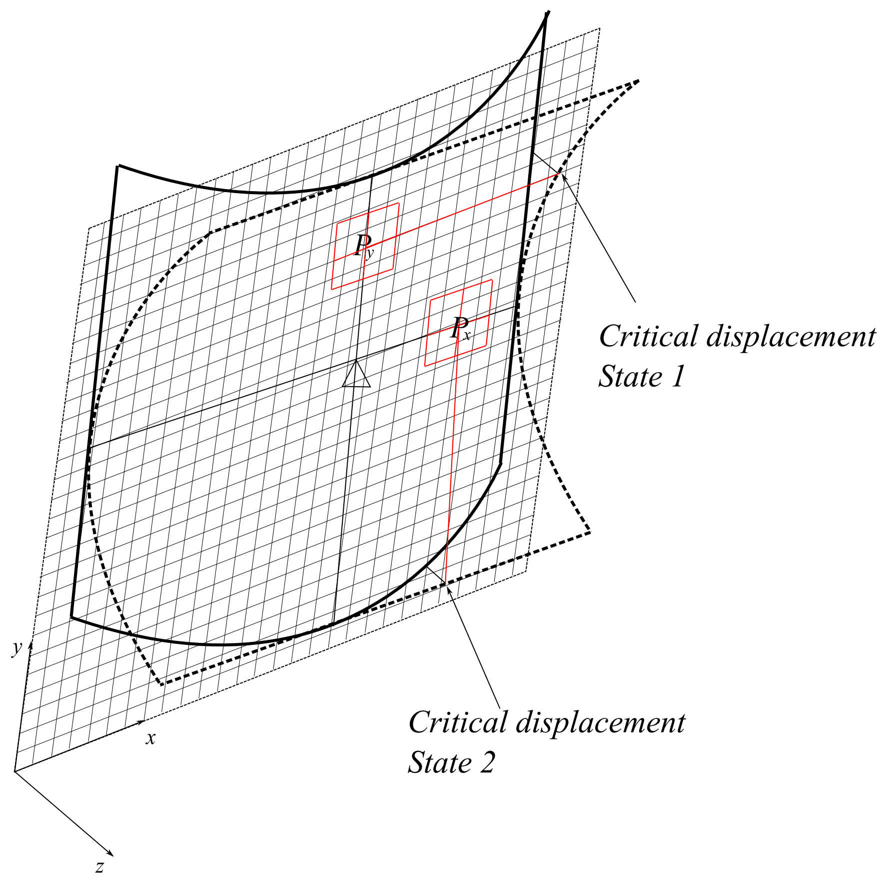


Figure 9: Critical displacement for a bi-stable composite.

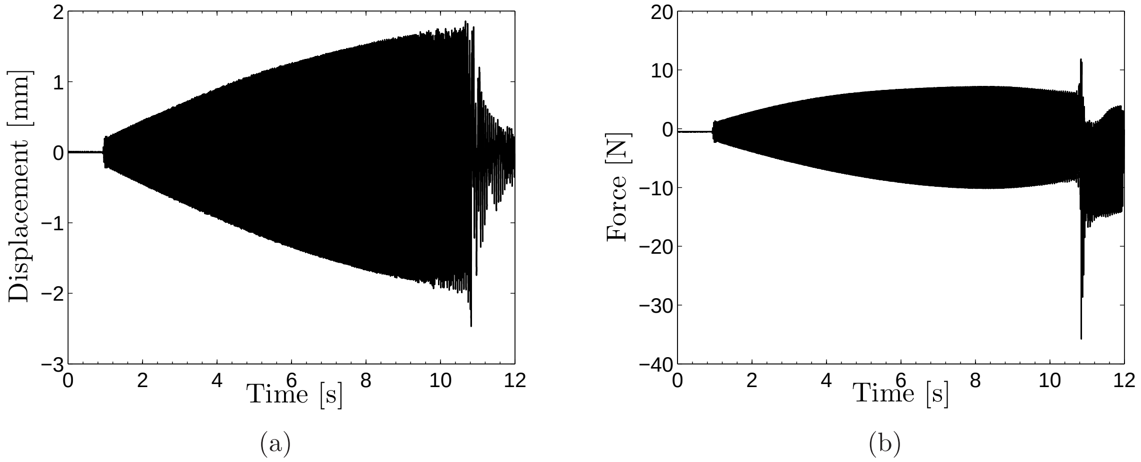


Figure 10: Experimental response showing a dynamically induced snap-through from state 2 to state 1 for a 8-Ply plate with MFC. Shaker forcing frequency Ω=35.5 Hz. (a) Displacement time series. (b) Force time series.

The concept of dynamically inducing snap-through was studied employing a shaker to excite the 8-ply bi-stable specimen past its critical displacement for a range of forcing frequencies. Sinusoidal forcing inputs with linearly increasing amplitudes were employed to drive the specimen until snap-through was triggered. The signals used to excite the plate were fine tuned such that the snap-through for each frequency was triggered approximately 10 s after the start of the linearly growing amplitude input. A time series for the displacement of point P_s on the plate and the measured force amplitude for a dynamically induced snap-through with the shaker as actuator are presented in Figure 10. The measured snap-through force from stable state 2 to stable state 1 for the bi-stable plate-MFC actuator under the sole action of the shaker is shown in Figure 11. The regions of lower snap-through force coincide with the linear (primary) and subharmonic (secondary) resonances of the bi-stable plate. This can be deduced by comparing the frequency response diagram in Figures 5 and 6 with the snap-through force frequency relation given in Figure 11. The snap-through force relationship enables us to find the frequency ranges requiring less actuation to induce snap-through, providing the target actuation frequencies most favourable for achieving morphing of the bi-stable plate-MFC actuator structures.

The dynamic critical displacement is measured as the deflection just before snap-through is triggered as a function of the forcing frequency. Figure 12 shows the results for point P_s in stable state 2 for 8-ply specimen. It can be seen that away from the resonances the critical displacement agrees with the previously measured static deflection. At both primary and secondary resonances the critical displacement increases quite dramatically. This is explained by the dominant effect of the inertial forces on the response around these frequencies, effectively increasing the critical displacement from its static value. Away from resonance and in the static case the elastic forces govern the deflection of the plate. Comparing the achievable deflections obtained with the MFC actuator shown in Figure 7 with the dynamic critical displacement we conclude that it lacks the required actuation authority to induce snap-through as the critical displacement cannot be achieved. Nevertheless, the authority of the actuator can be enhanced by taking advantage of external vibrations acting on the structure. The energy of external forcing

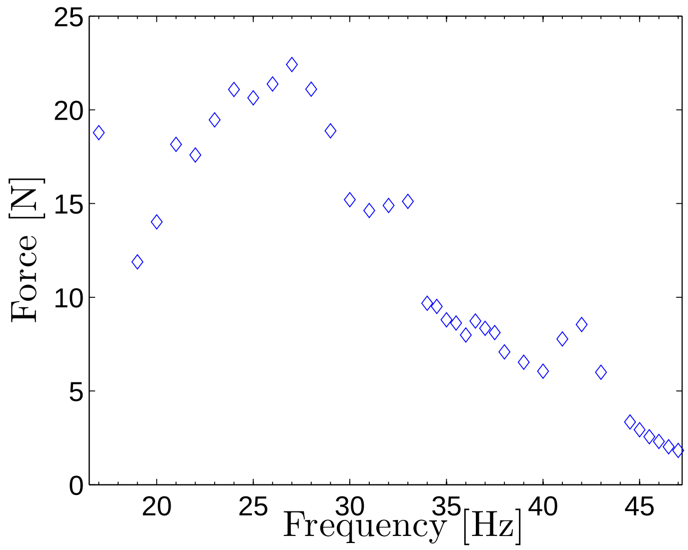


Figure 11: Experimental forcing amplitude required to dynamically induce snap-through from state 2 to state 1 for a 8-Ply plate with MFC actuator attached, but not active during the tests.

can be exploited by coupling the action of the MFC actuator to it and exploiting the dynamic characteristics of the structure, such as the resonant frequencies. Furthermore, the morphing structure can be devised in order to maximize this effect by designing the resonances, or adapting them by active means, to be close to the excitation frequencies. This form of energy harvesting is employed as an actuation strategy for morphing of the bi-stable plate-MFC structure.

Experiments using an external shaker are carried out to simulate the external energy provided by perturbations loading the structure. The actuation strategy consisted of combining the actuation of the MFC with excitation from the external shaker, targeting the frequencies identified to require minimum force to trigger snap-through, shown in Figure 11. As seen in Figure 11 the frequency ranges where minimum force is required to trigger a snap-through are around mode w_3 and the subharmonic oscillations. In order to excite mode w_3^{8P} an MFC large enough such that its edges lie close to those of the plate is required, however these type of large actuators are impractical. Therefore, for the current configuration the most efficient strategy is to exploit the dynamic characteristics of the subharmonic resonances. A subharmonic resonance is induced with the external shaker and the MFC targeted the subharmonic frequency of the plate response (at half the forcing frequency of the shaker). As a result, the energy of the MFC is directly exciting the frequency causing large amplitude response in the subharmonic resonance mechanism, effectively achieving a mechanical advantage (Arrieta et al., 2009a; Nayfeh, 2000).

A measure for comparison between the force required to dynamically induce snap-through with the shaker and with the combined action of the external shaker and the MFC actuator is required. To obtain this, the specimen is excited using a single frequency sinusoidal input with linearly increasing amplitude up to a steady state amplitude in a period of 10 s. Once the steady state amplitude is reached after about 3s, a sinusoidal signal of amplitude 500 V targeting the mode causing the subharmonic resonance is applied with the MFC triggering the snap-through, as seen in Figure 13.

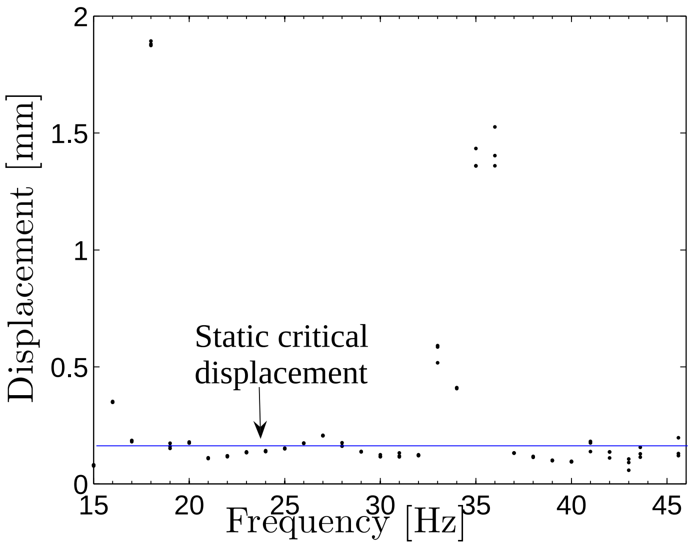


Figure 12: Experimental displacement of the point P_s just before the triggering of a dynamically induce snap-through from state 2 to state 1 for a 8-Ply plate with MFC actuator attached (not active during the tests). Away from linear and nonlinear resonance the *dynamic critical displacement* is very close to 0.2 mm which corresponds to the measured *static critical displacement*.

As the subharmonic oscillation consists of two frequency components it is difficult to measure the relative phase of the response, in a linear sense. The experiments show that the action of the MFC produces a complex interaction with the dynamic response of the plate which after a brief transient period becomes phase locked, causing a significant increase in amplitude. At this point the dynamic critical displacement threshold is surpassed and snap-through is triggered as seen in Figure 13(a).

The maximum force exerted by the shaker before the initiation of the snap-through process is chosen as the comparison measure for the results shown in Figure 14. The maximum mechanical advantage is observed at around twice the natural frequency of mode w_2^{8p} for stable state 2, this is at 36.2 Hz, of the test specimen. As the forcing frequency moves away from this range the actuation capability of the MFC actuator diminishes considerably as the subharmonic mechanism loses its effectiveness. Moreover, due the difference in the deflection shape as the forcing frequency diverts from the subharmonic of mode w_2^{8p}, the effect of the MFC action actually increases the snap-through force. This mechanism could be employed to enhance the stability of a desired configuration avoiding undesired snap-through, however this is beyond the scope of this paper.

For the 8-ply-MFC plate a maximum reduction of 1.46 N or 15.8 % of the shaker force to induce snap-through is achieved compared to the measured force under the lone action of shaker at a forcing frequency of 36.2 Hz. In principle if a powerful enough MFC or an arrangement of several MFC were available, a dynamically triggered snap-through could be induced, achieving a self morphing structure.

The 4-ply bi-stable specimen with smaller dimensions and stiffness is tested with an MFC attached to provide a second example of inducing dynamically snap-through. Figure 15 shows frequency response diagram of the 4-ply bi-stable plate-MFC actuator system. The basic response is similar to the previously studied 8-ply plates with some quantitative variations in the dominant modes now found at $w_1^{(4p)}$ at 22.8 Hz, $w_2^{(4p)}$ at

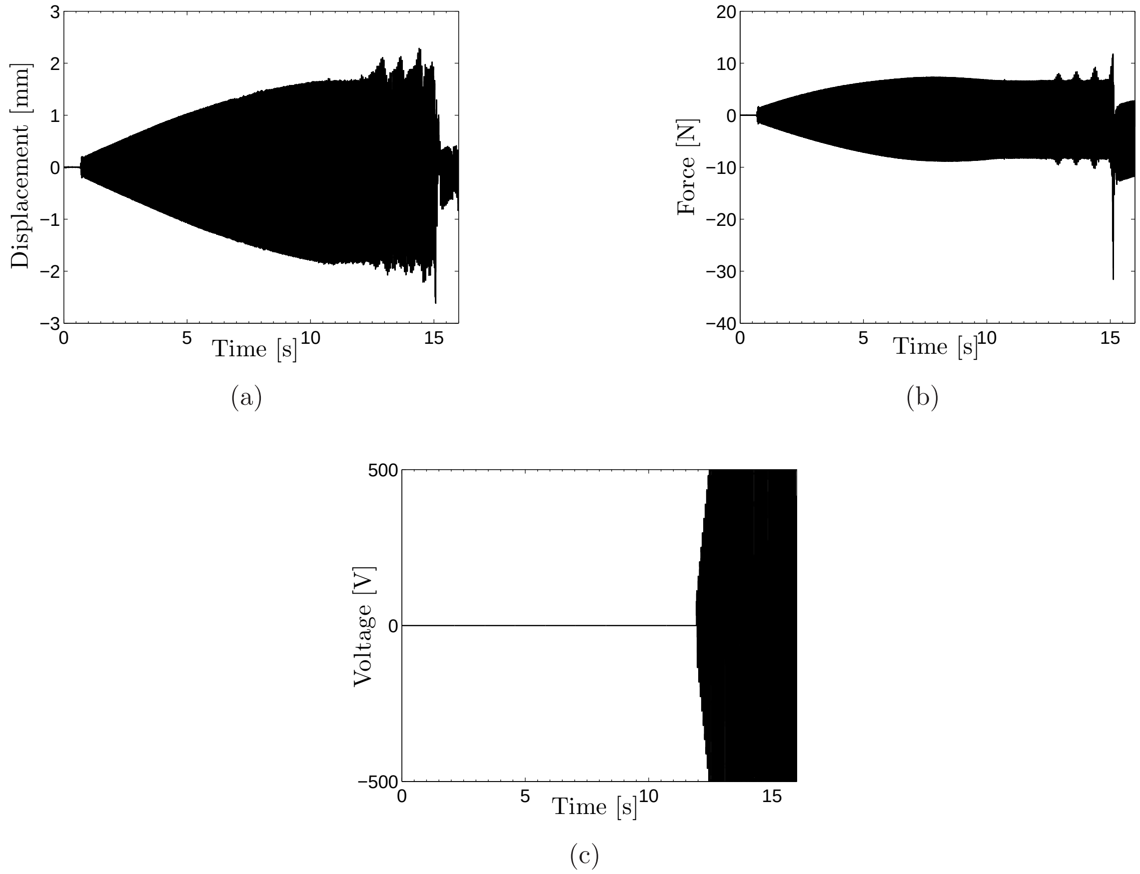


Figure 13: Experimental response showing a dynamically induced snap-through from state 2 to state 1 at 15 s. Forcing frequency Ω=36.4 Hz. (a) Displacement time series. (b) Force time series. (c) MFC signal targeting mode w_2 causing the subharmonic. Forcing frequency Ω=18.1 Hz. Voltage amplitude 500 V.

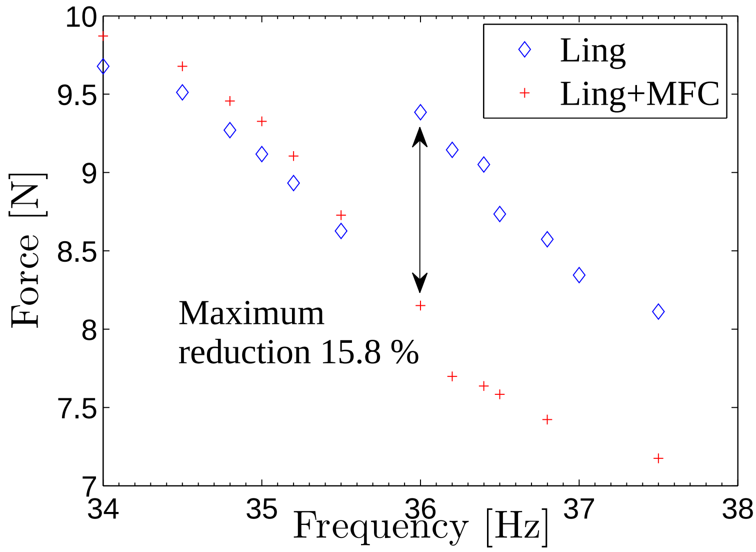


Figure 14: Dynamic snap-through shaker force comparison for 8-Ply bi-stable plate with MFC. $\diamond$ shaker induced. + shaker plus MFC induced. MFC signal at the subharmonic frequency, amplitude 500 V.

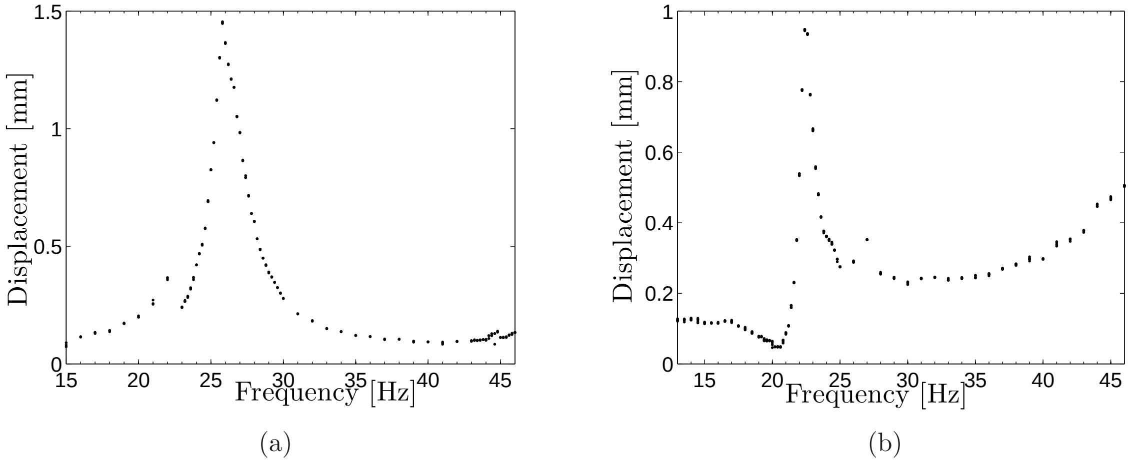


Figure 15: Experimental frequency response $[0_2 - 90_2/MFC]_T$ 4-ply plate for state 2. (a) For point P_x, forcing amplitude $F_o = 1.5\ N$. (b) For point P_y, forcing amplitude $F_o = 0.5\ N$.

25 Hz and $w_3^{(4p)}$ at 50 Hz, as seen in the frequency response diagrams given in Figure 15. The superscript $^{(4p)}$ is used in the notation to differentiate the modes of the 4-ply plate from those of the 8-ply plates. The observed quantitative difference in the modal frequencies between the 4-ply and 8-ply plates, does not affect the mechanisms causing the observed nonlinear dynamic behaviour. A frequency response diagram obtained using the MFC as the only external actuator, shown in Figure 16, indicates that with the current arrangement the highest actuation authority is obtained around the mode at 22.8 Hz.

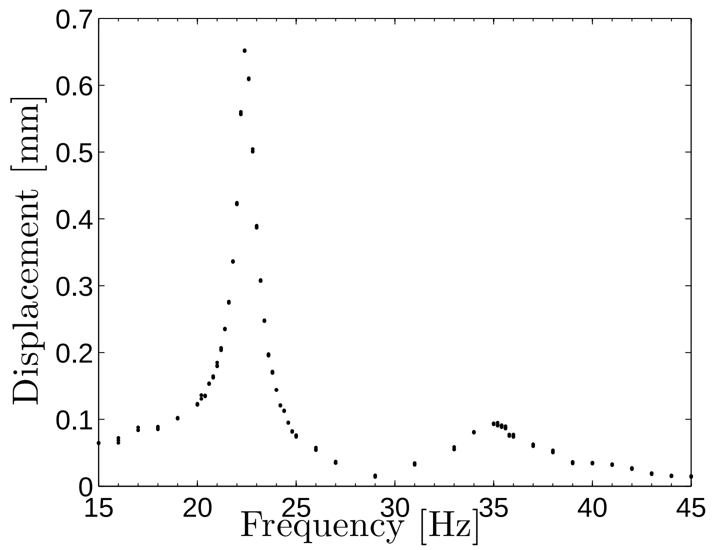


Figure 16: Experimental frequency response $[0_2 - 90_2/MFC]_T$ 4-ply plate for state 2, point P_y. Forcing amplitude $F_o = 1.0\ N$, frequency range Ω=[16, 50].

A subharmonic response of mode $w_1^{(4p)}$ at around 45 Hz can be observed in Figure 15(a). Thus the actuation strategy employed for the 8-ply plate can be replicated for the 4-ply plate, in this case targeting a frequency around 22.8 Hz with the MFC to maximize the actuation effect. The same comparison procedure for the force required to induce snap-through as for the 8-ply plates is implemented for the new 4-ply plate specimen. The results for the required force to induce snap-through with the shaker compared to those of the combined action of the MFC actuator and the shaker are shown in Figure 17. It can be seen that for the values closer to twice the natural fre-

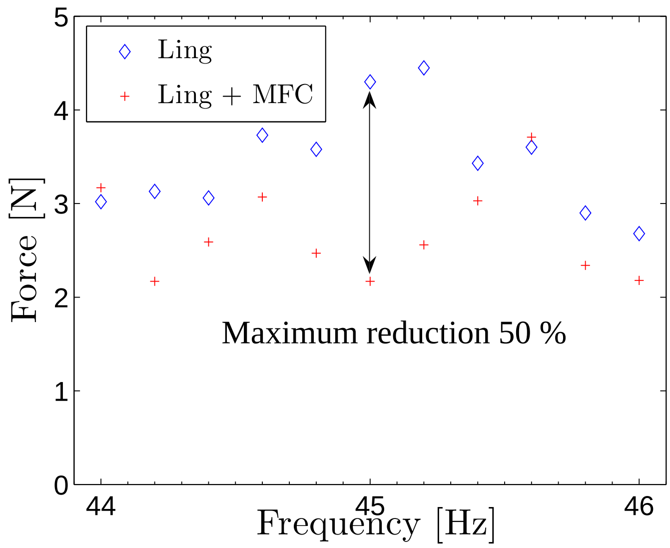


Figure 17: Dynamic snap-through force comparison for a 4-Ply Plate. ◊ shaker induced. + shaker plus MFC induced. MFC signal amplitude 500 V.

quency of mode $w_1^{(4p)}$ the effect of the MFC is maximised. The maximum reduction of 2 N or 47.6 % of the original force required to trigger snap-through under the sole action of the shakeris is obtained for a forcing frequency of 45 Hz.

Figures 18(a),18(b), and 18(c) show the displacement, force and MFC actuation signal time series respectively. It can be seen that the snap-through is triggered 0.1 after the maximum amplitude of the MFC signal is reached. Therefore, the dynamic actuation strategy may be seen as a pulse signal acting only over reduced time intervals on the morphing component minimising interaction with the rest of the structure

With the actuation strategy developed here almost 50 % of the actuation required to induce snap-through was obtained with the MFC actuator. Previous studies testing 2-ply $[0 - 90]$ 200x200mm bi-stable plates of the same material as the specimens in this work have successfully induced snap-through with piezo-electric actuators using a static DC voltage signal (Schultz and Hyer, 2003; Schultz et al., 2006). These 2-ply specimens required a voltage amplitude of 1700 V to induce snap-through. The experiments presented herein showed that taking advantage of the response of bi-stable plates, and the operational conditions which lead to dynamic perturbations, smaller voltage amplitudes to induce the snap-through can be achieved, even for much stiffer plates than previously presented. Therefore, in the context of adaptive structures the potential of exploiting the dynamic characteristics of a device as an actuation strategy for configuration control offers potential for augmenting the authority of the actuators to achieve the objective of morphing.

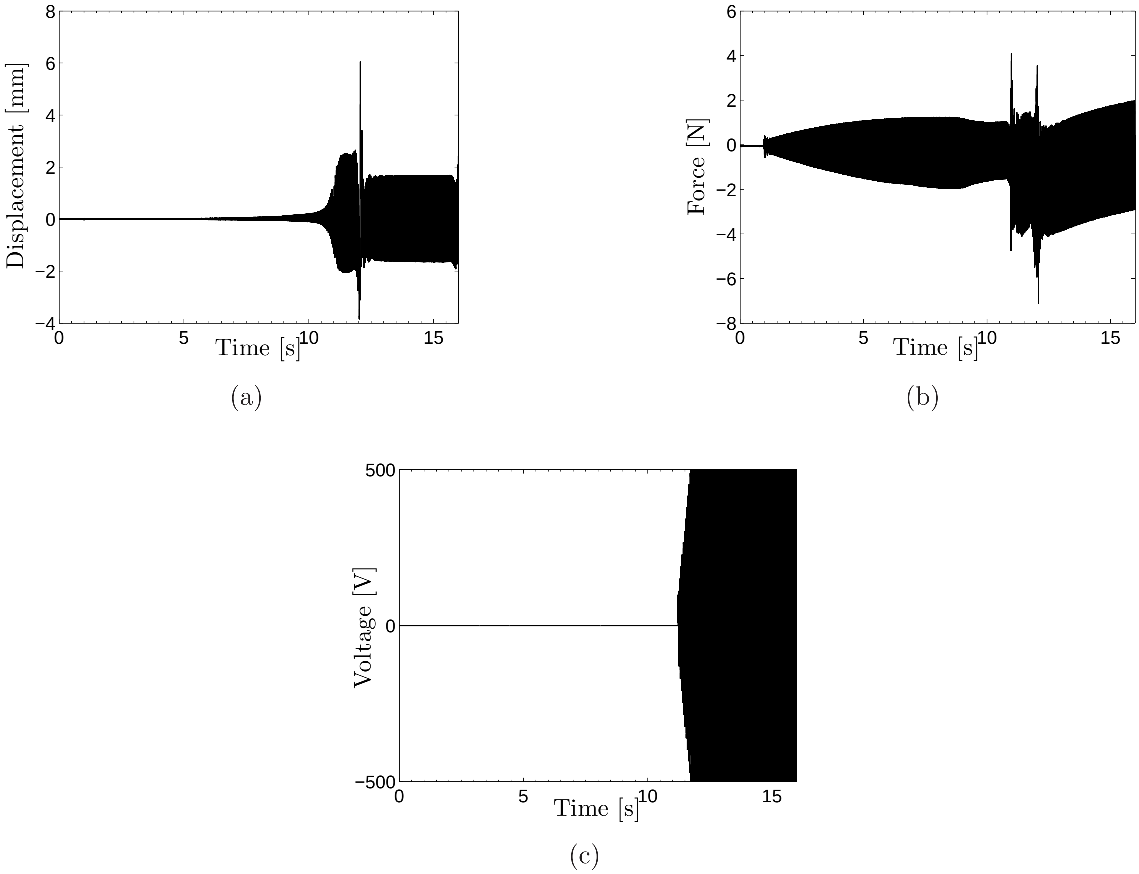


Figure 18: Experimental response showing a dynamically induced snap-through from state 2 to state 1 at 12 s. Forcing frequency Ω=44.8 Hz. (a) Displacement time series. (b) Force time series. (c) MFC signal amplitude 500 V.

4 CONCLUSIONS

The concept of dynamically inducing snap-through on bi-stable composites as a morphing strategy has been investigated. Bi-stable plate-MFC actuator morphing specimens were constructed to carry out this work. An actuation strategy based on dynamically inducing snap-through exploiting the linear and nonlinear dynamics of a bi-stable plate-MFC actuator structure is developed. The dynamic response of the specimens is exploited in order to choose the most efficient morphing strategy, taking into account the response of the structure and the MFC actuators capabilities.

A change of shape, or snap-through, using a dynamic excitation was successfully induced both with an external shaker and under the combined action of an MFC actuator and the external shaker simulating the dynamic operational environment these structures will be subject to. The developed morphing strategy significantly reduced the force required to induce snap-through when compared to the level needed with external shaker as sole actuation source. This strategy offers the possibility of lowering actuation voltages required to induce snap-through on structures incorporating bi-stable composites taking advantage of the dynamic characteristics and external perturbations these may be subject to during operation. Future developments will seek to test the concept by using more powerful MFC actuators and the combination of several MFC actuators to investigate new possible mechanisms for self-morphing structures.

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