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Inerter-based Vibration Suppression Systems for Laterally and Base-Excited Structures

Irina Lazar_1[1], S.A.Neild_2[1], D.J.Wagg_3[1]
[1]Department of Mechanical Engineering, Queens Building, University of Bristol,Bristol BS81TR, UK
email: Irina.Lazar@bristol.ac.uk, Simon.Neild@bristol.ac.uk, David.Wagg@bristol.ac.uk

ABSTRACT: The paper discusses the design and performance of a novel type of passive control system used for suppressing unwanted vibrations in civil engineering structures subjected to both lateral and base excitation. The new control system is inspired by the traditional tuned mass damper (TMD) with the modification that the mass is replaced by an inerter. An inerter has a two-terminal flywheel device capable of generating high apparent mass and its application is now extended from Formula 1 car suspension systems to train suspensions and building base isolation systems. The new control device is named the tuned inerter damper (TID). The tuning of the TID is based on existing tuning guidelines for damped vibration absorbers. We are assessing the performance of the TID in comparison to an equivalent TMD and an equivalent viscous damper, to show the advantages brought by the inerter's capacity of generating extra apparent mass. The analysis shows that the TID is capable of suppressing the response of higher vibration modes, while the TMD can only control the single mode targeted during the tuning of the device. Moreover, the TID is most efficient when located at the bottom of the structure, which is a potential advantage compared with TMD installation. A multi-degree-of-freedom (DOF) structure is presented as a numerical study to verify our theoretical results. The structure was subjected to base excitation in the form of unit impulse and earthquake load, and to lateral excitation based on wind tunnel tests data. Its performance was similar or superior to that of an equivalent TMD or viscous damper. Therefore, the TID represents a potentially attractive alternative to traditional passive control techniques.

KEY WORDS: inerter; tuned inerter damper; vibration suppression.

1 INTRODUCTION

The present paper is focused on the application of TID systems for vibration suppression in multi-storey laterally and base excited structures.

The TID, introduced by the authors in [1], is a passive control system consisting of a spring and a damper mounted in parallel and connected in series with an inerter.

The inerter was introduced by Smith [2] and it completes the force-current analogy between mechanical and electrical networks The force generated by the inerter, the equivalent of a capacitor in an electrical network, is

$$\mathbf{F} = b(\ddot{\mathbf{x}}_i - \ddot{\mathbf{x}}_j) \tag{1}$$

where b, the constant of proportionality and is named inertance and is measured in kilograms and $\ddot{\mathbf{x}}_i - \ddot{\mathbf{x}}_j$ represents the relative acceleration between its terminals.

The TID has initially been used in Formula 1 racing cars and then its use was extended to several types of suspension systems in vehicles [3], [4], [5], trains [6] and buildings [7], [8]. The optimality of inerter- based vibration isolation systems is studied in [9], [10], [11].

More recently, inerter-like mechanisms have been employed in vibration suppression systems for base excited structures. The authors of [12] study the performance of a device called tuned viscous mass damper (TVMD) and the modal response characteristics of a multiple-degree-of-freedom system incorporated with TVMDs [13]. TVMD systems have also been installed in a multi-storey steel structure built in Japan [14].

The TID has a similar layout to that of a passive TMD where the mass element has been replaced by an inerter. This change is aimed at overcoming the TMD mass limitation to $5-10\%$ of the mass concentrated on the targeted vibration mode, by exploiting the inerter's ability of generating an apparent mass that can be much greater than their physical mass. This can be realised through a range of mechanisms such as rack and pinions or ball-screw mechanisms. More recently, inertial hydraulic devices, with a helical tube providing "gearing", have also been patented [15].

An analytical tuning method for TID systems is given in [1] and their performance is assessed in comparison to that of equivalent TMD and viscous damper systems. It was shown through modal analysis that the device is most efficient when located at bottom storey level.

The existing literature is mostly focused on inerter-based vibration suppression systems installed in base excited structures. In [16], the authors study the behaviour of TID systems installed in a laterally-excited structure. The lateral load is represented by sine waves distributed triangularly along the building's height. It was shown that the similar performance is obtained for both TID and TMD systems. However, the TID had the advantage of being installed at bottom storey level and was capable of suppressing the response of superior modes of vibration.

Building on the results obtained in [16], this paper is studying the performance of TID systems when the host structure is subjected to realistic load patterns in the form of earthquake and wind loads.

In Section 2, we recall the n-DOF structure presented in [16], alternatively controlled using a TMD, a TID or a viscous damper. The structure is subjected to unit impulse and earthquake excitation and wind excitation based on the Tokyo Polytechnic University aerodynamic database, [17], as shown in Fig. 1. The TMD system parameters are based on Den Hartog's guidelines for tuned vibration absorbers [18]. The TID system is then finely tuned using the iterative approach described in [19], leading to the numerical values used in [16]. Sections 3 and 4 are dedicated to the assessment of the various systems performance under base and lateral excitation. Conclusions are drawn in Section 5.

2 BACKGROUND & STRUCTURAL SYSTEM

The structural system considered is a $n = 5$ storeys plane frame described in [16]. The system will be reduced to a shear beam building model with 5 DOFs. Three types of control are studied with the aim of assessing and comparing their performances. The structural system and the control systems are shown in Figure 1.

In our previous theoretical studies we have generally considered that the uncontrolled structure has null structural damping. This was done in order to preserve the similarities between the design of TID and of the damped vibration absorbers introduced by Den Hartog in [18]. Since this study is focused on realistic base and lateral excitation inputs, the structural damping is assumed to be 2%.

Building on the results obtained by the authors in [16], the tuning of the control systems components is carried out following the same steps. The values of all parameters are given in Tables 1 and 2. For the TID we use the values obtained through fine tuning, ensuring that the device performs optimally.

The equations of motion of each of the systems are expressed in state space form

$$\begin{cases} \dot{z}(t) = & Az(t) + B_d F \\ y(t) = & Cz(t) \\ z(0) = & z_0 \end{cases} \tag{2}$$

where A is the system matrix of dimensions $(2n \times 2n)$, B_d is the disturbance matrix of dimensions $(2n \times n)$, $y(t)$ represents the output, C is the $(2 \times 2n)$ output matrix, $z(0)$ represents the initial condition and $z(t)$ of dimension $(2n \times 1)$ is the state vector. $F(n \times 1)$ represents the exterior disturbance that can be either base or lateral excitation.

The state vector has the form $z(t) = [x(t), \dot{x}(t)]$, where $x(t)$ is the displacement vector of dimensions $(n \times 1)$,

$$A = \begin{bmatrix} O_{n,n} & I_{n,n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \text{ and } B_d = \begin{bmatrix} O_{n,n} \\ M^{-1} \end{bmatrix} \tag{3}$$

where $O_{n,n}$ and $I_{n,n}$ are the $(n \times n)$ null and unit matrices respectively. M $(n \times n)$, $C(n \times n)$ and $K(n \times n)$ are the mass, damping and stiffness matrices respectively. These matrices are particularised for each structural system (uncontrolled and controlled using either a TMD, a TID or a viscous damper), following the notations in Figure 1 and the assumptions valid for shear beam building models. Please note that in case of the TMD and TID systems, the number of DOFs increases to $n+1$.

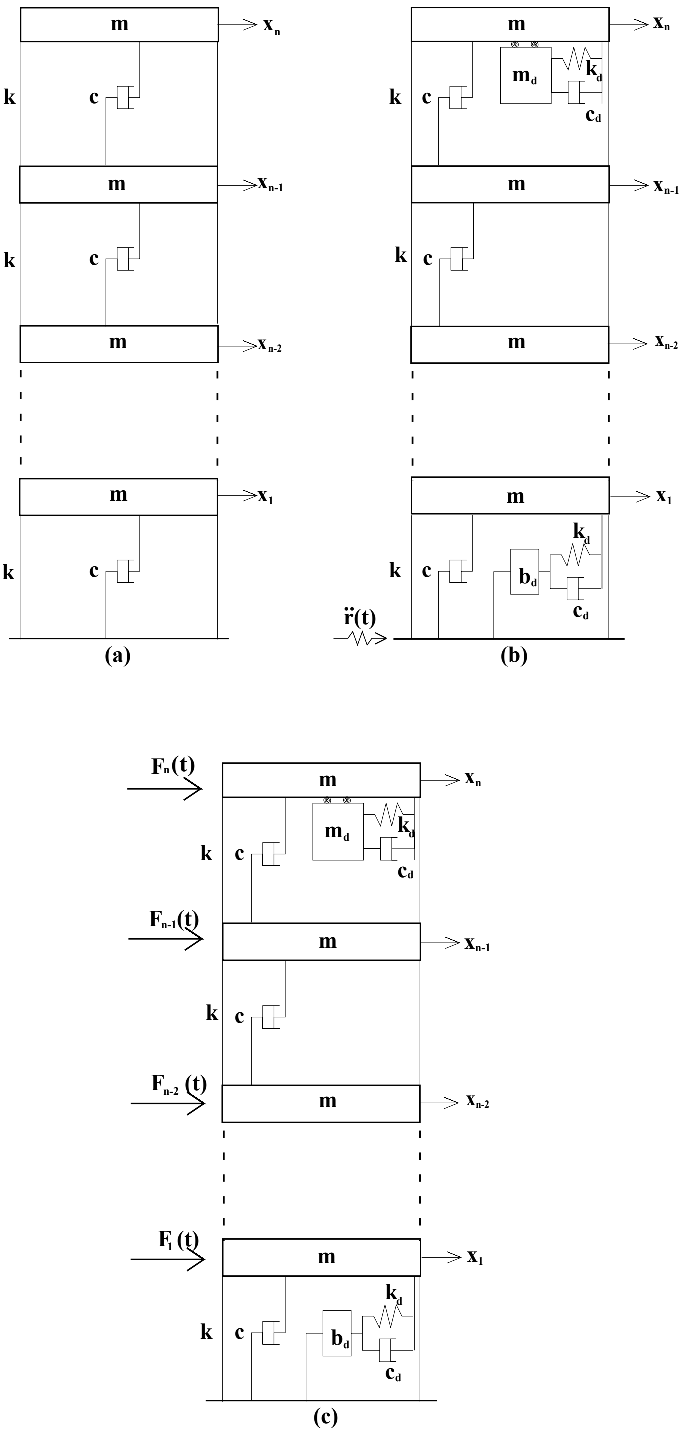

Figure 1. (a) Uncontrolled system; (b) Base excited system with a TMD installed at top storey level and a TID installed at bottom storey level; (c) Laterally excited system with a TMD installed at top storey level and a TID installed at bottom storey level.

The uncontrolled system parameters given in Figure 1 are $m = 1\text{kNs}^2/\text{m}$ and $k = 1500\text{kN/m}$. The damping matrix, C, was evaluated using Rayleigh's method for 2% structural damping in the first and second vibration modes. Using this set of parameters, the fundamental frequencies obtained are

$\omega_1 = 1.75$Hz, $\omega_2 = 5.12$Hz, $\omega_3 = 8.07$Hz, $\omega_4 = 10.37$Hz and $\omega_5 = 11.83$Hz.

For each type of excitation we will show the response in terms of displacement time histories and (or) Fourier spectra.

3 BASE EXCITATION

For base excited structures it is convenient to write the equation of motion in relative coordinates. Therefore, the disturbance force F in Equation 2 becomes $F = -MI_{n,1}\ddot{r}(t)$, where $\ddot{r}(t)$ represents the ground acceleration. After the evaluation of the M, C and K matrices for each of the four systems, we can calculate matrices A and B_d for each control system and find the structural response by solving Equation 2.

The tuned parameters evaluated in [16] for the base-excited structure are given in Table 1. Please note that the damping value of the viscous damper was selected such that the response in the first mode of vibration is similar to the TID and the TMD when the structure is subjected to sinusoidal ground excitation. Further details can be found in [16].

Control system	TMD	TID	Damper
$\mu_m(\mu_b)$ (-)	0.03	0.37	-
$m_d(b_d)(\mathrm{kNs^2/m})$	0.13	1.63	-
$k_d(\mathrm{kN/m})$	14.89	196.6	-
$c_d(\mathrm{kNs/m})$	0.3	4.5	43.72

Table 1. Control systems tuning parameters for base-excited structures.

3.1 *Unit impulse input*

In order to assess the control systems performance, the structure has first been subjected to a unit impulse input over a period of 50 seconds. Figure 2 shows the displacement response obtained at top storey level for each of the four systems. It can be seen that the TMD and TID controlled systems response is similar and superior to those of the uncontrolled and damper controlled structures.

Figure 3 shows the Fourier spectra of the displacement response described above. As expected, these are matching the results obtained in [16] where a sinusoidal base excitation was considered.

While all control system are capable of improving the uncontrolled structure response, the TMD and TID system outperform the damper controlled system in the vicinity of the first fundamental frequency. In addition, due to the fine tuning procedure applied for the TID system, the two split peaks obtained by targeting the first vibration mode have equal amplitudes leading to a better structural performance. As concluded in our previous work, the TID and damper systems are capable of suppressing the response of upper modes of vibration.

3.2 *Earthquake load*

The same structural systems have also been subjected to earthquake excitation using the NS acceleration recording of El Centro, shown in Figure 4.

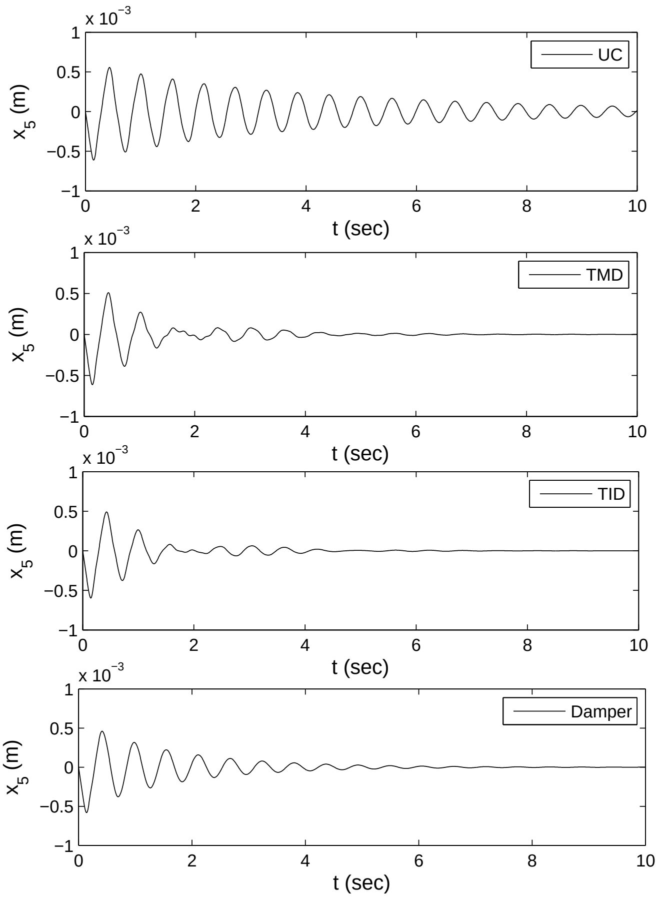

Figure 2. Impulse displacement response at top storey level.

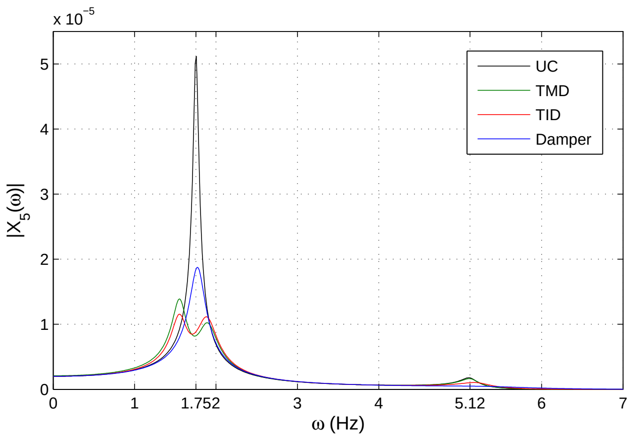

Figure 3. Fourier spectra of the displacements obtained at top storey level under impulse base excitation.

Figure 5 shows the displacement time histories at top storey level. Two zoomed plots are included for a better evaluation of the response in the high displacement regions. Once again, the TMD and TID systems show similar performance, while the damper system is less efficient.

The remarks above are verified by means of the Fourier spectra in Figure 6. Looking at the zoomed plot, it can be seen that the damper system does not affect the first fundamental frequency of $1.75Hz$ and has a poorer performance in its

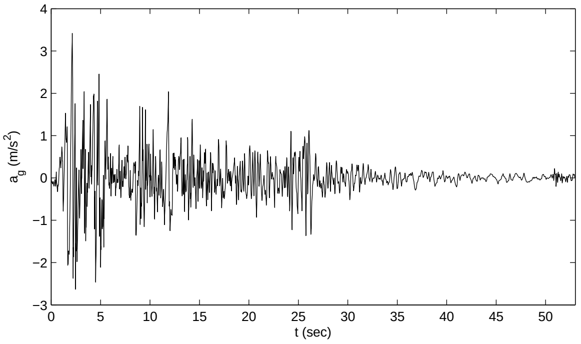

Figure 4. El Centro ground acceleration time history.

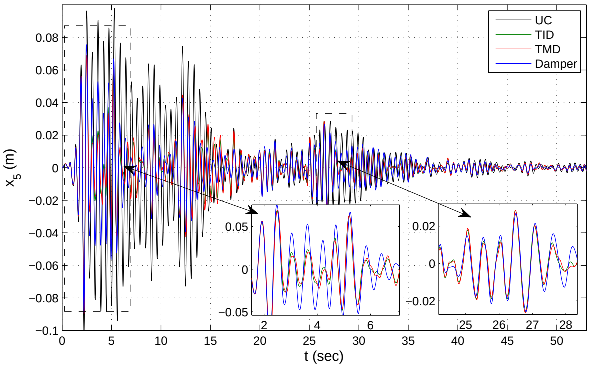

Figure 5. Displacement time history at top storey level.

vicinity. The TID and TMD systems create two lower amplitude peaks in this region (see Figure 3), leading to an improved performance of the controlled structure.

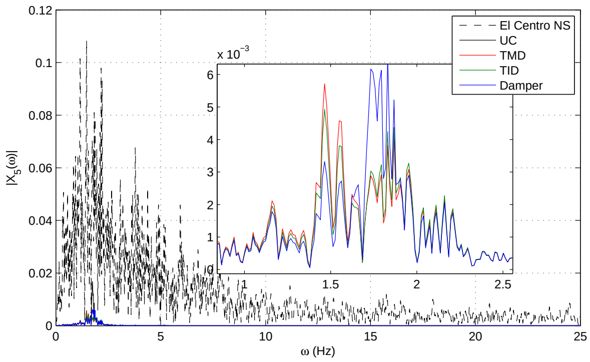

Figure 6. Fourier spectra of the displacements obtained at top storey level under earthquake excitation.

4 WIND LOAD

In the case of lateral excitation, the disturbance force, F, in Equation 2 becomes $F = [F_1, F_2, \ldots, F_n]$, where $F_i, i = 1 : n$ represents the force applied at each storey level.

The structural response is evaluated following the procedure described in the previous section.

The tuned parameters obtained in [16] for laterally-excited structures are given in Table 2. It can be noted that the values are similar to those obtained for base-excited structures. This is beneficial as in practice only one set of parameters may be used. However, for this numerical study we will consider slightly different parameters, as described in [16].

Control system	TMD	TID	Damper
$\mu_m(\mu_b)$ (-)	0.03	0.37	-
$m_d(b_d)$(kNs^2/m)	0.13	1.63	-
k_d(kN/m)	15.11	199.6	-
c_d(kNs/m)	0.29	4.5	43.72

Table 2. Control systems tuning parameters for laterally-excited structures.

The wind load considered in this paper is based on a set of data obtained through wind tunnel simulations on small scale models described in detail in [17] . Using the database provided by the authors of [17], we selected all necessary parameters and obtained a realistic wind-type excitation time history. Figure 7 shows the generalised model of a low-rise building without eaves having a flat roof subjected to an along-wind component acting on face 1 under the angle θ. D and B represent the depth and the breadth of the building, while H represents its height. For this study, we have chosen B=D=H=16m as given in [17]. This is convenient as it leads to a realistic storey height of 3.2m for the 5 storey building considered. The angle θ is null and therefore, the wind is perpendicular on face 1 of the building.

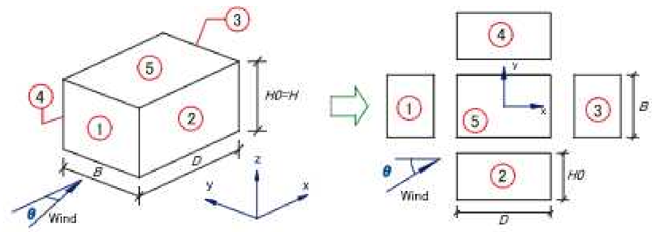

Figure 7. Wind direction and building plan as given in [17] .

Figure 8 shows the distribution of the points where the pressure was measured on each face of the building.

For a better understanding, Figure 9 shows five such time histories obtained from the tests in points situated on each face of the building. As expected, face 1, where the wind load is applied, is subjected to pressure, while all others are subjected to suction.

For convenience, we only take into consideration the coefficients obtained for faces 1 and 3. In order to evaluate the most unfavourable situation, their effect will be superimposed.

Using the wind pressure coefficient time histories in the database, we were able to determine the wind pressure in all points based on the formulas in Equations 4 and 5, given in [20].

$$p(t) = c_p(t)\frac{1}{2}\rho u^2 \tag{4}$$

$$u = u_{max}\left(\frac{Z}{Z_0}\right)^{\alpha} \tag{5}$$

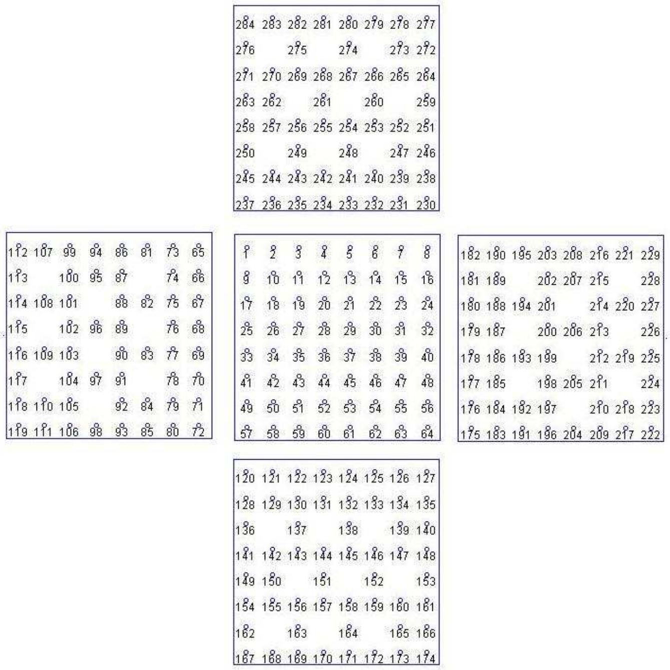

Figure 8. Measuring points distribution on the building surface [17] .

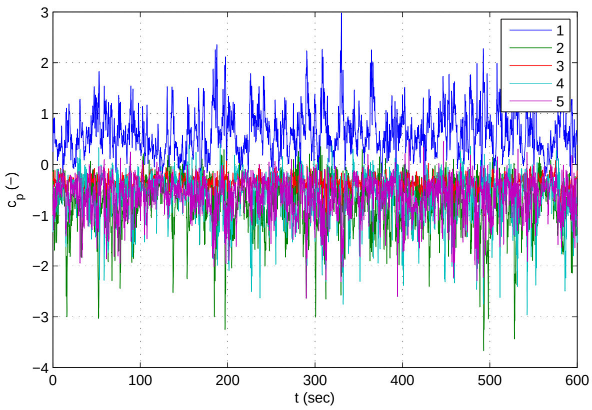

Figure 9. Wind pressure coefficients time history.

where c_p is the wind pressure coefficient in one point, $\rho = 1.229\text{kg}/\text{m}^3$ is the air density and u is the wind velocity calculated using Equation 5. u_{max} is the maximum wind velocity measured at reference hight Z_0. It is common practice to consider Z_0 equal to the roof height and therefore we considered $Z_0 = 16m$. Z represents the height of the point where we want to evaluate the wind velocity, u. $\alpha = 0.25$ is a constant coefficient called power-law exponent depending on the building location (urban or rural) and stability class. This value in appropriate for stability class D. The maximum wind speed at reference height was chosen as $u_{max} = 15\text{m}/\text{s}$.

The wind pressure time histories obtained are concentrated at each story level. The force time histories, F_i, are evaluated by multiplication with the afferent area of each storey. As the wind load frequency content is low, we have scaled it such that it excites the structural system under investigation. Figure 10 shows the Fourier spectra of the top storey displacements for the four systems. It can be seen that the TMD and TID systems performance is higher than that of the viscous damper system.

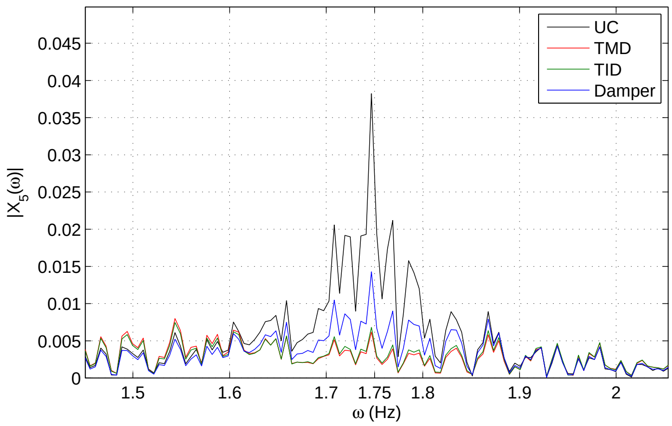

Figure 10. Fourier spectra of the displacements obtained at top storey level under lateral excitation.

5 CONCLUSION

The present study was focused on the analysis of a five storey structure subjected to base and lateral excitation. Building on previous results obtained by the authors in [16], the performance of four structural systems is assessed (uncontrolled structure, structure having a TMD installed at the top, structure having a TID installed at the bottom and structure having a viscous damper installed at the bottom). All systems are carefully tuned such that the best outcome is obtained.

First, they were subjected to impulse ground excitation which offers a basic understanding of the frequency response of each structure. Then, an earthquake recording is used in order to evaluate the structure's performance under realistic loading. In both cases the behaviour of the TMD and TID controlled structures was superior to that of the damper controlled structure. However, only the TID and the damper are capable of suppressing the vibration of upper modes. In the last section, the same structures were subjected to a realistic wind load based on wind tunnel tests performed at Tokyo Polytechnic University in Japan, with near identical performance obtained using the TID or TMD devices and poorer performance of the viscous damper. Although the tuning was done separately for each type of excitation, the TID parameters remain almost unchanged, implying that once tuned for either forcing, its performance will be near to optimal for both cases.

The potential advantage of the TID system over the TMD is that a similar performance can be obtained using a lighter device due to the gearing in the inerter and that the control system is placed at the bottom of the building and that it acts as a damper at higher frequencies. Future research is focused on experimental testing and implementation of TID systems.

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