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Use of the Riccati Equation On-Line for Adaptively Controlling a CSTC Distillation Column

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Abstract

A third-order CSTC model for the separation dynamics of packed distillation columns is converted to a form expressed in terms of deviations of input and output from steady-state. This form has been found to be necessary for the application of the Banks' method of On-line Riccati Control. The state-dependent coefficient matrices of the process are derived analytically in terms of the fundamental length, volatility and capacitance parameters of the process. Having successfully tested the modified model against the original, whole-value, simulation model and against analytic behavioural prediction, simulation experiments are conducted, comparing the responses and costs of the Banks'-controlled system with linear output feedback. The optimal control is found to be significantly superior to linear control for equal input excursions, in terms of cost-function-value, energy consumption and, for large-turn-down ratios, speed-of-response. The results offer promise for the control method when applied to tubular columns in the future.

1. Introduction

This report is the fourth of the series ACSE RR573[1], 576[2], 630[3] devoted to the analytical modelling and optimal control of distillation columns and chemical reactors. The Banks' method[4] of on-line application of the Riccati equation to current values of the state-dependent coefficient matrices to produce state-varying feedback coefficients has been the control target of this series. RR576 applied the method to a 1st-order, isothermal model of a chemical reactor and Rowlands[5] extended the application to a 2nd-order reactor model that included varying temperature effects.

Since the On-Line Riccati equation demands a model from which to compute the current process coefficient matrices, it is important that the model to be used is validated a priori. In RR630 therefore, the parametric model for the CSTC column derived analytically in RR573, was comprehensively tested using simulation under open-loop and linear closed-loop control to confirm the behavioural predictions derived from the model. In particular, its closed-loop stability margins, etc. were checked against analytic prediction for a wide range of the basic process parameters L, the normalised length of the column, α, related to the mixture's relative volatility and T, the normalised time-constant of the end vessels. The model was validated succesfully.

For the Banks method[4], a first-order matrix differential equation representation is required, (the state- and input-coefficient matrices being allowed to be state-dependent). Although the original report (RR573) was aimed primarily at parametric transfer-function derivation, a parametric matrix d.e. representation of the unlinearised source equations was also derived in preparation for attemps at future On-Line Riccati Control; together with an approriate integral quadratic cost function. The model, however, contained a feed disturbance term that is not readily accomdated. Buiding on experience from the CSTR chemical reactor[2],[5] therefore, the model is first modified in Section 2 of this report into a more readily applicable form. Before proceeding with the application in Section 4, the modified model was first tested alongside the original parametric representation of RR573 and 630 as described in Section 3 in order to ensure the absence of errors in manipulation to which analytical work of this sort is always prone.

Conclusions are drawn in Section 5.

2. Conversion to Error Co-ordinates

2.1 The Original Model

In ACSE research report 573 [1], the CSTC column model was derived in the form :

$$\begin{bmatrix} \dot{S} \\ \dot{S}_e \\ \dot{S}_e(1) \end{bmatrix} = \underline{A} \begin{bmatrix} S \\ S_e \\ S_e(1) \end{bmatrix} + \underline{B}\, V_s + \underline{J}\, F \qquad (1)$$

in which S(t) denotes the product separation, $S_e(t)$ its associated equilibrium value, and $S_e(1)(t)$ that of the fluids recycled through the column from the end vessels. $V_s(t)$ is the manipulable vapour rate in the stripping section and F(t) the feed flow rate of liquid (= also to that of the feed vapour).

State coefficient matrix $\underline{A}$ was shown to be constant at :

$$\underline{A} = \begin{bmatrix} -\dfrac{L\alpha+\varepsilon}{L\alpha} & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (2)$$

where L is the normalised column section length given by :

$$L = \frac{k\,L'}{V} \qquad (3)$$

in which L' is the actual length, k the evaporation constant (expressed in mols p.u. length p.u. time p.u. departure from equilibrium mol-fraction) and V (here a constant) is the designed vapour rate for the rectifier section. Parameter α is the slope of the linearised equilibrium curve in the stripping section and

$$\varepsilon = \alpha - 1 \qquad (4)$$

The designed value for feed F is related to V thus :

$$V = \frac{\alpha F}{\varepsilon} \qquad (5)$$

State-dependent input coefficient matrix $\underline{B}$ was derived to be :

$$\underline{B} = \begin{bmatrix} \dfrac{\varepsilon - S - \alpha S_e}{LV} \\ \dfrac{\alpha\left[S_e(1) - S_e\right]}{LV} \\ \dfrac{\left[S - \alpha S_e(1) + \varepsilon\right]}{\alpha V T} \end{bmatrix} \qquad (6)$$

where T is the normalised time constant of the end-vessels, whilst disturbance coefficient matrix $\underline{J}$ was shown to be given by :

$$\underline{J} = \begin{bmatrix} \dfrac{\varepsilon}{(\alpha+1)LV} \\ 0 \\ \dfrac{\left[S - \alpha S_e(1) + \varepsilon\right]}{\alpha T V} \end{bmatrix} \qquad (7)$$

In the light of experience gained in applying the Banks' on-line Riccati method[4] to a CSTR chemical reactor[2][5], the proposed cost function was expressed in terms of deviations :

$$\Delta S = S - S_r \qquad (8)$$

and $$\Delta V = V_s - \frac{V}{\alpha} \qquad (9)$$

from reference steady-state separation, S_r, and its associated design vapour rate, $\frac{V}{\alpha}$, for the stripping section respectively. Formulae for the steady state separation value and its associated equilibrium values were derived in terms of L, α, ε and V in the ealier report[1].

The proposed cost function was, and remains :

$$C = \int_0^\infty \left\{ (\Delta S)^2 + \lambda \left(\frac{\alpha \Delta V}{V} \right)^2 \right\} dt \qquad (10)$$

where λ is some weighting factor, possible values for which were estimated from derived gain formulae.

The Bank's method as presently formulated[4] does not admit the disturbance term $\underline{J}$ F in eqn. (1) and it is therefore necessary to attempt to reformulate the model to avoid such a term. We therefore derive the model in terms of the deviations ΔS and ΔV rather than the absolute values of the variables S and V.

2.2 Obtaining a Model in Error Coordinates : Generalised Treatment

Consider the system :

$$\underline{\dot{x}} = \underline{A}(\underline{x})\ \underline{x} + \underline{B}(\underline{x})\ \underline{u}' + \underline{J}(\underline{x})\ \underline{d} \qquad (11)$$

in which $\underline{d}$ is the disturbance vector.

Let $\underline{A}(\underline{x})$ be constant but let $\underline{B}(\underline{x})$ and $\underline{J}(\underline{x})$ take the linear, state-dependent forms :

$$\underline{B}(\underline{x}) = \underline{B}_0 + \underline{B}_1\ \underline{x} \qquad (12)$$

and $$\underline{J}(\underline{x}) = \underline{J}_0 + \underline{J}_1\ (\underline{x}) \qquad (13)$$

in which $\underline{B}_0$, $\underline{B}_1$, $\underline{J}_0$, $\underline{J}_1$ are constant (as in the case of the column model of eqns. (1) - (7)). We confine attention to scalar $\underline{u}'$ and $\underline{d}$ (also as with the column model) so that, for an n-th order process, $\underline{A}$, $\underline{B}_1$ and $\underline{J}_1$ are n x n matrices whilst $\underline{B}_0$ and $\underline{J}_0$ are n x 1.

Now if $\underline{x}_r$ and $\underline{u}'_r$ are associated steady state values in the presence of a constant disturbance d, then :

$$0 = \underline{A}\underline{x}_r + \underline{B}_0\underline{u}'_r + \underline{J}_0\underline{d} + \underline{B}_1\underline{x}_r\underline{u}'_r + \underline{J}_1\underline{x}_r\underline{d} \qquad (14)$$

whilst, in general :

$$\underline{\dot{x}} = \underline{A}\underline{x} + \underline{B}_0\underline{u}' + \underline{J}_0\underline{d} + \underline{B}_1\underline{x}\underline{u}' + \underline{J}_1\underline{x}\underline{d} \tag{15}$$

If we set deviations :

$$\underline{\Delta x} = \underline{x} - \underline{x}_r \tag{16}$$

$$\underline{\Delta u}' = \underline{u}' - \underline{u}'_r \tag{17}$$

the solution of eqn. (14), from (15) yields :

$$\underline{\Delta\dot{x}} = \underline{A}\underline{\Delta x} + \underline{B}_0\underline{\Delta u}' + \underline{J}_1\underline{\Delta x}\underline{d} + \underline{B}_1\left(\underline{x}\underline{u}' - \underline{x}_r\underline{u}'_r\right) \tag{18}$$

Eliminating whole values $\underline{x}$ and $\underline{u}'$ in favour of deviations $\underline{\Delta x}$ and $\underline{\Delta u}'$, together with constant references $\underline{x}_r$ and u'_r we therefore obtain :

$$\underline{\Delta\dot{x}} = \left(\underline{A} + \underline{u}'_r\underline{B}_1 + \underline{d}\underline{J}_1\right)\underline{\Delta x} + \left(\underline{B}_0 + \underline{B}_1\underline{x}_r + \underline{B}_1\underline{\Delta x}\right)\underline{\Delta u}' \tag{19}$$

Thus, the model may be expressed as a bilinear deviation system :

$$\underline{\Delta\dot{x}} = \underline{A}'\underline{\Delta x} + \left(\underline{B}'_0 + \underline{B}'_1\underline{\Delta x}\right)\underline{\Delta u}' \tag{20}$$

where the constant coefficient matrices are given by :

$$\underline{A}' = \underline{A} + \underline{u}'_r\,\underline{B}_1 + \underline{d}\,\underline{J}_1 \tag{21}$$

$$\underline{B}'_0 = \underline{B}_0 + \underline{B}_1\,\underline{x}_r \tag{22}$$

and $$\underline{B}'_1 = \underline{B}_1 \tag{23}$$

2.3 The CSTC Model in error coordinates

In this application we set :

$$u'(t) = V_s(t) \tag{24}$$

$$u'_r = \frac{V}{\alpha} \tag{25}$$

$$\underline{x} = \begin{bmatrix} S & S_e & S_e(1) \end{bmatrix}^T \tag{26}$$

$$d = F \tag{27}$$

Thus, from eqns. (21), (2), (6) and (7) as shown in Appendix 1 we deduce the constant coefficient matrix $\underline{A}'$ to be given by :

$$\underline{A}' = \begin{bmatrix} -\frac{(L+1)}{L} & \frac{L-1}{L} & 0 \\ 1 & -\frac{(L+1)}{L} & \frac{1}{L} \\ \frac{1+\varepsilon}{\alpha^2 T} & 0 & -\frac{1+\varepsilon}{\alpha T} \end{bmatrix} \tag{28}$$

We distinguish the steady-sate (reference) values of separation by suffix r so that :

$$\underline{x}_r = \begin{bmatrix} S_r & S_{er} & S_{er}(1) \end{bmatrix} \tag{29}$$

which may be obtained from the separation formulae of the original research report[1]. Hence, again as shown in Appendix A1 :

$$\underline{B}'_0 = \underline{B}_0 + \underline{B}_1 \underline{x}_r = \begin{bmatrix} \frac{\varepsilon - S_r - \alpha S_{er}}{L V} \\ \frac{\left[S_{er}(1) - S_{er}\right]\alpha}{LV} \\ \frac{S_r - \alpha S_{er}(1) + \varepsilon}{\alpha V T} \end{bmatrix} \tag{30}$$

But, from eqn. (38) of the original report[1] we find :

$$\alpha\, S_{er}(1) - S_r = \varepsilon \tag{31}$$

and also from that report we note, {from the eqn. preceding (39)}, that

$$S_{er} = \frac{S_r(L+1)(\alpha+1) - 2\varepsilon}{(L-1)(\alpha+1)} \tag{32}$$

so that, as shown in Appendix 1, substituting the derived steady state formula (39)[1] for S_r, i.e.

$$S_r = \frac{\varepsilon}{(\alpha+1)}\left[\frac{(3\alpha+1)L + \alpha-1}{(3\alpha-1)L + \alpha+1}\right] \tag{33}$$

in the expression for B'_{01} in eqn. (30) above, we get

$$\text{element } B'_{01} = \frac{-\ 2\varepsilon\left[\varepsilon+L(\alpha+1)\right]}{LV\,(\alpha+1)\left[(3\alpha-1)L + \alpha + 1\right]} \tag{34}$$

Similarly, as shown in Appendix 1, we deduce element B'_{02} to be :

$$B'_{02} = \frac{\alpha}{V}\left(S_{er}-S\right) = \frac{4\alpha\varepsilon}{V\,(\alpha+1)\left[(3\alpha-1)L + \alpha + 1\right]} \tag{35}$$

and $\qquad B'_{03} = 0 \qquad (36)$

Hence, in terms of plant parameters, the matrix $\underline{B}_0$ may be expressed :

$$\underline{B}'_0 = \begin{bmatrix} \dfrac{-2\alpha\varepsilon\left[L(\alpha+1)+\varepsilon\right]}{LV(\alpha+1)\left[(3\alpha-1)L+\alpha+1\right]} \\[2ex] \dfrac{4\alpha\varepsilon}{V(\alpha+1)\left[(3\alpha-1)L+\alpha+1\right]} \\[2ex] 0 \end{bmatrix} \tag{37}$$

Finally, we extract the state dependent coefficients of matrix $\underline{B}$ (eqn. (6)) to give B_1, i.e.

$$\underline{B}_1 = \begin{bmatrix} -\dfrac{1}{LV} & -\dfrac{\alpha}{LV} & 0 \\[2ex] 0 & -\dfrac{\alpha}{LV} & \dfrac{\alpha}{LV} \\[2ex] \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix} \tag{38}$$

Thus the CSTC model has been expressed as a bilinear representation involving only deviations in separation and input flow $V_s(t)$ with all coefficients algebraically related to plant coefficients, L, α, ε, T and V as required. As an essential check on the algebra, exhaustive numerical checks on parametric eqn. (28), (37) and (38) have been made by comparing the elemental values of matrices $\underline{A}'$, $\underline{B}'_0$ and $\underline{B}'_1$ computed (a) from these derived equations and (b) from the source equations for S_r, S_{er} and $S_{er}(1)$ together with eqns. (16) to (23) of RR630.

3. Testing the Error-Coordinate Model

The above-derived model was simulation tested using the scheme illustrated in Fig. 1 which is merely an implementation of eqn. (20) with $\underline{\Delta x}$ set equal to $[\Delta S, \Delta S_e, \Delta S_e(1)]^T$ and $\underline{\Delta u}' = \Delta V_s$, and using the derived parametric equations (28), (37) and (38) for computing respectively the elements $\underline{A}'$, $\underline{B}'_0$ and $\underline{B}'_1$ $(=\underline{B}_1)$ from given plant parameters L, α, ε $(=\alpha-1)$ and T. The whole values of S, S_e, $S_e(1)$ and V_s were derived from the computed error signals by addition of the associated steady-state-consistent reference values S_r, S_{er}, $S_{er}(1)$ and V_{sr} which were computed from the known steady-state solution formulae (again from prespecified values of L, α, ε) i.e. eqns. (33), (A1.6), (A1.10) and $V_{sr}=F/\varepsilon$ respectively. [The absolute value of F being unimportant as regards the dimensionless separation variables, this was kept at unity throughout as in the testing of the whole-value model of RR630 reported previously[3]].

A wide range of parameter space was explored and the error-coordinate model was found to give identical results, for open-loop step tests and for initial condition recovery, as were obtained for the whole-value model. Since a comprehensive survey of these responses was presented in RR630 and these were succesfully checked against analytical prediction, those obtained with error-coordinate model, being identical, are not presented again here.

A trivial point but worth noting is that, in RR630, the pertubing input u was a dimensionless fractional value defined as :

$$u = \frac{2\nu}{V} = \frac{2\left(V_s - V_{sr}\right)}{V} = \frac{2\varepsilon\left(V_s - V_{sr}\right)}{\alpha F} \tag{39}$$

where ν is the deviation of vapour flow rate (in the stripping section) from its desired reference level V_{sr} whereas u is its value normalised as shown. Here whole-value input u' is defined as V_s and pertubation

$$\Delta u' = \nu = V_s - V_{sr} \tag{40}$$

Hence
$$\Delta u' \equiv \frac{uV}{2} = \frac{u\alpha F}{2\varepsilon} \tag{41}$$

The relationship (41) and distinction between the process input u, (as defined and used in RR573 and RR630) and process input $\Delta u'$, (as defined here), is important for the correct interpretation of gain values used for linear control comparisons with optimal control of given input weights, λ.

Having completely tested the reformulated SIMULINK model, it was then used in conjunction with MATLAB Riccati solving routine at every simulation time step, and the updated control law coefficients of feedback matrix D used for optimal control in line with Banks' method[4] as also indicated in Fig. 1 and as discussed more fully in Section 4.

4. Application of On-Line Riccati Control

The CSTC column deviation model of eqn. (20), was simulated using SIMULINK representation based on Fig. 1 and the whole values of the observed output and input variables S(t) and $V_s(t)$ reconstructed by the addition of the known references Sr and V_{sr}. [From entry of normalised parameters F (=1), L and α, S_r was obtained from known steady-state solution (33) and V_{sr} set to the steady state consistent value of $F/(\alpha-1)$]. For optimal- (i.e. On-Line Riccati-) Control, the SIMULINK model was connected on-line to the MATLAB Riccati Solver as indicated merely by using state deviations ΔS, ΔS_e and $\Delta S_e(1)$, at each simulation update, to compute the new driving coefficient matrix B from $\underline{B}'_0 + \underline{B}'_1 \left[\Delta S,\ \Delta S_e,\ \Delta S_e(1)\right]^T$ as indicated in Fig. 1, before passing this to the Riccati Solver. From this, the updated control coefficients of the D matrix were computed and returned for use in the m.v. simulation feedback loop as shown. The matrices A (here set = A'), B'_0 and B'_1 were calculated from eqns. (28), (37) and (38).

For comparison purposes a constant coefficient, linear output - proportional feedback control law

$$V_s = V_{sr} + \left(\frac{KV}{2}\right)\left(S_r - S\right) \tag{42}$$

could also be switched in, K being the proportional given as previously defined in RR630 and elsewere.

4.1 Preliminary Results and Discussion

Fig. 2. shows the response of separation S(t) and associated input response $V_s(t)$ for L=15, α=5 [i.e. for case (d) as specified in earlier reports] from initial conditions set at feed separation z-Z = $\varepsilon/(\alpha+1)$, here = 0.6667 with reducing input weighting coefficient λ in the cost function. The results are all non-oscillating and increase in speed as λ is reduced as

would be expected because the increasing excursion in V_s that results from the reducing λ value. A limit on the maximum attainable $V_s = 3V_{sr}$ (here = 3 x 0.25 = 0.75), i.e. a turn-down ratio of 3:1 is set in the series of tests which only the lowest two values of λ impinge, as seen. Clearly faster responses could be obtained by further reduction in λ, but only at the expense of higher transient values of $V_s(t)$. Fig. 4 shows the responses from the extreme case of zero initial conditions i.e. no initial separation whatsoever and in this case larger values of λ are needed, as would be expected, in order to prevent the larger initial error in separation causing the limit $3V_{sr}$ to be seriously impinged for excessive time periods.

Fig. 4 shows the responses of optimal control from initial separation (S(0)=0.7200) much closer to the target separation of 0.7531. It is interesting to note that a non-minimum phase response now results, somewhat similar to the predicted (and observed) open-loop responses in RR573.

The response of linear control within the same turn-down ratio of 3:1 are shown in Figs. 5, 6 and 7 for feed, zero and near steady-state initial conditions respectively for increasing values of control gain K. Again the expected increase of speed of response (here with increasing gain) is observed but with the V_s limit imposing an envelope on the maximum speed attainable as with optimal control. Figs. 8, 9 and 10 are for similar conditions but show the speed-up effect on increasing the V_s limit from $3\overline{V}_S$ to $10\overline{V}_S$ using linear control whilst Figs. 11, 12 and 13 show a similar effect of this relaxation under optimal control.

4.2 Linear v Optimal Control Comparison

A potential pitfall of linear, output-feedback control revealed in RR630[3] (in addition to the prediction of small signal instability for large gains predicted in RR573[1] and demonstrated in RR630) was the risk of process moving to an undesired operating condition for long periods (i.e. the "dwell" phenomenon) before reverting to the target condition. This was the result of large initial offsets and large gains (> around 50% critical gain) driving the value of $V_s(t)$ beyond the peak of the steady state S versus V_s separation characteristic. This has not arisen here however as a result of the V_s-limit imposed (on both linear and optimal control). The lack of robustness of linear (versus optimal) control only becomes apparent in the absence of such a limit when K is increased (in the linear case) in an attempt to achieve a very fast response. The higher speed is achievable with optimal control by reducing λ, but only by the demand for very large input excursions. We therefore confine ourselves here to comparing linear and optimal controls set for similar speeds of response with similar V_s excursions within the limit $0 \leq V_s \leq 3\overline{V}_S$.

Responses for the optimal and linear cases are compared in Figs. 14, 15 and 16 for feed, zero and near steady-state initial conditions with K and λ set to give near equal maximum V_s excursion. As can be seen, output response shapes and speeds are very similar. There are, however, some improvements (a) in the computed cost as defined by eqn. (10) and (b) in the area under the $V_s(t)$ curve which is proportional to column energy consumption over the 50-unit time-interval chosen. Their values are tabulated below :

Table 1 **Cost and Energy Consumptions of Linearly and Optimally Controlled CSTC**						
Response Initial Conditions	Cost			Energy		
	Linear	Optimal	% Improvement	Linear	Optimal	% Improvement
Feed Conditions	0.0496	0.0351	23.23	15.610	15.071	3.455
Zero IC's	3.1248	2.1843	30.10	15.886	14.930	6.020
Near Steady State	0.0101	0.0072	28.71	15.291	14.822	3.070

An outstanding improvement is apparent when the V_S-limit is relaxed to $0 \le V_S \le 10\overline{V}_S$ as shown in Fig. 17 for near steady state initial conditions. The calculated cost function of linear control is 0.0192 and that of optimal control is 0.0059, which gives 69.36% improvement. Energy consumption is also remarkably improved by 28.365% (c.f. only 3.070% when the limit is $0 \le V_S \le 3\overline{V}_S$, as in Table 1) i.e. energy consumption in linear control = 23.0490 and in optimal control = 16.5111. There is a significant improvement also in the speed of response under optimal control.

Fig. 18 shows that with the $10\overline{V}_S$ limit imposed, the dwell phenomenon (under linear control only and demonstrated in RR630[3]) begins to manifest itself when the gain is increased to 70. Note that this instability starts to show when K > 50% critical gain value (of 110.25, calculated from eqn. (36) in RR630 for case (d) (i.e. L=20, α=2)), which accords well with that of case (a) (i.e. L=20, α=2) as presented in RR630[3]. The calculated cost function of linear control is 0.0263 and that of optimal control is 0.0057, which give 78.33% improvement whilst energy consumption of linear control is 30.766 and that of optimal control is 16.845, i.e. 45.25% better. Even when control signal hits the limit, optimal control can perform better still and is stable.

5. Conclusions

As would be expected optimal control generates a significant percentage improvement in the cost (as defined) of the process operation and a noticeable though reduced improvement in energy consumption when compared to linear output control set for similar peak excursion in the manipulable vapour rate and at turn-down ratios of around 3:1. Response times are similar. Improvements produced by optimal control in terms of cost function, energy and speed are enhanced vey considerably (compared to linear controls set for the same transient input excursion) when the turn-down ratio is increased to 10:1.

For the lower turn-down ratio, results for case (d) (i.e. L=15, α=5) have been chosen rather than case (a) (i.e. L=20, α=2 which shows less relative improvement through optimal control). The reason is that case (d) runs at a considerably larger steady-state separation (0.7531) than does case (a) (0.4563) giving the improved control less scope to demonstrate its potential. This observation is encouraging, however, in that the more realistic distributed system models of tubular columns should reveal greater performance improvements under the optimal (On-Line Riccati) Control strategy since such models generate even higher separation for given length and volatility parameters. However, the development is awaited of (a) a suitably simplified model for incorporation in the on-line Riccati controller for tubular columns and (b) a simulation of tubular columns. These tasks should now be undertaken, this report having shown that substantial performance benefits might reasonably be expected.

Finally, a cost function based on whole values of input and output variables seems likely to generate further improvements in the true cost of the process (of which energy is an important component). This, however, requires some modification to the zero steady-state $\dot{P}$ matrix limitation that the present Banks' formulation requires for the on-line Riccati method.

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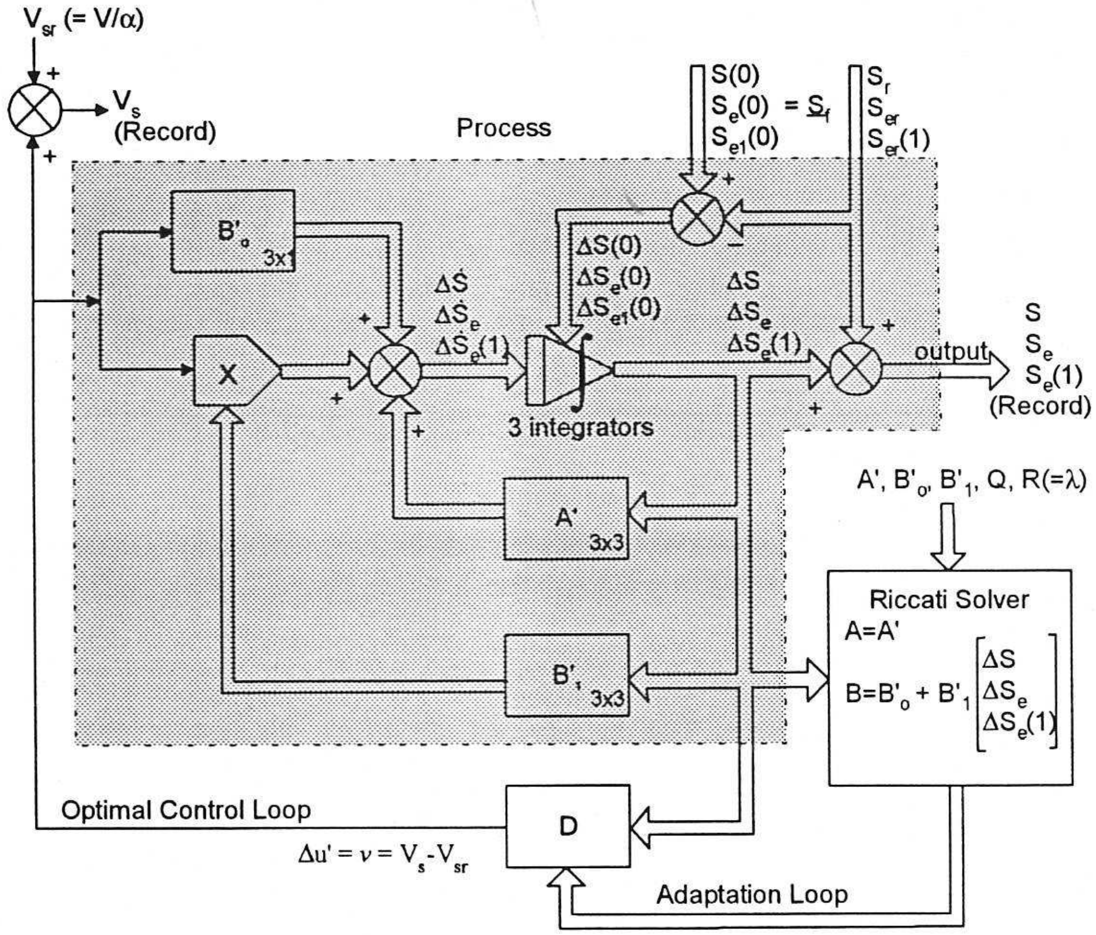

Fig. 1 Simulation scheme for Error Coordinate Model

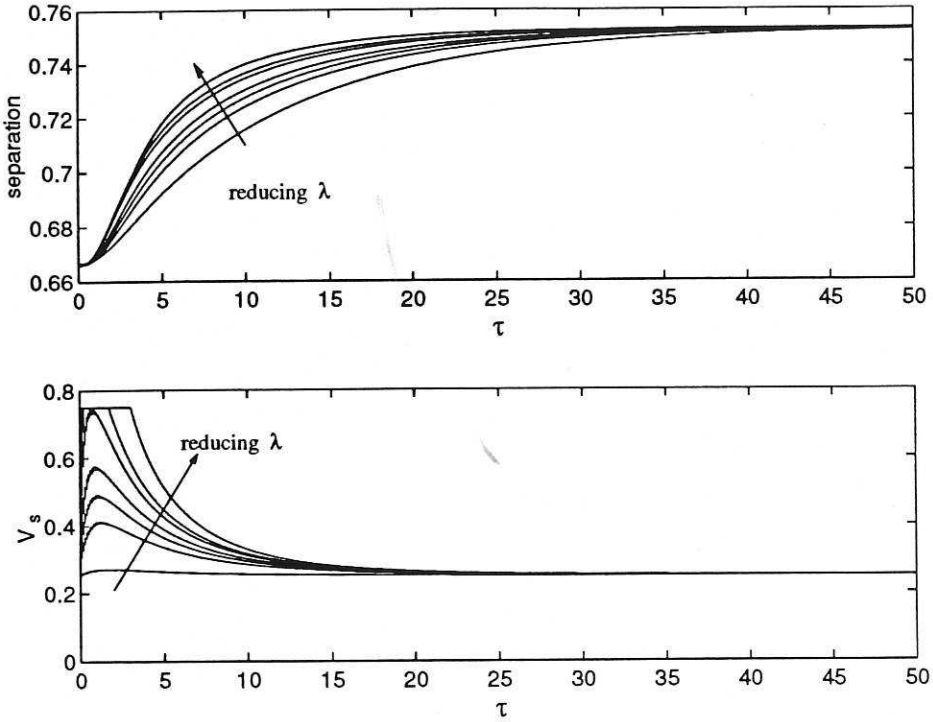

Fig. 2. Responses of optimal control with I.C's set at feed condition.
λ=0.5, 0.05, 0.03, 0.011, 0.008, 0.006, 0.004

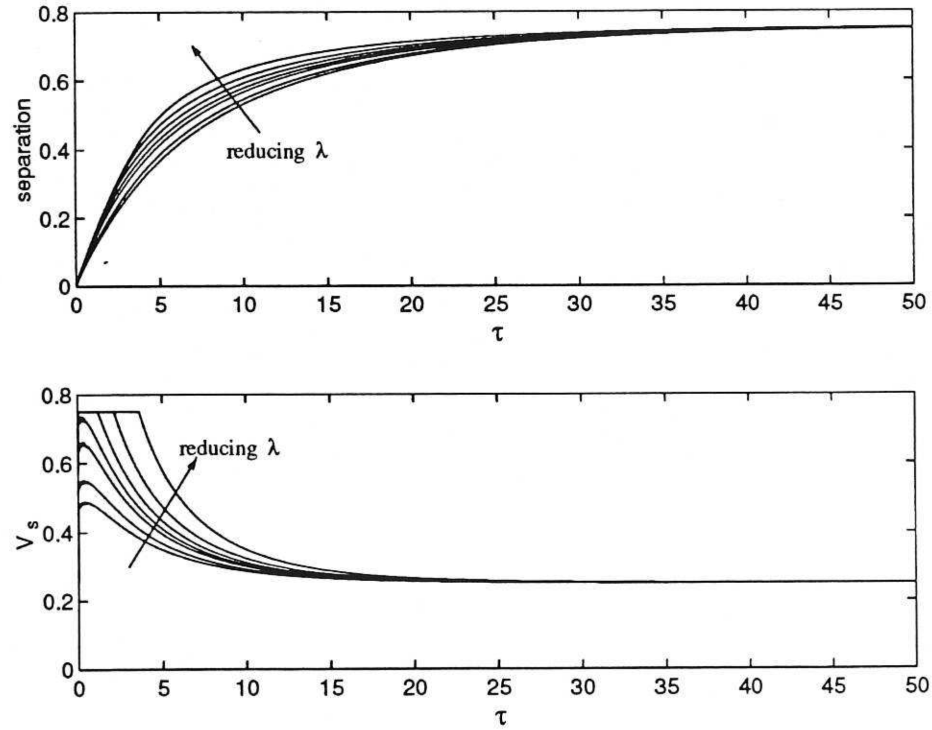

Fig. 3. Responses of optimal control from zero initial separation.
λ=2.0, 1.5, 1.0, 0.8, 0.6, 0.4

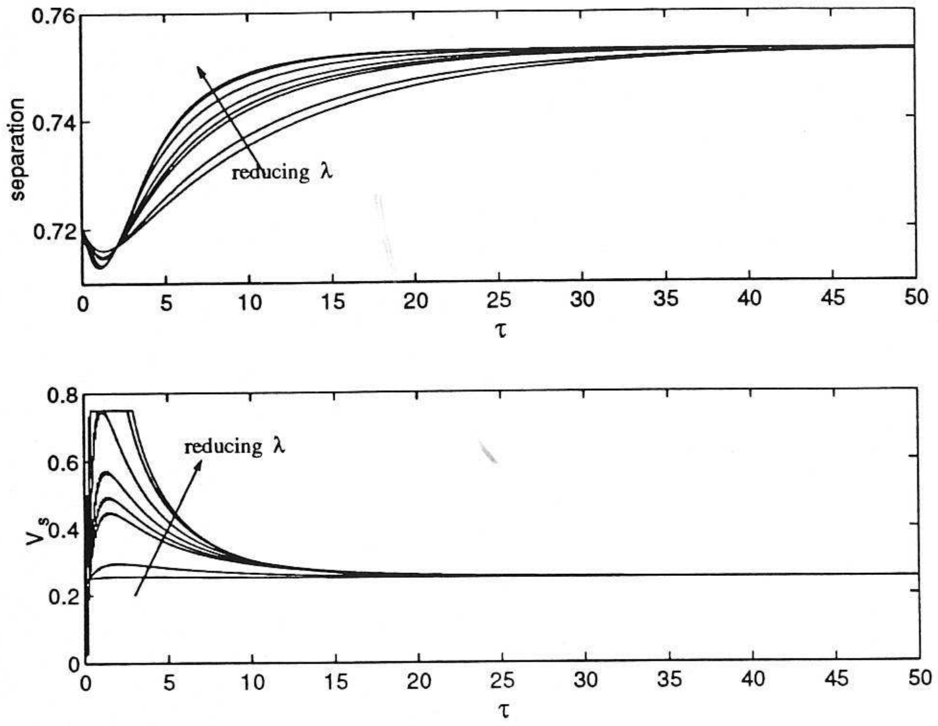

Fig. 4. Responses of optimal control with I.C's close to steady-state.
λ = 0.5, 0.05, 0.008, 0.006, 0.004, 0.002, 0.0009, 0.0007

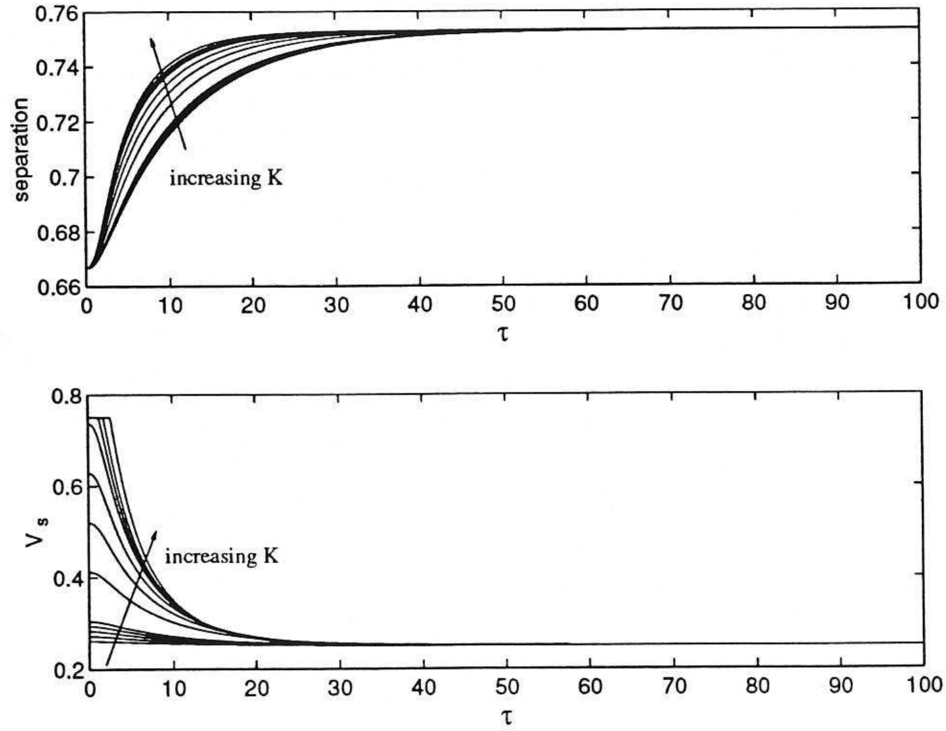

Fig. 5. Responses of linear control with I.C's set at feed condition.
K = 0.2, 0.4, 0.6, 0.8, 1.0, 3.0, 5.0, 7.0, 9.0, 10.0, 11.0, 13.0

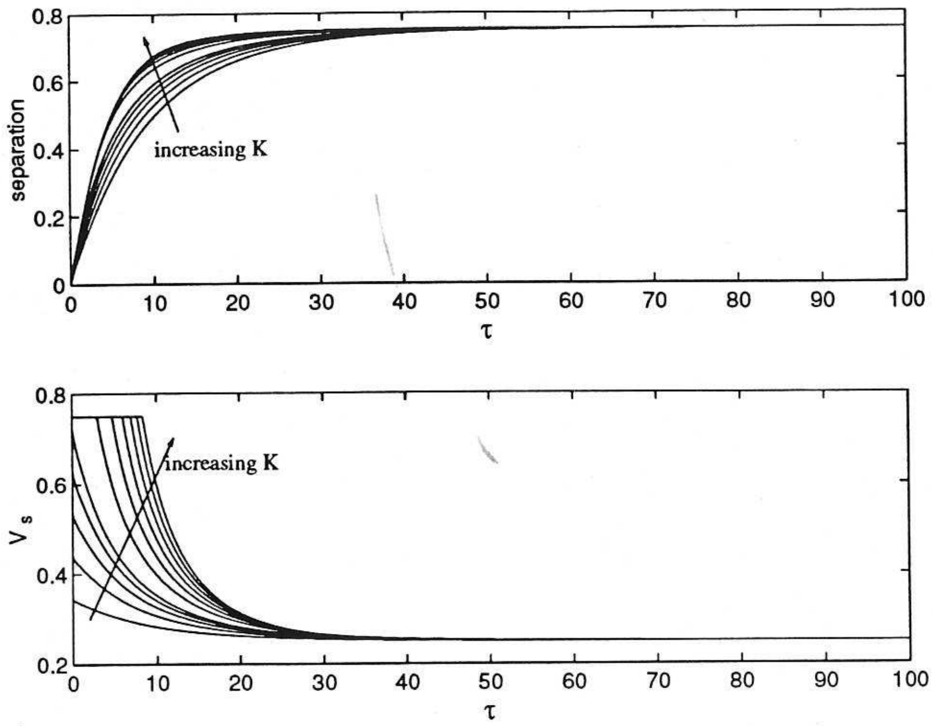

Fig. 6. Responses of linear control from zero initial separation.
K = 0.2, 0.4, 0.6, 0.8, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0

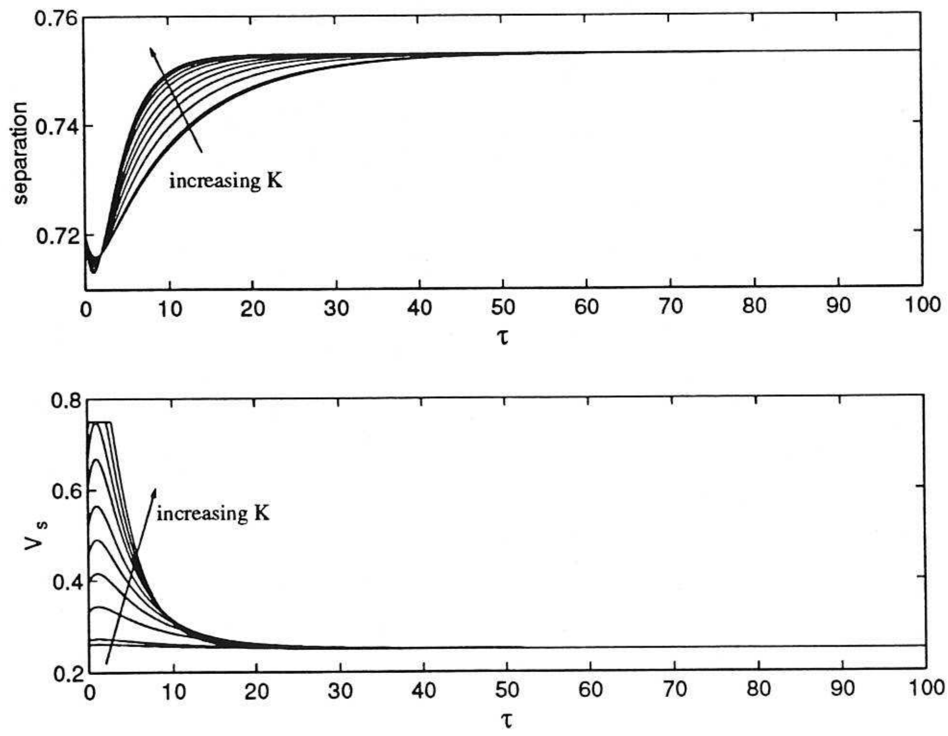

Fig. 7. Responses of linear control with I.C's close to steady state.
K=0.5, 1.0, 4.0, 7.0, 10.0, 13.0, 17.0, 20.0, 23.0, 26.0

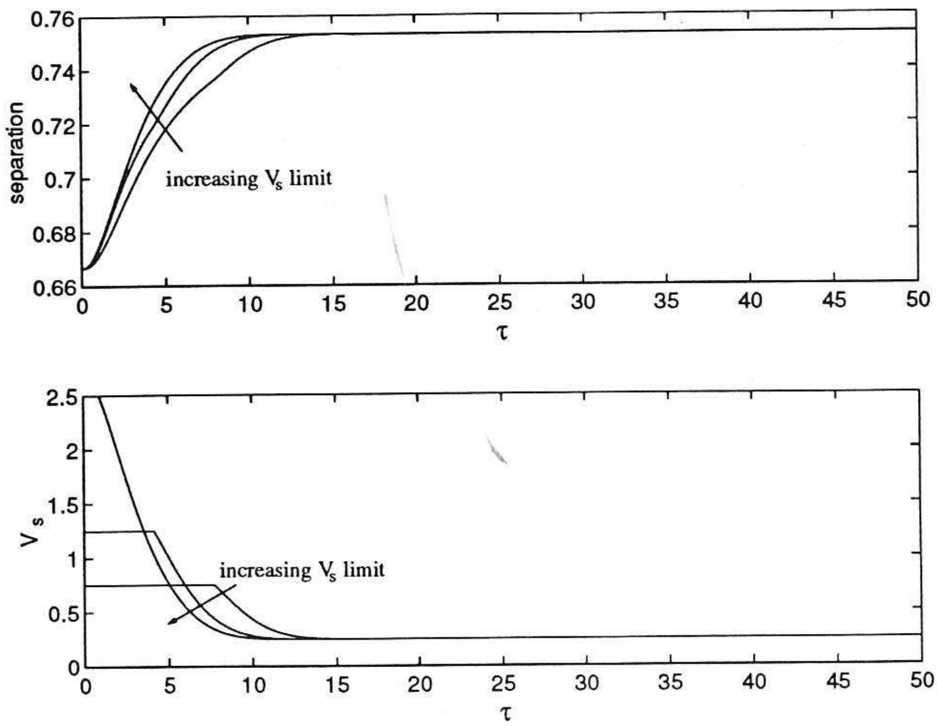

Fig. 8. Effect of increasing the V_s-limit ($= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s$) : Feed I.C's, linear control, K=45.0.

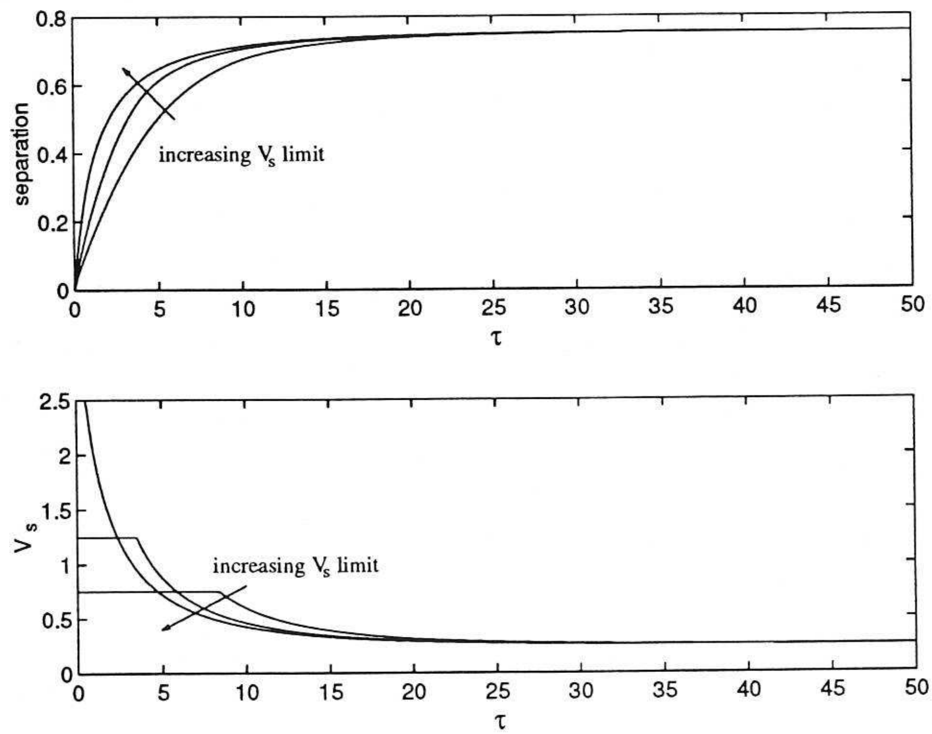

Fig. 9. Effect of increasing the V_s-limit ($= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s$) : Zero I.C's, linear control, K=7.0.

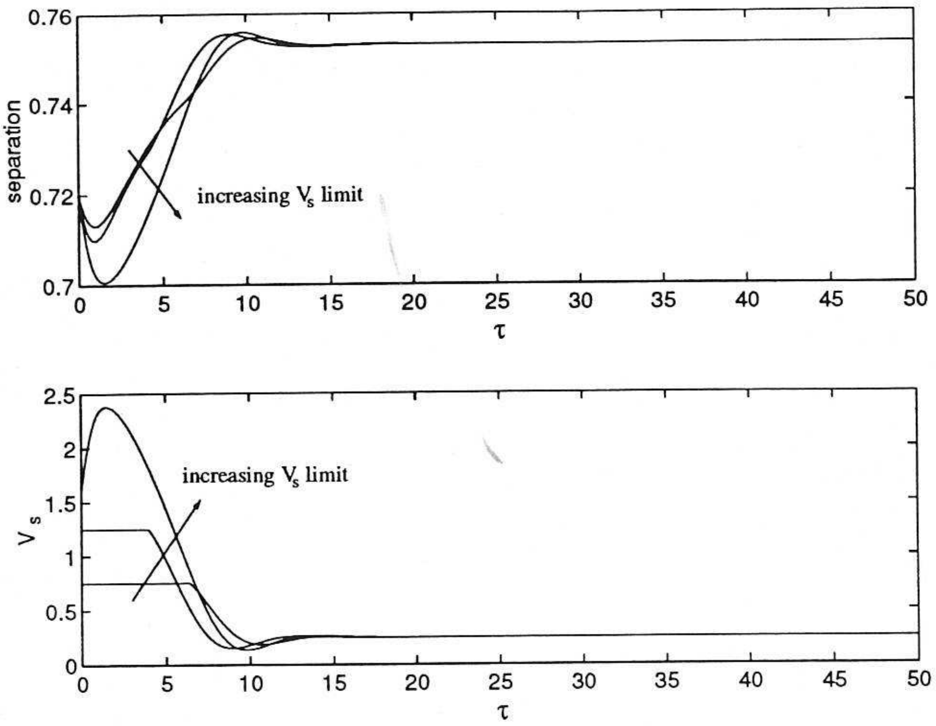

Fig. 10. Effect of increasing the V_s-limit $(= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s)$: Near steady-state I.C's, linear control, K=65.

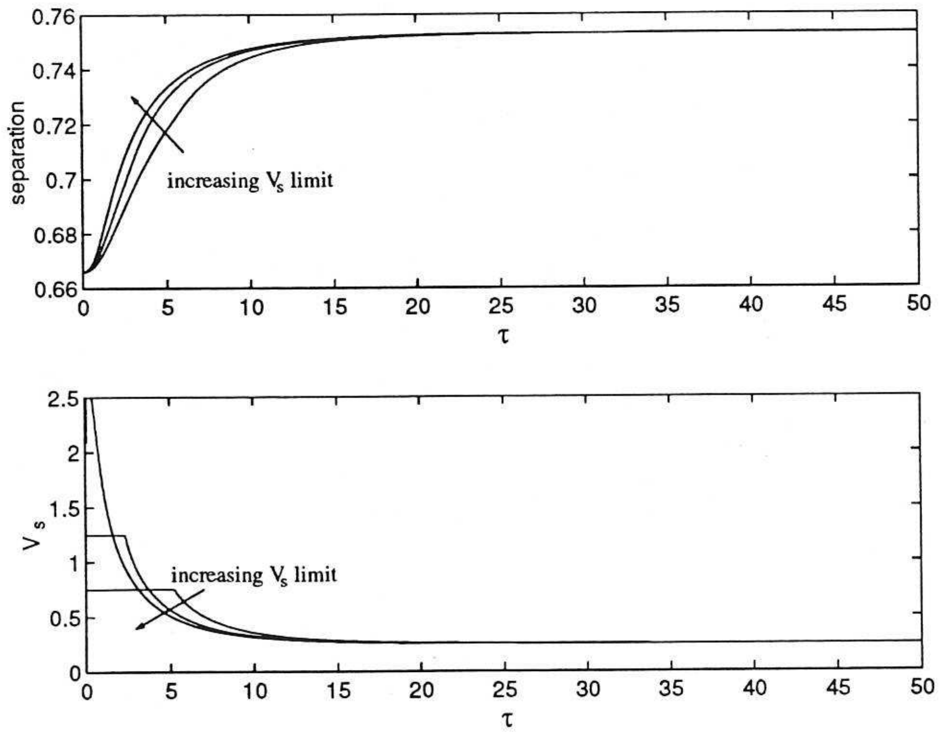

Fig. 11. Effect of increasing the V_s-limit $(= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s)$: Feed I.C's, Optimal control, λ=0.0009

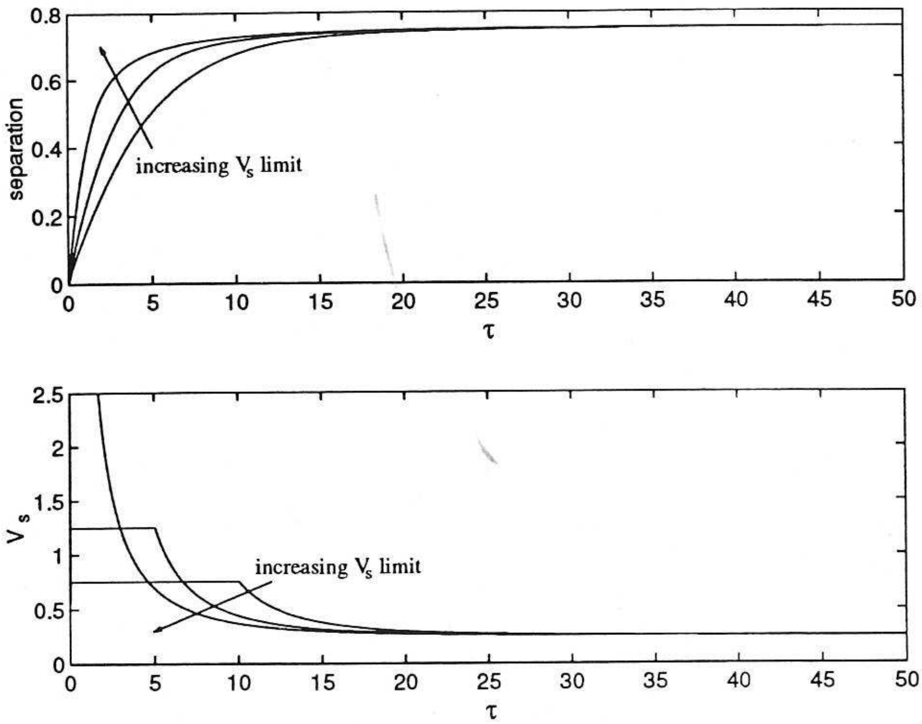

Fig. 12. Effect of increasing the V_s-limit $(= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s)$: Zero I.C's, optimal control, λ=0.008

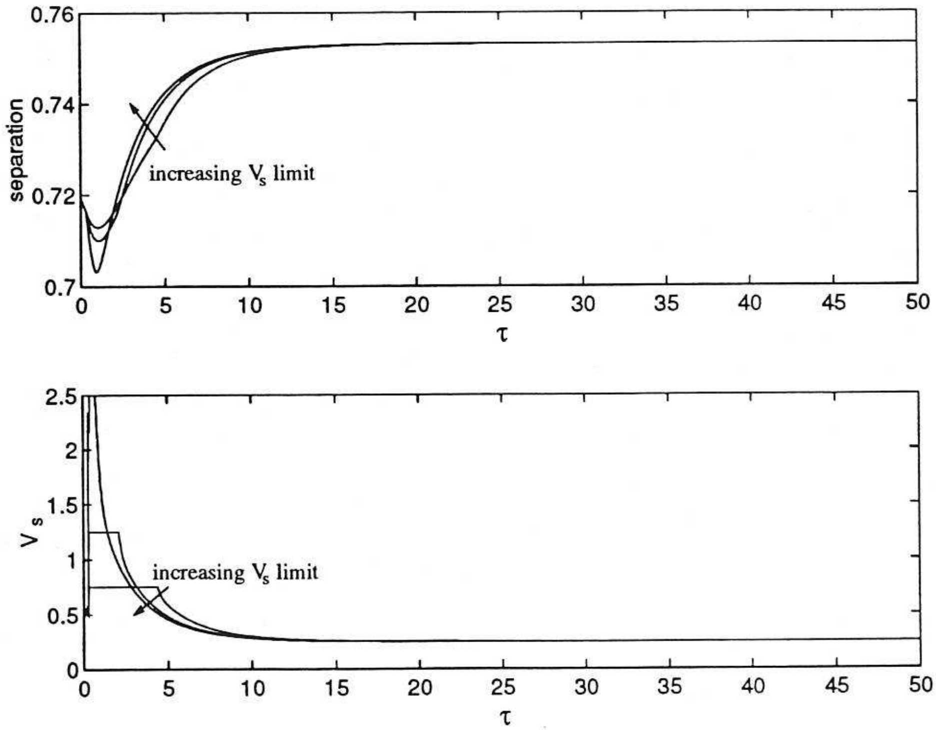

Fig. 13. Effect of increasing the V_s-limit $(= 3\overline{V}_s,\ 5\overline{V}_s,\ 10\overline{V}_s)$: Near steady-state I.C's, optimal control, λ=0.0001

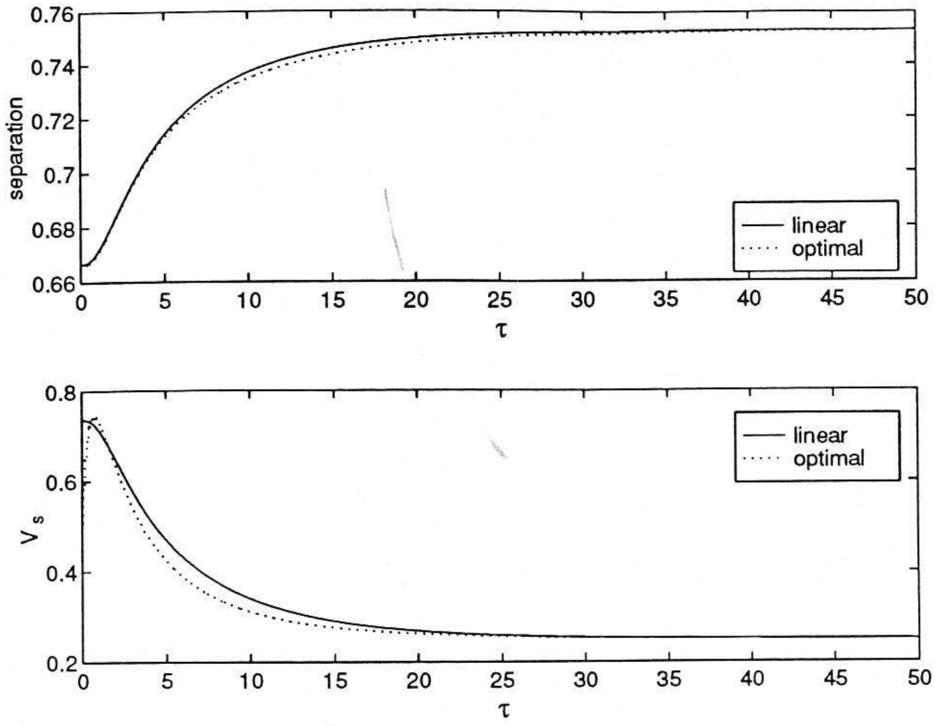

Fig. 14. Comparison between linear and optimal control performance under similar maximum input excursion, V_s : Feed I.C's, K=9.0, λ=0.011.

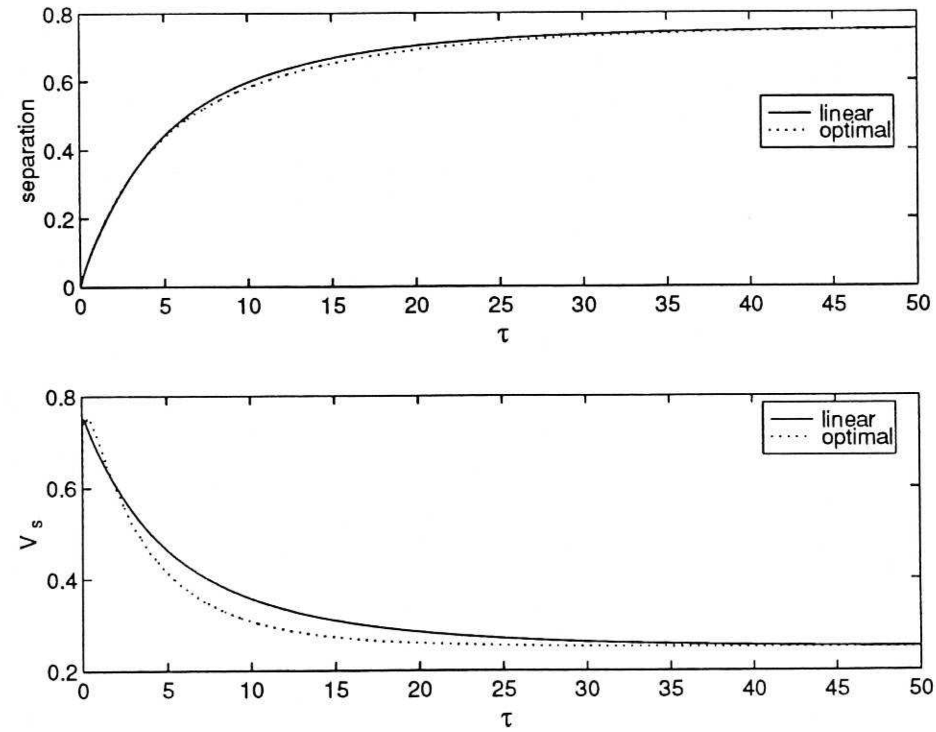

Fig. 15. Comparison between linear and optimal control performances under similar maximum input excursion, V_s : Zero I.C's, K=1.1, λ=0.75

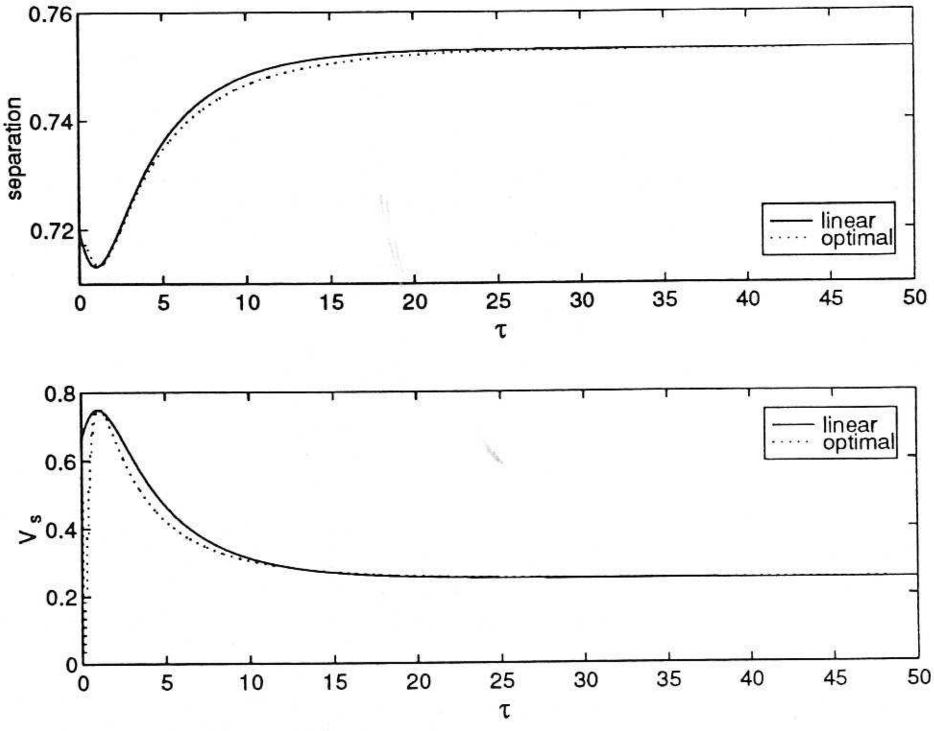

Fig. 16. Comparison between linear and optimal control performances under similar maximum input excursion, V_S : Near steady-state I.C's, K=20, λ=0.002

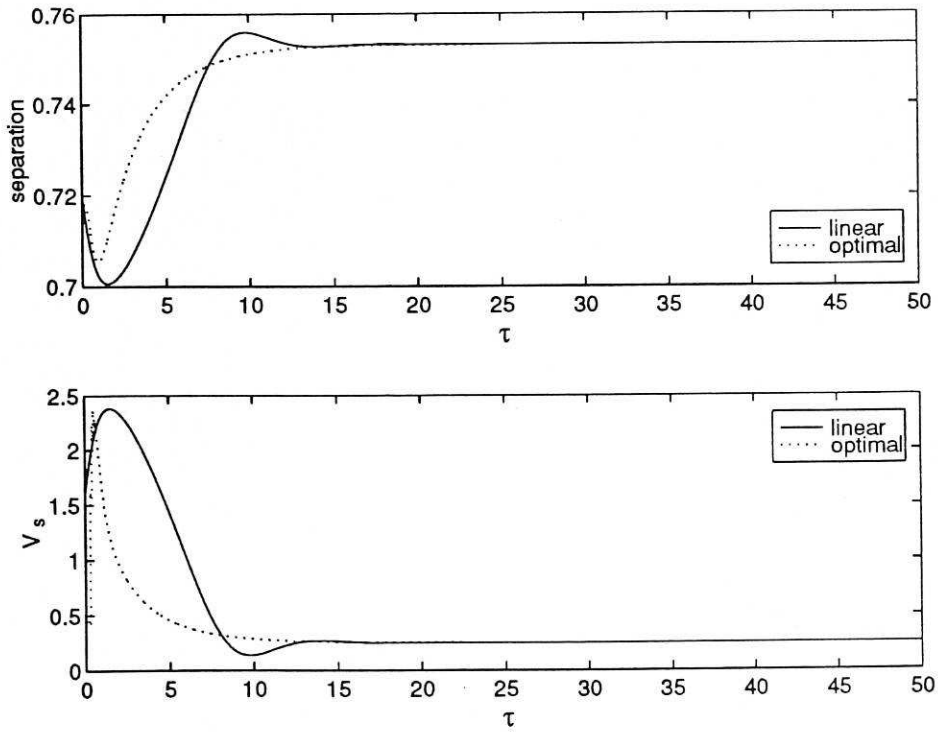

Fig. 17. Comparison between linear and optimal control performances under similar maximum input excursion, V_S : Near steady-state I.C's, K=65, λ=0.00017, with relaxed V_S-limit to $0 \leq V_S \leq 10\overline{V}_S$

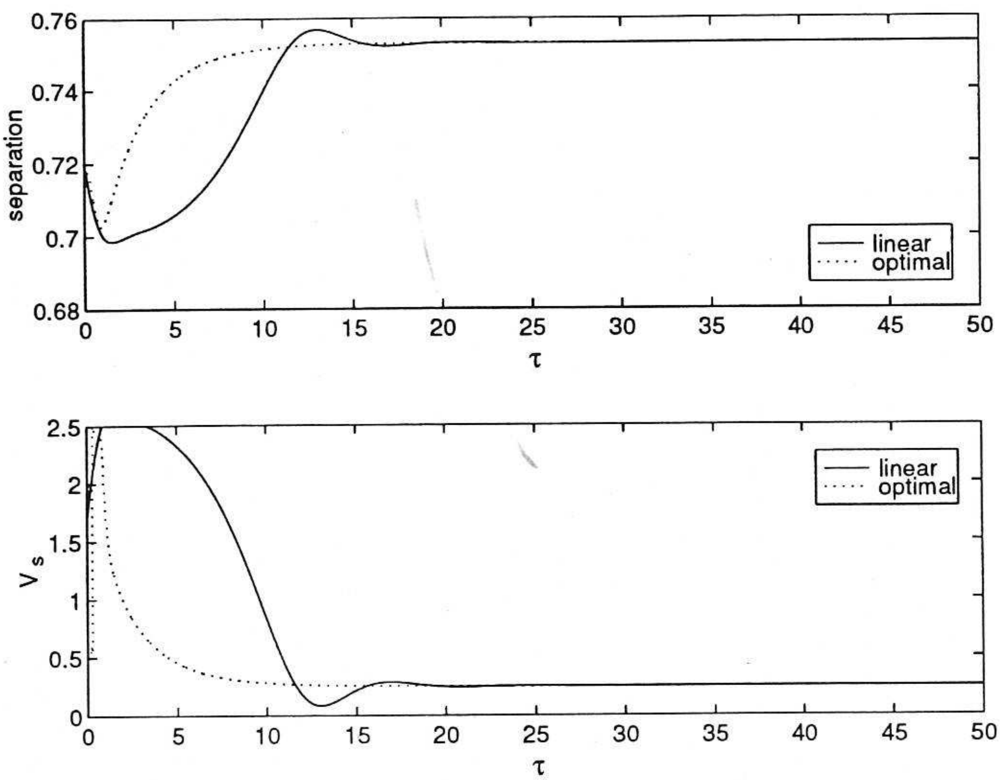

Fig. 18 Comparison between linear and optimal control performances under similar excursion, V_S, where the limit of $10\overline{V}_S$ is hit : Near Steady State I.C's, K=70, λ=0.00008.

Appendix 1

Derivation of Coefficient Matrices $\underline{A}'$, $\underline{B}'_0$ and $\underline{B}'_1$ for the Bilinear CSTC Model Based on Error Coordinates

Firstly it is necessary to split matrix $\underline{B}$ into its constant and state-proportional components, i.e.

$$\underline{B} = \underline{B}_0 + \underline{B}_1 \underline{x}$$

and from eqn. (6) therefore :

$$\underline{B} = \begin{bmatrix} \dfrac{\varepsilon - S - \alpha S_e}{LV} \\ \dfrac{\alpha\{S_e(1) - S_e\}}{LV} \\ \dfrac{S - \alpha S_e(1) + \varepsilon}{\alpha VT} \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon}{LV} \\ 0 \\ \dfrac{\varepsilon}{\alpha VT} \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{LV} & -\dfrac{\alpha}{LV} & 0 \\ 0 & -\dfrac{\alpha}{LV} & \dfrac{\alpha}{LV} \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix} \begin{bmatrix} S \\ S_e \\ S_e(1) \end{bmatrix}$$

Therefore, $\underline{B}_0 = \begin{bmatrix} \dfrac{\varepsilon}{LV} \\ 0 \\ \dfrac{\varepsilon}{\alpha VT} \end{bmatrix}$ and $\underline{B}_1 = \begin{bmatrix} -\dfrac{1}{LV} & -\dfrac{\alpha}{LV} & 0 \\ 0 & -\dfrac{\alpha}{LV} & \dfrac{\alpha}{LV} \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix}$

Similarly, for matrix $\underline{J}$, from eqn. (7), we deduce :

$$\underline{J} = \underline{J}_0 + \underline{J}_1 \underline{x}$$

$$\underline{J} = \begin{bmatrix} \dfrac{\varepsilon}{LV(\alpha+1)} \\ 0 \\ \dfrac{S - \alpha S_e(1) + \varepsilon}{\alpha VT} \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon}{LV(\alpha+1)} \\ 0 \\ \dfrac{\varepsilon}{\alpha VT} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix} \begin{bmatrix} S \\ S_e \\ S_e(1) \end{bmatrix}$$

Therefore $\underline{J}_0 = \begin{bmatrix} \dfrac{\varepsilon}{LV(\alpha+1)} \\ 0 \\ \dfrac{\varepsilon}{\alpha VT} \end{bmatrix}$ and $\underline{J}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix}$

From eqn. (21)

$$\underline{A}' = \underline{A} + u_r \underline{B}_1 + d\underline{J}_1 \qquad \text{(A1.1)}$$

using eqns. (2), (6) and (7), we get :

$$\underline{A}' = \begin{bmatrix} -\dfrac{\alpha L + \varepsilon}{\alpha L} & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{V}{\alpha}\begin{bmatrix} -\dfrac{1}{LV} & -\dfrac{\alpha}{LV} & 0 \\ 0 & -\dfrac{\alpha}{LV} & \dfrac{\alpha}{LV} \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix} + \frac{V\varepsilon}{\alpha}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{\alpha VT} & 0 & -\dfrac{1}{VT} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{\alpha L+\varepsilon}{\alpha L} - \dfrac{V}{\alpha}\left(\dfrac{1}{LV}\right) & 1 - \dfrac{V}{\alpha}\left(\dfrac{\alpha}{LV}\right) & 0 \\ 1 & -1 - \dfrac{V}{\alpha}\left(\dfrac{\alpha}{LV}\right) & \dfrac{V}{\alpha}\left(\dfrac{\alpha}{LV}\right) \\ \dfrac{V}{\alpha}\left(\dfrac{1}{\alpha VT}\right) + \dfrac{V\varepsilon}{\alpha}\left(\dfrac{1}{\alpha VT}\right) & 0 & -\dfrac{V}{\alpha}\left(\dfrac{1}{VT}\right) - \dfrac{V\varepsilon}{\alpha}\left(\dfrac{1}{VT}\right) \end{bmatrix}$$

$$\text{Therefore, } \underline{A}' = \begin{bmatrix} -\dfrac{L+1}{L} & \dfrac{L-1}{L} & 0 \\ 1 & -\dfrac{L+1}{L} & \dfrac{1}{L} \\ \dfrac{1}{\alpha T} & 0 & -\dfrac{1}{T} \end{bmatrix} \qquad \text{(A1.2)}$$

This is reproduced as eqn. (28) in the main text.

For the calculation of expression for the elements of matrices $\underline{B}'_0$ and $\underline{B}'_1$, we introduce reference vector

$$\underline{x}_r = \begin{bmatrix} S_r \\ S_{er} \\ S_{er}(1) \end{bmatrix} \tag{A1.3}$$

and note from eqn. (31) {eqn. (38) in RR573}, that $\alpha S_{er}(1) - S_r = \varepsilon$ so that, separating the constant and state-dependent components of $\underline{B}$ in eqn. (6) we obtain from eqn. (22) :

$$\underline{B}'_0 = \underline{B}_0 + \underline{B}_1 \underline{x}_r$$

$$= \begin{bmatrix} \frac{\varepsilon}{LV} \\ 0 \\ \frac{\varepsilon}{\alpha VT} \end{bmatrix} + \begin{bmatrix} -\frac{1}{LV} & -\frac{\alpha}{LV} & 0 \\ 0 & -\frac{\alpha}{LV} & \frac{\alpha}{LV} \\ \frac{1}{\alpha VT} & 0 & -\frac{1}{VT} \end{bmatrix} \begin{bmatrix} S_r \\ S_{er} \\ S_{er}(1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\varepsilon}{LV} - \frac{S_r}{LV} - \frac{\alpha S_{er}}{LV} \\ -\frac{\alpha S_{er}}{LV} + \frac{\alpha S_{er}(1)}{LV} \\ \frac{\varepsilon}{\alpha VT} + \frac{S_r}{\alpha VT} - \frac{S_{er}(1)}{VT} \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon - S_r - \alpha S_{er}}{LV} \\ \frac{\alpha\{S_{er}(1) - S_{er}\}}{LV} \\ \frac{S_r - \alpha S_{er}(1) + \varepsilon}{\alpha VT} \end{bmatrix} \tag{A1.4}$$

This result is reproduced as eqn. (30) in the main text.

For calculating the elements of $\underline{B}'_0$ in terms of just L, α and ε, we restate eqns.(30), (32) and (33) respectively as :

$$B'_{01} = \frac{\varepsilon - S_r - \alpha S_{er}}{LV} \tag{A1.5}$$

$$S_{er} = \frac{S_r(L+1)(\alpha+1) - 2\varepsilon}{(L-1)(\alpha+1)} \tag{A1.6}$$

$$S_r = \frac{\varepsilon}{(\alpha+1)} \left[\frac{(3\alpha+1)L + \alpha - 1}{(3\alpha-1)L + \alpha + 1} \right] \tag{A1.7}$$

Hence,

$$\varepsilon - S_r - \alpha S_{er} = \varepsilon - S_r - \alpha\left[\frac{S_r(L+1)(\alpha+1)-2\varepsilon}{(L-1)(\alpha+1)}\right]$$

$$= \frac{\varepsilon(L-1)(\alpha+1) - S_r(L-1)(\alpha+1) - \alpha S_r(L+1)(\alpha+1) + 2\alpha\varepsilon}{(L-1)(\alpha+1)}$$

$$= \frac{\varepsilon(L-1)(\alpha+1) + 2\alpha\varepsilon - S_r(\alpha+1)[(L-1)+\alpha(L+1)]}{(L-1)(\alpha+1)}$$

$$= \frac{\varepsilon(L-1)(\alpha+1) + 2\alpha\varepsilon - S_r(\alpha+1)[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}$$

$$= \frac{\varepsilon L\alpha + \varepsilon L - \varepsilon\alpha - \varepsilon + 2\alpha\varepsilon - S_r(\alpha+1)[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}$$

$$= \frac{\varepsilon L(\alpha+1) + \varepsilon^2 - S_r(\alpha+1)[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}$$

$$= \frac{\varepsilon[L(\alpha+1)+\varepsilon] - S_r(\alpha+1)[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}$$

$$= \frac{[L(\alpha+1)+\varepsilon][\varepsilon - S_r(\alpha+1)]}{(L-1)(\alpha+1)}$$

Finally, substituting for S_r using eqn. (A1.7), we conclude that the term :

$$\varepsilon - S_r - \alpha S_{er} = \frac{[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}\left[\varepsilon - \varepsilon\left\{\frac{(3\alpha+1)L+\alpha-1}{(3\alpha-1)L+\alpha+1}\right\}\right]$$

$$= \frac{[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}\,\varepsilon\left[1 - 1\left\{\frac{(3\alpha+1)L+\alpha-1}{(3\alpha-1)L+\alpha+1}\right\}\right]$$

$$= \frac{[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}\,\varepsilon\left[\frac{(3\alpha-1)L+\alpha+1-(3\alpha+1)L-\alpha+1}{(3\alpha-1)L+\alpha+1}\right]$$

$$= \frac{[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}\,\varepsilon\left[\frac{L\{(3\alpha-1)-(3\alpha+1)\}+2}{(3\alpha-1)L+\alpha+1}\right]$$

$$= \frac{[L(\alpha+1)+\varepsilon]}{(L-1)(\alpha+1)}\,\varepsilon\left[\frac{-2L+2}{(3\alpha-1)L+\alpha+1}\right]$$

$$= \frac{-2\varepsilon[L(\alpha+1)+\varepsilon](L-1)}{(L-1)(\alpha+1)[(3\alpha-1)L+\alpha+1]}$$

Therefore $$B'_{01} = \frac{-2\varepsilon[L(\alpha+1)+\varepsilon]}{LV(\alpha+1)[(3\alpha-1)L+\alpha+1]} \qquad \text{(A1.8)}$$

This result is restated as eqn. (34) in the main text.

Similarly, for elements B'_{02}, from eqn. (A1.4)

$$B'_{02} = \frac{\alpha\left\{S_{er}(1) - S_{er}\right\}}{LV} \qquad \text{(A1.9)}$$

and from the RHS of eqn. (35) in RR573

$$S_{er}(1) - S_{er} = L\left(S_{er} - S_r\right) \qquad \text{(A1.10)}$$

Hence, it follows from eqns. (A1.9), (A1.10) and (32) that :

$$B'_{02} = \frac{\alpha}{V}\left(S_{er} - S_r\right)$$

$$= \frac{\alpha}{V}\left[\frac{S_r(L+1)(\alpha+1) - 2\varepsilon}{(L-1)(\alpha+1)} - S_r\right]$$

$$= \frac{\alpha}{V}\left[\frac{S_r(L+1)(\alpha+1) - S_r(L-1)(\alpha+1) - 2\varepsilon}{(L-1)(\alpha+1)}\right]$$

$$= \frac{\alpha}{V}\left[\frac{S_r(\alpha+1)\left\{(L+1) - (L-1)\right\} - 2\varepsilon}{(L-1)(\alpha+1)}\right]$$

$$= \frac{\alpha}{V}\left[\frac{2S_r(\alpha+1) - 2\varepsilon}{(L-1)(\alpha+1)}\right]$$

Substituting for S_r using (A1.7) we get :

$$= \frac{\alpha}{V(L-1)(\alpha+1)}\left[\frac{2\varepsilon\left\{(3\alpha+1)L + \alpha - 1\right\}}{\left\{(3\alpha-1)L + \alpha + 1\right\}} - 2\varepsilon\right]$$

$$= \frac{2\varepsilon\alpha}{V(L-1)(\alpha+1)}\left[\frac{3\alpha L + L + \alpha - 1 - 3\alpha L + L - \alpha - 1}{(3\alpha-1)L + \alpha + 1}\right]$$

$$= \frac{2\varepsilon\alpha}{V(L-1)(\alpha+1)}\left[\frac{2(L-1)}{(3\alpha-1)L + \alpha + 1}\right]$$

Therefore $$B'_{02} = \frac{4\varepsilon\alpha}{V(\alpha+1)\left[(3\alpha-1)L+\alpha+1\right]} \qquad \text{(A1.12)}$$

This is restated as eqn. (35) in the main text.

Finally, for the matrix B'_{03}, we note from eqn. (A1.4) that

$$B'_{03} = \frac{\varepsilon + S_r - \alpha S_{er}(1)}{\alpha VT}$$

and since from eqn. (31) {eqn. (38) in RR573}

$$\alpha S_{er}(1) - S_r = \varepsilon$$

we obtain :

$$B'_{03} = -\frac{-\varepsilon+\varepsilon}{\alpha VT}$$

$$B'_{03} = 0 \qquad \text{(A1.14)}$$

This equation is restated as eqn. (36) in the main text.

Finally we note that, from eqns. (23) and (38) that :

$$\underline{B}'_1 = \underline{B}_1$$

$$= \begin{bmatrix} -\frac{1}{LV} & -\frac{\alpha}{LV} & 0 \\ 0 & -\frac{\alpha}{LV} & \frac{\alpha}{LV} \\ \frac{1}{\alpha VT} & 0 & -\frac{1}{VT} \end{bmatrix}$$