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Borehole water level response to barometric pressure as an indicator of aquifer vulnerability

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1. Introduction

[1] The response of borehole water levels to barometric pressure changes in semiconfined aquifers can be used to determine barometric response functions from which aquifer and confining layer properties can be obtained. Following earlier work on barometric response functions and aquifer confinement, we explore the barometric response function as a tool to improve the assessment of groundwater vulnerability in semiconfined aquifers, illustrated through records from two contrasting boreholes in the semiconfined Chalk Aquifer, East Yorkshire, UK. After removal of recharge and Earth tide influences on the water level signal, barometric response functions were estimated and aquifer and confining layer properties determined through an analytical model of borehole water level response to barometric pressure. A link between the thickness and vertical diffusivity of the confining layer determined from the barometric response function, and groundwater vulnerability is proposed. The amplitude spectrum for barometric pressure and instrument resolution favor determination of the barometric response function at frequencies to which confining layer diffusivities are most sensitive. Numerical modeling indicates that while the high frequency response reflects confining layer properties in the immediate vicinity of the borehole, the low frequency response reflects vertical, high diffusivity pathways though the confining layer some hundreds of meters distant. A characteristic time scale parameter, based on vertical diffusivities and thicknesses of the saturated and unsaturated confining layer, is introduced as a measure of semiconfined aquifer vulnerability. The study demonstrates that the barometric response function has potential as a tool for quantitative aquifer vulnerability assessment in semiconfined aquifers.


1. Introduction

[2] Aquifer vulnerability assessment is an important tool in the protection of aquifers from surface contamination. For semiconfined aquifers, quantitative vulnerability assessment is frequently hindered by a lack of information on the nature and spatial distribution of the confining layer hydraulic properties. It is proposed here that quantitative information on confining layer properties derived from barometric response functions can be used as an aid to quantitative aquifer vulnerability assessment. It is well known that water levels in boreholes tapping perfectly confined aquifers (where the confining layer has negligible conductivity) fluctuate in response to changes in barometric pressure while those in unconfined aquifers where the

unsaturated zone is thin and/or has a high conductivity, show a shallow water table response with no reaction to changes in barometric pressure [e.g., Jacob, 1940]. These two cases represent “end members” of the wide range found in nature and most aquifers are in fact either semi-confined (where confining layer has significant permeability) or unconfined, showing a deep water table response (response to barometric pressure change due to a thick or low permeability unsaturated zone). In case of a perfectly confined aquifer, changes in barometric pressure are transmitted instantaneously to the aquifer where they are distributed between the aquifer skeleton and pore waters while the same changes are transmitted in total to the water surface in the borehole [Batu, 1998]. This results in a pressure imbalance between an open borehole and the aquifer so that an increase in barometric pressure causes a decrease in borehole water level and vice versa. In the case of a perfectly confined aquifer, the ratio of change in borehole water level to change in barometric pressure is a constant, termed the static barometric efficiency of the aquifer [Jacob, 1940; Rojstaczer, 1988; Rasmussen and Crawford, 1997; Spane, 2002]. However, when the aquifer is semi-confined or unconfined with a thick unsaturated zone and/or unsaturated zone of low permeability, barometric efficiency becomes frequency dependent [Weeks, 1979; Furbish, 1991] and the response of borehole water level to
barometric pressure is described by a barometric response function which may be determined in either the time or the frequency domain. System properties including the vertical diffusivities (unsaturated zone pneumatic diffusivity and saturated zone hydraulic diffusivity) can be estimated from barometric response functions by fitting to an appropriate model [e.g., Weeks, 1979; Robins, 1998; Boland et al., 1991; Butler et al., 2011].

Intrinsic vulnerability is commonly defined as a function of the nature and thickness of overlying confining layer, depth to the water table, and the characteristics of the aquifer and overlying materials [e.g., Robins, 1998; Boland et al., 1999]. The heterogeneity of overlying material plays an important role as more highly conductive lithologies which form connected vertical pathways from surface to aquifer limit the capacity of any confining layer to protect the aquifer. Traditional pumping and slug tests give predominantly horizontal hydraulic conductivity. Estimates of vertical hydraulic conductivity of confining layers can be obtained from aquifer pumping tests when leakage from the confining layer occurs [e.g., Hantush, 1956]. However, to clearly see such leakage effects and accurately estimate vertical hydraulic conductivity often requires pumping tests lasting tens of hours which are not routinely undertaken for observation boreholes. It is here suggested that vertical diffusivity (ratio of conductivity to specific storage) of the confining layer, derived from barometric response functions, can be used to characterize the potential for contaminant transport through the confining layer to the aquifer, providing a potential link with intrinsic groundwater vulnerability. The link between barometric response functions and aquifer vulnerability has been tentatively suggested in the past [Rojstaczer, 1988; Landmeyer, 1996] while others have used barometric response functions to distinguish between confined, unconfined, well skin and combined responses [e.g., Rasmussen and Crawford, 1997; Spane, 2002] and to assess the degree of confinement [e.g., Butler et al., 2011]. To the authors’ knowledge, however, application of these techniques as an aid to quantifying aquifer vulnerability has not yet been demonstrated. In this paper, the application of the barometric response function to the assessment of aquifer vulnerability in confined to semiconfined aquifers is described through application to the semiconfined Chalk Aquifer of East Yorkshire, UK.

2. Previous Work on Barometric Response Functions

Work during the late 1980s and early 1990s established time domain [Weeks, 1979; Furbish, 1991; Rasmussen and Crawford, 1997] and frequency domain [Welch, 1967; Galloway and Rojstaczer, 1988; Evans et al., 1991] techniques for determining barometric response functions from borehole water level and barometric pressure records. Estimation of the barometric response function is improved by prior removal of Earth and ocean tides for which time domain methods [Rasmussen and Crawford, 1997; Rasmussen and Mote, 2007; Toll and Rasmussen, 2007] and frequency domain methods [Rojstaczer, 1988; Galloway and Rojstaczer, 1988; Rojstaczer and Riley, 1990] have been developed. To date, little attention has been given to the impact of interference from recharge and many locations have been chosen in part for their lack of recharge signal [e.g., Galloway and Rojstaczer, 1988; Rojstaczer and Riley, 1990; Quilty and Roeloffs, 1991; Beavan et al., 1991; Evans et al., 1991] while other authors have restricted their analysis to selected time periods where such interference is minimal or where effects can corrected by simple linear trend removal [e.g., Butler et al., 2011].

Barometric response functions have been used to correct borehole water level response for barometric pressure effects where they mask groundwater flow characteristics of interest [Quilty and Roeloffs, 1991; Rasmussen and Crawford, 1997; Spane, 2002; Toll and Rasmussen, 2007]. The sensitivity of barometric response functions to conditions in the immediate vicinity of the borehole has been shown in the time domain by Rasmussen and Crawford [1997] and Spane [2002] and in the frequency domain by Rojstaczer [1988].

Barometric response functions have been used by a number of previous workers to investigate degree of aquifer confinement which relates closely to aquifer vulnerability. Hare and Morse [1997, 1999] demonstrated that open boreholes within and outside the areal extent of a containment system consisting of a clay cap and impermeable cutoff wall showed different responses to barometric pressure variations and used this to monitor the performance of the containment system. Ackworth and Brain [2008] demonstrated that even a thin (<2 m) weathered zone in fractured granite can induce borehole water level responses to barometric pressure indicating confined behavior with implications for recharge and groundwater-surface water interactions (and by implication groundwater vulnerability). Rasmussen and Crawford [1997] showed how time domain barometric response functions reflect the degree of aquifer confinement and how comparison of response functions from adjacent wells can be used to gauge hydrostatic continuity of the confining layer. Butler et al. [2011] also showed how the form of the barometric response function is related to the degree of confinement and conditions in the vadose zone. By fitting observed response functions from three boreholes to a time domain model, they obtained values for aquitard hydraulic diffusivity and conductivity which were in agreement with pumping test results, and which demonstrated aquitard continuity over a distance of at least 700 m.

3. The Study Area and Data Collection

The study area (Figure 1) comprises the confined to semiconfined Chalk Aquifer in East Yorkshire, UK. The Chalk Aquifer occurs over approximately one quarter of England and is the principal aquifer of the UK supplying...
more than 50% of total groundwater abstractions. In East Yorkshire, the Chalk Aquifer is unconfined over the higher ground in the west (The Wolds) where recharge is around 300 mm/year. In the east (the Holderness Peninsula), the aquifer is confined by up to 50 m of highly heterogeneous glacial deposits of Quaternary age comprising clay-rich till, alluvium, and sands and gravels [see Smedley et al., 2004, Figure 1]. The hydraulic conductivity of the chalk matrix is very low at around $10^{-5}$ m/day [Hartmann et al., 2007] and flow in the aquifer is fracture dominated [e.g., Smedley et al., 2004]. In the confined aquifer, flow occurs principally in the upper 10 m of highly fractured chalk with transmissivities ranging up to 500 m$^2$/day [Smedley et al., 2004; Hartmann et al., 2007; Odling et al., 2013]. The UK Environment Agency (EA) maintains a network of some 100 monitoring boreholes distributed over the region. Aquifer vulnerability is of concern throughout the aquifer due to increasing nitrate levels in groundwater from the use of agricultural fertilizers since the 1950s [Wellings and Cooper, 1983; Stuart et al., 2007]. In the area of the confined aquifer, aquifer vulnerability is closely linked to the nature of the overlying glacial sediments.

The current designated zones of high, intermediate, and low vulnerability (Environment Agency, UK) are based on maps of the Quaternary glacial sediments constructed from sparse borehole log and outcrop data and reflect lithological variations on scale of 100 m or greater. High vulnerability status is based on outcrops of sands and gravels and low vulnerability on outcropping clay-rich tills. However, a recent study [Kilner et al., 2005] has demonstrated that sands and gravels may form highly conductive vertical pathways through the clay-rich sediments on the scale of meters to 10s meters. The vulnerability of the aquifer is enhanced by widespread downward head gradients in the confining sediments. Prolonged groundwater pumping for municipal water supplies over the last 100 years has lowered the water table along most of the western margin of the confined aquifer (see Figure 1), reducing the zone of natural artesian flow to its present extent [Smedley et al., 2004] and generating year-round downward gradients elsewhere. To the east, gradients are seasonal with upward gradients in summer and downward in winter when recharge occurs. Thus, the confined aquifer is potentially vulnerable to contamination from the surface over most of its extent.

In a recent study, water level and barometric pressure data at 15 min time intervals were collected from 12 boreholes across the confined Chalk Aquifer, using absolute (nonvented) automatic pressure transducers (manufacturers’ quoted resolution of ±0.09 to ±0.25 cmH$_2$O and range of 0–10 m), over periods of up to 799 days during September 2008 to April 2011 [Hussein, 2012]. At each borehole, a pressure transducer was placed below the water level to measure total pressure (water plus barometric pressure) and another placed in the air column to measure barometric pressure. The borehole water level signals were obtained by subtracting the barometric pressure from the total pressure, where both are expressed in units of equivalent water head (unit cmH$_2$O). The analyses of data from two boreholes representing the range in confining layer properties which are situated 13 km (Benningholme) and 4.5 km (Thornholme Moor) from the margin of the confined aquifer (see Figure 1), are presented here to illustrate the application of the barometric response function as an aid to aquifer vulnerability assessment. Details of borehole
construction are given in Table 1 and the time series data of borehole water levels and barometric pressure are shown in Figure 3.

4. Estimating the Barometric Response Function

4.1. Preprocessing the Water Level Signal

[10] The frequency range over which the barometric response function may be determined depends on length of monitoring record, the amplitude of the barometric pressure spectrum, instrument resolution, and the presence of other influences on borehole water level such as recharge, Earth tides, and groundwater pumping. The barometric pressure signal comprises aperiodic fluctuations at frequencies below ~0.8 cycles/day with the periodic atmospheric tides $S_1$ (diurnal component caused by ground heating) and $S_2$ (semidiurnal component caused by ozone heating) [Chapman and Lindzen, 1970], see Figure 2a. The accuracy of the barometric response function is greatly improved when any influences on the borehole water levels other than barometric pressure are removed. For the borehole records here, these comprise recharge and Earth tides [Batu, 1998; Künipfe, 1997]. As a preliminary step in the data processing, each signal is detrended (linear trend is removed and mean subtracted). This is common practice in spectral analysis and is done to avoid spectral leakage which can mask behavior at higher frequencies [e.g., Gubbins, 2004]. The coherence, $C_{WB}$, between borehole water level and barometric pressure signals is then used to assess the frequency range over which the barometric response function is valid [Evans et al., 1991]:

$$C_{WB}(f) = \frac{|X_{WB}(f)|^2}{X_{WW}(f) \cdot X_{BB}(f)}$$  \hspace{1cm} (1)

where $X_{WB}(f)$ is the cross spectrum of the water level and barometric pressure signals and $X_{WW}(f)$ and $X_{BB}(f)$ are the auto-spectra of the water level and barometric pressure signals, respectively [Bendat and Piersol, 2010]. Recharge and Earth tides can be seen to cause low coherence between water level and barometric pressure signals at low to intermediate frequencies and at the diurnal and semidiurnal frequencies, respectively (see Figure 2d).

[11] Aquifer recharge depends on rainfall and evapotranspiration and thus the exact nature of the recharge signal may vary from year to year. Water level records from boreholes in the unconfined aquifer (where barometric pressure has negligible effect) display a strong recharge signal (Figures 2b and 3c). The effect of recharge on the water level signal in the confined aquifer can be seen at frequencies below 0.1 cycles/day where water level amplitudes (maximum 25 cmH$_2$O) are higher than barometric pressure amplitudes (maximum 7 cmH$_2$O, see Figures 2a and 2c). However, in the confined aquifer, the recharge signal becomes progressively attenuated and lagged with distance from the margin of the confined aquifer resulting in a shift in the frequency limit affected by recharge to lower frequencies. A cutoff coherence of 0.5 was used to determine this limit which at Benningholme (13 km from the margin) is 0.015 cycles/day and at Thornholme Moor (4.5 km from the margin) is 0.03 cycles/day. The use of coherence thus allows the lower frequency limit to the barometric response function to be determined on a borehole by borehole basis, maximizing the extent of the barometric response.
response function at low frequencies. The influence of recharge was then removed from the signal using a high pass filter at the appropriate frequency for each borehole.

Earth tides comprising the five major diurnal ($O_1$, $K_1$) and semi-diurnal ($N_2$, $M_2$, $S_2$) components can clearly be identified in the water level signals (Figure 2c), causing low coherence at these frequencies (Figure 2d). Following the removal of recharge effects, the data was detrended and the effects of Earth tides removed by the method of Ras- mussen and Mote [2007] using time domain regression deconvolution (see Appendix A for more details). Water level fluctuations resulting from Earth tides were found to range up to 2 cm. Figure 4a illustrates the removal of recharge and Earth tides on the water level data from the Benningholme borehole. The final corrected signal shows a clear inverse relationship with barometric pressure characteristic of confined to semi-confined aquifers.

4.2. Barometric Response Functions

[13] The frequency domain form of the barometric response function (gain and phase) is obtained by deconvolution of the borehole water level signal by the barometric pressure signal. This is done using the technique of cross-spectral deconvolution by ensemble averaging [Welch, 1967] previously used by a number of investigators [Rojstaczer, 1988; Rojstaczer and Riley, 1990; Beavan et al., 1991; Quilty and Roeloffs, 1991]. In this technique, a number of partially independent barometric response functions are determined from overlapping segments of the water level and barometric pressure signals. These are then averaged to give the final barometric response function with one standard deviation errors. This technique smooths and optimizes the accuracy of the barometric response function. Further details of the technique are given in Appendix B and the Matlab code used to compute barometric response functions is available as supporting information accompanying this article.

[14] Figures 4b and 4c illustrate the estimation of the barometric response function for the Benningholme borehole where three segment lengths are used. As segment length decreases, the gain and phase curves become smoother but with less information at low frequencies. The use of three different segment lengths allows determination of the barometric response function at lower frequencies but with lower accuracy, while for higher frequencies the use of shorter segments improves accuracy. The final valid range of the barometric response function is determined from frequencies where coherence is greater than 0.5. At Benningholme, this gives a viable frequency range of 0.015–0.8 cycles/day plus one point at 1 cycle/day, shown in Figure 4c. At 0.8 cycles/day, the water level signal amplitude is around 0.03 cmH$_2$O (Figure 2c) which is considerably lower than the manufacturers’ stated resolution for the pressure transducers of 0.25 and 0.09 cm. However, a better signal-to-noise ratio is obtained through stacking of many observations in a long time series [Florsch et al., 1991; Merritt, 2004] with over 76,000 data records for Benningholme and almost 30,000 data records for Thornholme Moor. In addition, the manufacturers quote instrument resolutions as 0.25 cm (or better) and 0.09 cm (or better) and are likely to be somewhat conservative.

5. Estimating Aquifer and Confining Layer Properties

[15] Aquifer and confining layer properties are estimated by fitting the analytical model of Rojstaczer [1988], modified to include capillary fringe attenuation [Evans et al., 1991], to the barometric response function. An overview of model equations is given in Appendix C and full details of their derivation can be found in Rojstaczer [1988] and Evans et al. [1991]. The model considers three flow problems that arise from a change in barometric pressure: (a) vertical air flow in the unsaturated zone from the Earth’s surface to the water table, (b) vertical groundwater flow within the confining layer, and (c) horizontal groundwater flow between the aquifer and the borehole (see Figure 5a).
The model parameters are static barometric efficiency of the aquifer \( (BE) \), confining layer pneumatic and hydraulic diffusivities \( (D_{con} \text{ and } D_{unsat}) \), capillary fringe attenuation factor \( (T_{cf}) \), confining layer thickness \( (L_{con}) \), unsaturated zone thickness in the confining layer \( (L_{unsat}) \), transmissivity \( (T_{aqu}) \), storativities of the aquifer and confining layer \( (S_{aqu} \text{ and } S_{con}) \), and borehole radius \( (r_w) \).

[16] A typical model barometric response function, constructed using parameters from this study, is shown in Figure 5b. Phase is plotted according of the sign convention of Rojstaczer [1988] where phase advance is greater than, and phase lag less than, \(-180^\circ\). The barometric response function can be divided into three stages comprising low, intermediate and high frequency ranges within the barometric pressure signal [Rojstaczer, 1988]. At low frequencies of the barometric response function (stage A), gain increases and phase decreases with increasing frequency, controlled primarily by the properties of the confining layer. At very low frequencies, equilibrium is maintained between the confining layer, the aquifer and the borehole and the system behaves as if unconfined (gain approaches zero). At intermediate frequencies of the barometric response function (stage B) a plateau exists at a gain which represents the static barometric efficiency and a phase of \(-180^\circ\), and the behavior is similar to that of a fully confined aquifer. At high frequencies (stage C), the barometric response function is controlled by the rate

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**Figure 4.** (a) Segments of the time series data for barometric pressure (green) and water level for the Benningholme borehole showing the initial water level signal (red), after removal of recharge (magenta) and after removal of recharge and Earth tides (blue). (b) Gain and phase frequency plots showing estimates of barometric response functions for three overlapping frequency bands with number of segments N1, N2, and N3. (c) Final barometric response function gain and phase curves constructed from (b) with a viable frequency range of 0.017–0.8 cycles/day plus 1 cycle/day (dashed boxes).
Figure 5. (a) Diagrammatic cross section showing aquifer, confining layer, and borehole with key parameters and flow directions in response to changes in barometric pressure (WL—water level, WT—water table). (b) Illustrative barometric response function based on the model of Rojstacer [1988] showing (A) low, (B) intermediate, and (C) high frequency response stages constructed using input parameters typical for the semiconfined Chalk Aquifer in East Yorkshire: static $BE = 0.6$, $D_{con} = D_{unsat} = 35 \text{ m}^2/\text{day}$, $T_{aqu} = 20 \text{ m}^2/\text{day}$, $T_{cf} = 1$, $S_{con} = 10^{-4}$, $S_{aqu} = 10^{-4}$, $L_{unsat} = 1.3 \text{ m}$, $L_{sat} = 14.8 \text{ m}$ and $r_w = 0.098 \text{ m}$.

at which water can flow between aquifer and borehole, and both gain and phase decrease with increasing frequency, governed by borehole design and aquifer transmissivity and storativity. The sensitivity of the barometric response function to model parameters is illustrated in Figure 6. Low to intermediate frequencies of the barometric response function gain and phase are primarily sensitive to confining layer pneumatic and hydraulic diffusivities, $D_{unsat}$ and $D_{con}$ respectively, unsaturated zone thickness, $L_{unsat}$ (and therefore saturated confining layer zone thickness $L_{sat}$, for a given confining layer thickness), and capillary fringe attenuation factor, $T_{cf}$ (Figures 6b–6e). High frequencies are primarily sensitive to aquifer transmissivity, $T_{aqu}$ (Figure 6f). Variations in barometric efficiency, $BE$, simply scale the gain curve with no effect on the phase curve (Figure 6a) and, by comparison to the above parameters, the model shows little sensitivity to the storativities of the aquifer and confining layer, $S_{aqu}$ and $S_{con}$, consistent with observations on the sensitivity to $S_{aqu}$ by Furbish [1991] (Figures 6g and 6h).

The model is fitted to the barometric response function using six parameters: barometric efficiency ($BE$), pneumatic and hydraulic diffusivities of the confining layer ($D_{unsat}$, $D_{con}$), aquifer transmissivity ($T_{aqu}$), capillary fringe attenuation factor ($T_{cf}$), and unsaturated zone thickness ($L_{unsat}$). Four further parameters are treated as constants; total confining layer thickness ($L_{con}$), storage coefficients of the confining layer and aquifer ($S_{con}$ and $S_{aqu}$), and borehole radius ($r_w$). These parameters are either known ($L_{con}$, $r_w$) or do not significantly impact on model results ($S_{con}$, $S_{aqu}$). In addition, barometric efficiency ($BE$) and capillary fringe attenuation factor ($T_{cf}$) are constrained to lie between 0 and 1. The best fit solution is obtained by minimizing the sum of square differences of observed and model barometric response functions in the complex plane using a combined hybrid genetic (GA) and pattern search (PS) algorithm [Alsulma et al., 2010; Liuni et al., 2010]. This combines the advantages of the pattern search method, which is computationally efficient but requires an initial estimate, with those of the genetic algorithm which locates the global minimum without the need for a starting point, but is computationally intensive. The Matlab code used to compute the fit of the model to the barometric response function is available as supporting information accompanying this article.

6. Data Analysis and Parameter Estimation

6.1. Estimating Aquifer and Confining Layer Properties

Barometric response functions (gain and phase) with one standard error are shown with best fit model curves for two contrasting boreholes (Benningholme and Thornholme Moor) in Figure 7. Borehole lithology logs (Figure 8) show that the thickness of the glacial sediments varies from 16 m at Benningholme to 19 m at Thornholme Moor and contain significant proportions of clay rich material (40% at Benningholme and 84% at Thornholme Moor). The barometric response functions (Figure 7) are strong functions of frequency indicating that the confining layer in both cases has nonzero diffusivity. The gain for Benningholme (Figure 7a) shows a typical bell-shaped curve over a frequency range of 0.015 to 1 cycles/day with a plateau at intermediate frequencies in both gain and phase indicating a static confined barometric efficiency of around 0.5. The barometric response function for Thornholme Moor borehole (frequency range 0.045–2.0 cycles/day) shows overall increasing gain and
Figure 6. Barometric response functions calculated from the model of Rojstaczer [1988] showing sensitivity to (a) static barometric efficiency, $BE$, (b) vertical hydraulic diffusivity of the confining layer, $D_{\text{con}}$, (c) vertical pneumatic diffusivity of the confining layer, $D_{\text{unsat}}$, (d) thickness of the unsaturated zone (confining layer), $L_{\text{unsat}}$, (e) capillary fringe attenuation factor, $T_{\text{cf}}$, (f) aquifer transmissivity, $T_{\text{aqu}}$, (g) storativity of the aquifer, $S_{\text{aqu}}$, and (h) storativity of the confining layer, $S_{\text{con}}$. Model input parameters are as listed in Figure 5 with parameters varied as indicated. The plots show that confining layer properties $D_{\text{con}}$, $D_{\text{unsat}}$, $L_{\text{unsat}}$, and $T_{\text{cf}}$ are sensitive principally to low and intermediate frequencies, while $T_{\text{aqu}}$ is sensitive to intermediate and high frequencies. The model shows little sensitivity to storativities, $S_{\text{aqu}}$, and $S_{\text{con}}$. Double-headed arrows indicate range for which barometric response functions are determined in this study (0.015–2 cycles/day).

decreasing phase with increasing frequency over the range of 0.045–0.5 cycles per day (Figure 7b) above which the gain flattens indicating a static barometric efficiency of around 0.4.

[19] The barometric response functions in Figure 7 were fitted to the analytical model [Rojstaczer, 1988] from which the parameters barometric efficiency ($BE$), confining layer pneumatic and hydraulic diffusivities ($D_{\text{con}}$ and $D_{\text{unsat}}$), aquifer transmissivity ($T_{\text{aqu}}$), capillary fringe coefficient ($T_{\text{cf}}$), and thickness of the unsaturated zone ($L_{\text{unsat}}$) were determined. Confining layer thickness was set to the values indicated by the borehole logs (Figure 8) and specific storage coefficients were held constant at $10^{-4}$ m$^{-1}$ for the confining layer (typical for glacial sediments) and $10^{-5}$ m$^{-1}$ for the aquifer (typical for the Chalk Aquifer in East Yorkshire). Upper and lower bounds to parameters were estimated from the range of model curves that lie within the one standard deviation errors for the barometric response.
function (Figure 7). Best fit parameters with upper and lower bounds are listed in Table 2. The best fit model curve to the barometric response functions show overall good fits for the gain and phase curves with some discrepancy for the gain at low frequencies for the Benningholme borehole (Figure 7a).

The greatest difference between the results for the two boreholes is seen in the vertical hydraulic diffusivities, $D_{\text{con}}$, which range from 10 m$^2$/day at Benningholme to 310 m$^2$/day at Thornholme Moor. This is reflected in the position of the gain and phase curves (Figure 7) where the Benningholme curves are shifted toward lower frequencies relative to those of Thornholme Moor. The differing values of $BE$ reflect the positions of the gain curves (Figure 7) where the Benningholme curve ($BE$ 0.49) lies above that of Thornholme Moor ($BE$ 0.39). All parameters are well constrained by the upper and lower bounds except the pneumatic diffusivities at both boreholes and the unsaturated zone thickness at Thornholme Moor for which only lower bounds could be determined due to model insensitivity. The narrow range of unsaturated zone thickness (0.7–1.2 m) is compatible with water table depths recorded in a borehole screened within the confining layer at Benningholme (0.4–2.9 m) and the location of field drains at around...
Aquifer transmissivities (1.5 m$^2$/day at Benningholme and 10.0 m$^2$/day at Thornholme Moor) lie toward the low end of the range observed for the confined Chalk Aquifer (1–500 m$^2$/day) \cite{Smedley2004,Hartmann2007}. These estimates are also significantly less than transmissivities estimated from pumping tests \cite{Parker2009} which are 52 m$^2$/day at Benningholme (5 h pumping test) and 264 m$^2$/day at Thornholme Moor (3 h pumping test).

### 6.2. Aquifer Transmissivity From Barometric Response Functions

The variations in borehole water levels induced by barometric pressure changes over time periods of a few hours are of the order of 1 cm or less (see Figures 3a and 4a) whereas drawdown induced during pumping tests is of the order of a meter or more. To investigate the impact of head change on the estimation of aquifer transmissivity, a series of falling head slug tests inducing head changes from 2 to 41 cm (volumes 0.37–20 l) were conducted at Benningholme and Thornholme Moor. Slug tests were carried out in ascending order of initial head and then repeated in descending order, as recommended by \textit{Butler} \cite{Butler1998}. The slug tests were analyzed (AquiferWin32$^\circledR$ software) using the Hvorslev \cite{Hvorslev1951} method and the results are shown in Figure 9. The results of the slug tests in ascending order at Benningholme indicate a power-law relationship between initial head displacement and aquifer transmissivity with the barometric response function and pumping test values lying on the same trend. However, slug tests carried out in descending order give transmissivities similar to that of the largest slug test regardless of initial head displacement (see Figure 9). This suggests that the observed scaling in transmissivity is caused by a skin effect in the immediate vicinity of the borehole wall. It is thought likely that slugs of increasing volume progressively break seals developed in fractures adjacent to the borehole wall, which are likely to be of precipitated calcium carbonate. Repeat trials of slug tests show that this skin effect becomes re-established in 6–18 months. At Thornholme Moor, all slug tests gave values similar to the pumping test value of transmissivity, over an order of magnitude larger than that derived from the barometric response function (see Figure 9). This suggests that the smallest slug test with a displacement of 3.5 cm, around three times greater than head changes induced by barometric pressure, is sufficient to disrupt skin effects at this borehole. A dependence of transmissivity derived from slug tests on slug test initial displacement was found at a number of boreholes in the area of the confined aquifer showing that the development of such skin effects is a wide spread occurrence in this aquifer.

### 7. Discussion

#### 7.1. Estimating System Properties From Barometric Response Functions

Barometric response functions are here estimated over a frequency range of 0.015–2 cycles/day. This is

### Table 2. Best Fit Values and Ranges of Parameters$^a$

<table>
<thead>
<tr>
<th>Parameter/Borehole</th>
<th>Benningholme (m$^2$/day)</th>
<th>Thornholme Moor (m$^2$/day)</th>
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</thead>
<tbody>
<tr>
<td>$BE$ (—)</td>
<td>0.49 (0.49–0.49)</td>
<td>0.39 (0.38–0.4)</td>
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<tr>
<td>$D_{con}$</td>
<td>10.0 (8.0–13.0)</td>
<td>310.0 (250.0–370.0)</td>
</tr>
<tr>
<td>$T_{aq}$ (m$^2$/day)</td>
<td>10.0 (≥2.0)</td>
<td>50.0 (≥20)</td>
</tr>
<tr>
<td>$T_{cf}$ (m$^2$/day)</td>
<td>1.5 (1.2–1.9)</td>
<td>10.5 (5.0–90.0)</td>
</tr>
<tr>
<td>$T_{cf}$ (—)</td>
<td>0.82 (0.7–0.9)</td>
<td>0.95 (0.93–0.97)</td>
</tr>
<tr>
<td>$L_{unsat}$ (m$^2$/day)</td>
<td>1.2 (0.5–2.0)</td>
<td>0.7 (≤1.5)</td>
</tr>
</tbody>
</table>

$^a$BE, barometric efficiency; $D_{con}$, vertical hydraulic diffusivity of confining layer; $D_{unsat}$, vertical pneumatic diffusivity of confining layer; $T_{aq}$, aquifer transmissivity; $T_{cf}$, capillary fringe coefficient; $L_{unsat}$, thickness of unsaturated zone.
similar to the frequency ranges reported by others [Rojstaczer, 1988; Galloway and Rojstaczer, 1988; Rojstaczer and Riley, 1990]. Evans et al. [1991] obtained barometric response functions over the wider range of 0.02–50 cycles/day made possible by the use of high resolution, vented pressure sensors. The highest frequency in the barometric response function is controlled by the amplitude of the barometric pressure frequency spectrum, instrument resolution and record length with enhanced resolution resulting from long record lengths [Florsch et al., 1991; Merritt, 2004]. For the present case, frequencies are restricted to below 0.85 cycles/day with a power law relation between transmissivity, \( T \) and initial displacement, \( H \), while descending slug test results (open symbols) give constant transmissivity close to the pumping test value. At Thornholme Moor, slug tests give results similar to the pumping test (PT(Tho)) which are an order of magnitude greater than the barometric response function value (BRF(Tho)).

FIGURE 9. Slug tests analysis results (Hvorslev method) for Benningholme and Thornholme Moor boreholes. At Benningholme, ascending slug test results (filled symbols) together with the barometric response function (BRF(Ben)) and pumping test values (PT(Ben)) borehole show a power law relation between transmissivity, \( T \) and initial displacement, \( H \), while descending slug test results (open symbols) give constant transmissivity close to the pumping test value. At Thornholme Moor, slug tests give results similar to the pumping test (PT(Tho)) which are an order of magnitude greater than the barometric response function value (BRF(Tho)).

frequency of 2 cycles/day, a recording time interval of 250 min or less is required. The lowest frequency of the barometric response function is here constrained by recharge to 0.015 and 0.03 cycles/day. The minimum record length required to reach these frequencies is given by the number of segments used to determine the barometric response function multiplied by the proportion of overlap and divided by the lowest required frequency. For the data presented here, 20 segments with 50% overlap were required which, with a lower frequency limit of 0.03 cycles/day imposed by recharge, thus infers a minimum record length of 333 days. The accuracy and resolution of absolute transducers, which are cheaper and easier to install and maintain than vented instruments, is steadily increasing and this will extend the upper frequency limit for barometric response functions in the future.

[23] Extending the barometric response function to lower frequencies requires the development of a filter to remove the effects of recharge. The process of recharge is complex depending on many factors including rainfall intensity and duration, soil characteristics, and land use. Since the recharge signal is complex (Figure 3c), containing a range of frequencies whose amplitudes may vary from year to year, the method used to remove Earth tides [Rasmussen and Mote, 2007] which occur at known frequencies cannot be directly applied but a technique in which a recharge signal was determined by maximizing the coherence between water level and barometric pressure, conditioned by information from boreholes where the aquifer is unconfined, could possibly be developed. A recent study by Jimenez-Martinez et al. [2013] which investigates recharge in the frequency domain shows that the recharge is dominated by large low frequency events with some high frequency events of up to several cm at frequencies up to 0.5 cycles/day can occur. Such an approach could, in theory, be used to develop a filter to remove recharge signals and allow determination of the barometric response function to lower frequencies.

[24] A frequency range for the barometric response function of 0.015–2 cycles/day covers only the low and intermediate frequency response stages of model barometric response function curves in this study (Figure 6). This range favors estimation of confining layer properties which show greatest sensitivity to these lower frequencies (Figures 6b–6e). Thus, to determine confining layer properties from barometric response functions, time series data of borehole water level, and barometric pressure with a time interval of 250 min or less over a time period of 1 year or more is required giving a basic general recommendation for data collection where estimation of confining layer properties are the main aim. However a smaller recording time interval should be chosen if information at frequencies higher than 2 cycles/day are of interest, for instance if higher resolution instruments than those used here were available.

[25] The inclusion of capillary fringe attenuation in the model affects the slope of the gain and phase curves at low frequencies (Figure 6c). Evans et al. [1991] explain the capillary fringe attenuation effect in terms of a partial absorption of the air pressure wave due to volume changes of encapsulated air bubbles close to the water table. Encapsulated air content and capillary
fringe attenuation is greater in sandy than in clay-rich sediments [Honig and Murphy, 2001] and an attenuation of 20% (T <sub>f</sub> of 0.8) has been observed in silty loam soil [Turk, 1975], close to the values for T <sub>f</sub> for Benningholme (0.82). The unsaturated zone in the region of the confined aquifer in East Yorkshire ranges from 0.5 to 3.5 m, so that T <sub>f</sub> is controlled by the nature of the soil and glacial sediments at shallow depths. This is consistent with a soil layer of around 0.5 m and the presence of 0.65 m of sand beneath the soil layer at Benningholme (Figure 8).

[26] Discrepancies of up to two orders of magnitude between aquifer transmissivity derived from the barometric response function and from pumping tests raise questions about the validity of estimating aquifer transmissivity from barometric response functions. The relationship between slug initial head displacement and derived hydraulic transmissivity indicates the presence of skin effects, thought to be caused by precipitation of calcium carbonate in fractures close to the borehole wall. The small changes in borehole water level (here of the order of 1 cm) induced by barometric pressure changes over the time scale of the slug tests of 1–2 h are not sufficient to overcome these effects so that estimates of aquifer transmissivity from the barometric response function are sensitive to any skin effect present. Thus estimates of transmissivity from barometric response functions where such skin effects may be present should be regarded as lower bounds only.

7.2. Effect of Confining Layer Heterogeneity on Barometric Response Functions

[27] The analytical barometric response function models of Rojstaczer [1988] and Evans et al. [1991] assume that both aquifer and confining layers are laterally and vertically homogeneous. In nature, however, aquifers and their confining layers are heterogeneous. The confining layer is made of clay and confining layer grades representing a wide range of hydraulic conductivities and thus diffusivities. In such a system, the lateral extent of the area around a monitoring borehole that influences the barometric response function is important for interpreting estimated confining layer properties in terms of aquifer vulnerability. To investigate the potential effects of heterogeneity in the confining layer, a simple transient, 2-D, cross-sectional groundwater model has been constructed using Visual MODFLOW (Schlumberger). The model (Figure 10) represents a vertical cross section of 10 km by 20 m comprising 14,800 cells ranging in size from 20 × 0.5 m to 100 × 0.5 m. The model consists of two layers, each 10 m thick, representing the confining layer (hydraulic conductivity 0.01 m/day and specific storage 10<sup>-3</sup> m<sup>-1</sup>, typical of clay-rich sediments), overlying an aquifer (hydraulic conductivity of 10 m/day and specific storage 10<sup>-2</sup> m<sup>-1</sup>, typical of the Chalk Aquifer). The attenuation of barometric pressure in the unsaturated zone is here assumed to be negligible (since Modflow cannot model flow in the unsaturated zone) and the barometric pressure signal is modeled as a constant head boundary condition at the top of the model using the barometric pressure signal observed at Benningholme (September–October 2008). The model lateral and lower boundaries are represented by no flow boundaries. The transient model is run with stress periods of 4 h and a total simulation time of 60 days. The impact of initial conditions (zero head everywhere) was found to be negligible after 20 days of simulation. Four scenarios were created in which the confining layer contains a block with a large hydraulic conductivity of 10 m/day and a specific storage of 10<sup>-4</sup> (typical for sands) and the results are compared to those of the homogeneous scenario. This 2-D model represents the effect of a channel of high hydraulic conductivity material that fully penetrates the confining layer which is of infinite lateral extent in the direction perpendicular to the plane of the model. This geometry is similar to that formed by the glaciofluvial sediments (sand and gravels) which form channel-like bodies in the clay-rich tills in the confining layer of the Chalk Aquifer in East Yorkshire. Figures 10b and 10d show the results from a simulation period of 10 days (days 50–60).

[28] In the first scenario, (Figure 10a) a highly conductive block of 500 m width fully penetrates the confining layer. Figure 10b shows that in the aquifer immediately below the high conductivity block, the head signal is virtually unaltered from the imposed barometric pressure signal, indicating unconfined conditions. With increasing distance, D, from the center of the block, the signal observed in the aquifer is progressively damped and lagged until, at distances greater than about 750 m, it becomes indistinguishable from that observed the homogeneous case. In addition, it can be seen in Figure 10b that high frequencies are more severely damped than low frequencies. Figure 11 shows a plot of the average difference over the ten day period depicted in Figure 10, between the homogeneous and heterogeneous models versus distance from the heterogeneity edge. Taking twice the pressure transducer resolution (i.e., 0.5 cm) as a conservative estimate of the head difference between homogeneous and heterogeneous cases that is needed to modify the barometric response function, the signal observed in the aquifer becomes indistinguishable from the homogeneous case when the observation borehole is at about 600 m distant from the heterogeneity. In scenario 2, where the width of the high conductivity block is reduced to 20 m, very similar results were obtained (see Figure 11).

[29] In the third heterogeneous scenario (Figure 10c), the highly conductive block of 500 m width penetrates only half of the confining layer. The results in Figure 10d show that the signal in the aquifer immediately below the heterogeneity is now significantly damped, particularly at high frequencies. Using the same criteria as above, the presence of the heterogeneity can be detected for distances of up to 300 m from the heterogeneity (Figure 11). Simulations in which the width of the partially penetrating high conductivity block is reduced to 20 m (scenario 4), showed that in this case results are not significantly different from the homogeneous case even directly under the heterogeneity itself (Figure 11).

[30] The numerical model, generated using Modflow, models only the propagation of barometric pressure through the pore waters of the confining layer and aquifer and does not include the grain-to-grain loading effects of changes in barometric pressure that are transmitted to the aquifer. These models thus correspond to the situation where the static barometric efficiency, BE, equals 1 and the loading efficiency is therefore zero. This will maximize the

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head changes in the aquifer pore waters and in reality within the Chalk Aquifer which has a static barometric efficiency of around 0.5, the head changes will be around half those recorded in the Modflow model. Conversely, the estimate of the head difference required to distinguish between simulations with and without the heterogeneity of twice the pressure transducer resolution (0.5 cm) is very conservative and therefore likely to underestimate the distance at which a heterogeneity may be detected. Without using model results to calculate the full barometric response function (beyond the scope of the present article), it is not possible to be more precise and the distances given by the model can be taken as rough guide only to the impact of a heterogeneity. However, this preliminary modeling suggests that the presence of vertically continuous, highly conductive pathways through a low hydraulic conductivity confining layer at distances up to several hundreds of meters from the monitoring borehole will exert a significant influence on the barometric response function. When highly conductive material is present but does not provide a connected flow pathway through the confining layer, the impact on the barometric response function is markedly less. This shows that the head signal observed in the aquifer, and thus the barometric response function, is more sensitive to the vertical connectivity of high conductivity heterogeneities than to their lateral dimensions. The greater damping and lagging of high compared to low frequencies indicates that the high frequency response of borehole water level to barometric pressure is controlled by confining layer properties in the immediate vicinity of the borehole, while the low frequency response reflects confining layer properties over a region of considerable lateral extent around the borehole. Thus, high and low frequencies in the barometric response function may reflect different confining layer properties. This provides one possible explanation for the relatively poor fit between estimated and model gain curves at low frequencies for the Benningholme borehole (Figure 7a) which may reflect varying characteristics of the confining layer with increasing distance from the monitoring borehole.

A good illustration of these effects is seen at the Thornholme Moor borehole where the high values of estimated confining layer hydraulic diffusivity (310 m$^2$/d) is in conflict with the borehole lithology log (Figure 8) which is strongly clay-dominated suggesting a low hydraulic diffusivity. However, this borehole is situated 100 m from a large outcrop (1200 by 500 m) of glaciofluvial sand and...
from other sources. However, a log-log plot (Figure 12) of specific storage ($S_s$) versus hydraulic conductivity ($K$) for glacial sediments from the literature [Urish, 1981; Younger, 1993; Martin and Frind, 1998; Batu, 1998; Kilner, 2004; Quinn, 2009] shows that while hydraulic conductivity varies over 11 orders of magnitude, specific storage varies only over two orders and thus hydraulic diffusivity is most sensitive to hydraulic conductivity. In addition, a modeling study by Knudby and Carrera [2006] indicates that hydraulic diffusivity in heterogeneous media is largely controlled by the connected, highly conductive pathways. This with the results of numerical modeling described in this study (section 7.2), indicates that barometric response functions will reflect the presence of such connected high flow pathways. In addition, the low frequencies of the barometric response function which are those most easily determined, are also those that are most useful in estimating vertical diffusivity of the confining layer. The above suggests that vertical hydraulic diffusivity derived from barometric response functions can be used to provide a quantitative measure of intrinsic aquifer vulnerability.

A number of methods exist to characterize vertical contaminant transport and pressure propagation in porous media. The ratio of the square of thickness to the vertical diffusivity has been used to define a characteristic time scale for the vertical diffusion of aquifer pore pressure to the water table [Roeloffs, 1996; Foster et al., 1993]. Kruseman and de Ridder [1994] suggested the use of hydraulic resistance, defined as the ratio of confining layer thickness to hydraulic conductivity ($L/K$), as a measure of vertical transport potential. Building on the arguments above, we suggest a characteristic time scale, $C_{ts}$, that is a function of the unsaturated and saturated confining layer vertical diffusivities and their thicknesses, as a measure of intrinsic aquifer vulnerability:

$$C_{ts} = \frac{L^2}{D_{unsat}} + \frac{L^2}{D_{sat}}$$  \hspace{1cm} (2)

Low values of $C_{ts}$ reflect thin and/or highly diffusive confining layers, implying high vulnerability, and vice versa. In the case of the boreholes presented here, the thickness of the unsaturated zone is small and $C_{ts}$ is dominated by the second term in equation (2). Values of $C_{ts}$ with thicknesses of clay-rich sediment for the two boreholes are listed in Table 3. The Benningholme borehole shows the largest $C_{ts}$ value (22.5 days) and therefore least vulnerability, while the Thornholme Moor borehole indicates the most vulnerable location with a $C_{ts}$ value of only 1.1 days. This difference is reflected in positions of barometric response function curves where the gain and phase curves for Benningholme lie to the left (toward lower frequencies) with respect to those for Thornholme Moor (Figure 7), reflecting lower pneumatic and hydraulic diffusivities (see Figures 6b and 6c). Note that $C_{ts}$ does not correlate well with the percentage of clay-rich sediments seen in the borehole logs (Table 3) where the Thornholme Moor borehole gives the lowest value of $C_{ts}$ (1.1 days), and therefore highest vulnerability, but the highest percentage of clay-rich sediments from the borehole log (84%).

The characteristic timescales derived from the barometric response functions can be compared to the existing groundwater vulnerability designations at the localities of

---

**Figure 11.** Plot of the average difference between heads observed in the aquifer over a 10 day period in heterogeneous and homogeneous models with distance from the heterogeneity edge. 500 FP and 20 FP—fully penetrating heterogeneities, 500 m and 20 m across, respectively, 500 PP and 20 PP—partially penetrating heterogeneities, 500 m and 20 m across, respectively. Fully penetrating heterogeneities, regardless of width, have a much greater impact on the head signal observed in the aquifer than partially penetrating heterogeneities.
sus hydraulic conductivity, varies over 11 orders of magnitude, deposits composed of sands and gravels (100 m away) vul-
uifer is classified as low. However, at nearby glaciofluvial
poses of clay-rich till where vulnerability of the major aq-
Major aquifer at this location is low. The Thornholme
the confining layer and confirms that the vulnerability of
these two boreholes. The Benningholme borehole where
the confining layer comprises alluvial sediments is situated
in a zone classified as low vulnerability for the major aqui-ers. The low characteristic time derived from the
barometric response function indicates high vulnerability,
confirming that the glaciofluvial sands and gravels pene-
trate vertically to the contact with the Chalk Aquifer
beneath. In these two cases, therefore, the barometric
response functions provide additional information on the
degree to which highly conductive sediments within the
confining layer provide connected, vertical flow pathways
from the surface to the aquifer.

The increasing use of pressure transducers to auto-
mate routine monitoring of borehole water levels at hourly
or more frequent time intervals is replacing the traditional
manual dipping methods at monthly or larger time inter-
vals. Thus, many data sets that are suitable for the calcula-
tion of barometric response functions already exist. This,
with the abundance of monitoring boreholes in many major
aquifers, suggests that estimation of vertical diffusivities
from barometric response functions has the potential to
improve the quantitative assessment of aquifer vulnerabil-
ity in semiconfined aquifers.

Table 3. Vertical Hydraulic Diffusivity ($D_{v,con}$) From Barometric Response Functions, Thickness and Percentage of Clay-Rich Sediments in the Confining Layer, and Characteristic Time Scales ($C_v$)

<table>
<thead>
<tr>
<th>Parameter/Borehole</th>
<th>Benningholme</th>
<th>Thornholme Moor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness clay (m)</td>
<td>6.8</td>
<td>16</td>
</tr>
<tr>
<td>% Clay-rich sediments</td>
<td>42</td>
<td>84</td>
</tr>
<tr>
<td>$D_{v,con}$ (m²/day)</td>
<td>10</td>
<td>310</td>
</tr>
<tr>
<td>$C_v$ (days)</td>
<td>22.5</td>
<td>1.1</td>
</tr>
</tbody>
</table>

8. Conclusions

[37] Confining layer and aquifer properties have been estimated from barometric response functions and related to aquifer vulnerability. The main conclusions are:

[38] 1. As previous studies have shown [e.g., Weeks, 1979; Rojstaczer, 1988; Spane, 2002; Butler et al., 2011], barometric response functions can be used to estimate vertical hydraulic and pneumatic diffusivities of the confining layer. These diffusivities are of greater relevance to intrinsic aquifer vulnerability than horizontal hydraulic properties determined by the more traditional methods of hydraulic testing (pumping and slug testing).

[39] 2. The limitations of barometric pressure signal amplitude at high frequencies and instrument resolution means that, in this study, only the lower to intermediate frequencies (0.015–2 cycles/day) of the barometric response function are readily determined. However, this frequency range allows the estimation of confining layer pneumatic and hydraulic diffusivities which are most sensitive to frequencies in this range.

[40] 3. Previous work [e.g., Furibish, 1991; Rasmussen and Crawford, 1997; Spane, 2002] has shown the effect of borehole storage and skin effects on the response of borehole water levels to barometric pressure change. The present study confirms that estimates of aquifer transmissivity from barometric response functions are highly sensitive to
skin effects which can lead to underestimation of one to two orders of magnitude.

[4] 4. Head signals in a semiconfined aquifer are more sensitive to vertical connectivity of highly conductive heterogeneities than they are to their lateral dimensions. This indicates that barometric response functions will be sensitive to the presence of highly conductive, vertically connected flow pathways through the confining layer at distances of up to several hundred meters from the borehole.

[5] 5. Higher frequencies in the barometric response function are dominated by confining layer properties in the immediate vicinity of the borehole, while low frequencies reflect properties over a wider area.

[6] 6. A characteristic time scale, $C_{ts}$, which is a function of vertical diffusivities and thicknesses of the unsaturated and saturated zones in the confining layer is suggested as a quantitative measure of intrinsic aquifer vulnerability for semiconfined aquifers.

[7] 7. With increasing use of pressure transducers to routinely record borehole water levels, records with sufficient temporal resolution for determination of the barometric response functions will become increasingly available. This, with the abundance of monitoring boreholes throughout most major aquifers, suggests that the barometric response function provides a potential tool to improve the quantitative assessment of vulnerability in semiconfined aquifers.

**Appendix A: Removal of Earth Tide Components Water Level Signals**

[45] The contribution of Earth tides in water level signals is removed using the method of Rasmussen and Mote [2007] comprising time domain regression deconvolution of the water level and barometric pressure time series. Before applying this method, the influence of recharge on the borehole water level data is removed using a high pass Butterworth filter and the borehole water level and barometric pressure data are detrended (linear trend removed and mean subtracted). The observed change in borehole water level, $\Delta WL$, over time interval, $t$, is then given by:

$$\Delta WL = \sum_{\tau=0}^{n} \mu(\tau) \cdot \Delta B_p(t - \tau) + \xi(t) \quad (A1)$$

where $\Delta B_p$ is the change in barometric pressure over time interval $t$, $\mu(\tau)$ is the unit barometric response function at lag $\tau$, and $\xi(t)$ is the sum of the Earth tide components given by:

$$\xi(t) = \sum_{j=1}^{S} (a_j \cos \omega_j t + b_j \sin \omega_j t) \quad (A2)$$

where $a_j$ and $b_j$ are the Earth tide component coefficients and $\omega_j$ is angular frequency of the $j$th component. The first term on the right-hand side of equation (A1) represents the influence of barometric pressure and the second term represents the contribution from the Earth tide components (equation (A2)). The coefficients $a_j$ and $b_j$ are determined and the influence of Earth tides removed from the water level signal using time domain regression deconvolution.

This procedure allows the separation of the contributions from Earth tides and barometric pressure at frequencies of 1 and 2 cycles/day. For full details of the method we refer to Rasmussen and Mote [2007].

**Appendix B: Estimation of the Barometric Response Function**

[46] The barometric response function (BRF) is obtained from the borehole water level and barometric pressure signals using cross-spectral deconvolution by ensemble averaging [Welch, 1967] used by previous investigators [Rojstaczer, 1988; Rojstaczer and Riley, 1990; Beavan et al., 1991; Quilty and Roeloffs, 1991]. The barometric response function is given by:

$$BRF(f) = \frac{X_{WB}(f)}{X_{WB}(f)} \quad (B1)$$

where $X_{WB}(f)$ and $X_{WB}(f)$ are the cross spectra of water level and barometric pressure signals and the autospectrum of the barometric pressure signal, respectively. Details of the cross-spectrum $X_{WB}(f)$ and the autospectrum $X_{WB}(f)$ estimation methods can be found in Bendat and Piersol [2010]. BRF($f$) is a complex function which can be expressed as the barometric response function gain, $A_{BRF}(f)$, and phase, $\theta_{BRF}(f)$, given by the modulus and the argument of BRF($f$), respectively:

$$A_{BRF} = |BRF(f)| \quad (B2)$$

$$\theta_{BRF}(f) = \arctan \left( \frac{\text{imag}(BRF(f))}{\text{real}(BRF(f))} \right) \quad (B3)$$

[47] Welch’s technique is applied as follows: the time series for water level and barometric pressure (corrected for recharge and Earth tides) are divided into a number of segments, $N$, with 50% overlap, $\delta$ [Bendat and Piersol, 2010]. Each segment is detrended, high frequency noise above three cycles/day removed using a Butterworth filter [Bendat and Piersol, 2010] and a tapering periodic Hanning window applied. A barometric response function is computed for each pair of segments using equation (B1) and these $N$ partially independent estimates of the barometric response function are then averaged. This technique is applied to five overlapping frequency bands, each with its own number of segments, $N$, following the method of Beavan et al. [1991]. Using a small number of segments extends the BRF to lower frequencies at lower accuracy, whereas using a larger number of segments gives increased accuracy at higher frequencies. The frequency range of the final barometric response function is determined using
coherence and signal amplitude. Frequencies where the coherence between barometric pressure and water level signals is lower than 0.5 and/or where the amplitude of the water level signal is less than 0.03 cm are excluded, following Rojstaczer and Riley [1990].

Standard errors for gain, \( \sigma(A) \), and phase, \( \sigma(\theta) \), were determined using average coherences between barometric pressure and water level signals, \( C_{WB}(f) \), over \( N \) segments [Beavan et al., 1991; Bendat and Piersol, 2010]:

\[
\sigma(A) = \sigma(f) \times A^{BRF(f)} \\
\sigma(\theta) = \sigma(f) \times \frac{180}{\pi}
\]

where the normalized standard error, \( \sigma(f) \), is given by:

\[
\sigma(f) = \left[ \frac{1}{2p} \left( \frac{1}{C_{WB}(f)^2} - 1 \right) \right]^{1/2}
\]

and \( p = N - (N - 1) \times \delta \). From equation (B6), it can be seen that the error reduces with increasing coherence, \( C_{WB}(f) \), and number of segments, \( N \).

Appendix C: Calculation of Theoretical Barometric Response Functions

The analytical model of Rojstaczer [1988] is fitted to the barometric response functions and used to estimate aquifer and confining layer properties. This model is formulated in terms of three-dimensionless frequencies, \( R \), \( Q \), and \( W \):

\[
R = \frac{L_{\text{unsat}}^2 \omega}{2D_{\text{unsat}}} , \quad Q = \frac{L_{\text{sat}}^2 \omega}{2D_{\text{sat}}} , \quad \text{and} \quad W = \frac{r_{\text{sat}}^2 \omega}{T_{\text{sat}}}
\]

where \( \omega \) is angular frequency, \( L_{\text{unsat}} \) is the unsaturated zone thickness, \( L_{\text{sat}} \) is the depth from water table to top of the aquifer, and \( D_{\text{unsat}} \) and \( D_{\text{sat}} \) are the vertical pneumatic and hydraulic diffusivities of the unsaturated and saturated zones, respectively. Pore water pressure in the aquifer, \( P_0 \), far from the borehole is given by:

\[
P_0 = [(M + iN) - (1 - BE)] \cdot \exp [-i(i + 1) \cdot Q^0.5] + 1 - BE
\]

where \( BE \) is the static barometric efficiency of the aquifer, \( i \) is the imaginary number, and \( M \) and \( N \) are given by:

\[
M = T_{cf} \left[ \frac{2\cosh (R^0.5)}{\cosh (2R^0.5) + \cosh (R^0.5)} \right] \quad \text{and} \quad \quad N = T_{cf} \left[ \frac{2\sinh (R^0.5)}{\cosh (2R^0.5) + \cosh (R^0.5)} \right]
\]

where

\[
Z = K_0 \left\{ \left[ W^2 \left( S_{\text{con}}^2 + \frac{S_{\text{sat}}}{2Q} \right)^2 \right]^{1/25} \cdot \exp [0.5 \cdot i \cdot \tan^{-1}(2Q)] \right\}
\]

\[
\text{References}
\]

[49] The analytical model of Rojstaczer [1988] is fitted to the barometric response functions and used to estimate aquifer and confining layer properties. This model is formulated in terms of three-dimensionless frequencies, \( R \), \( Q \), and \( W \):

\[
R = \frac{L_{\text{unsat}}^2 \omega}{2D_{\text{unsat}}} , \quad Q = \frac{L_{\text{sat}}^2 \omega}{2D_{\text{sat}}} , \quad \text{and} \quad W = \frac{r_{\text{sat}}^2 \omega}{T_{\text{sat}}}
\]

\[
Z = K_0 \left\{ \left[ W^2 \left( S_{\text{con}}^2 + \frac{S_{\text{sat}}}{2Q} \right)^2 \right]^{1/25} \cdot \exp [0.5 \cdot i \cdot \tan^{-1}(2Q)] \right\}
\]

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