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Use of Control to Maintain Period-1 Motions During Wind-up or Wind-down Operations of an Impacting Driven Beam

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Abstract

We consider the dynamical response of a thin beam held fixed at one end while excited by an external driving force. A motion limiting constraint, or stop, causes the beam to impact. During wind-up or wind-down operations, in which the driving frequency is continuously altered, the system can undergo complicated motions close to the value of frequency at which impacts may first occur, the grazing bifurcation. In this region the beam may experience several impacts within a long period repeating solution or even chaotic behavior which, in practical terms, may be undesirable to the long-term integrity of the system. The first task is to identify the zones in the space of parameters (forcing amplitude, or alternatively the gap between the beam and the stop) in which period-1 motions can be guaranteed. In this paper, in the areas in which complicated or chaotic motion occurs, a control strategy is proposed which stabilises unstable period-1 motions. As a consequence, numerical simulations indicate that for any choice of parameter, simple period-1 motions can be maintained, limiting the number of impacts (together with their velocity).

1 Introduction

We examine here a particular configuration for a driven beam whose free end is allowed to impact with a stop during dynamical motions which falls into a class of systems called impact oscillators. While this allows comparison with earlier works and experimental studies [1] the results can be considered representative of a wide class of impacting systems. Following earlier work by Shaw and Holmes [2] a great many research studies have been carried out on impact oscillators (also known as vibro-impacting systems), with particular emphasis on highlighting the complex behavior that can follow as a result of a grazing

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bifurcation. Such a bifurcation occurs when an orbit just touches the constraint, or stop, leading to a zero (or very low) velocity impact. To explain this behavior Nordmark [3] derived a nonlinear mapping which was valid close to grazing. The non-smooth nature of the mapping results in an array of behavior and new bifurcations [4] which correspond well with experiments [5, 6, 7].

There is therefore much that we know about impact oscillators and yet in engineering terms many of the complicated motions may be considered undesirable, reducing our ability to predict the number of impacts or the forces exchanged during interaction. Of interest here is the behavior of the system during operations in which the frequency of driving is either increased from zero, at fixed amplitude, or decreased to zero which may typically occur in start-up or shut-down procedures. Based on a simple model, for a specific placement of the restraining stop, it is theoretically possible to determine a critical value of forcing amplitude for which a smooth transition occurs between non-impacting and impacting period-1 motion [8, 9]. On one side of this critical value a finite jump between period-1, non-impacting and period-1 impacting is possible, while the other side produces complicated and chaotic behavior. In a typical engineering environment it may be hard to achieve suitable precision or choice of parameter so that an unwanted and unpredictable response cannot be avoided [10].

In separate studies the control of chaos has been at the forefront of recent research following the seminal work of Ott, Grebogi and Yorke [11]. Motivated by this concept the aim of the current research is to determine the parameters for which the grazing bifurcation produces complex, chaotic motion and then implement a control strategy which stabilises period-1 impacting motions as the driving frequency is varied so that simple, period-1 motions can be maintained.

2 Mathematical model and parameter variations

The physical system we consider consists of a stiff vertical beam, clamped at the base but free to vibrate in response to a driving electro-magnetic force. A stop is fixed in position which inhibits motion so that under certain conditions the beam may impact with the stop. For driving frequencies close to the fundamental natural frequency it has previously been demonstrated that a single degree of freedom mathematical model adequately captures much of the qualitative and quantitative behavior of the beam [1].

Experimental evidence of the control process is not given in this paper and will form the basis of future work but the model is selected for study since some experimental evidence is already available from earlier research.

We thus focus our mathematical attention on the dynamic behavior of the single degree of freedom impact oscillator. The displacement of the oscillator's mass is represented by a nondimensional value $x(t)$. At a displacement $x = a$ a rigid impact stop restricts the motion of the mass. Thus away from the stop $x < a$ the model is governed by the equation of motion for a one degree of

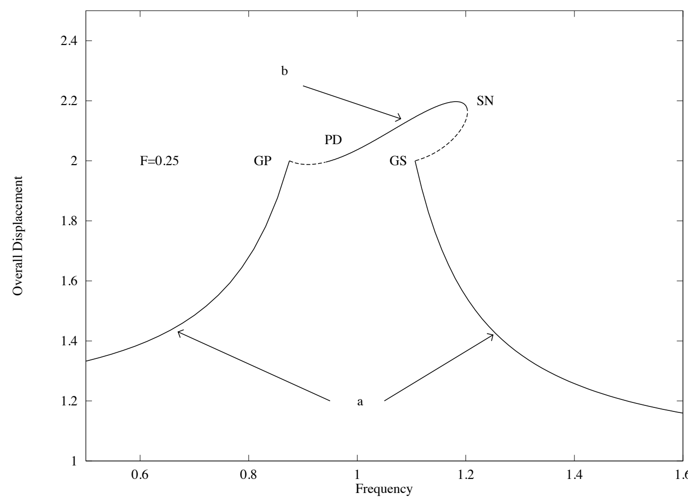

Figure 1: Analytically determined response curve for impact oscillator; Overall displacement *vs* Frequency. Solid lines; labelled (a) represent stable non-impacting period-1 solutions, those labelled (b) represent impacting (1,1) solutions. Dashed lines represent unstable (1,1) impacting solutions.

freedom forced linear oscillator.

$$\ddot{x} + 2\alpha\dot{x} + x = F\cos\omega t \qquad x(t) < a \tag{1}$$

Where $\ddot{x}(t)$ and $\dot{x}(t)$, are the acceleration and velocity of the mass. ω is the ratio of the forcing and natural frequencies and t is nondimensional time, scaled from real time via natural frequency. 2α and F are the nondimensionalised damping and forcing respectively. An overdot represents differentiation with respect to dimensionless time t.

Equation (1) has been nondimensionalised in such a way that an impact occurs when $x(t) = a = 1$. At impact a simple instantaneous coefficient of restitution rule is applied.

$$\dot{x}(t^{+}) = -r\dot{x}(t^{-}) \qquad x(t) = a \tag{2}$$

Where $\dot{x}(t^{-})$ is the velocity before impact , $\dot{x}(t^{+})$ is the velocity after impact and r is the coefficient of restitution which has a range between 0 and 1 depending on the material properties of the system. To perform numerical simulations we select parameters which are representative of other experimental work [1, 12]. We note that, for the steel components used in the experimental impacting beam the coefficient of restitution has been estimated to be in the range $0.7 - 0.9$. However past investigations [1, 12] have demonstrated that to match experimental and numerical results values around 0.2 have to be used. In this way the decreased value of coefficient of restitution is used to additionally account for energy loss due to the beams response in higher modes. In addition

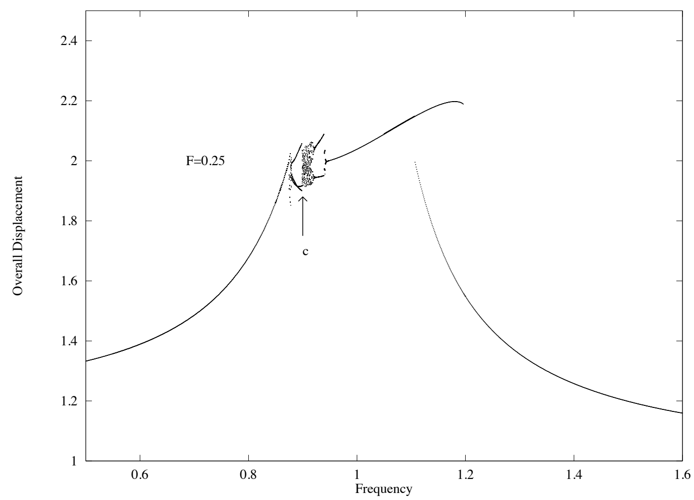

Figure 2: Numerically determined response curve for impact oscillator; overall displacement *vs* frequency. The complex motion between first grazing and stable (1,1) impacting motion is labelled (c).

these studies indicate that a damping value in the range $0.05 - 0.1$ is a close approximation for the actual damping of the beam.

The forcing amplitude F and frequency ω are chosen as parameters which can be varied during the experiment. The stop distance a can also be used as a control parameter to obtain similar results, see for example [8, 13].

Periodic impacting solutions are denoted as (m, n) solutions with m impacts in n periods of $2\pi/\omega$ of the forcing. For this much simplified mathematical system analytic solutions can be determined for the $(1, n)$ motions [2, 14] and used to compute amplitude response bifurcation diagrams.

An example of this is shown in figure 1. Here the forcing amplitude F has been fixed at 0.25 and forcing frequency ω is used as the bifurcation parameter. The response is measured as maximum displacement minus minimum displacement, so that, with a stop distance at $a = 1$, grazing will occur when the overall displacement $(x_{max} - x_{min}) = 2$. Solid lines represent stable period-1 solutions; those labelled (a) are non-impacting and those labelled (b) have one impact per period. Dashed lines represent unstable impacting solutions. From the analytical expressions we see that a stable period-1 non-impacting motion exists as the driving frequency ω is increased from zero. As ω is increased the amplitude of the oscillating periodic orbit grows approaching the stop at which point the first grazing occurs at the point GP, $\omega \approx 0.875$, producing a zero velocity impact. After grazing the motion loses its stability. From other detailed numerical evidence the subsequent impacting motion may either be chaotic or undergo a complicated sequence of bifurcations (see figure 2). At the point PD the motion stabilises onto a $(1, 1)$ impacting solution via a period doubling bifurcation.

This motion persists as frequency is increased, until a saddle node bifurcation occurs at SN and the motion jumps to a non-impacting period-1 solution.

Winding down the frequency (from $\omega = 1.6$, say), the non-impacting period-1 solution grows in amplitude until grazing at GS, $\omega \approx 1.11$. Here the motion jumps to the $(1,1)$ impacting solution thus forming a hysteresis loop between GS and SN. For a further decrease the motion then continues as the reverse of the wind-up behaviour.

The labels GP and GS denote grazing bifurcations of different type [15]. GP is a grazing bifurcation of the *period doubling* type, and GS is a grazing bifurcation of the *saddle node* type.

The numerical simulation of the system (using the same parameter values as used in figure 1) given in figure 2 shows the complex behavior in the unstable $(1,1)$ region (c) between GP and PD of figure 1. In this region an unstable $(1,1)$ impacting solution exists and is shown as the dashed line between GP and PD in figure 1.

Again, due to the simple nature of the mathematical model, it is possible to produce analytical expressions for the value of forcing amplitude at which a grazing bifurcation will first occur for any chosen frequency via the relation;

$$F_{graze} = a\sqrt{(1-\omega^2)^2 + 4\alpha^2\omega^2} \tag{3}$$

The locus of these first grazing bifurcations can be plotted in the (F,ω) parameter space. In addition the locus of the saddle node, SN and period doubling PD bifurcations can also be followed in the same parameter space. Combining this space with the response diagram of figure 1 gives the parameter space-response diagram shown in figure 3.

Here the locus of first grazing bifurcations is shown as a solid line in the frequency *vs* forcing amplitude plane. The dashed line between the points $C1$ and $C3$ is the locus of period doubling bifurcations. The other dashed lines in this plane represent loci of saddle node bifurcations. The structure of the response curve from figure 1 can be seen in the overall displacement *vs* frequency plane. Dashed lines are used to show how bifurcation points on this curve relate to the bifurcation loci in the (F,ω) plane. The locus of first grazing bifurcations contains both the GP and GS type. The solid line between points $C1$ and $C3$ is a locus of GP type bifurcations, other solid lines are of the GS type.

In figure 3 the points $C1$ and $C3$ denote the points where grazing, period doubling and saddle node loci meet, forming a codimension 2 bifurcation. The structure of this bifurcation has been discussed in Ivanov [9] and Foale [14].

Away from these codimension 2 points, the singularity in the derivative of the grazing bifurcation [3] leads to one of the eigenvalues tending to infinity when a zero velocity impact occurs, and hence a high degree of instability. The codimension 2 points points occur when $2(\sqrt{(1-\alpha^2)})/\omega = 1, 2, 3, \ldots$ at which values the singularity in the derivative is cancelled by the term $(\sin[2(\sqrt{(1-\alpha^2)})\pi/\omega])$ becoming zero.

We label the points accordingly as $C1, C2, C3, \ldots$ so that for a system with fixed damping α we can identify the corresponding values of ω for each Ci

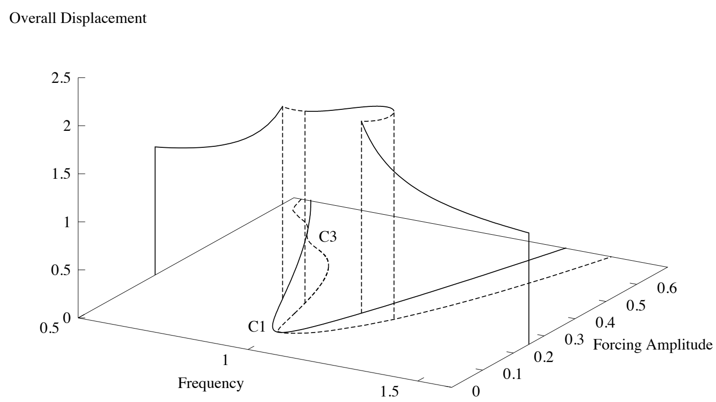

Figure 3: Parameter space response diagram for impact oscillator. Frequency *vs* forcing amplitude *vs* overall displacement.

value. Then the corresponding F_C value can be found by substituting the ω value associated with the point Ci into equation 3, thus locating the point Ci in (F, ω) plane.

In the parameter range shown in figure 3 only the points $C1$ and $C3$ are present. The region around $C3$ is schematically shown in close up in figure 4, indicating regions of different dynamic behavior. The inset diagrams illustrate a typical phase plane $(x, \dot{x})$ for each of the regions. The impacting and non-impacting regions are separated by the GP and saddle node line. Within the impacting region there are three separate areas;

1. Stable period $(1,1)$ impacting.
2. Unstable period $(1,1)$ impacting, where other periodic and chaotic behavior may be present.
3. Hysteresis; non impacting period-1 and stable $(1,1)$ impacting solutions coexist in a hysteretic loop.

Within the window of parameters given in figure 4. it can be seen how at the point $C3$ the period doubling and saddle node lines coalesce with the GS and GP lines. As discussed in Peterka [8] and Ivanov [9], selecting an appropriate value of forcing amplitude F_C so that as ω is increased we pass directly through the point $C3$ produces a motion which moves continuously from non-impacting to impacting at the graze.

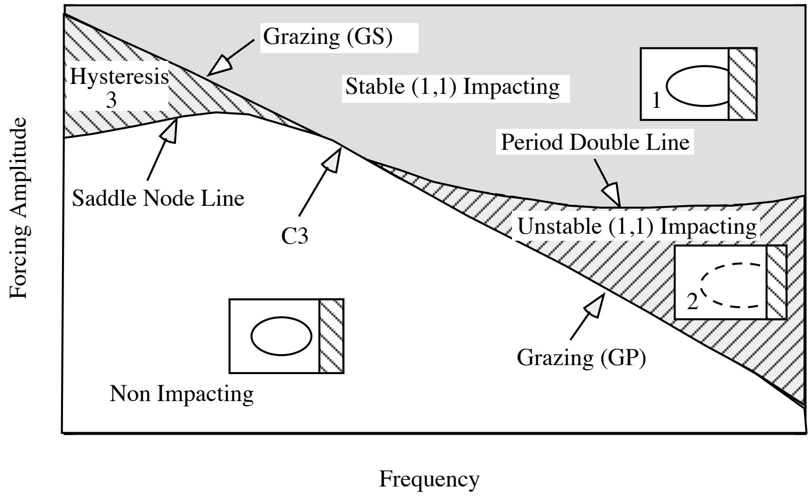

Figure 4: Close up around $C3$ in the parameter space; frequency *vs* forcing amplitude for the impact oscillator. Inset diagrams represent typical phase portraits $(x, \dot{x})$ for each of the regions of dynamical behaviour.

This scenario is numerically demonstrated in figure 5 where we see that our aim of producing only stable period-1 motion as ω increases is achieved. The point labelled $GC3$ is the codimension 2 grazing point associated with F_{C3}. Jumps will still occur at SN for increasing ω and GS for decreasing ω.

From close investigation of the behavior at $C3$, seen in figure 4, for values of forcing amplitude larger than F_C the saddle node bifurcation occurs before the grazing (when increasing ω from zero) so that the complete picture of the period-1 motions when followed for increasing and decreasing frequency is given in figure 6. Thus for typical values just above F_C our aim of producing only period-1 motions is achieved although now jumps will occur at grazing and saddle node bifurcations for both increasing and decreasing ω with hysteresis between SN and GS.

3 Control strategy

From our earlier discussion we note that for values of forcing amplitude less than F_C, chaotic motions or periodic orbits of long periods cannot be avoided. This may create severe problems in a practical engineering environment since the number and velocity of impacts can no longer be predicted. However, motivated by studies on the control of chaos [11], we note that while the period-1 impacting motion is no longer stable after grazing at GP an unstable solution does exist. Our control strategy is then one based on a stabilisation of this unstable motion using control in conjunction with a tracking technique to follow the orbit as ω

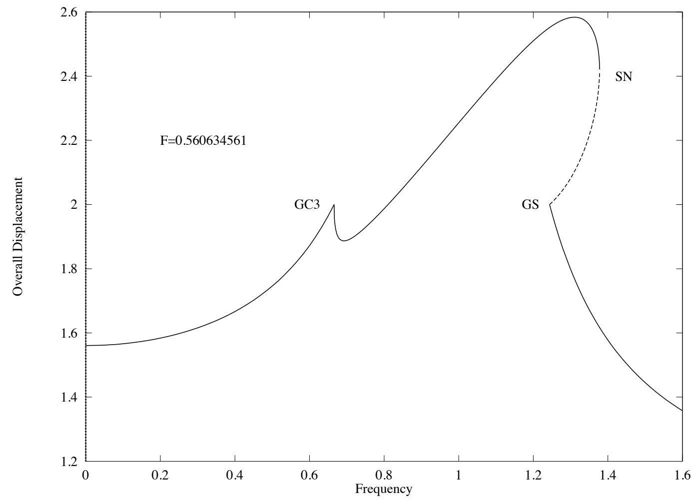

Figure 5: Analytical response curve for the impact oscillator for $F = F_C$. Overall displacement *vs* frequency.

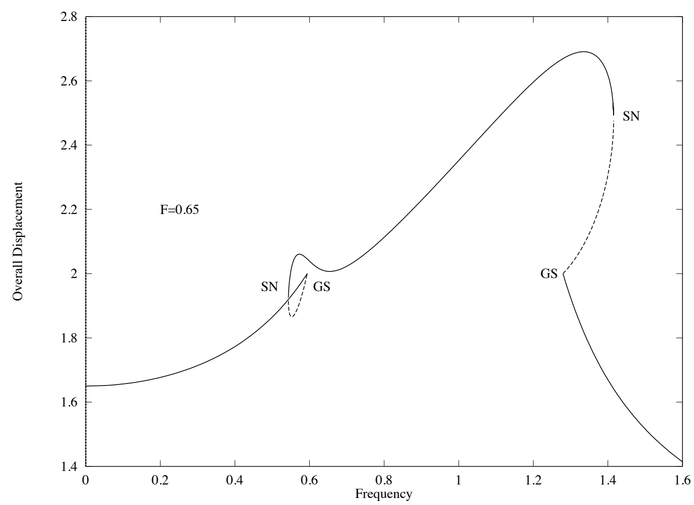

Figure 6: Analytical response curve for the impact oscillator for $F > F_C$. Overall displacement *vs* frequency.

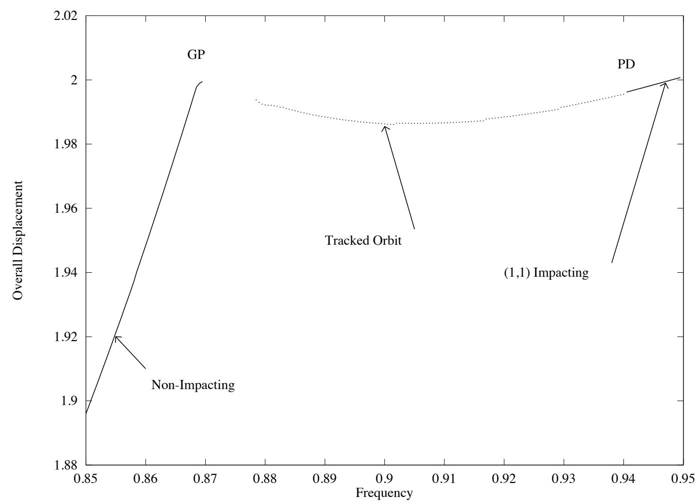

Figure 7: Control tracking unstable (1,1) impacting motion as frequency is decreased towards grazing (GP). Overall displacement *vs* frequency

is varied.

To carry out the control a variety of methods could be applied, however we use here a self-locating scheme based on the Newton root finding algorithm which incorporates the feedback of an output sequence on accessible parameters. More details of the method can be found in [16]. Before applying control only an approximate location of the position of the unstable orbit is required which is readily available from our analytic studies. In the control and tracking process we select the amplitude of the driving force F as the accessible control parameter though numerically the damping or gap between the beam and the stop could equally be used. The control input is adjusted once during each period of the tracked orbit. Figure 7 indicates the path of the tracked period-1 impacting motion as ω is decreased towards the grazing bifurcation (GP). To perform the tracking ω is adjusted once the difference between successive amplitudes of the periodic orbit is within 0.0001. For successful tracking, particularly close to GP very small increments in frequency must be taken. For this numerical study we chose $\Delta\omega = 0.0001$ under which conditions we can approach to within a distance of 0.009 of the grazing. Close to grazing the magnitude of the eigenvalues increases dramatically so that in a small zone of frequency the control process fails but this zone is small when compared to the original unstable zone.

4 Conclusion

We have focused our attention on the period-1 solutions for a simple vibrating beam. From analytical and numerical investigations we have demonstrated that the parameter space around the natural frequency for such a beam can be divided into two main regimes of behavior separated by a codimension 2 bifurcation. At this point we have shown that a smooth transition from non-impacting to period $(1, 1)$ impacting is possible by winding frequency up or down through the critical value F_C. Above this value jump phenomena exist between impacting and non-impacting solutions. Below F_C control of chaos techniques can be used to stabalise onto period (1,1) impacting solutions after grazing. Thus for wind-up or wind-down operations from a non-impacting state through the impacting zone close to resonance, period-1 non-impacting and impacting (1,1) solutions can be maintained.

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