A review of nonlinear structural control techniques

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Abstract

In this paper we present a review of nonlinear structural control techniques. This is an area of growing importance in a range of engineering applications, where nonlinear behaviour is encountered. Structural control is usually divided into three main areas (i) passive (ii) semi-active and, (iii) active control. This paper follows this convention, and highlights in each section the relevant state of the art for nonlinear systems, with additional references to related linear approaches.

Keywords: Structural control, passive, semi-active, active, modal, adaptive.

1 Introduction

Controlling nonlinear structural vibrations is becoming increasingly important in a range of engineering applications. In particular, the increasing use of flexible structures in application areas such as aerospace engineering, medicine and robotics,

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means that control methods are sometimes employed to carry out tasks such as (i) limit unwanted vibration, (ii) detect damage and (iii) actuate part of the structure. The use of flexible structures leads naturally to geometric nonlinearities from large deflections. Material nonlinearities may also be present for some systems. Therefore, understanding the nonlinear structural dynamics is an important first step from both a design and control perspective.

This paper presents a review of a range of control techniques which can be applied to nonlinear structural dynamics. We will focus primarily on structural systems made up from continuous elements such as beams, cables and plates which have a series of actuators and sensors to carry out control tasks. However, even within this remit the literature is large and any review will be somewhat selective. As a result we have tried to focus on highlighting key papers in the literature and application areas which are currently topical.

Control of structural vibrations presents some particular challenges which are not usually encountered in the majority of control problems. For example there are typically multiple lightly damped resonances in the system response. In addition, actuators have limited power, and sensors can be non-collocated. These, amongst other factors, can make for very challenging control tasks. A selection of the techniques which can be used to approach these type of control problems are discussed below. An excellent starting point for consulting background literature on structural control in general is the review by Housner *et. al.* [1].

2 Control design for nonlinear structural vibrations

The traditional way of approaching control design for structural vibrations has been to use *passive redesign* [2, 3]. This is a technique typically used to reduce unwanted vibrations by using special materials or adding physical damping devices [4]. Passive redesign techniques, such as the classical tuned mass damper [5] have been extended to nonlinear systems — see [6, 7] and references therein. Furthermore, in some applications, the nonlinear characteristics can be exploited to improve the vibration isolation [8–12]. Passive solutions are often preferred in practice as they can be built into the system and there is no control element, which eliminates any issues with stability or robustness. Engineering applications include helicopters [13], space structures [14], buildings/structures [15–17] and automotive applications [18, 19]. However, for a growing class of structures for which reduced weight and flexibility are important features, passive redesign is not always an effective design solution.

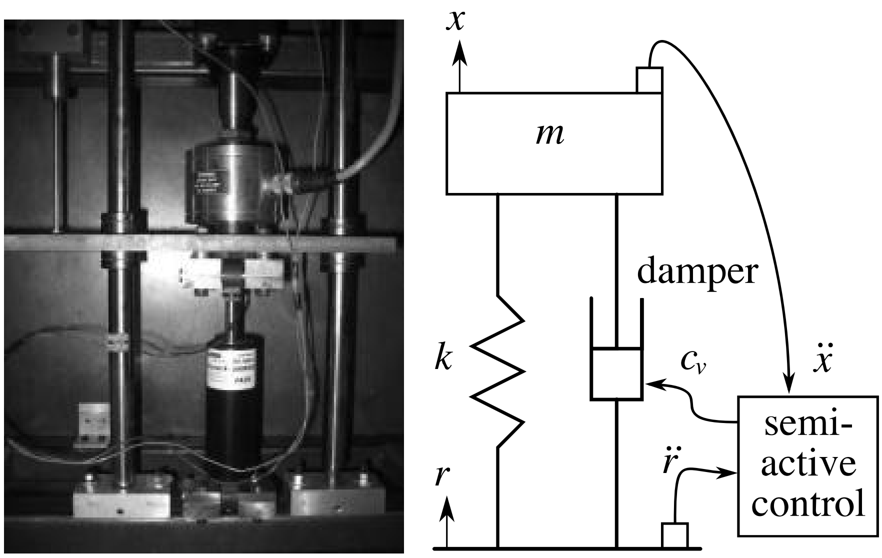
It is also interesting to note that *passive actuation* methods have received considerable attention for use in morphing aircraft structures [20–26]. In this scenario the flight control surface is designed such that it will morph (i.e. change shape) in response to specific aerodynamic loads. In some of these designs the nonlinear characteristics of particular structures are deliberately exploited, such as bi-stable shells [27]. This is still an active area of research, and the passive actuation method is often designed to work in conjunction with an active control element [26].

The alternative to passive design are to use either active control or semi-active control techniques which are discussed next.

3 Semi-active control

If specific actuation systems are used to control the structural dynamics, the structure is said to be *actively controlled*. Semi-active control can be thought of as a method which is a compromise between passive and active control [28]. In fact, it is normally implemented without using control actuators at all. Instead, a semi-active element, e.g. a damper, is used to effect change in the system [29]. The controlling effect is applied by including within the semi-active element a physical process which acts like a system parameter is being varied. For example, varying the viscosity of a fluid inside a damper system or the size of the damper orifice.

This ability to vary a system parameter makes it possible to design the system to achieve a specific control task. However, the fact that the system is not fully actively controlled, puts constraints on the level of control which can be achieved. An important difference between semi-active and active control is that, semi-active cannot add energy to the system, and therefore is normally an unconditionally stable form of control (assuming actuators and sensors are collocated). This is its most



(a) MR damper being tested (b) SDOF oscillator with semi-active damper

Figure 1: Semi-active vibration control.

significant advantage over active control, and is why it is almost always preferred in practice.

Semi-active control methods have been used extensively in many civil/structural engineering applications [30–35]. A review of these techniques as applied to structures such as bridges and tall buildings is given by [36]. Applications to structural engineering include modifications of the classic tuned mass damper [50] and for base-isolation of structures [51–57]. More general discussions of the application of semi-active control techniques can be found in [37, 38]

Semi-active control has also been used extensively for automotive semi-active suspension systems [39–49]. One of the most popular methods for implementing semi-active control is to use magneto-rheological (MR) dampers [58, 59]. In fact MR dampers are highly nonlinear, and are typically modelled using hysteretic models such as the Bouc-Wen and Dahl models [30, 60]. MR dampers have been successfully used for applications such as seat and vehicle suspensions [61–64], helicopter dampers [65, 66], buildings and other large structures [67–72]. They have also been used to control nonlinear structural systems [73, 74]

As a technical example, an MR damper is shown in Fig. 1(a), being tested in an experimental test rig. This type of damper can be incorporated into a single-degree-of-freedom oscillator to provide semi-active vibration control, as shown schematically in Fig. 1(b). In the example shown in Fig. 1(b), the mass-spring-damper system can be thought of as representing a suspension model that is excited by a road input of $r(t)$, and the control task is to minimize the displacement, x, of the supported mass, m. To decide how to select c_v, the damping value of the variable damper, information is needed about the relative velocity of the mass and the road input. This can be achieved by using accelerometers to measure the acceleration of the mass and the support, $\ddot{x}$ and $\ddot{r}$ respectively, which can be integrated in real-time to give approximations to $\dot{x}$ and $\dot{r}$.

One of the most common semi-active control strategies is sky-hook [28, 44, 75]. The concept is that the mass can be be isolated from the support input by getting the semi-active damper to behave as if it is a grounded passive damper, resulting in a damping force which resists the absolute velocity of the mass. To implement this in a semi-active element a very simple approach is to switch between a high and a low damping value — see [76] and references therein. It should be noted that however, that introducing switching into a system introduces nonlinear effects such as chatter [77–79]. Typically, the high damping is selected when the damper force is resisting the direction of motion of the mass, and the low damping force is used when this is not the case. Practically this can be achieved, for example, by implementing a control which has the effect of switching between high and low viscosity in the MR damper.

For this example the governing equation of motion for the oscillator is given by

$$m\ddot{x} + c_v(\dot{x} - \dot{r}) + k(x - r) = 0, \quad (1)$$

where the displacement of the mass, m, is given by x, k is the spring stiffness and c_v is a variable damping parameter which can be controlled by the semi-active controller. The control objective is to isolate the mass, or minimise the absolute acceleration of the mass $\ddot{x}$.

Using the sky-hook strategy to achieve this, the damper needs to be in the high

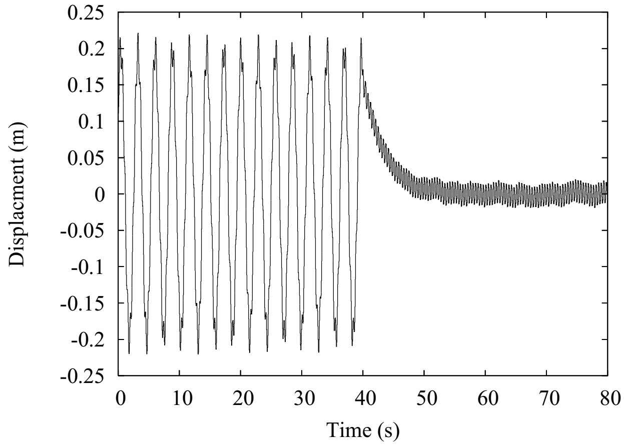


Figure 2: Time simulation of on-off sky hook control.

damping state when opposing the motion of the mass, and in the low damping state when aiding the motion of the mass. This can be defined by noting that when the relative velocity has the same sign as the absolute velocity $\dot{x}$ then the damper is opposing the mass. So the semi-active control law applied to the single degree-of-freedom system can be written as

$$c_v = \begin{cases} c_{high} & (\dot{x} - \dot{r})\dot{x} > 0, \\ c_{low} & \text{otherwise.} \end{cases} \quad (2)$$

The semi-active control will act like additional damping in the linear oscillator, which in turn will reduce the height of the resonance peak.

A time simulation for the sky-hook example when $r = \sin(15t)$ is shown in Fig. 2 with $x(0) = 0.11$m and $\dot{x}(0) = 0.03$m/s, $m = 1$kg, $k = 5$N/m and $c_v = 0.001$kg/s. Initially the sky hook control is switched off, and then at time $t = 40s$ the control is switched on, with $c_{high} = 1.9$kg/s and $c_{low} = 0.001$kg/s. The maximum vibration amplitude per period of oscillation is reduced by more than a factor of ten within 10 seconds of the skyhook control being switched on. Notice also that the reduced signal has high frequency oscillations, a typical characteristic of this type of switching system [79].

Like sky-hook, it is common for semi-active strategies to involve switching off the control action when the conditions are not favourable for control. As a result the system behaviour is uncontrolled for significant portions of time. For a range of applications, at some point, this becomes unacceptable and active control must be considered as an alternative. We also note that systems with multiple degrees of freedom, such as continuous structural elements it becomes increasingly difficult to apply semi-active control methods like sky-hook, unless the behaviour is limited to a very low number (usually one) of modes of vibration. Usually, active vibration control is required to tackle these types of application.

4 Active vibration control

An introduction to the basic ideas of feedback control is a useful starting point for understanding active structural control. A number of text books can be found which discuss linear vibration with control, such as [2, 80–82]. There are also a range of texts which discuss the vibration and control of smart/adaptive structures more specifically, such as [83–88]. A good introduction to the concepts of linear control can be found in [89], and for nonlinear control, [90] or [91] give good introductions.

Techniques from control theory can often be applied to low dimensional systems (typically single-degree-of freedom). For example a good discussion on velocity feedback and other linear techniques is given by [83]. An introduction to applying nonlinear techniques for single-degree-of freedom systems such as Lyapunov control design and feedback linearization is given by [88]. Single/low degree-of-freedom systems are often used in models for active suspension systems, which has seen a considerable amount of research and [92] gives a review of this area. Linear techniques such as optimal control [93–95], pole placement [96] and H-infinity [46] have been modified to be used in active structural control. Nonlinear problems have also been considered using techniques such as sliding mode control [98, 99, 102], Lyapunov control design [73], particle filters [100] and adaptive approaches [97] (discussed in Sec. 6) — see [101] for a review of techniques used for active aeroelastic control. Adaptive approaches have also been applied to vibration absorber applications using other actuation systems such as shape memory alloys (SMA), see for example [103] or magnetostrictive alloys [104].

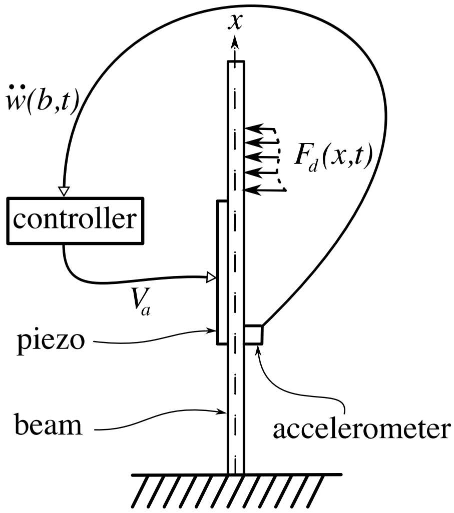
For nonlinear control, the objectives fall broadly into the two main classes of either stabilization or tracking. A stabilization problem is concerned with finding a control signal which forces the system to a stable equilibrium point (typically zero) for any initial conditions and parameter values in the required range. Tracking is concerned with getting particular system state, or output variables, to follow a predefined reference trajectory.

Active vibration control can be thought of as being in the stabilization category. Whereas adaptive structures, requiring shape change or morphing would fall into the tracking category. As with any control design, the other criteria which need to be considered, are (i) stability, (ii) robustness, (iii) performance, and (iv) cost.

Structures which cannot be easily modelled as lumped parameter, with low numbers of degrees-of-freedom are more problematic from a control perspective. For example, structural control examples for a cantilever beam are shown in Fig. 3. Figure 3(a) shows a macro fibre composite (MFC) piezo actuator attached to a bi-stable composite cantilever beam — see [26] for a discussion of bi-stable composite structures. In this scenario, the actuator is being tested to control the shape change from one stable state to another.

A similar configuration can be used for active vibration control, and this is shown schematically in Fig. 3(b). Here the cantilever beam is being controlled by a piezoelectric actuator. The beam is subject to an external disturbance force, F_d, and acceleration at a point $x = b$ along the beam, $\ddot{w}(b, t)$, is measured using an accelerometer. The active control task is to minimise $|\ddot{w}(b, t)|$ using the voltage signal, V_a input to the piezo actuator. Piezoceramic materials have been used extensively in structural control and other smart structure applications, see for example [31, 104–115]. Of particular note is the increasing development and use of Macro Fibre Composite actuators (MFC) [112, 116] which allow large curvature deflections to be measured and actuated. In this example underlying vibrating system (i.e. the cantilever beam) is infinite dimensional, but is acted on by only a small number of actuators, and measured with a small number of sensors. This is a typical scenario in active structural control. When a modal decomposition is carried out, the effect of discrete point

(a) MFC actuator on adaptive composite beam



(b) Beam with piezo used for control

Figure 3: Active vibration control for a beam with piezo actuator/sensors.

forces from the control actuators is captured by the modal participation factors — see for example [88]. The modal participation factor gives a measure of the effect of the applied force on each of the vibration modes in the structure. As a result, controllability of a particular mode will depend directly on the associated modal participation factor.

Using a sensor to measure at a discrete point has a similar effect on observability, because the transverse displacement, $w(x,t)$, at point A on the beam is typically approximated as $w(A,t) = \sum_{j=1}^{\infty} \phi_j(A) q_j(t)$, where the $\phi_j(A)$ values are the beam modes evaluated at point A and the $q_j(t)$ values are the modal coordinates. Typically N modes are taken as a truncated model for a continuous system (see [2] for a discussion on the effects of modal truncation), but the observability and controllability only relate to the controlled part of the system.

For example, if the control objective is just to control the first two modes of vibration of a beam, then it is only of interest to know if these modes are controllable or observable. However, the modes other than the controlled modes may still have significant dynamics. For example, if the controlled modes run from $1, 2, ..., N_c$ and

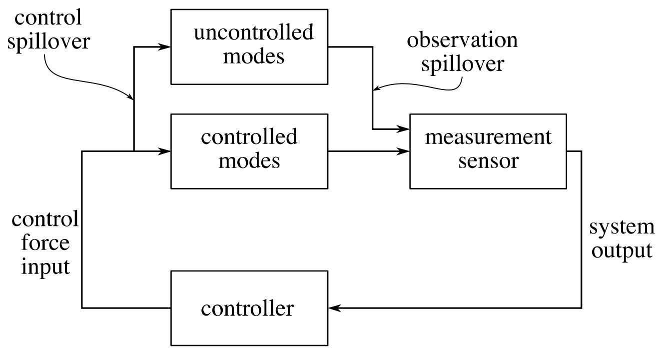


Figure 4: Block diagram of a typical modal control scenario.

the uncontrolled modes from $N_c+1, ...\infty$, then when making a measurement at point A, the response is actually given by $w(A,t) = \sum_{j=1}^{N_c} \phi_j(A)q_j(t) + \sum_{j=N_c+1}^{\infty} \phi_j(A)q_j(t)$. As a result if the response of the uncontrolled modes, $N_c+1, ...\infty$, is significant then the measurement of $w(A,t)$ will be corrupted by their contribution. This effect is usually referred to as observation spillover. A similar effect occurs when the control force is applied at a single point, because in general the modal participation factors are non-zero for the uncontrolled modes, so the effect of the control force is to excite the uncontrolled modes. This is called control spillover [117, 118], which can lead to unwanted excitation of the uncontrolled modes. The scenario is shown as a control block diagram in Fig. 4.

The position of the sensors and actuators is important, because for many modes both the mode-shape, ϕ, and the modal participation factor are zero (or close to zero) at some points along the beam. For linear modal systems, in-depth analysis of the effect of actuator and sensor placement has been developed, see for example [119].

Assuming the sensor and actuator positions can be selected as part of the control design, the issue of finding whether the controlled modes are controllable and observable remains. Typically, the system is linearized and linear observability and controllability conditions are tested [89]. A more general discussion on controllability and observability in nonlinear systems, particularly those in which the underlying linear system is not necessarily controllable or observable can be found in [120–122].

Note that in Fig. 3 (b) the base of the piezo actuator is collocated (in terms of position) with the accelerometer to give a moment-acceleration control relationship. However, in some situations, measurements from non-collocated sensors may be the only form of feedback available. Such a situation is called non-collocated control, which is more difficult to deal with, see for example the discussion and references in [83, 123]. Note also that in some situations the actuator-sensor positions are predetermined, and in other cases their position can be chosen to give the best control effect.

5 Modal control

For structures with multiple degrees-of-freedom it is usual to model the vibration of the structure using a modal decomposition. Typically using the underlying linear modes of the system as the modal basis [2, 88]. Once a modal model has been obtained, specific vibration control objectives can be defined in terms of the modes [2, 83]. For example reducing a vibration response which is dominated by a particular mode, could result in a control objective to reduce the effect of the dominant mode. Example applications include using active tendon control to reduce vibrations of cable-stay bridges [124, 125] controlling the effect of actuator saturation for large tuned mass dampers [126], active isolation of a helicopter gearbox [127] and mitigating earthquake response in buildings [128].

In some applications, established control techniques such as H-infinity [129] or sliding mode [128] are modified and applied to the modal control problem. In addition control using neural networks and genetic algorithms [130–135] has also been employed for systems with multiple modes of vibration. However, specialist techniques for modal control have also been developed. These include linear methods such as positive position feedback (PPF) [136, 137] and hybrid wave/mode active vibration control [138]. Nonlinear problems have been tackled using methods such as modal coupling control [106], PPF [88] and wavelet methods [109].

Now consider a pinned-pinned beam example with two collocated actuators and sensors shown in Figure 5. The transverse displacement of the beam is taken as

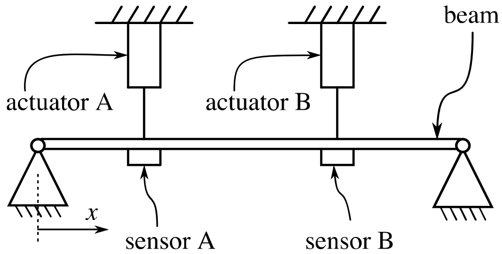


Figure 5: Vibration control of a beam with two collocated actuators and sensors.

$w(x,t)$, where x is the length along the beam. The displacements at points A and B are measured using sensors and taken as the control outputs for the system, $y_A = w(A,t)$ and $y_B = w(B,t)$. Assuming that modes 2 and 3 are the controlled modes, the outputs can be (linearly) related to the modal displacements q_2 and q_3 by a modal matrix, so that

$$\begin{bmatrix} y_A(t) \\ y_B(t) \end{bmatrix} = \begin{bmatrix} w(A,t) \\ w(B,t) \end{bmatrix} = \begin{bmatrix} \phi_2(A) & \phi_3(A) \\ \phi_2(B) & \phi_3(B) \end{bmatrix} \begin{bmatrix} q_2 \\ q_3 \end{bmatrix}, \tag{3}$$

where $\phi_i(x)$, $i = 2, 3$, are the corresponding mode-shapes for the pinned-pinned beam. Assuming a negligible contribution to the response from modes $1, 4, 5, ..., \infty$ i.e. no observation spillover, the output vector from equation (3) is $\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{q}$, where $\boldsymbol{\Phi}$ is the 2×2 modal matrix, and $\boldsymbol{q} = [q_2, q_3]^T$. The modal displacement vector can be estimated directly from $\boldsymbol{q} = \boldsymbol{\Phi}^{-1}\boldsymbol{y}$. Notice that having a different number of outputs to assumed modes requires a pseudoinverse to find $\boldsymbol{q}$.

In some situations it may be possible to place the sensors so that $\phi_3(A) \approx 0$ and/or $\phi_2(B) \approx 0$, then a direct relationship can be obtained between at least some of the outputs and the modal displacements (assuming no other modes have a significant response). The modal velocities, $\dot{q}_2$ and $\dot{q}_3$ will also need to be estimated from the outputs, using the fact that $\dot{\boldsymbol{y}} = \boldsymbol{\Phi}\dot{\boldsymbol{q}}$. The velocity of the output measurements can be estimated using a variety of numerical techniques, for example, the Savitsky-Golay filter, see [139].

Now we will show how feedback linearization [88, 90, 91] can be applied to the Example in Figure. 5 when coupling exists between vibration modes. We assume that modes 2 and 3 are to be targeted and the nonlinear modal model is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} q_2 \\ q_3 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_{n2}^2 & 0 & -\zeta_2\omega_{n2} & 0 \\ 0 & -\omega_{n3}^2 & 0 & -\zeta_3\omega_{n3} \end{bmatrix} \begin{bmatrix} q_2 \\ q_3 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \mu_2 q_2^3 + \delta_2 q_2^2 q_3 \\ \mu_3 q_3^3 + \delta_3 q_3^2 q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \alpha_2 p_1 \\ \alpha_3 p_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ \beta_2 p_2 \\ \beta_3 p_2 \end{bmatrix} u_2, \tag{4}$$

where δ_2 and δ_3 are constant terms which determine the level of nonlinear cross-coupling between modes 2 and 3. In addition, for $i = 2, 3$, ω_{ni} are the modal natural frequencies, ζ_i are the modal damping parameters, μ_i are the cubic nonlinearity parameters, α_i are the modal participation factors for the actuator at point A, β_i are the modal participation factors for the actuator at point B, p_i are the actuator constants and u_i are the control signals for the actuators at A and B respectively. We will assume that both observation and control spillover are negligible, which corresponds to $\alpha_3 = \phi_3(A) \approx 0$ and $\beta_2 = \phi_2(B) \approx 0$ such that the outputs are the modal displacements $y_A = \phi_2(A)q_2$ and $y_B = \phi_3(B)q_3$ from equation (3).

Direct feedback linearization can be applied by choosing control signals which cancel the nonlinear terms. This can be done by inspection of equation (4), from which it can be seen that setting

$$u_1 = \frac{1}{\alpha_2 p_1}(\mu_2 q_2^3 + \delta_2 q_2^2 q_3) \quad \text{and} \quad u_2 = \frac{1}{\beta_3 p_2}(\mu_3 q_3^3 + \delta_3 q_3^2 q_2) \tag{5}$$

will linearize each mode directly. Care needs to be taken with this approach, as it assumes that the underlying linearized system is stable. More details on this approach can be found in [91].

An related approach is to obtain an input-output linearization — see also [91] for an introduction. To do this, first define the outputs as $y_A = \phi_2(A)q_2$ and $y_B = \phi_3(B)q_3$, assuming $\phi_2(A)$ and $\phi_3(B)$ are not small. Then differentiate the outputs twice to get the relationship with the control inputs which from equation (4) with $\alpha_3 = \beta_2 \approx 0$ gives

$$\ddot{y}_A = \phi_2(A)\ddot{q}_2 = \phi_2(A)(-\omega_{n2}^2 q_2 - \zeta_2\omega_{n2}\dot{q}_2 - \mu_2 q_2^3 - \delta_2 q_2^2 q_3 + \alpha_2 p_1 u_1),$$

$$\ddot{y}_B = \phi_3(B)\ddot{q}_3 = \phi_3(B)(-\omega_{n3}^2 q_3 - \zeta_3\omega_{n3}\dot{q}_3 - \mu_3 q_3^3 - \delta_3 q_3^2 q_2 + \beta_3 p_2 u_2).$$

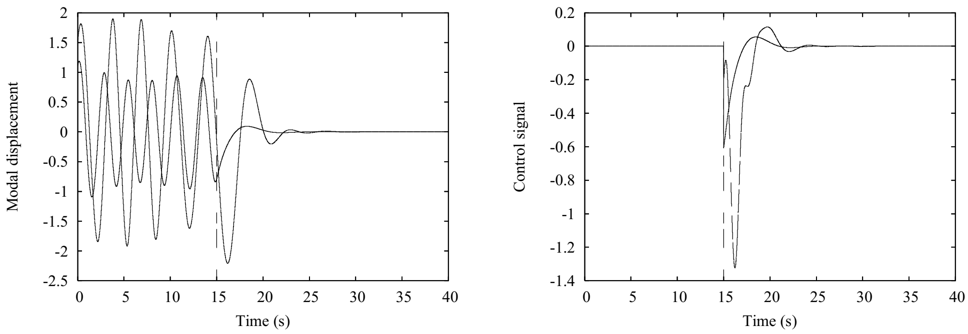
Then choosing

$$\begin{aligned} u_1 &= \frac{1}{\alpha_2 p_1}(v_2(t) + \omega_{n2}^2 q_2 + \zeta_2\omega_{n2}\dot{q}_2 + \mu_2 q_2^3 + \delta_2 q_2^2 q_3), \\ u_2 &= \frac{1}{\beta_3 p_2}(v_3(t) + \omega_{n3}^2 q_3 + \zeta_3\omega_{n3}\dot{q}_3 + \mu_3 q_3^3 + \delta_3 q_3^2 q_2), \end{aligned} \tag{6}$$

will give an input-output linearization with the result that $\ddot{q}_2 = v_2(t)$ and $\ddot{q}_3 = v_3(t)$, where $v_2(t)$ and $v_3(t)$ are the new control signals, which can be chosen to give the desired linear system response. Notice the similarity between the direct and input-output linearization results. The input-output linearization is preferable because it removes the underlying linear dynamics, which can be replaced with a desired behaviour.

Note that to apply this control technique, detailed knowledge of the modal equations is required, and access to all modal states needs to be assumed. Note also that the nonlinear cross-coupling terms typically give rise to nonlinear resonance phenomena, which can dominate the vibration response.

A numerical simulation of the cantilever beam example is shown in Fig. 6, with numerical parameters $\omega_{n2}^2 = 5$, $\zeta_2 = 0.006$, $\omega_{n3}^2 = 1.5$, $\zeta_3 = 0.025$, $\mu_2 = \mu_3 = 0.8$, $\delta_2 = 0.5$, $\delta_3 = 0.5$, $\alpha_2 p_1 = 7$ and $\beta_3 p_2 = 5$. The system is uncontrolled until time $t = 15$ seconds, when the input-output linearization control is switched on. In this example $v_i(t) = -q_i - \dot{q}_i$ which is a stable linear oscillator with large damping in each mode $i = 2, 3$. In Fig. 6(a) the modal displacement responses are shown. A clear change can be seen from the distorted non-harmonic response occurring before 15 seconds to a strongly damped response immediately after 15 seconds. In Fig. 6(b) the control signals for each actuator are shown. It can be seen that the control rapidly damps out the modal vibrations.



(a) Modal displacement response

(b) Control signal

Figure 6: Feedback linearization for the cantilever beam example (a) displacement response, (b) control signal, where solid and dotted lines refer to the modal displacements q_2, q_3 and modal control inputs u_1, u_2 respectively.

6 Adaptive control

Finally in this review, we briefly mention adaptive control techniques. When there is a significant level of uncertainty in the system, then adaptive control techniques can be applied. Typically slow variation of parameters over time, is the form of uncertainty for which adaptive control is designed to deal with. In this type of situation, a control design based on fixed parameter values will become increasing inaccurate as time increases. Allowing the control signal to adapt, based on some measurements of the changing parameters in the system, is one way of solving this problem. It should be noted that adaptive control can introduce additional problems, particularly gain drift and a reduced level of robustness. A good introduction to the subject of adaptive control is given by [140], and a discussion of adaptive control for nonlinear systems can be found in [90, 141]. However, is should be noted that most adaptive control techniques have not been developed with structural control in mind, and will therefore require appropriate modification.

Adaptive control is traditionally based on the use of a feedback signal, for a linear system. Even in this case the resulting dynamics of the system are highly nonlinear — see for example [142, 143]. However, techniques for nonlinear control, such as feedback linearization can be modified to be adaptive [88, 144–146]. Information on robustness of adaptive control is discussed, for example, by [147–150]. Adaptive control applications include active automotive suspensions [97], vibration absorbers [103, 104] and aeroelastic systems [144].

An alternative approach to dealing with uncertainty is using neural networks and genetic algorithms [130–133], which as previously mentioned, has been applied to systems with multiple modes of vibration. Structural control of nonlinear systems with uncertainty is one of the most challenging aspects of the current research field, and is beyond the scope of the current review. However, a good introduction to this area is given by [151], and a comprehensive review of the system identification aspects are given by [152].

7 Conclusions

In this review we have briefly outlined some of the current methods for controlling systems which have nonlinear structural dynamics. The nonlinearities in these systems typically arise from different forms of geometric nonlinearity, such as large deflection, curvature or axial/in-plane loading. We have grouped the methods into the conventional categories of (i) passive, (ii) semi-active, and (iii) active control. Passive methods typically involve some form of redesign using special materials or adding physical damping devices. Passive solutions are preferred in practice as they can be built into the system and there is no control element. Currently there is considerable interest in passive actuation for use in morphing aircraft structures. This and other passive techniques have been highlighted in Section 2.

When passive redesign is not applicable, semi-active control is typically the next method to try. It has been used extensively in structural and automotive engineering to enhance the performance of vibration dampers. Currently there is a great deal of interest in using magneto-rheological (MR) dampers to perform this task. In Section 3 we highlighted the relevant literature and showed how sky-hook control can be implemented for a single-degree-of-freedom system.

The final category of active control has been covered in Section 4. Following a general introduction on active control techniques we have selected the topics of modal and adaptive control for more detailed discussion. Modal control techniques are applied to systems where many vibration modes are present, and usually one (or more) are targeted for vibration reduction. In Section 4 we showed an example of how this can be achieved using feedback linearization techniques. Adaptive control is a large subject in its own right, and is usually only required when there is a significant degree of uncertainty in the system — typically parameter drift with time. In the last part of Section 4 we highlight some of the adaptive control literature which is relevant to structural control.

Nonlinear structural control is a rapidly developing area, and many techniques from other applications are being applied and/or modified for use on these type of control problems. While active control can usually offer the best performance, concerns about stability and robustness often make this unsuitable for certain applications. In this case, the semi-active area offers the most potential, and the continuing development of smart materials such as peizoceramics and MR fluid means that there will be significant advances in this area in the near future.

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References

[1] **Housner, G. W.**, **Bergman, L. A.**, **Caughey, T. K.**, **Chassiakos, A. G.**, **Claus, R. O.**, **Masri, S. F.**, **Skelton, R. E.**, **Soong, T. T.**, **Spencer, B. F.**, and **Yao, J. T. P.** Structural control: Past, present, and future. *Journal of Engineering Mechanics—ASCE*, **123**(9):897–971, 1997.

[2] **Inman, D. J.** *Vibration with control.* Wiley, 2006.

[3] **Mottershead, J. E.** and **Ram, Y. M.** Inverse eigenvalue problems in vibra-

tion absorption: Passive modification and active control. *Mechanical Systems & Signal Processing*, **20**(1):5–44, 2006.

[4] **Soong, T. T.** and **Dargush, G. F.** *Passive energy dissipation systems in structural engineering*. Wiley, 1997.

[5] **Den Hartog, J. P.** *Mechanical Vibrations*. McGraw-Hill: New York, 1934.

[6] **Ibrahim, R. A.**, Recent advances in nonlinear passive vibration isolators. *Journal of Sound & Vibration*, **314**(3-5):371–452, 2008.

[7] **Alexander, N. A.** and **Schilder, F.** Exploring the performance of a nonlinear tuned mass damper. *Journal of Sound & Vibration*, **319**(1-2):445–462, 2009.

[8] **Semercigil, S. E.**, **Collette, F.**, and **Huyni, D.** Experiments with tuned absorber — impact damper combination. *Journal of Sound & Vibration*, **256**(1):179–188, 2002.

[9] **Vyas, A.**, **Bajaj, A. K.**, and **Raman, A.** Dynamics of structures with wideband autoparametric vibration absorbers: experiment. *Proceedings of the Royal Society of London A.*, **460**:1857–1880, 2003.

[10] **Shoeybi, M.** and **Ghorashi, M.** Nonlinear vibration control of a system with dry friction and viscous damping using the saturation phenomenon. *Nonlinear Dynamics*, **45**(3-4):249–272, 2006.

[11] **Mikhlin, Y. V.** and **Reshetnikova, S. N.** Dynamical interaction of an elastic system and a vibro-impact absorber. *Mathematical Problems in Engineering*, pages 1–15, 2006. Article ID 37980.

[12] **Carrella, A.**, **Brennan, M. J.**, and **Waters, T. P.** Static analysis of a passive vibration isolator with quasi-zero-stiffness characteristic. *Journal of Sound & Vibration*, **301**(3-5):678–689, 2007.

[13] **Panda, B.**, **Mychalowycz, E.**, and **Tarzanin, F. J.** Application of passive dampers to modern helicopters. *Smart Materials & Structures*, **5**(5):509–516, 1996.

[14] **Nair, P. B.** and **Keane, A. J.** Passive vibration suppression of flexible space structures via optimal geometric redesign. *AIAA Journal*, **39**(7):1338–1346, 2001.

[15] **Kasai, K.**, **Fu, Y. M.**, and **Watanabe, A.** Passive control systems for seismic damage mitigation. *Journal of Structural Engineering—ASCE*, **124**(5):501–512, 1998.

[16] **Wang, J.**, **Lin, C.**, and **Chen, B.** Vibration suppression for high-speed raitway bridges using tuned mass dampers. *International Journal of Solids & Structures*, **40**(2):465–491, 2003.

[17] **Ghosh, A.** and **Basu, B.** Seismic vibration control of nonlinear structures using the liquid column damper. *Journal of Structural Engineering—ASCE*, **134**(1):146–153, 2008.

[18] **Cole, D. J.** and **Cebon, D.** Truck suspension design to minimize road damage. *Proceedings of the Institution of Mechanical Engineers Part D—Journal of Automobile Engineering*, **210**(2):95–107, 1996.

[19] **Tamboli, J. A.** and **Joshi, S. G.** Optimum design of a passive suspension system of a vehicle subjected to actual random road excitations. *Journal of Sound & Vibration*, **219**(2):193–205, 1999.

[20] **Bharti, S.**, **Frecker, M.**, **Lesieutre, G.**, and **Ramrakhyani, D.** Active and passive material optimization in a tendon actuated morphing aircraft structure. *Proceedings of SPIE*, Vol. 5390, pp 247–257, 2004. Smart Structures and Materials Conference, San Diego, CA, March 15–18, 2004.

[21] **Lucato, S.**, **Wang, J.**, **Maxwell, P.**, **McMeeking, R.**, and **Evans, A.** Design and demonstration of a high authority shape morphing structure. *International Journal of Solids & Structures*, **41**(13):3521–3543, 2004.

[22] **Spadoni, A.** and **Ruzzene, M.** Static aeroelastic response of chiral-core airfoils. *Journal of Intelligent Materials Systems & Structures*, **18**(10):1067–1075, 2007.

[23] **Wang, J.**, **Nausieda, A.**, **Lucato, S. L. d. S. E.**, and **Evans, A. G.** Twisting of a high authority morphing structure. *International Journal of Solids & Structures*, **44**(9):3076–3099, 2007.

[24] **Thill, C.**, **Etches, J.**, **Bond, I.**, **Potter, K.**, and **Weaver, P.** Morphing skins. *Aeronautical Journal*, **112**(1129):117–139, 2008.

[25] **Baker, D.** and **Friswell, M. I.** Determinate structures for wing camber control. *Smart Materials & Structures*, **18**(3), 2009.

[26] **Daynes, S.**, **Nall, S. J.**, **Weaver, P. M.**, **Potter, K. D.**, **Margaris, P.**, and **Mellor, P. H.** Bistable Composite Flap for an Airfoil. *Journal of Aircraft*, **47**(1):334–338, 2010.

[27] **Wagg, D.**, **Bond, I.**, **Weaver, P.**, and **Friswell, M.**, editors. *Adaptive Structures: Engineering Applications*. Wiley, 2007.

[28] **Karnopp, D.** Active and semiactive vibration isolation. *Journal of Mechanical Design*, **117**(Sp. Iss. B):177–185, 1995.

[29] **Ahmadian, M.** On the isolation properties of semiactive dampers. *Journal of Vibration & Control*, **5**(2):217–232, 1999.

[30] **Yoshida, O.** and **Dyke, S. J.** Seismic control of a nonlinear benchmark building using smart dampers. *Journal of Engineering Mechanics—ASCE*, **130**(4):386–392, 2004.

[31] **Chen, G. D.** and **Chen, C. Q.** Semiactive control of the 20-story benchmark building with piezoelectric friction dampers. *Journal of Engineering Mechanics—ASCE*, **130**(4):393–400, 2004.

[32] **Ohtori, Y.**, **Christenson, R. E.**, **Spencer, B. F.**, and **Dyke, S. J.** Benchmark control problems for seismically excited nonlinear buildings. *Journal of Engineering Mechanics—ASCE*, **130**(4):366–385, 2004.

[33] **Yoshida, O.** and **Dyke, S. J.** Response control of full-scale irregular buildings using magnetorheological dampers. *Journal of Structural Engineering—ASCE*, **131**(5):734–742, 2005.

[34] **Xu, Z.**, **Agrawal, A. K.**, and **Yang, J. N.** Semi-active and passive control of the phase I linear base-isolated benchmark building model. *Structural Control & Health Monitoring*, **13**(2-3):626–648, 2006.

[35] **Agrawal, A.**, **Tan, P.**, **Nagarajaiah, S.**, and **Zhang, J.** Benchmark structural control problem for a seismically excited highway bridge-Part I: Phase I Problem definition. *Structural Control & Health Monitoring*, **16**(5):509–529, 2009.

[36] **Spencer, B. F.** and **Nagarajaiah, S.** State of the art of structural control. *Journal of Structural Engineering-ASCE*, **129**(7):845–856, 2003.

[37] **Casciati, F.**, **Magonette, G.**, and **Marazzi, F.** *Semiactive devices and applications in vibration mitigation*. Wiley, 2006.

[38] **Preumont, A.** and **Seto, K.** *Active Control of Structures*. Wiley Blackwell, 2008.

[39] **Besinger, F. H.**, **Cebon, D.**, and **Cole, D. J.** An experimental investigation into the use of semi-active dampers on heavy lorries. *Vehicle System Dynamics*, **20**:57–71, 1992.

[40] **Hrovat, D.** Survey of advanced suspension developments and related optimal control applications. *Automatica*, **33**(10):1781–1817, 1997.

[41] **Zaremba, A.**, **Hampo, R.**, and **Hrovat, D.** Optimal active suspension design using constrained optimization. *Journal of Sound & Vibration*, **207**(3):351–364, 1997.

[42] **Fialho, I. J.** and **Balas, G. J.** Design of nonlinear controllers for active vehicle suspensions using parameter-varying control synthesis. *Vehicle System Dynamics*, **33**(5):351–370, 2000.

[43] **Kitching, K. J.**, **Cole, D. J.**, and **Cebon, D.** Performance of a semi-active damper for heavy vehicles. *Journal of Dynamic Systems Measurement & Control—Transactions of the ASME*, **122**(3):498–506, 2000.

[44] **Hong, K. S.**, **Sohn, H. C.**, and **Hedrick, J. K.** Modified skyhook control of semi-active suspensions: A new model, gain scheduling, and hardware-in-the-loop tuning. *Journal of Dynamic Systems Measurement & Control-Transactions of the ASME*, **124**(1):158–167, 2002.

[45] **Jalili, N.** A comparative study and analysis of semi-active vibration-control systems. *Journal of Vibration & Acoustics-Transactions of the ASME*, **124**(4):593–605, 2002.

[46] **Sammier, D.**, **Sename, O.**, and **Dugard, L.** Skyhook and H-infinity control of semi-active suspensions: Some practical aspects. *Vehicle System Dynamics*, **39**(4):279–308, 2003.

[47] **Verros, G.**, **Natsiavas, S.**, and **Papadimitriou, C.** Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation. *Journal of Vibration & Control*, **11**(5):581–606, 2005.

[48] **Shen, Y.**, **Golnaraghi, M. F.**, and **Heppler, G. R.** Semi-active vibration control schemes for suspension systems using magnetorheological dampers. *Journal of Vibration & Control*, **12**(1):3–24, 2006.

[49] **Giorgetti, N.**, **Bemporad, A.**, **Tseng, H.**, and **Hrovat, D.** Hybrid model predictive control application towards optimal semi-active suspension. *International Journal of Control*, **79**(5):521–533, 2006.

[50] **Casciati, F.** and **Ubertini, F.** Nonlinear vibration of shallow cables with semiactive tuned mass damper. *Nonlinear Dynamics*, **53**(1-2):89–106, 2008.

[51] **Barbat, A. H.**, **Rodellar, J.**, **Ryan, E. P.**, and **Molinares, N.** Active control of nonlinear base-isolated buildings. *Journal of Engineering Mechanics—ASCE*, **121**(6):676–684, 1995.

[52] **Papoulia, K. D.** and **Kelly, J. M.** Visco-hyperelastic model for filled rubbers used in vibration isolation. *Journal of Engineering Materials & Technology-Transactions of the ASME*, **119**(3):292–297, 1997.

[53] **Yoshioka, H.**, **Ramallo, J. C.**, and **Spencer, B. F.** "smart" base isolation strategies employing magnetorheological dampers. *Journal of Engineering Mechanics—ASCE*, **128**(5):540–551, 2002.

[54] **Ramallo, J. C.**, **Johnson, E. A.**, and **Spencer, B. F.** "smart" base isolation systems. *Journal of Engineering Mechanics—ASCE*, **128**(10):1088–1099, 2002.

[55] **Bani-Hani, K. A.** and **Sheban, M. A.** Semi-active neuro-control for base-isolation system using magnetorheological (mr) dampers. *Earthquake Engineering & Structural Dynamics*, **35**(9):1119–1144, 2006.

[56] **Nagarajaiah, S.** and **Narasimhan, S.** Seismic control of smart base isolated buildings with new semiactive variable damper. *Earthquake Engineering & Structural Dynamics*, **36**(6):729–749, 2007.

[57] **Bahar, A.**, **Pozo, F.**, **Acho, L.**, **Rodellar, J.**, and **Barbat, A.** Hierarchical semi-active control of base-isolated structures using a new inverse model of magnetorheological dampers. *Computers & Structures*, **88**(7-8):483–496, 2010.

[58] **Jansen, L. M.** and **Dyke, S. J.** Semiactive control strategies for mr dampers: Comparative study. *Journal of Engineering Mechanics—ASCE*, **126**(8):795–803, 2000.

[59] **Yang, G. Q.**, **Spencer, B. F.**, **Jung, H. J.**, and **Carlson, J. D.** Dynamic modeling of large-scale magnetorheological damper systems for civil engineering applications. *Journal of Engineering Mechanics—ASCE*, **130**(9):1107–1114, 2004.

[60] **Ikhouane, F.** and **Dyke, S. J.** Modeling and identification of a shear mode magnetorheological damper. *Smart Materials & Structures*, **16**:605–616, 2007.

[61] **Besinger, F. H.**, **Cebon, D.**, and **Cole, D. J.** Force control of a semiactive damper. *Vehicle System Dynamics*, **24**(9):695–723, 1995.

[62] **Choi, S. B.** and **Han, Y. M.** MR seat suspension for vibration control of a commercial vehicle. *International Journal of Vehicle Design*, **31**(2):202–215, 2003.

[63] **Lai, C.** and **Liao, W.** Vibration control of a suspension system via a magnetorheological fluid damper. *Journal of Vibration & Control*, **8**(4):527–547, 2002.

[64] **Simon, D.** and **Ahmadian, M.** Vehicle evaluation of the performance of magneto rheological dampers for heavy truck suspensions. *Journal of Vibration & Acoustics—Transactions of the ASME*, **123**(3):365–375, 2001.

[65] **Ngatu, G. T.**, **Hu, W.**, **Wereley, N. M.**, and **Kothera, C. S.** Adaptive Snubber-Type Magnetorheological Fluid-Elastomeric Helicopter Lag Damper. *AIAA Journal*, **48**(3):598–610, 2010.

[66] **Hu, W.** and **Wereley, N. M.** Magnetorheological fluid and elastomeric lag damper for helicopter stability augmentation. *International Journal of Modern Physics B*, **19**(7-9):1471–1477, April 2005.

[67] **Qu, W. L.** and **Xu, Y. L.** Semi-active control of seismic response of tall buildings with podium structure using er/mr dampers. *Structural Design of Tall Buildings*, **10**(3):179–192, 2001.

[68] **Wang, J. Y.**, **Ni, Y. Q.**, **Ko, J. M.**, and **Spencer, B. F.** Magneto-rheological tuned liquid column dampers (mr-tlcds) for vibration mitigation of tall buildings: modelling and analysis of open-loop control. *Computers & Structures*, **83**(25-26):2023–2034, 2005.

[69] **Sahasrabudhe, S. S.** and **Nagarajaiah, S.** Semi-active control of sliding isolated bridges using mr dampers: an experimental and numerical study. *Earthquake Engineering & Structural Dynamics*, **34**(8):965–983, 2005.

[70] **Lee, S. H.**, **Min, K. W.**, **Lee, R.**, **Lee, H. G.**, **Sung, K. G.**, and **Kim, D. K.** Preliminary design procedure of mr dampers for controlling seismic response of building structures. *International Journal of Modern Physics B*, **19**(7-9):1492–1498, 2005.

[71] **Jin, G.**, **Sain, M. K.**, and **Spencer, B. F.** Nonlinear blackbox modeling of MR-dampers for civil structural control. *IEEE Transactions on Control Systems Technology*, **13**(3):345–355, 2005.

[72] **Maiti, D. K.**, **Shyju, P. P.**, and **Vijayaraju, K.** Vibration control of mechanical systems using semi-active mr-damper. *Smart Structures & Systems*, **2**(1):61–80, 2006.

[73] **Kuehn, J. L.** and **Stalford, H. L.** Stability of a Lyapunov controller for a semi-active structural control system with nonlinear actuator dynamics. *Journal of Mathematical Analysis & Applications*, **251**(2):940–957, 2000.

[74] **Ying, Z. G.**, **Ni, Y. Q.**, and **Ko, J. M.** A semi-active stochastic optimal control strategy for nonlinear structural systems with MR dampers. *Smart Structures & Systems*, **5**(1):69–79, JAN 2009.

[75] **Preumont, A.**, **Francois, A.**, **Bossens, F.**, and **Abu-Hanieh, A.** Force feedback versus acceleration feedback in active vibration isolation. *Journal of Sound & Vibration*, **257**(4):605–613, 2002.

[76] **Potter, J. N.**, **Neild, S. A.**, and **Wagg, D. J.** Generalisation and optimisation of semi-active, onoff switching controllers for single degree-of-freedom systems. *Journal of Sound & Vibration*, **329**:2450–2462, 2010.

[77] **Brogliato, B.** *Nonsmooth mechanics: Models, dynamics and control.* Springer-Verlag, 1999.

[78] **Awrejcewicz, J.** and **Lamarque, C. H.** *Bifurcation and Chaos in Nonsmooth Mechanical Systems.* World Scientific Publishing, 2003.

[79] **di Bernardo, M.**, **Budd, C.**, **Champneys, A. R.**, and **Kowalczyk, P.** *Piecewise-smooth dynamical systems: Theory and applications.* Springer, 2008.

[80] **Beards, C. F.** *Vibration analysis and control system dynamics.* Ellis Horwood, 1981.

[81] **Fuller, C. R.**, **Elliot, S. J.**, and **Nelson, P. A.** *Active control of vibration.* Academic Press, 1996.

[82] **Moheimani, S. O. R.**, **Halim, D.**, and **Fleming, A. J.** *Spatial control of vibration.* World Scientific, 2003.

[83] **Preumont, A.** *Vibration control of active structures.* Kluwer Academic Publishers, 1997.

[84] **Clark, R. L.**, **Saunders, W. R.**, and **Gibbs, G. P.** *Adaptive structures; dynamics and control.* John Wiley, 1998.

[85] **Srinivasan, A. V.** and **McFarland, D. M.** *Smart structures.* Cambridge, 2001.

[86] **Worden, K.**, **Bullough, W. A.**, and **Haywood, J.** *Smart Technologies.* World Scientific, 2003.

[87] **Leo, D. J.** *Smart material systems.* Wiley, 2007.

[88] **Wagg, D. J.** and **Neild, S. A.** *Nonlinear Vibration with Control.* Springer-Verlag, 2009.

[89] **Goodwin, G. C.**, **Graebe, S. F.**, and **Salgado, M. E.** *Control System Design.* Pearson, 2000.

[90] **Slotine, J.-J. E.** and **Li, W.** *Applied nonlinear control.* Prentice Hall, 1991.

[91] **Khalil, H. K.** *Nonlinear Systems.* Macmillan: New York, 1992.

[92] **Williams, R. A.** Automotive active suspensions - Part 1: basic principles. *Proceedings of the Institution of Mechanical Engineers Part D—Journal of Automobile Engineering*, **211**(6):415–426, 1997.

[93] **El Beheiry, E. M.** and **Karnopp, D. C.** Optimal control of vehicle random vibration with constrained suspension deflection. *Journal of Sound & Vibration*, **189**(5):547–564, 1996.

[94] **Zhu, W. Q.**, **Ying, Z. G.**, and **Soong, T. T.** An optimal nonlinear feedback control strategy for randomly excited structural systems. *Nonlinear Dynamics*, **24**(1):31–51, 2001.

[95] **Cai, X. S.** and **Han, Z. Z.** Inverse optimal control of nonlinear systems with structural uncertainty. *IEE Proceedings-Control Theory & Applications*, **152**(1):79–83, 2005.

[96] **Butcher, E.** and **Lu, R.** Constant-gain linear feedback control of piecewise linear structural systems via nonlinear normal modes. *Journal of Vibration & Control*, **10**(10):1535–1558, OCT 2004.

[97] **Alleyne, A.** and **Hedrick, J. K.** Nonlinear adaptive-control of active suspensions. *IEEE Transactions on Control Systems Technology*, **3**(1):94–101, 1995.

[98] **Yoshimura, T.**, **Kume, A.**, **Kurimoto, M.**, and **Hino, J.** Construction of an active suspension system of a quarter car model using the concepts of sliding mode control. *Journal of Sound & Vibration*, **239**(2):187–199, 2001.

[99] **Sam, Y. M.**, **Osman, J. H. S.**, and **Ghani, M. R. A.** A class of proportional-integral sliding mode control with application to active suspension system. *Systems & Control Letters*, **51**(3-4):217–223, 2004.

[100] **Sajeeb, R.**, **Manohar, C. S.**, and **Roy, D.** Use of particle filters in an active control algorithm for noisy nonlinear structural dynamical systems. *Journal of Sound & Vibration*, **306**(1-2):111–135, 2007.

[101] **Librescu, L.** and **Marzoeca, P.** Advances in the linear/nonlinear control of aeroelastic structural systems. *Acta Mechanica*, **178**(3-4):147–186, 2005.

[102] **Alleyne, A.**, **Neuhaus, P. D.**, and **Hedrick, J. K.** Application of nonlin-

ear control — theory to electronically controlled suspensions. *Vehicle System Dynamics*, **22**(5-6):309–320, 1993.

[103] **Williams, K.**, **Chiu, G.**, and **Bernhard, R.** Nonlinear control of a shape memory alloy adaptive tuned vibration absorber. *Journal of Sound & Vibration*, **288**(4-5):1131–1155, 2005.

[104] **Ashour, O. N.** and **Nayfeh, A. H.** Adaptive control of flexible structures using a nonlinear vibration absorber. *Nonlinear Dynamics*, **28**(3-4):309–322, 2002.

[105] **Preumont, A.**, **Francois, A.**, **De Man, P.**, and **Piefort, V.** Spatial filters in structural control. *Journal of Sound & Vibration*, **265**(1):61–79, 2003.

[106] **Khajepour, A.** and **Golnaraghi, M.** Experimental control of flexible-structures using nonlinear modal coupling: Forced and free vibration. *Journal of Intelligent Materials Systems & Structures*, **8**(8):697–710, 1997.

[107] **Preumont, A.**, **Dufour, J. P.**, and **Malekian, C.** Active damping by a local force feedback with piezoelectric actuators. *Journal of Guidance Control & Dynamics*, **15**(2):390–395, 1992.

[108] **Schultz, M. R.** and **Hyer, M. W.** Snap-through of unsymmetric cross-ply laminates using piezoceramic actuators. *Journal of Intelligent Material Systems & Structures*, **14**(12):795–814, 2003.

[109] **Zhou, Y. H.** and **Wang, J. Z.** Vibration control of piezoelectric beam-type plates with geometrically nonlinear deformation. *International Journal of Non-linear Mechanics*, **39**(6):909–920, 2004.

[110] **Moheimani, S. O. R.** and **Vautier, B. J. G.** Resonant control of structural vibration using charge-driven piezoelectric actuators. *IEEE Transactions on Control Systems Technology*, **13**(6):1021–1035, 2005.

[111] **Preumont, A.**, **Francois, A.**, **De Man, P.**, **Loix, N.**, and **Henrioulle,**

K. Distributed sensors with piezoelectric films in design of spatial filters for structural control. *Journal of Sound & Vibration*, **282**(3-5):701–712, 2005.

[112] **Song, G.**, **Sethi, V.**, and **Li, H. N.** Vibration control of civil structures using piezoceramic smart materials: A review. *Engineering Structures*, **28**(11):1513–1524, 2006.

[113] **Moheimani, S. O. R.** and **Fleming, A. J.** *Piezoelectric transducers for vibration control and damping.* Springer, 2006.

[114] **de Marneffe, B.** and **Preumont, A.** Vibration damping with negative capacitance shunts: theory and experiment. *Smart Materials & Structures*, **17**(3), 2008.

[115] **Harari, S.**, **Richard, C.**, and **Gaudiller, L.** New Semi-active Multi-modal Vibration Control Using Piezoceramic Components. *Journal of Intelligent Material Systems & Structures*, **20**(13):1603–1613, 2009.

[116] **Deraemaeker, A.**, **Nasser, H.**, **Benjeddou, A.**, and **Preumont, A.** Mixing Rules for the Piezoelectric Properties of Macro Fiber Composites. *Journal of Intelligent Material Systems & Structures*, **20**(12):1475–1482, AUG 2009.

[117] **Balas, M. J.** Feedback control of flexible systems. *IEEE Transactions on Automatic Control*, **23**(4):673–679, 1978.

[118] **Balas, M. J.** Stable feedback control of linear distributed parameter systems: Time and frequency domain conditions. *Journal of Mathematical Analysis & Applications*, **225**(1):144–167, 1998.

[119] **Gawronski, W. K.** *Advanced Structural Dynamics and Active Control of Structures.* Springer, 2004.

[120] **Nijmeijer, H.** and **van der Schaft, A.** *Nonlinear Dynamical Control Systems.* Springer-Verlag, 1990.

[121] **Vidyasagar, M.** *Nonlinear Systems Analysis.* Society for Industrial and Applied Mathematics: SIAM, 2002.

[122] **Sastry, S.** *Nonlinear systems:Analysis, stability and control.* Springer-Verlag: New York, 1999.

[123] **Gawronski, W.** Actuator and sensor placement for structural testing and control. *Journal of Sound & Vibration*, **208**(1):101–109, NOV 20 1997.

[124] **Warnitchai, P.**, **Fujino, Y.**, **Pacheco, B. M.**, and **Agret, R.** An experimental study on active tendon control of cable-stayed bridges. *Earthquake Engineering & Structural Dynamics*, **22**(2):93–111, 1993.

[125] **Bossens, F.** and **Preumont, A.** Active tendon control of cable-stayed bridges: a large-scale demonstration. *Earthquake Engineering & Structural Dynamics*, **30**(7):961–979, 2001.

[126] **Chase, J. G.**, **Smith, H. A.**, and **Suzuki, T.** Robust H infinity control considering actuator saturation .2. Applications. *Journal of Engineering Mechanics—ASCE*, **122**(10):984–993, 1996.

[127] **Sutton, T.**, **Elliott, S.**, **Brennan, M.**, **Heron, K.**, and **Jessop, D.** Active isolation of multiple structural waves on a helicopter gearbox support strut. *Journal of Sound & Vibration*, **205**(1):81–101, 1997.

[128] **Yang, J. N.**, **Wu, J. C.**, and **Agrawl, A. K.** Sliding mode control for nonlinear and hysteretic structures. *Journal of Engineering Mechanics—ASCE*, **121**(12):1330–1339, 1995.

[129] **Chase, J. G.**, **Breneman, S. E.**, and **Smith, H. A.** Robust H-infinity static output feedback control with actuator saturation. *Journal of Engineering Mechanics—ASCE*, **125**(2):225–233, 1999.

[130] **Bani-Hani, K.** and **Ghaboussi, J.** Nonlinear structural control using neural networks. *Journal of Engineering Mechanics—ASCE*, **124**(3):319–327, 1998.

[131] **Kim, J. T.**, **Jung, H. J.**, and **Lee, I. W.** Optimal structural control using neural networks. *Journal of Engineering Mechanics—ASCE*, **126**(2):201–205, 2000.

[132] **Li, Q. S.**, **Liu, D. K.**, **Leung, A. Y. T.**, **Zhang, N.**, **Tam, C. M.**, and **Yang, L. F.** Modelling of structural response and optimization of structural control system using neural network and genetic algorithm. *Structural Design of Tall Buildings*, **9**(4):279–293, 2000.

[133] **Sahani, N.** and **Horn, J.** Adaptive model inversion control of a helicopter with structural load limiting. *Journal of Guidance Control & Dynamics*, **29**(2):411–420, 2006.

[134] **Chen, S.-H.**, **Meng, L.-A.**, and **Chou, J.-H.** A mixed robust/optimal active vibration control for uncertain flexible structural systems with nonlinear actuators using genetic algorithm. *Journal of Vibration & Control*, **13**(2):185–201, 2007.

[135] **Chen, C.-W.** Modeling and control for nonlinear structural systems via a NN-based approach. *Expert Systems with Applications*, **36**(3, Part 1):4765–4772, 2009.

[136] **Fanson, J.** and **Caughey, T.** Positive position feedback control for large space structures. *AIAA*, **28**:717–724, 1990.

[137] **Creasy, M. A.**, **Leo, D. J.**, and **Farinholt, K. M.** Adaptive positive position feedback for actively absorbing energy in acoustic cavities. *Journal of Sound & Vibration*, **311**:461–472, 2008.

[138] **Mei, C.**, **Mace, B. R.**, and **Jones, R. W.** Hybrid wave/mode active vibration control. *Journal of Sound & Vibration*, **247**(5):765–784, 2001.

[139] **Press, W. H.**, **Teukolsky, S. A.**, **Vettering, W. T.**, and **Flannery, B. P.** *Numerical recipes in C.* Cambridge University Press, 1994. 2nd Ed.

[140] **Åström, K. J.** and **Wittenmark, B.** *Adaptive Control.* Addison Wesley, 1995.

[141] **Fradkov, A. L.**, **Miroshnik, I. M.**, and **Nikiforov, V. O.** *Nonlinear and adaptive control of complex systems.* Kluwer, 1999.

[142] **Colonius, F.** and **Kliemann, W.** *The dynamics of control.* Birkhauser, 2000.

[143] **Ronki Lamooki, G. R.**, **Townley, S.**, and **Osinga, H. M.** Bifurcations and limit dynamics in adaptive control systems. *International Journal of Bifurcation & Chaos*, **15**:1641–1664, 2005. International Journal of Bifurcation and Chaos.

[144] **Zeng, Y. M.** and **Singh, S. N.** Output feedback variable structure adaptive control of an aeroelastic system. *Journal of Guidance Control & Dynamics*, **21**(6):830–837, 1998.

[145] **Ko, J. H.**, **Strganac, T. W.**, and **Kurdila, A. J.** Adaptive feedback linearization for the control of a typical wing section with structural nonlinearity. *Nonlinear Dynamics*, **18**(3):289–301, 1999.

[146] **Wagg, D. J.** Partial synchronization of non-identical chaotic systems via adaptive control, with applications to modelling coupled nonlinear systems. *International Journal of Bifurcation and Chaos*, **12**(3):561–570, 2002.

[147] **Anderson, B. D. O.**, **Bitmead, R. R.**, **Johnson Jr, C. R.**, **Kokotovic, P. V.**, **Kosut, R. L.**, **Mareels, I. M. Y.**, **Praly, L.**, and **Riedle, B. D.** *Stability of adaptive systems: Passivity and averaging analysis.* MIT press: Cambridge Massachusetts, 1986.

[148] **Mareels, I. M. Y.** and **Bitmead, R. R.** Bifurcation effects in robust adaptive control. *IEEE Transactions on Circuits and Systems*, **35**(7):835–841, 1988.

[149] **Rey, G. J.**, **Bitmead, R. R.**, and **Johnson Jr, C. R.** Qualitative classification of the dynamics of adaptive feedback systems. *IEEE Transactions on Circuits and Systems I*, **41**(8):529–538, 1994.

[150] **Neild, S. A.**, **Yang, L.**, and **Wagg, D. J.** Modified model reference adaptive control approach for systems with noise or unmodelled dynamics. *Proceedings of the IMechE, Part I: Journal of Systems and Control Engineering*, **222**:197–208, 2008.

[151] **Worden, K.** and **Tomlinson, G. R.** *Nonlinearity in structural dynamics.* IOP, 2000.

[152] **Kerschen, G.**, **Worden, K.**, **Vakakis, A. F.**, and **Golinval, J. C.** Past, present and future of nonlinear system identification in structural dynamics. *Mechanical Systems & Signal Processing*, **20**(3):505–592, 2006.

9 List of notation

b Distance along a beam

c_v Variable damping

F_d External disturbance force

k Spring stiffness

m Mass

N Total number of modes

N_c Number of controlled modes

p_i The ith actuator constant

q_j The jth modal displacement

$\boldsymbol{q}$ Modal displacement vector

r Road input

t Time

v_i The ith linearized control signal

V_a Voltage input to the piezo actuator

w Beam displacement

x Displacement of mass or length along beam

$\dot{x}$ Velocity

$\ddot{x}$ Acceleration

y_i The ith control output

$\boldsymbol{y}$ The control output vector

α_i The ith modal participation factor for actuator at point A

β_i The ith modal participation factor for actuator at point B

δ_i The ith nonlinear coupling parameter

μ_i The ith cubic nonlinearity parameter

ϕ_j The jth beam mode shape

$\mathbf{\Phi}$ Modal matrix

ω_j The jth modal natural frequency

ζ_j The jth modal damping ratio