



This is a repository copy of *Harold Hazen and the Theory and Design of Servomechanisms*.

White Rose Research Online URL for this paper:  
<http://eprints.whiterose.ac.uk/76934/>

---

**Monograph:**

Bennett, Stuart (1984) *Harold Hazen and the Theory and Design of Servomechanisms*. Research Report. Acse Report 270 . Dept of Automatic Control and System Engineering. University of Sheffield

---

**Reuse**

Unless indicated otherwise, fulltext items are protected by copyright with all rights reserved. The copyright exception in section 29 of the Copyright, Designs and Patents Act 1988 allows the making of a single copy solely for the purpose of non-commercial research or private study within the limits of fair dealing. The publisher or other rights-holder may allow further reproduction and re-use of this version - refer to the White Rose Research Online record for this item. Where records identify the publisher as the copyright holder, users can verify any specific terms of use on the publisher's website.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.

**Harold Hazen and the Theory and Design of Servomechanisms**

by

**Stuart Bennett**

**Department of Control Engineering  
University of Sheffield  
Mappin Street  
SHEFFIELD S1 3JD**

**Research Report No. 270**

**November 1984**

## 1. Introduction

Fifty years ago, in September and November 1934, Harold Hazen published two papers in the Journal of the Franklin Institute, one on the theory of servomechanisms (Hazen 1934a) and the other on the design of a high performance servomechanism (Hazen 1934b). The first of these papers - the theory paper - must be ranked alongside the papers of Maxwell (1868), Routh (1877), Hurwitz (1895), Minorsky (1922) and Nyquist (1932) in its influence on the development of control engineering. Nyquist's work stemmed from the demands of the burgeoning telephone network and was based on the need for a theory to predict the stability of electronic feedback amplifiers: Hazen's work arose out of the need to design high-performance electro-mechanical equipment for the differential analyzer and other calculating machines. However, in both cases, the underlying requirements represented a change in direction for control engineering in that the systems with which they were dealing had to respond to, and follow, rapidly changing signals. Prior to this time the emphasis had been on regulation: even Minorsky's automatic steering mechanism was concerned with maintaining a ship on a preset course. The problem of automatic following presents greater difficulties than that of regulation in that the follow-up device '... not only must accelerate at the proper rate and move at the right speed, but also must be in the correct position' (Hanna et al. 1945).

In comparison with Nyquist's paper, Hazen's was the more readily accessible to his contemporaries in that he worked in the time domain and used the familiar (to electrical engineers) operator techniques, whereas Nyquist's work was based on frequency response ideas which were, in the early 1930s, familiar only to a small group of communications specialists. As a consequence the paper by Hazen had the more immediate impact. A major feature of the paper was that it clearly classified, for the first time, the various types of control system in use. Hazen's schema was:

- (a) relay servomechanisms
- (b) definite correction servomechanisms (i.e. sampled data systems)
- (c) continuous correction servomechanisms.

He also very clearly expressed the amplification involved in a servo-mechanism and explained that because it operated on the difference between the input and output it was not dependent, as is a simple amplifying device, on the constancy of the amplifier parameters for its linearity (Hazen 1934a, p.282). He reached this conclusion independently from H.S. Black, whose paper on the feedback amplifier was published in the same year (Black 1934), however, he does not seem to have been as aware as Black was of the reduction in noise and distortion arising from the use of feedback.

Hazen's two papers arose out of his work in the Department of Electrical Engineering at the Massachusetts Institute of Technology (MIT) on various mechanical calculating machines, including Vannevar Bush's differential analyzer: and they formed the basis for a course on servomechanisms which was started in 1939 (see below). A question which then arises is - why, over a period of almost 20 years, was there such a concentration on calculating machines within the Electrical Engineering Department at MIT? There had been, throughout the nineteenth century, a gradual development of mechanical aids for calculation, from simple integrators through harmonic analyzers (and of course difference engines). At the Napier Tercentenary Celebrations, in Edinburgh, in 1914, an extensive range of calculating machines was exhibited (Horsborough 1914) and the early part of the twentieth century saw the rapid development, by electrical engineers and physicists of many types of harmonic analyzers (Cockcroft 1925), but such work was hardly the mainstream of electrical engineering. It might have been expected that a major academic department such as that at MIT would have concentrated on, for example, the development in communications, or power systems, or motor design.

An important influence on the direction of the work at MIT must have been the strength of personality and the drive of Bush: it would have been difficult to resist any development supported by him - a 'cagey, shrewd, Yankee' was how Hazen (1976) described him [1]. Bush's interest in calculating machines arose partly through his experience of working on design studies for power system networks and transmission lines, an



activity which involved large amounts of tedious calculations, and partly because he saw that the development of calculating machines was necessary for engineering to progress:

Engineering can proceed no faster than the mathematical analysis on which it is based. Formal mathematics is frequently inadequate for numerous problems pressing for solution, and in the absence of radically new mathematics, mechanical solution offers the most promising and powerful attack wherever a solution in graphical form is adequate for the purpose. This is usually the case in engineering problems. (Bush and Hazen 1927, p.615).

Bush had been interested in calculating devices from an early age: while still at college he had invented a surveying machine, the patent for which was issued on December 3 1912 (No. 1,048,649). This device incorporated a mechanical integrator and a spring operated servomechanism (Bush 1970). He developed a further simple mechanical integrator in 1920 for a Harmonic Analyzer which, he also noted, could be used as an aid to solving 'Carson's extension of Heaviside's expansion theorem for determining the alternating from the constant potential solution of transients in a network' (Bush 1920).

Another factor was that the Head of the Department, Dugald C. Jackson, was also the senior partner in a firm of consulting engineers who, among other things, were involved in design studies for power system networks and transmission lines, an activity which required large amounts of tedious calculations. There was thus considerable incentive for Jackson to support the development of calculating devices.

## 2. Hazen's early work on the Network Analyzer

Harold Locke Hazen was born in Philo, Illinois on 1 August 1901 and he grew up in Three Rivers, Michigan. He began the transition from 'a naive small-town youth' to a career as engineer, educator and administrator, in the fall of 1920 when he went to MIT. He was to remain associated, except for some short breaks, with MIT for the rest of his life. The department which Hazen entered, the Electrical Engineering Department, was under the direction of Dugald C Jackson, a man who held firm views on the need for the involvement of staff - and of students, for he fought hard to establish co-operative courses - in industrial

work. He himself was the senior partner in a firm of consulting engineers. He believed that young engineers should be faced with the challenging problems of the time, but only when they were ready - in his terms this was at graduate level: the purpose of the undergraduate course '... was not to make engineers but was to make young men with a great capacity for becoming engineers...' (Jackson 1939, Barker 1939). Prior to moving to MIT in 1907, Jackson had spent 16 years at the University of Wisconsin, Madison, where he had created an exemplary department for electrical engineering education. It was during this time that he formulated his views that only through personal experience of grappling with the unknown can a student truly gain knowledge and fully appreciate its significance. (Rosenberg 1984). Hazen must have shown this capacity and high ability for he was invited to work under Vannevar Bush for his S.B. thesis. Eleven years older than Hazen, Bush was at this time a Professor of Electrical Engineering at MIT. The period 1924-25 was a crucial time in Hazen's life and it was the beginning of a twenty year period in which his life and work were closely interwoven with that of Bush (Brown 1981, Johnson 1980).

One of Bush's interests was the steady-state and dynamic behaviour of large interconnected electric power systems. The reliable operation of such systems required, and still requires, a quantitative understanding of the currents, voltages and power flows through the various elements which make up the network. The idea of networking was still in its infancy, as was the idea of system control [2]. The move towards interconnection had arisen out of the rapid increase in demand brought about by the First World War, for it was realized that by joining distribution networks with a diversity of loading an overall increase in load factor could be achieved and hence an increase in available power. Government pressure to interconnect decreased after the war but interconnection schemes continued to be developed, albeit at a slower rate (Hughes 1983, pp. 285-322). The specific problem involving in 1924 Bush was the proposal to build a 500 mile transmission line to bring Canadian hydro-electric power into the New England and New York areas. Many of the ablest engineers in the USA were analyzing or debating this problem and Bush worked with Ralph Booth of the firm Jackson and Moreland on the

project (Bush and Booth 1925).

The theory and the formulation of the network equations was well understood, but the quantity of numerical calculation required severely limited that which could be practically achieved; as a consequence resort was made to experimental methods and several computing devices - in essence miniature power systems - were developed. The simplest and earliest of these was the dc short-circuit calculating table, valuable in providing information on current magnitudes but, because it used direct current, unable to provide any information on phase angles (Lewis 1920, Fortescue 1925). The first miniature system to employ alternating current was that of Gray (1917) but this, like the dc table, was limited to a static representation of the system.

From 1919 onwards several systems using three-phase generators, motors, static loads, and lumped three-phase artificial lines, were developed. Schurig (1923) of the General Electric Company produced a system which used machines of 3.75 kVA rating with a voltage of 440V, while Evans and Bergvall (1924) of the Westinghouse Company developed a system using 200 to 600 kVA machines and voltages of 2300V. The larger system was necessary for stability investigations since these involved mechanical momentum and electrical damping, and the normal relationship between such factors cannot be maintained as the model scale is reduced. This was not clearly understood when Hazen began work on his S.B. thesis and his first system, based on the use of 5kVA generators to represent 50000 to 100000 kVA units, was found to be unstable when tried out by the General Electric Company engineers - the low mechanical momentum of the small machines in relation to the electrical characteristics resulted in very large transient power flows in the model. To avoid this problem Bush suggested replacing the rotating machines by static phase-shifting transformers. Hazen, together with Hugh H Spencer, who was a co-operative-course student (his industrial base was the General Electric Company plant at Lynn, Mass.), pursued this approach and the scheme was found to work well. Hazen graduated in June 1924 and a paper based on the work with Spencer was published in 1925 (Hazen and Spencer 1925).

On graduation Hazen joined the General Electric Company, Schenectady, 'on test' and was invited to join the group working in the office of Robert E Doherty, who was then the Chief Consulting Engineer for the company and a close friend of Bush. Spencer, Hazen's collaborator on the power systems model, was also transferred to this office where the main problem being studied was the Canadian-New England transmission line. Doherty suggested to Hazen and Spencer that they should follow up the work which they had done as undergraduates. Consequently they returned to Cambridge, Mass, where they were able to demonstrate that the system, which in their thesis they had shown could be used for steady-state studies, was also capable of being used for the analysis of transient stability behaviour. Involvement in this work stimulated Hazen to continue his academic studies and he decided to return to MIT to do graduate work under the guidance of Bush. His graduate work was to be done on a part-time basis, and he was appointed as a research assistant in the Electrical Engineering Department in September of 1925 and then as an Instructor in 1926. As a consequence of this part time study he did not obtain his S.M. degree until 1929 and his doctorate until 1931 (Brown 1981). The work for his master's degree was an extension of his undergraduate work on the power system models and resulted in the construction, with the support of the General Electric Company, of the MIT Network Analyzer, which in the early 1930s was the most advanced system for transmission network analysis in the world (Hazen et al. 1930).

### 3. An Introduction to Servomechanisms

In his work on the analysis of transmission networks Bush had been making use of J.R. Carson's (1917,1919) work on transients in electrical networks. This work necessitated lengthy calculations to evaluate integrals of the form

$$E\omega\cos(\omega t+\theta) \int_0^t \cos(\omega\lambda)A(\lambda)d\lambda$$

that is, the evaluation of the convolution integral. The general form

is

$$f_3(x) \int_a^x f_1(x)f_2(x)dx$$

where  $f_1, f_2$  and  $f_3$  are given functions. In 1924, Herbert Stewart, one of Hazen's undergraduate classmates, started work on a master's thesis under the tutelage of Bush, with the task of developing a machine to evaluate integrals of the above form. Mechanical integrators were not, of course, new but as Bush and Stewart remarked later, they '... usually evaluate the definite integral between given fixed limits. There has been a need for a machine which would continuously evaluate and plot the integral as a function of a variable upper limit.' (Bush, Gage and Stewart 1927, p.64). The result of this work was the Product Integrator.

A key idea in using the Product Integrator to solve differential equations - and hence a step towards the Differential Analyzer - was the use of 'back-coupling':

It has been found that by an expedient fairly well described as 'back-coupling' it [the Product Integrator] may be used to solve certain types of integral equations. As an example the equation

$$\phi(x) = \int_a^x f(x)\phi(x)dx$$

may be solved for an unknown  $\phi$  when  $f$  is known' (Bush, Gage and Stewart 1927, p.64).

The use of 'back-coupling' was the result of the work of another of Bush's graduate students, King E Gould, who used the Product Integrator to investigate the temperature distribution along a thermionically emitting filament (Bush and Gould 1927).

One of the courses which Hazen took as part of the graduate student program, in 1926, was Bush's course on operational circuit analysis. Bush set as a homework problem the analysis of an RL coupled amplifier: with the aid of the Product Integrator, Hazen found the output of such an amplifier - including in the solution the use of the actual nonlinear vacuum tube characteristic. Bush then wanted to try something more complicated, so he suggested that Hazen attempt the analysis of a simple

oscillator circuit. Hazen (1976) recalls that he

... played with that at home for an evening or two and, by golly, if we had a second integrator we could handle the oscillator circuit, so I made a little sketch of the mechanical wheel and disc integrator as a simpler one than the Watt hour meter and took it to Bush. He came back next morning with a sheaf of 'Bush scrawl' showing how it would not only solve the equation I had now, but a whole family of second order differential equations.

Hazen was told to design a wheel and disc integrator and to add it to the Product Integrator. This he did and the two-stage Product Integrator and eventually the Differential Analyzer, was on its way. At this time neither Bush nor Hazen knew of William Thomson's work using wheel and disc integrators, in which 'back-coupling' and linked units were proposed as a means of solving the general second order differential equation (Thomson 1876a, 1876b, 1876c). Neither were they aware of James Thomson's integrator design. (Hazen discovered the Thomson papers in 1927 while preparing a term paper as part of his graduate course.)

#### 4. The Product Integrator

The original single stage form of the Product Integrator was described in a paper by Bush, Gage and Stewart (1927). The principle of operation of this machine is shown schematically in figure 1. The table is driven at constant speed and mounted on it are graphs of the two known functions  $f_1(x)$ ,  $f_2(x)$ , arranged so that the x axis of the graphs are parallel to the movement of the table. Fixed across the table at A and B are two centre-tapped potentiometers. Two operators for the machine were required and they had to move the pointers attached to the slides of the potentiometers in order to follow the plotted curves. The voltages from the two potentiometers were fed respectively to the potential and the current coils of the a modified Thomson direct-current integrating watt-hour meter. The rotation of the watt-hour meter represents the integral of the product of the two functions and this motion was used to move a pen, located at C, across the table to continuously record the value of the integral.



The success of this integrator was dependent on a number of factors: the accuracy of 'tracking' achieved by the operators; the accuracy of the potentiometers; the performance of the watthour meter and the performance of the mechanism used to follow the movement of the watthour meter disc (it was important that this mechanism did not load the disc).

The follow-up mechanism used is shown schematically in figure 2. Attached to the watthour meter disc were three platinum tipped contact screws and a bakelite table on which were fixed metal contact strips was mounted quarter of an inch below the disc. The contact screws could be adjusted to ride just above the contact strips and contact between the two was established by wetting the tips of the contact screws with mercury. The centre contact on the table was used as the ground return and the outer contact was connected to a two-way relay ( $A_1, A_2$ ), the remaining contact operated what was known as the high-speed relay (B). The bakelite table could be driven, through a gear box, from a dc motor. When the watthour meter disc rotated slowly, the outermost contact operated relay A, the main motor was energized and turned the bakelite table. The motor speed would not have exactly matched that of the watthour meter disc and hence there would have been relative motion between the screw and the contact strip. Eventually the screw would move into the gap between the contact strips and then onto the other contact strip. When this occurred the direction of the motor was reversed. The bakelite table thus followed the movement of the contact mounted on the watthour meter disc, but superimposed on the normal motion was an oscillatory movement which was found to have a period of about 0.5 seconds. If the meter speed exceeded the normal motor speed the second, high speed, contact system came into use. It worked by short circuiting a resistance in the armature circuit and increasing the motor speed to a value greater than the maximum speed of the meter disc. The movement of the bakelite table was used to drive the pen P3.

The oscillatory motion produced by the follow-up mechanism was utilized to reduce the friction in the meter support, for the meter disc spindle was supported in a jewel bearing mounted on the bakelite table. The use of an oscillating contact with feedback to obtain proportional

action from a relay system was common practice at this time, for example Sperry's 'phantom' ) worked in this way (Bennett 1979, p.126, Hughes 1971, p.113-4, as did the Tirrill voltage regulator (Bennett 1979, p.168.). The use of 'dither' to reduce static friction was also well established - it had been patented by Charles Parsons in 1892 (British Patent 15677).

This single stage integrator was used as the basis for the two-stage unit and hence the watthour meter mechanism was retained and a wheel and disc integrator added as the second stage. It was also now necessary to have two recording shafts, one for each stage and it was clearly recognised by Bush and Hazen that the operation of the device was dependent on the recording being carried out without loading the integrators:

It is essential that these integrator shafts - in the first stage, the watthour meter rotor; in the second, the wheel shaft - be free from all friction and load torque, and hence they cannot directly furnish energy to drive the recording shafts. A servo-motor follower mechanism is therefore used to drive each recording shaft, and this not only reduces the necessary energy output of the integrator shafts to a negligible value, but, as mentioned above, practically eliminates bearing friction on these shafts at the same time. This mechanism is really the key to the success of the machine from the practical point of view (Bush and Hazen 1927, p.585-588).

The servo-followers used were very simple: a light electrical contact was used to detect the position of the shaft to be followed, the signal from this contact was used to turn on and off the follow-up motor. In this simple form, as used on the single stage integrator, the system has velocity lag. The velocity lag error is of no consequence if the device is being used simply to integrate a function, but if the result is to be fed to a second integrator and the output of the second integrator is fed back to input of the first integrator, the error becomes cumulative. Hazen and Bush carefully studied the sources of error in the system and identified three principle sources:

- (a) backlash
- (b) lag in the servomechanism
- (c) lag due to the inertia of the watthour meter rotor.



Backlash was eliminated by careful design. The velocity lag in the servo-system was eliminated by adding a second servo-motor to the system. The method which they used is shown schematically in figure 3. If the speed of the main motor A matches the watthour meter disc speed, the auxiliary motor B merely oscillates about a mean position. Changes in the watthour meter speed cause motor B to make unequal oscillations and hence cause a change to the voltage supplied to the main motor armature, thus correcting the speed of the main motor. The resulting servo-system was of type 2 and hence had zero velocity error. As can be seen from the block diagram (figure 4) there was an additional method of reducing lag: the motion of the auxiliary control motor was fed forward and added to the motion of the main motor. The angle added was thus proportional to the speed of rotation of the watthour meter disc and was adjusted to compensate for the error associated with the mechanical time constant of the watt-hour meter - it in effect cancelled the inertia of the watt-hour meter disc [3].

Bush was very pleased with the performance of this machine - its importance was recognised by the award to Bush, in May 1928, of the Levy Gold Medal by the Franklin Institute (the award cited the work of Hazen and Stewart) - and was anxious to build a larger version. He was successful in gaining support, a draughtsman was hired and with Hazen began work on the design of a six-integrator machine, a 'real machine' was how Bush referred to it. This was to be the 'Differential Analyzer' (Bush 1931) which, by 1930, had been successfully used in the solution of many complicated problems. Hazen's contribution to the six-integrator differential analyzer project was mainly to the design and construction, particularly with respect to the wheel and disc integrators with their torque amplifiers and frontlash units.

As with the earlier machines the crucial problem was to avoid loading the output shafts of the integrators. The solution which Hazen found came to him almost by chance. In the summer of 1928 while on vacation in his home town of Three Rivers, Michigan, his former Sunday School teacher, Adam Armstrong, gave him a couple of papers written by

C.W. Nieman (1927a, 1927b). One of these papers described the two-stage torque amplifier, the other a 'lashlock' a device to eliminate backlash. Hazen adopted the torque amplifier to provide the coupling between the wheel of the integrator and the mechanical shafts of the rest of the analyzer, and when adjusted the device worked well. The nuisance was in the need for careful adjustment without which oscillations could occur: '... presumably caused by a small part of the output being fed back in one way or another into the input' was the diagnosis of Bush (1931,p.465) [4]. His solution was to add a vibration damper to the output shaft.

A device based on Nieman's 'lashlock', the so called 'frontlash' unit, which used a differential gear to compensate for the backlash in a gear train and the elasticity of shafting, was invented during one weekend spent at Bush's house. Hazen's main memory of that weekend was of seeing Bush's mind in operation '... he struggled just like the rest of us, just like a dog with a bone.' (Hazen 1976).

The six-integrator differential analyzer was a great success and in 1930 a full set of engineering drawings of the machine were prepared and copies went to the Aberdeen Proving Ground, Maryland, to the University of Pennsylvania and to Douglas Hartree at the University of Manchester, England. As a consequence three more machines were built all of which saw extensive use (Goldstine 1972).

While the development of the differential analyzer was in progress, interest on the part of Doherty of General Electric in the development of a power system simulator revived. An agreement was reached between the General Electric Company and MIT to design and build a simulator in which all elements would be adjustable, thus providing for the simulation of a wide range of power systems, and Hazen was placed in charge of the project. The outcome was the MIT Network Analyzer (Hazen, Schurig and Gardener 1930). This machine was used extensively by power companies and electric equipment manufacturers over a period of twenty five years and about forty Network Analyzers of this type were built.

From the early 1930s onwards Hazen became much more involved in the teaching activities of the department than previously and he gradually withdrew as the key participant in the development of the Network Analyzer and the Differential Analyzer. He did, however, become interested in a problem which was common to the whole range of analogue computing machines, that of designing and building a follow-up servo-mechanism, activated by an electrical signal, with a fast and accurate response. The impetus towards this work came from two problems, one which arose in connection with yet another computing machine - the Cinema Integrator - the other from the design of an automatic curve follower to reduce the labour involved and increase the accuracy of inputting data to the Differential Analyzer.

## 5. Cinema Integrator

The Cinema Integrator was Norbert Wiener's 'baby' and was an alternative to the Product Integrator. Wiener had joined the mathematics department at MIT in 1919 after a somewhat strange childhood and early manhood (Wiener 1953,1956; Heims 1980). It was at this time that he began working to extend the Lebesgue measure concept from sets of points to sets of trajectories, work which was eventually to lead to the mathematical theory of stochastic processes. Wiener was a mathematician who needed constant contact with actual phenomena in physics, biology or engineering in order to stimulate and refresh his mathematical intuition and was a constant visitor to other departments. His visits, however, were not always mathematical, for he was a very immature person and needed constant reassurance, so that anyone who was sympathetic - providing that they had some mathematical understanding - would end up on his 'weeping circuit'. There were rewards, for Wiener did not consider it beneath his dignity to deal with the practical mathematical problems of an engineer looking for results, Hazen commented 'In fact, I always felt he was rather tickled to be asked and of course with the quality of mind he had, the kind of insight he provided was superlative.' (Hazen 1976, see also Heims 1980, pp.372-391).

Once having seen the early Product Integrator, Wiener rapidly pro-

duced an alternative basis for a machine to evaluate the convolution integral and also to evaluate integrals of the form

$$\int_A^B f(\lambda) \frac{\sin(n\lambda)}{\cos} d\lambda$$

as a function on  $n$ , where  $A$  and  $B$  are any fixed limits. Evaluation of this integral, providing that  $f$  converges with sufficient rapidity, provides the approximate evaluation of

$$\int_0^\infty f(\lambda) \frac{\sin(n\lambda)}{\cos} d\lambda$$

Wiener's method was based on the passage of radiation through apertures whose shapes represented the functions to be multiplied and integrated; it was investigated by one of Bush's students, King E Gould in 1927-28 (Gould 1928). The method used is illustrated in figure 5. A line source of radiation (in Gould's case infra-red, in later machines visible light)  $SS'$ , having uniform brightness, is placed at some distance  $\underline{x}$  from a mask shaped to represent some function  $f(\lambda)$ , the distance  $\underline{x}$  being sufficiently large that the intensity of the radiation falling on the face of the mask from any point on  $SS'$  varies by less than 1%. Placed an equal distance behind the mask is a second mask which is shaped to represent a function  $g(\lambda)$  (drawn to twice the scale of function  $f(\lambda)$ ). The amount of radiation which passes through a vertical strip of width  $\delta$  at point  $\underline{a}$  of mask 1 is proportional to the ordinate of  $f(\lambda)$  at  $\lambda_a$ . Similarly the amount of radiation which passes through a vertical strip of mask 2 at point  $b$  is proportional to the ordinate of  $g(\lambda)$  at  $\lambda_b$ , but since the radiation incident on mask 2 is dependent on  $f(\lambda)$ , and if  $\underline{a}=\underline{b}/2$ , where  $\underline{a}$  and  $\underline{b}$  represent the distance from the origin for masks 1 and 2 respectively, then the radiation transmitted through mask 2 is the product  $f(\lambda)g(\lambda)$  and hence the total transmitted radiation over the range  $\lambda=0, \lambda=\lambda_1$  is

$$I_s = \int_0^{\lambda_1} f(\lambda)g(\lambda)d\lambda$$

The evaluation of the integral involving the variable parameter  $t$

can be performed by moving the second mask horizontally so that the radiation passing through a strip at point  $\lambda_a$  of mask 1 falls on the strip corresponding to  $(t+\lambda_a)$  on mask 2. The method is also easily extended to enable the value of the integral as a function of  $t$  to be recorded. A table is attached to mask 2 and moves with it as  $t$  is varied. Providing some mechanism is available to instantaneously measure the light transmitted, a graph of the value of the integral can be obtained.

Although the method is simple in concept, implementation of it required the solution of many practical problems. Gould did not give any precise figures for the accuracy which he obtained from his system, but it is known that he experienced great difficulty in excluding extraneous radiation and in accurately measuring the radiation transmitted (Gray 1931, p. 81). Development of the photo-electric cell led Bush to try a different approach and this work was done by another student, Truman S Gray. He adopted a null balance approach to the measurement of the transmitted light and made use of a high impedance vacuum tube which had recently been developed by the General Electric Company. He was able to obtain an accuracy of between 2% and 5%.

Gray's machine used a manual method to obtain the null balance necessary for the measurement and hence was somewhat slow and laborious to use; it would have been much improved if the null balancing system and the movement of the masks could have been made automatic. (Both of these operations were eventually made automatic through the use of Hazen's high speed servo-system, but this was not until the late 1930s with the development of the Cinema Integrator by Hazen and Brown (1940).) The significance of the device in 1930-1 was that it represented a computing mechanism which would benefit from the development of a fast, accurate position control mechanism, and was a device which would provide an electrical not a mechanical input signal for the servo-systems.

As inevitably happens, once a reliable computing device of reasonable accuracy becomes available, the demands on it grow rapidly and it

is used to solve problems which could not have been imagined when the machine was designed. This happened with the Differential Analyzer and studies were made of ways of reducing the set-up time and the solution time of the machine. Little could be done to quicken the setting up of the machine since this involved the physical interconnection of shafts and gears. The solution time was largely dependent on the speed at which the machine could be run - the motor speed controlled the rate of change of the independent variable and the limiting factor on this speed was the ability of the operator to trace the curve of the input function with the required degree of accuracy - hence a balance had to be achieved between speed and accuracy. It was in an attempt to speed up the machine while still retaining accuracy that Hazen began work on designing and building an automatic curve follower. An essential feature of such a unit was a high performance servomechanism. There were, therefore, two potential applications waiting for the development of a suitable servomechanism.

## 6. The High Speed Servomechanism

A detailed account of the analysis of the requirements and the design of the servomechanism used in the Cinema Integrator were given in the second of the two papers which Hazen published in 1934 (1934b). The input signal to the servomechanism was the difference between the outputs of two photo-electric cells, a signal with a power level of the order of  $10^{-10}$  watts; the servo output had to operate (i) a shutter to adjust the light balance and (ii) the recording unit - a total power requirement of the order of 10 to 100 watts giving an overall power amplification requirement of  $10^{11}$  or  $10^{12}$ . The servomechanism was also required to have a non-oscillatory response and the ability accurately to follow rapid changes in the input signal. A further requirement was the conversion of electrical energy into mechanical energy since the input was an electrical signal and the required output was a mechanical position. Hazen decided that the essential elements would be an electrical power amplifier, an electric motor and possibly a mechanical power amplifier.



Before attempting to decide on the use of the mechanical amplifier he considered the 'dynamic' elements which would be introduced by each of the units. He concluded that the use of a resistance-coupled dc amplifier would not introduce friction or inertia effects; that an electric motor would introduce inertia and damping and that a mechanical amplifier would introduce only inertia. He further concluded that this inertia, together with the inertia and friction of the load (suitably reduced by the amplification factor of the torque amplifier), could be added to the inertia of the motor. He thus reduced the system to that shown schematically in figure 6.

The subsequent stages of the design were based on the use of a 'figure of merit'. This was chosen to be

$$M = \frac{\omega_m^2}{\Theta_m}$$

where  $\omega_m$  = maximum speed of the servo

$\Theta_m$  = steady state error at speed  $\omega_m$

In the paper explaining the theory of servomechanisms which had been published two months earlier it was shown that

$$M = \frac{\tau_m}{4\gamma^2 J}$$

where  $\tau_m$  = maximum driving torque

$J$  = total moment of inertia (including load inertia)

$\gamma$  = '...factor characterizing the oscillatory tendency of the output' i.e. in modern terminology  $\xi$ , the damping ratio.

The arguments for using this 'figure of merit' and the relationship between  $M$ ,  $\omega_m$ ,  $\gamma^2$ , and  $J$  were discussed in detail in the theory paper, together with a detailed analysis of the system shown in figure 6.

According to Hazen's recollection the papers as published 'had the appendices at the front'. The work had begun as an attempt to build a practical high performance servo, this must have been in 1931-32 (an automatic curve follower designed and built by Hazen and Gordon S Brown was exhibited at the Chicago World's Fair in 1932, and one of Hazen's students, Harold A Traver described a curve follower in his S.M. thesis

of 1933 (Wildes 1975)), during 1932-3 Hazen worked on an analysis of the continuous control system. It was during the latter work that he found the Minorsky paper of 1922 with its analysis of ship steering control. The study of the continuous servo-system and the description of the servo-motor design were written up as one paper and shown to Bush in the early part of the summer of 1933. Bush suggested that during the summer Hazen should visit him at his summer cottage at South Dennis (Cape Cod) where

... they could spend half a day on it in pleasant circumstances. ...So we went down there, went to work, chewed it over - it was about the third or fourth thing that we'd done together....We discussed the fact that there would be off/on relay type controls and then the definite correction controls.... This was early summer and he [Bush] said 'I think you want to go back and put some stuff together on the overall treatment of these as well as the continuous control. That might just be something if you did that'. So I went back to work on it.

The comprehensive coverage of the theory paper is therefore due to the suggestion and encouragement of Vannevar Bush.

Returning to the practical work, it is important to realize that in the early 1930s Hazen could not turn to a manufacturer's catalogue to choose a suitable servomotor. He had to start from first principles. The theory of continuous controls and the figure of merit which he had determined told him that a high torque to inertia ratio was required: his next step was to try to decide between amplification using an electric motor and mechanical amplification. This decision he approached by calculating the electromagnetic force that could be developed in copper and the force that could be transmitted by a unit mass of steel one centimetre in length. From these calculations he concluded that '...a device which develops forces mechanically could be made much lighter i.e. with much less mass, than one which develops forces electromagnetically.' (1934b, p.549). He therefore decided to use a mechanical torque amplifier with a small, low inertia, dc motor.

He imposed a further restriction on the motor design in that he required the motor to provide the damping necessary to give an overall aperiodic response for the system. The arrangement of the motor is shown in figure 7, to reduce the inertia it had a stationary iron core



and incorporated an eddy current damper. The motor characteristics achieved were as follows:

Armature resistance	1590 ohms
Moment of inertia	$0.34 \times 10^{-5} \text{ kg m}^2$
Torque constant	1.12 Nm/A
Mechanical time constant	0.0146 s
Damping coefficient	$0.465 \times 10^{-3} \text{ Nms}^{-1}$
(including eddy current damper)	

The inclusion of damping within the motor design procedure suggests that the work must have been well advanced before Hazen discovered Minorsky's work and became aware that the overall system characteristics could be modified by using the time derivatives of the input-output deviation, for he simply comments that these methods were not used. However, by the time he came to write the theory paper, Hazen had fully absorbed this idea and stated that

...the servo operation can be made aperiodic or oscillatory in any desired degree by the damping effect introduced by the component of restoring torque depending on the first derivative of  $\theta$ ... Physically the torque corresponding to the [damping] term... is readily introduced, when a d-c. vacuum-tube amplifier is used, by an inductance component of interstage coupling. (Hazen 1934a, pp.324-5).  
He later found that it was not so simple.

## 7. The Theory of Servo-mechanisms Paper

In the theory paper we see evidence of the change in Hazen from the practical, experimental, engineering research worker and designer to the teacher, for in this paper ten years of experience are codified and presented with great clarity. In the introduction to the paper the reader is led to appreciate the difference between open and closed loop control (closed-cycle control in Hazen's terminology), before being given some examples of closed loop control. It should be noted that by this time it was not difficult to provide references describing control systems (30 out of the 32 references in the paper are to articles which describe control systems), but there was still the need to clearly

explain the difference between open and closed loop systems; the word 'control' was still associated only with contactor systems used in open loop sequencing controllers. Some idea of the need for careful explanation and education at this time is given by Black's experience: it took him nine years to obtain the American patent for his feedback amplifier and several years to obtain the British patent. The British Patent Office, according to Black, treated the device in the same way that it treated applications for perpetual motion devices and demanded a working model (Black 1977).

In a section dealing with the general characteristics of servo-mechanisms the following definition is given:

A servo-mechanism may thus be defined as a power-amplifying device in which the amplifier element driving the output is actuated by the difference between the input to the servo and its output. At this time Hazen was unaware of Black's work on the feedback amplifier, he remembers being vaguely aware of Nyquist's and Bode's work at BTL, but mentally associated it only with communication network theory 'I did not recognize at the time the intimate and fundamental interconnection between this and the transient analysis approach.' (Hazen 1975). However, he reached the same conclusions as Black about the effects of feedback. He explained that for a simple vacuum tube amplifier the linear response '...was due to the constancy of the parameters within the amplifier. Any departure from constancy of these parameters affects the relation between input and output directly.' (Hazen 1934a, p.282) For a servomechanism, however, the '...only function of the servo amplifier element is to apply sufficient force to the servo output to bring it rapidly to correspondence with the servo input. Such an amplifier can be a relatively crude affair.' (Hazen 1934a, pp.282-3).

As part of the section on general characteristics Hazen introduced his classification scheme: relay type, definite correction, and continuous control servomechanisms, and in the rest of the paper he deals with each in turn. The order in which he dealt with the three types reflects the practical significance of each at that time.

The types of relay control analysed are shown in table 1. The

method used in the analysis was to assume that the output would follow a particular form and hence determine the equation of motion and the terminal conditions for particular segments of the output path. Using classical operator techniques the equations were then solved in terms of the known terminal conditions; the solution gives the conditions which have to be satisfied for the the original assumptions as to the form of the solution to be valid. For example for an ideal relay (i.e. no dead space and no time delay) the assumption made was that the output would settle to a steady oscillation. On this assumption the solution of the system equation gives

$$\ln \frac{1 - \frac{\omega_o}{\omega_s}}{1 + \frac{\omega_o}{\omega_s}} + 2 \frac{\omega_o}{\omega_s} = 0$$

This equation is satisfied only for vanishingly small values of  $\omega_o/\omega_s$  and since  $\omega_s$  is finite then  $\omega_o$  must be vanishingly small; i.e. the output oscillates with infinitesimal amplitude and infinite frequency. The implication which Hazen drew was, of course, that any deviation from the assumed conditions, e.g. a time delay, would result in a finite amplitude and frequency of oscillation.

The importance of the work on relay type controllers lay not so much in the results obtained, although they were of use to designers, but in the approach to the solution and in the encouragement they gave to others to attempt an analysis. A.L. Whiteley (1976) recalled seizing eagerly on Hazen's work when faced with a problem with a control system for the register control of a colour printing press.

Hazen did not attempt any analysis of the definite correction servo-mechanism, except to comment that if such a device was to follow a high speed input then either the maximum amount of correction which could be applied at any one instance must be high, or the interval between applying successive corrections must be small, or both. At the time this case was largely of theoretical interest since the common use of definite correction servomechanisms was in the process industries where the rate of change of the input signal was usually very small. He also noted

that such devices had long life, whereas there was a danger with a relay system - which would always tend to oscillate in search of a balance - of high wear and hence a short life.

As an example of continuous control Hazen chose the system used in the Cinema Integraph which is shown schematically in figure 6. This system, because of the assumptions made, reduces to the standard second order system; and the method of analysis was to solve the system equations using operator techniques and to consider the three cases which result i.e. critically damped, underdamped and oscillatory. The important ideas which Hazen extracted from the analysis were (a) the use of 'relative damping factor', i.e. damping coefficient, to characterize the form of the response; (b) the importance of the time constant in determining the speed of response; (c) the establishment of a 'figure of merit' as a basis for design.

As in the 'Design' paper Hazen discussed the use of derivatives of error instead of damping to obtain the desired form of transient response. In 1934-5 attempts were made by Hazen's students to use inter-stage coupling networks to introduce the first three time derivatives of error into the system. Hazen noted that it was difficult to get derivatives of the required magnitude and he reported that 'None of the mechanisms having the more complex control signals has evidenced the complete stability and independence of small variations in the parameters observed in the uncorrected mechanism [the automatic curve follower]...' (Hazen, Brown and Jaeger 1936, p.357).

For the two papers on servomechanisms Hazen was, in 1935, awarded the Levy Medal of the Franklin Institute, thereby following in the steps of his mentor Vannevar Bush. The Levy Medal is awarded annually to the paper appearing in the Journal of the Franklin Institute for the preceding year which is adjudged to have the highest merit in terms of both research and literary style.

## 8. The Influence of Hazen's Work on the Development of Control Engineering

In the 1920s and 1930s the major constraint on the rapid development (and application) of control engineering was the lack of a system - a servomechanism - capable of following a rapidly changing signal [5]. For Black, working in the telephone industry, the signal was the electrical voltage representing the speech pattern. The development of the electrical filter led to the use of the carrier system of transmission which enabled several different conversations to be transmitted over a single pair of wires. The consequences were, with the increase in carrier frequencies, higher cable losses and hence the need for a greater number of repeater amplifiers; by the early 1920s the bandwidth and freedom from distortion of the existing amplifiers were not adequate. Black's achievement was to find, in the feedback amplifier, the solution to this problem. The discovery was made in 1927 but not reported until 1934 (Black 1934, 1977).

In the mechanical field the crucial problem was not recognised so early. The main reason for this was that there was not a pressing commercial problem to stimulate thought, although there was a military problem in the remote control of guns and searchlights. Both in the USA (Barnes 1929) and in England the military research establishments began work on servomechanisms for these applications in the 1920s (Gairdner 1947; PRO 1930; Wood 1965). However, the military requirements did not translate into the need for very high performance, this came only with the increase in aircraft speeds in the 1930s. It was the problems involved in extending the Product Integrator and those arising in the design of the Cinema Integrator which led Hazen to formulate the crucial problem, the design of a high performance electromechanical servomechanism.

Hazen's first solution, as developed for the Product Integrator, was to use the existing technology and approach and to find a way to compensate for its faults. The solution which he found was the addition of the

second motor to provide a feedforward signal to compensate for the lags in the system. He was able to proceed further using existing techniques through the development of the mechanical torque amplifier, for this is an excellent power amplifier with low drift and good stability. However, systems based on the use of contactors and relays were complicated, prone to wear and difficult to adjust and maintain. The breakthrough came in the early 1930s when he began to think in general about servomechanisms and established that the 'critical problem' was the development of a high performance servomechanism; he now escaped from the established techniques and began to think along new lines.

The key to the change was the commercial development of the photoelectric cell. This was a device which could convert mechanical movements into electrical signals with much less disturbance to the motion being measured than any other device then available (dc potentiometer, crude synchros). It was also a device which in the early 1930s was capturing the imagination of engineers and the public. The photoelectric vacuum tube had been developed commercially for use in producing the sound for 'talkies', but manufacturers were interested in finding other markets. In 1929 Hull of the General Electric Company developed a circuit in which the photoelectric vacuum tube was used to operate a Thyatron valve (Hull 1929) and in 1930 GEC announced a photoelectric relay (Anon. 1930). A.L. Whiteley (1976) recalled being told in 1930, having just joined the British Thomson Houston Company as a graduate, to find applications for the 'magic eye'. Hazen himself was, however, still faced with the problem of taking power from the cell. The solution was a dc amplifier, but he was not aware of Black's work (this was still a commercial secret of the Bell Laboratories and the American Telegraph and Telephone Company). He was fortunate though in that several people within the Electrical Engineering Department at MIT were working on this problem, including Gordon Brown and S.H. Caldwell (Caldwell 1932). It was in fact Brown's design which was used (Wildes 1975). Unlike Black, Hazen did not have to think about introducing feedback, for he was trying to automate manual null balancing systems and hence the presence of a feedback loop was implicit. He did, however, appreciate the need for damping and he determined the amount of



damping required to give the desired response.

Hazen's work marks the change in emphasis from relay systems to continuous control and the beginnings of a method for the design of a control system with a specified response. It is the latter which is Hazen's most significant achievement for others were also beginning to make use of continuous controllers: for example R.M. Zabel and R.R. Hancox (1934) used a photoelectric cell and a thyatron for temperature control and noted that the system provided '...continuous modulation of the heating current and hence a reduction in the tendency of the system to alternate'. Similar changes were also beginning to take place in the Process Control field. In the mid-to-late 1930s the pneumatic three-term controller was developed (Ziegler 1964) and there was a growing body of literature concerned with the dynamic analysis of process and controller behaviour - examples are the work of Ivanoff (1934), Mitereff (1935), Mason (1938) and perhaps the most ambitious, the work of Callender, Hartree and Porter (1936). This work, however, was still concerned with regulation: it was through the military requirements at the beginning of the second world war that the major developments based on Hazen's work took place.

The immediate impact Hazen's work was, of course, on his colleagues and students, Gordon S Brown began working for his Master's degree under Hazen in 1932, working on the automatic curve follower and then worked for his doctorate with Hazen on the Cinema Integrator. In 1939 he became the first person to teach a course on Servomechanisms. This course arose out of a request from the US Navy, Office of the Chief of Ordnance, in 1936/7, who had realized that the work of Hazen at MIT was relevant to the problems which were arising in the development of fire-control equipment to cope with the increased speeds of ships and aircraft. The original intention was that the course was to be taught by Bush, Hazen and Caldwell. On Bush's departure to Washington in 1938 Hazen took over the planning of the course and he remembers that he was looking forward to teaching it.

In September of 1939 four lieutenants (all eventually became

Admirals) joined the special course which had been organized and under Gordon Brown studied Hazen's and Minorsky's papers in depth. Brown was assisted by John W Anderson and George C Newton (then a third year student), and later by A.C. Hall. In the second term Brown began to get together a laboratory and this was the beginnings of the Servomechanisms Laboratory. Although Brown seized this opportunity eagerly and made the most of it, it cannot have been an easy task to teach a new course to four experienced and clever, Naval Officers, particularly as Brown must have been aware of the close interest that Hazen, as Head of Department, was taking in the course. Brown was, however, grateful for the opportunity and in recent years has paid generous tributes to Hazen and to his work:

Later he passed to me the opportunity to exploit all that he had created in the field of servomechanisms and control....For his expression of confidence in me, for his willingness to help me whenever necessary, and for the sacrifice he made in forgoing the joy of teaching a course he would have loved, I am eternally grateful. (Brown 1980)

There was also a rapid transfer of Hazen's ideas to England in that Douglas Hartree of the University of Manchester was a frequent visitor to MIT and (as was noted above) received one of the sets of drawings of the Differential Analyzer, Arthur Porter, a student of Hartree's, spent a year in the Electrical Engineering Department of MIT before working with Hartree on the application of the Differential Analyzer. Porter later became secretary to the wartime Servopanel of the Ministry of Supply and worked at ADRDE on servomechanisms (Porter 1969). Work on the design of an automatic curve follower also introduced the idea of feedback control to F.C. Williams in 1938-9 (Bennett 1979).

It is easy to underestimate Hazen's contribution to the development of control engineering: this is particularly so if knowledge of his work is confined to the two papers of 1934. The material contained in them is so familiar and so deceptively simple and understandable that it is easy to dismiss the papers as a workmanlike account of current practice and thus not as important as the papers of, say, Nyquist or Black. Of course Hazen did not provide a new method for determining stability and



his servomechanism was not of such immediate importance as Black's feedback amplifier. What he did do, though, was to design and build a servomechanism with a performance far beyond that of any previous design, a servomechanism which also involved electromechanical energy conversion. To achieve this he not only had to understand the operation of feedback, but had to design and build all the components, the dc amplifier, the servomotor and the mechanical amplifier. These were of course not Hazen's achievements alone; the Electrical Engineering Department at MIT was a hot bed of ideas and it was often difficult, as he would have been the first to admit, to correctly apportion credit for them.

These first attempts, by Hazen and others, to automate the tracking or following of a signal mark a significant step in technological development in that they represent a major change in the level at which human skill and ability was to be replaced by machine action. The tracking or curve-following problem itself was of enormous importance during the Second World War, and continues to be an important military problem. What was not realized fully at the time, but has gradually emerged since, is that although it was possible to transfer the manual skill to the machine, it was and still is, much more difficult to transfer the mental skills which the human tracker employed. These mental skills include the ability to smooth and extrapolate data and to adapt to changing conditions. Automation at a conceptual level not significantly different than that introduced by Hazen - the technology has changed to include microprocessors, integrated circuits, stepping motors - is now part of the normal industrial world. The theory and design techniques are of course much refined and extended. However, we are now, with the so-called knowledge based systems, seeing a further change in the level at which human skills will be transferred to machines.

## 9. Epilogue

The publication of the 1934 papers marked the end of Hazen's career as an engineer personally involved in technical work. With the retire-

ment of Dugald Jackson in 1935 and the appointment of Bush as Dean of Engineering and Edward Moreland (Jackson's business partner) as Head of Department, Hazen became much more involved in teaching and administration; in 1937 he became responsible for the Graduate Study and Research program of the department and in 1938, following Bush's appointment as President of Carnegie Institute in Washington, Hazen became Head of Department and Moreland Dean of Engineering. Hazen's tenure as Head of Department lasted 14 years and saw a doubling of the size of the Department (it became the largest in MIT). It is estimated that during this period some 5000 students passed through the Department. In 1952 he was appointed as Dean of the Graduate school, a post he held until his retirement in 1967.

Harold Hazen had two other careers which are perhaps not well known: one as head of Division 7, Fire Control, of the National Defense Research Committee (NRDC) from December 1942 until 1946; the other in the enhancement of engineering education abroad through his work with engineering institutions in Japan and his trusteeship of Robert College, Istanbul and the University of Petroleum and Minerals, Dhahran, Saudi Arabia.

It is in the work for the NDRC that Hazen's abilities as an administrator and his own personal qualities were put to the greatest test. The task of Division 7 was to promote the development of systems for controlling the aiming and firing of guns, in military terminology 'fire control'. The major problems were not technical - although the technical problems were difficult enough - but were in the resolving of the many misunderstandings and personal conflicts. As might have been expected, the armed services considered that 'fire control' was their speciality and that they were the 'experts'. However, because of the need for secrecy the groups which had been working on 'fire control' had become inbred, and although they had produced fire control equipment of great ingenuity and high craftsmanship, they did not have any input from any other related -or even unrelated - areas of knowledge. A major task therefore was to educate the services so that they could understand what resources and possibilities for new methods of fire control were avail-

who had been developing and supplying equipment that new electronic techniques might have some use, and another to convince academics that conditions on the battlefield were different than conditions in the laboratory. Harold Hazen somehow managed to do this and with his ability to understand the other persons point of view, his patience, his strength and his intellectual ability he was able to get Division 7 functioning and still find time to make the occasional technical contribution to the work. He was awarded the President's Certificate of Merit by President Truman in 1948 for this work.

Harold Locke Hazen, Dean Emeritus of the Graduate School at the Massachusetts Institute of Technology, died in Belmont, Massachusetts on February 21 1980.

#### **Acknowledgements**

The work was begun during a sabbatical term spent in the Division of Mechanisms, the National Museum of American History, the Smithsonian Institution, Washington DC and the help given by the staff of the Division and the Museum is greatly appreciated. I am grateful to Dr. Michael Cable for providing me with a copy of the addresses given at the Memorial Service for Harold Hazen and to Professor Karl Wildes for a copy of part of his then unpublished history of the Electrical Engineering Department of MIT. Financial support for the work has been received from the Research Fund of the University of Sheffield, and the interviews with Hazen, Whiteley and Porter were made possible through a grant from The Royal Society.

#### **Notes**

1. Hazen delighted in telling the story of Bush's doctorate - 'Bush was short of money as a student and wanted to complete his doctorate as quickly as possible, within the year, this was just not done and his supervisor Arthur Kennelly was a proper gentleman who would not agree to any deviation from the normal procedures. Bush sized him up thoroughly and wrote out a proposal for a thesis project and got Kennelly to agree that when this result was achieved he was done. During the year he submitted progress reports, but the gap between

the reported progress and the actual progress gradually stretched out until at the end of the year his actual progress had reached the terminal point. He had carefully got Kennelly's signature to the original proposal and so that there was no way out when the result was presented, and Bush got his degree in the year.' (Hazen 1976).

2. At this time system and network control was largely manual, automatic voltage control was well established, boiler feedwater control was customary and combustion control was being introduced, but system control relied on speed governors and manual intervention. Automatic frequency control was introduced in 1927 by the New England Power Company (they were forced into it since they had contracted to sell 'time' and the task of maintaining the frequency, with the accuracy necessary to provide a time reference, by manual control proved to be too arduous) and only towards the end of the 1920s did the application of load frequency control become widespread (Cohn 1984).
3. A similar method for eliminating lag was used by the Admiralty Research Laboratory in England for the oil-hydraulic searchlight stabilization system which was developed in the late 1920s (Gairdner 1947, PRO 1930). It is believed that the feedforward idea was devised by J.M. Ford and H. Clausen (Coales 1976). The designers of the system also took great care to eliminate backlash and used 'dither' to reduce the effects of stiction. The ARL system went into service in 1929 (Wood 1965).
4. Arthur Porter, who worked with the Manchester University version of the Differential Analyzer recalls that transverse vibrations of the bands could occur and that these were eliminated by coating the drums with vaseline - the decrease in friction necessitated an increase in the number of turns on the drum (Porter 1975).
5. This constraint was perhaps closer to a 'design impasse' in the sense used by Duffy (1984) than the 'reverse salient' as used by Hughes (1983, pp.14-17), for control applications as a whole could not at this time have been said to be moving forward rapidly while

leaving this one area behind. The position was rather that engineers had been elaborating designs based on contactor mechanisms and that new and different techniques were needed.

6. This point was made by Hazen in his introduction for the history of Division 7 prepared immediately after the war; it was made in much more vigorous terms by Ivan Getting in his draft for the history of section 7.6, Navy Fire Control with Radar, where he makes cogent comments about the problems which arise in closed technical societies: overconfidence, conservatism, a view that by definition all the 'experts' are inside and the opinions of outsiders are worthless. The final, official, published history (Boyce 1947) presents a heavily edited and anodyne version of Getting's contribution - the records themselves suggest that Getting's original version was closer to reality.

## REFERENCES

- ANON, 1930, New device uses light to control machinery, General Electric Review, 33, 398
- BARKER, J.W., 1939, Dugald Caleb Jackson - 1938 Edison Medallist, Electrical Engineering, 64-6.
- BARNES, G.M., 1929, Antiaircraft artillery, Army Ordnance, 10, 103-111
- BENNETT, S., 1979, F.C. Williams : his contribution to the development of automatic control, Electronics and Power, 800-3
- BLACK, H.S., 1934, Stabilized feedback amplifiers, Bell System Technical Journal, 13, 1-18.
- BLACK, H.S., 1977, Inventing the negative feedback amplifier, IEEE Spectrum, 14, 55-60.
- BOYCE, J.C., 1947, New Weapons for Air Warfare: Fire Control Equipment, Proximity Fuzes and Guided Missiles, (Boston, Mass.: Little Brown and Co.)
- BROWN, G.S., 1980, Address at the Harold L Hazen Memorial Service, 25 Feb 1980
- BROWN, G.S., 1981, Eloge: Harold Hazen 1901-1980, Annals of the History of Computing, 3, 4-12
- BUSH, V., 1920, A simple harmonic analyzer, Journal of the American Institute of Electrical Engineers, 39, 903-8.
- BUSH, V., 1931, The differential analyzer, Journal of the Franklin Institute, 212, 447-88
- BUSH, V., 1970, Pieces of the Action, (New York: Morrow)
- BUSH, V., BOOTH, R.D., 1925, Power system transients, Transactions of the American Institute of Electrical Engineers, 44, 80-97.
- BUSH, V., GAGE, F.D., STEWART, H.R. 1927, A continuous integrator,

- Journal of the Franklin Institute, 211, 63-84
- BUSH,V., GOULD,K.E. 1927, Temperature distribution along a filament, Physics Review 29, 337-45
- BUSH,V., HAZEN, H.L., 1927, Integrator solution of differential equations, Journal of the Franklin Institute, 211, 575-615
- CALDWELL,S.H., OLER,C.B., PETERS,J.C., 1932, An improved form of electrocardiograph, Review of Scientific Instruments, 3, 277-86
- CALLENDER,A., HARTREE,D.R., PORTER,A., 1936, Time-lag in a control system, Philosophical Transactions of the Royal Society of London, 235, 415-44
- CARSON,J.R., 1919, Theory of transient oscillations of electrical networks and transmission systems, Transactions of the American Institute of Electrical Engineers, 38, 345-427.
- CARSON,J.R., 1917, General expansion theorem for transient oscillations of a connected system, Physical Review, 14, 115-36
- COALES, J., 1976, Interview with author.
- COCKCROFT,J.D., COE,R.T., TYACKE,J.A., WALKER,M., 1925, An electric harmonic analyser, Journal of the Institution of Electrical Engineers, 63, 69-113
- COHN,N., 1984, Recollections of the evolution of real-time control applications to power systems, Automatica, 20, 145-62
- DUFFY,M.C., 1983, Technomorphology and the Stephenson traction system, Transactions of the Newcomen Society, 54, 55-78
- EVANS,R.D., BERGVALL,R.C., 1924, Experimental analysis of stability and power limitations, Transactions of the American Institute of Electrical Engineers, 43, 39-58.
- FORTESCUE,C.L., 1925, Transmission stability, analytical discussion of some factors entering into the problem, Transactions of the American Institute of Electrical Engineers, 44, 984-1003
- GAIRDNER, J.O.H., 1947, Some servo mechanisms used by the Royal Navy,



- Journal of the Institution of Electrical Engineers, 94(part IIA), 161-221
- GOLDSTINE, H.H., 1972, The Computer from Pascal to von Neumann, (Princeton, NJ.: Princeton University Press)
- GOULD, K.E., 1928, A new machine for integrating a functional product, Journal of Mathematics and Physics, 3, 309
- GRAY, G.H., 1917, Design, construction and tests of an artificial power transmission line for the Telluride Co. of Provo, Utah, Transactions of the American Institute of Electrical Engineers, 36, 789
- GRAY, T.S., 1931, A photo-electric integrator, Journal of the Franklin Institute, 212, 77-102
- HANNA, G.R., OSBON, W.O., HARTLEY, R.A., 1945, Tracer-controlled position regulator for propeller milling machine, Transactions of the American Institute of Electrical Engineers, 64, 201-5.
- HAZEN, H.L., 1931, The MIT Network Analyzer, Department of Electrical Engineering Report (Cambridge, Mass.: MIT,)
- HAZEN, H.L., 1934a, Theory of servo-mechanisms, Journal of the Franklin Institute, 218, 279-331
- HAZEN, H.L., 1934b, Design and test of a high-performance servo-mechanism, Journal of the Franklin Institute, 218, 543-80
- HAZEN, H.L., 1975, letter to author.
- HAZEN, H.L., 1976, Interview with author.
- HAZEN, H.L., BROWN, G.S. 1940, The cinema integrator, Journal of the Franklin Institute 230, 19-44, 183-205
- HAZEN, H.L., JAEGER, J.J., BROWN, G.S. 1936, An automatic curve follower, Review of Scientific Instruments 7, 353-57
- HAZEN, H.L., SCHURIG, O.R., GARDENER, M.F., 1930, The MIT network analyzer: design and application to power station problems, Transactions of the American Institute of Electrical Engineers, 49,



1102-13.

- HAZEN, H.L., SPENCER, H.H., 1925, Artificial representation of power systems, Transactions of the American Institute of Electrical Engineers, 44, 72-
- HEIMS, S.J., 1980, John von Neumann and Norbert Wiener, (Cambridge, Mass.: MIT Press)
- HORSBURGH, E.M., 1914, Handbook of the Napier Tercentenary Exhibition, (Edinburgh: The Royal Society of Edinburgh)
- HUGHES, T.P., 1971, Elmer Sperry: Inventor and Engineer, (Baltimore, MD: Johns Hopkins Press)
- HUGHES, T.P., 1983, Networks of Power: Electrification in Western Society 1880-1930, (Baltimore, MD: Johns Hopkins Press)
- HULL, A.W., 1929, Hot cathode thyratrons, General Electric Review, 32, 213-23, 390-4
- HURWITZ, A., 1895, Über die Bedingungen unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt, Mathematische Annalen 46, 273-84
- IVANOFF, A., 1934, Theoretical foundations of automatic regulation of temperature, Journal of the Institute of Fuel, 7, 117-30
- JACKSON, D.C., 1939, Electrical Engineering, 166.
- JOHNSON, H.W., Address at the Harold L Hazen Memorial Service 25 Feb 1980 (copy provided by Dr. M. Cable)
- LEWIS, W.W., 1920, A new short-circuiting calculating table, General Electric Review, 23, 669
- MASON, C.E., 1938, Quantitative analysis of process lags, Transactions of the American Society of Mechanical Engineers, 60, 327
- MAXWELL, J.C., 1868, On governors, Proceedings of the Royal Society of London 16, 270-83

- MINORSKY, N., 1922, Directional stability of automatically steered bodies, Journal of the American Society of Naval Engineers, 34, 280-309
- MITEREFF, S.D., 1935, Principles underlying the rational solution of automatic control problems, Transactions of the American Society of Mechanical Engineers, 57, 159-63
- NIEMAN, C.W. 1927, Bethlehem torque amplifier, American Machinist, 66, 895-7
- NIEMAN, C.W. 1927, Backlash eliminator, American Machinist, 66, 921-4
- NYQUIST, H., 1932, Regeneration theory, Bell Systems Technical Journal, 11, 126-47
- PADFIELD, P., 1974, Guns at Sea, (New York: St. Martin's Press)
- PORTER, A., 1965, The servo-panel - a unique contribution to control systems engineering, Electronics and Power, 330-3
- PORTER, A., 1975, Interview with author
- PRO, 1930, The A.R.L. System of Stabilised Searchlight Control, ADM 204/251, Public Record Office, London
- ROSENBERG, R., 1984, The origin of EE education: a matter of degree, Spectrum, Institute of Electronic and Electrical Engineers, 60-8
- ROUTH, E.J., 1877, A Treatise on the Stability of a Given State of Motion, (London: Macmillan)
- SCHURIG, O.R., 1923, A miniature ac transmission system for the practical solution of network and transmission problems, Transactions of the American Institute of Electrical Engineers, 42, 831-40.
- THOMSON, W., 1876, On an instrument for calculating  $(\int \phi(x) (\alpha) dx)$ , the integral of the product of two given functions, Proceedings of the Royal Society of London, 24, 266-9
- THOMSON, W., 1876, Mechanical integration of the linear differential equations of the second order with variable coefficients, Proceedings

- of the Royal Society of London, 24, 269-71
- THOMSON, W., 1876, Mechanical integration of the general linear differential equation of any order with variable coefficients, Proceedings of the Royal Society of London, 24, 271-5
- WHITELEY, A.L., 1976, Interview with author.
- WILDES, K.L., 1975, A History of Electrical Engineering at MIT (unpublished notes)
- WIENER, N., 1953, Ex-Prodigy, (New York: Simon and Schuster)
- WIENER, N., 1956, I am a Mathematician: the later life of a prodigy, (Garden City, NJ: Doubleday)
- WOOD, A.B., 1965, From board of invention and research to Royal Naval Scientific Service, Journal of the Royal Naval Scientific Service, 20, 59
- ZABEL, R.M., HANCOX, R.R., 1934, The use of thyratrons for temperature control, Review of Scientific Instruments, 5, 2

Table  
Movement

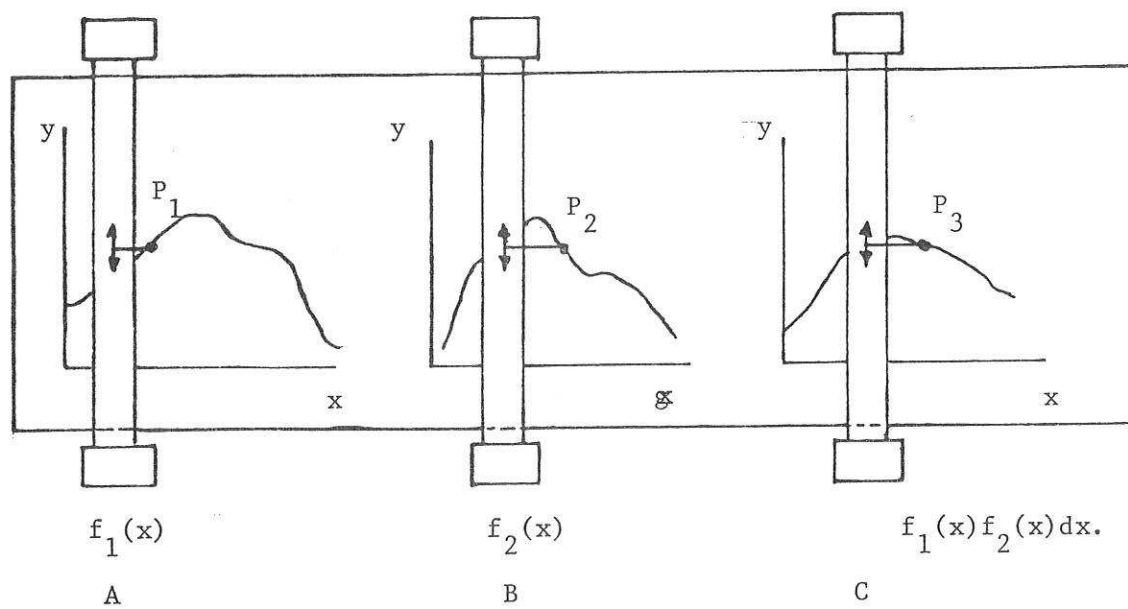


Figure 1.

C

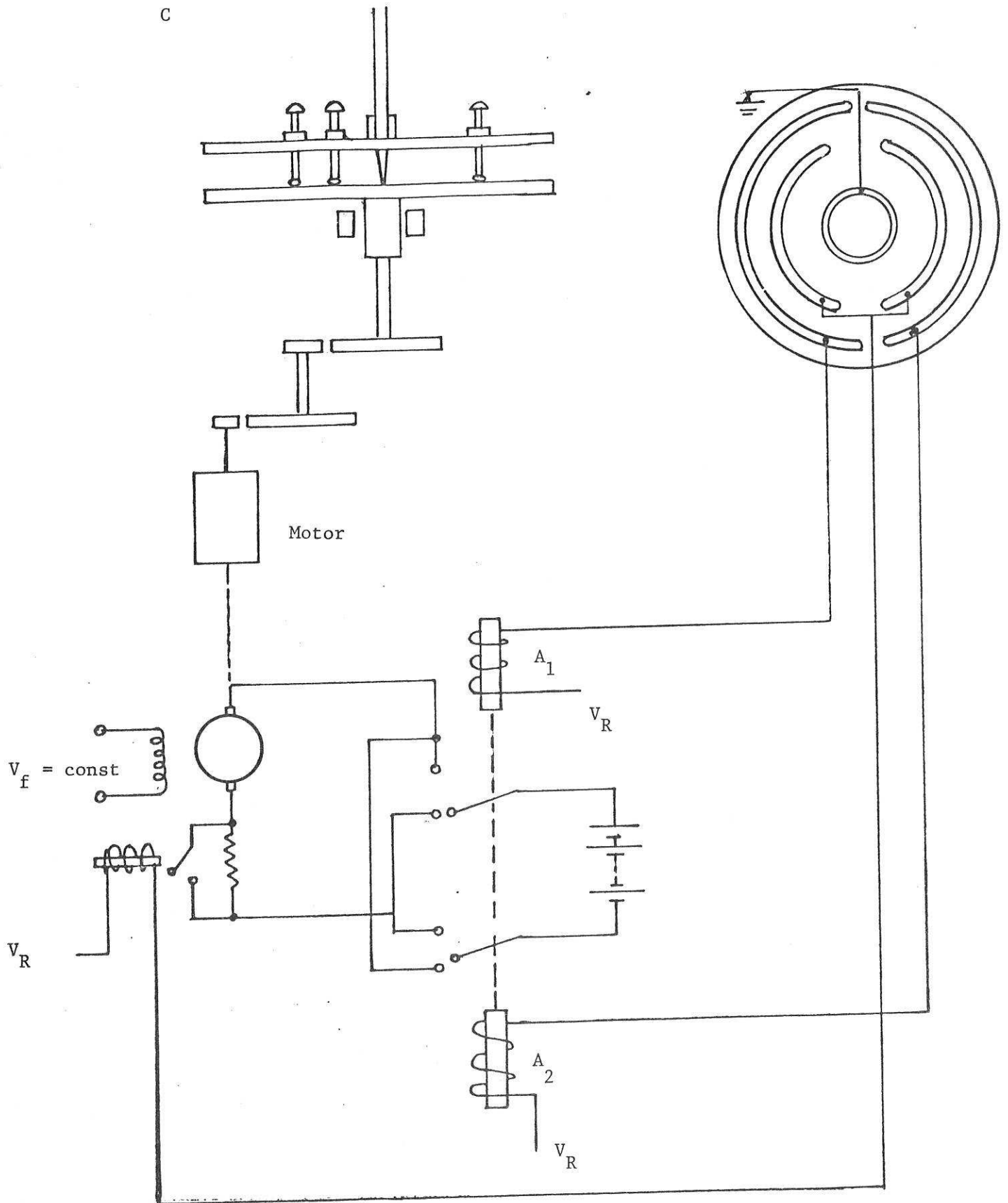


Figure 2.

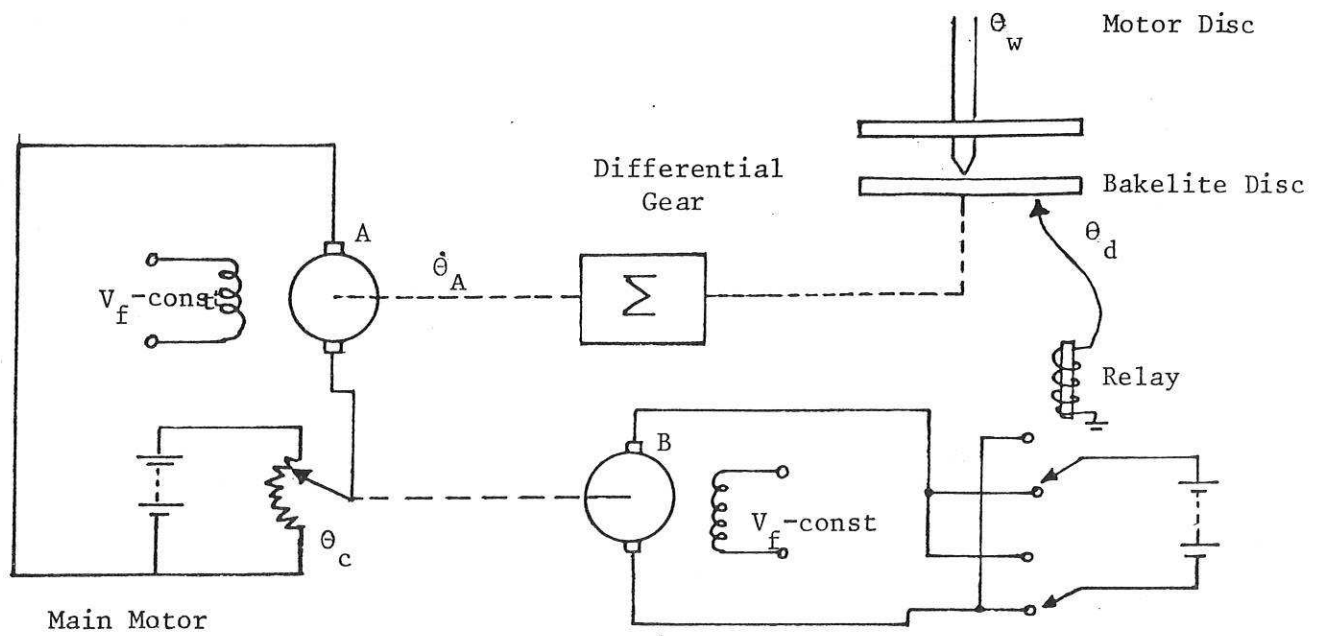


Figure 3.

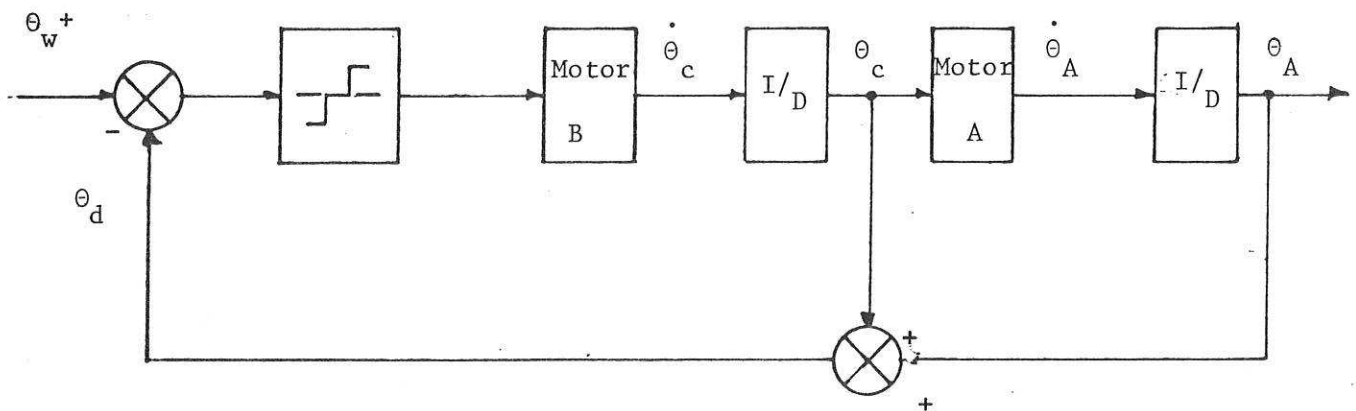


Figure 4





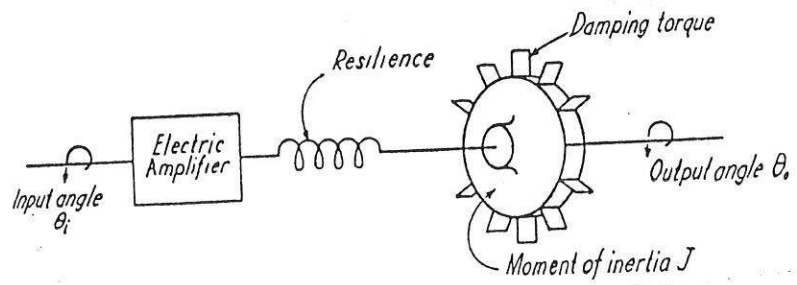


Figure 6.

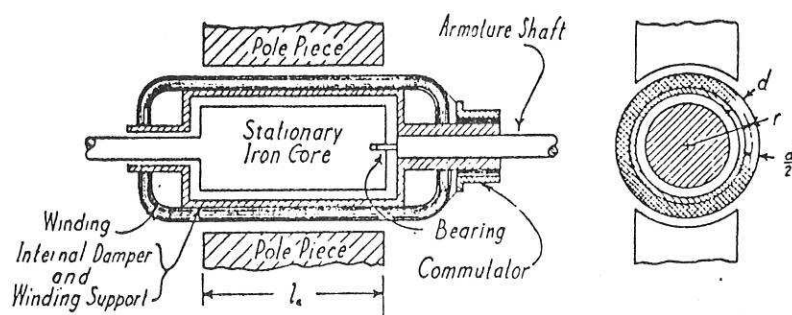


Figure 7.

	Stationary Input			Constant Velocity Input		
	Steady-state Oscillation		Average Lag Error	Steady-state Oscillation		Average Lag Error No Time Lag
	No Time Lag	Time Lag		No Time lag	Time lag	
A. No inactive zone						
1. Viscous friction...	Zero amplitude infinite frequency	Finite and function of time lag	Zero	Zero amplitude infinite frequency	Finite amplitude	Zero
2. Coulomb friction...	Zero amplitude infinite frequency	Finite and proportional to square of time lag	Zero	Indeterminate	Amplitude increases until postulated conditions fail to hold	Indeterminate
B. Finite inactive zone						
1. Viscous friction...	None	May or may not oscillate depending on parameters	$\pm$ half of inactive range with no time	Zero amplitude infinite frequency	Finite amplitude	$\pm$ half of inactive range
2. Coulomb friction...	None			Indeterminate	Amplitude increases until postulated conditions fail to hold	Indeterminate

Table 1.