Monograph:

Chotai, A. and Owens, D.H. (1984) Design of Mismatched Smith Predictors Using Plant Step Data. Research Report. Acse Report 263 . Dept of Automatic Control and System Engineering. University of Sheffield

Design of Mismatched Smith Predictors Using Plant Step Data

by

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Research Report No. 263

August 1984

To be presented at the IMC Symposium on Application of Multivariable System Techniques, Royal Naval Engineering College, Plymouth, October 1984.

This work was supported by SERC under grant GR/B/23250.

DESIGN OF MISMATCHED SMITH PREDICTORS USING PLANT STEP DATA

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Systematic design techniques for mismatched Smith control schemes are presented using the open-loop plant and model unit step responses. The approaches are all graphical in nature and are easily implemented in an interactive CAD mode.

1. Introduction

In the design of Smith control schemes (see, Owens and Raya [1], Chotai et al. [2] and Marshall [3]) for time-delay plant, it is common to include a mismatch between the plant and model dynamics either

(a) because the plant is high order and a reduced order model will reduce computational requirements or

(b) because the plant is subject to uncertainty in its structure, hence precluding the possibility of constructing an exact model.

In both cases, design will proceed assuming that the plant is modelled exactly by the model. On implementation however the plant/model mismatch will affect both the stability characteristics and the performance characteristics - possibly severely. This paper describes a systematic procedure for assessing the effect of mismatch by using information on the error in the modelling of the plant open-loop unit step response in (i) frequency-domain design and (ii) simulation-based design to produce estimates of stability characteristics and closed-loop performance deterioration.

The design procedures are illustrated by application to simple process plant using both time-domain and frequency-domain techniques.

2. Background Theory and Assumptions

Consider an ℓ-input/m-output linear, convolution plant expressed in the separable form $\tilde{T}G$ where the m-input/m-output element T represents output time-delays and the ℓ-input/m-output element G represents strictly proper delay-free dynamics. The destabilizing effect of the time-delay T can be offset with a considerable improvement in performance by the use of the Smith Predictor control scheme illustrated in Fig.1(a) where K represents a proper m-input/ℓ-output delay free controller and G_A and T_A represent convolution models of G and T respectively. The plant TG may not be known in detail but it is frequently the case that the plant step response matrix

$$Y(t) \triangleq \begin{pmatrix} Y_{11}(t) & \ldots\ldots & Y_{1\ell}(t) \\ \vdots & & \vdots \\ Y_{m1}(t) & \ldots\ldots & Y_{m\ell}(t) \end{pmatrix} \qquad \ldots(1)$$

is known from plant trials or complex model simulations. Here, $Y_{ij}(t)$ is the system response from zero initial conditions of the ith plant output to a unit step in the jth input, with all other inputs held to zero. Let $Y_A(t)$ be the step response matrix of the model T_AG_A and define the error matrix

$$E(t) \triangleq Y(t) - Y_A(t) \qquad \ldots(2)$$

The problems considered in this paper are (i) how the error E can be used to guarantee the stability of the implemented mismatch Smith scheme of Fig.1(a) if the predictor is designed off-line based on the assumption that the plant TG is equal to the model T_AG_A (see Fig.1(b)) and (ii) how the same information can be used to bound the deterioration in predicted transient performance due to the mismatch.

3. Stability and Performance Theory [2]

The stability theory is crucially dependent on the use of bounds on the modelling error as follows:

Proposition 3.1 (see ref [2])

$$\|T(s)G(s) - T_A(s)G_A(s)\|_P \leq N_\infty^P(E) \quad , \ \text{Re}s \geq 0 \qquad \ldots(3)$$

where the quantity $N_\infty^P(E)$ is the 'matrix total variation' of E,

$$N_\infty^P(E) \triangleq \begin{pmatrix} N_\infty(E_{11}) & \ldots\ldots & N_\infty(E_{1\ell}) \\ \vdots & & \vdots \\ N_\infty(E_{m1}) & \ldots\ldots & N_\infty(E_{m\ell}) \end{pmatrix} \qquad \ldots(4)$$

and $N_\infty(E_{ij})$ is the norm of E_{ij} regarded as a function of bounded variation on $[0,\infty)$.

(Remarks: (i) $\|M\|_P$ denotes the matrix obtained by replacing M_{ij} by $|M_{ij}|$, and

(ii) $A \leq B$ denotes the inequalities $A_{ij} \leq B_{ij}$ for all i,j).

This characterization leads to the result -

Theorem 3.1 (Frequency domain stability result (see [2]))

The mismatched Smith predictor of Fig.1(a) is input/output stable in the

L_2 sense if

(a) the Smith scheme of Fig.1(b) is stable,

(b) both G and G_A are stable, and

(c) the inequality

$$\lambda \triangleq \sup_{s \in D} r\left(\|(I_\ell + K(s)G_A(s))^{-1}K(s)\|_{p}N_\infty^{P}(E)\right) < 1 \qquad \ldots(5)$$

is satisfied, where D is the usual Nyquist contour in the complex plane, r(M) denotes the spectral radius of M.

The result given above can be interpreted as providing lower estimates of the largest permissible mismatch that retains stability. The special case (which includes the scalar case) of the above result has a useful graphical interpretation similar to that of the inverse Nyquist array technique [4].

If $m = \ell$ and G_A and K are diagonal of the form

$$G_A(s) = \mathrm{diag}\{g_j(s)\}_{1\le j\le m}, \quad K(s) = \mathrm{diag}\{k_j(s)\}_{1\le j\le m} \qquad \ldots(6)$$

then the above result remains valid with condition (c) replaced by the two conditions:

(i) The inequality

$$\limsup_{\substack{\mathrm{Re}\ s\ge 0 \\ |s|\to\infty}} |k_j(s)| < \frac{1}{\sum_{k=1}^{m} N_\infty(E_{jk})}$$

is satisfied for $1\le j\le m$ ...(7)

and

(ii) the 'confidence band' generated by plotting the inverse Nyquist loci of $g_j(s)k_j(s)$, $1\le j\le m$, for $s = i\omega$, $\omega\ge 0$, with superimposed 'confidence circles' at each point of radius

$$r_j(i\omega) \triangleq |g_i^{-1}(i\omega)| \sum_{k=1}^{m} N_\infty(E_{jk}) \qquad \ldots(8)$$

does not contain or touch the (-1,0) point of the complex plane. Note that a graphical interpretation of condition (ii) is given in Fig.2.

Stability can also be approached using simulation methods:

Theorem 3.2 (Time domain stability result (see [2])

Suppose that

(a) the Smith control scheme of Fig.1(b) is stable,

(b) both G and G_A are stable and that simulations are undertaken to reliably calculate the matrix

$$W_A(t) \triangleq [W_A^{(1)}(t),\ldots,W_A^{(\ell)}(t)] \qquad \ldots(9)$$

where $W_A^{(j)}(t)$ is the response from zero initial conditions of the 'delay-free' system $(I+KG_A)^{-1}K$ to the input vector $E^{(j)}(t)$. ($E^{(j)}(t)$ is the jth column of $E(t)$). Then the Smith scheme of Fig.1(a) is input/output stable in the L_∞ sense if the following inequality holds:

$$r(N_\infty^P(W_A)) < 1 \qquad \text{...(10)}$$

An advantage of simulation based methods is the ability to predict performance deterioration:

Theorem 3.3 (Input performance assessment result due to mismatch (see [2])

Suppose that the time domain stability result holds and that

(a) $u_A(t)$ is the input response of the Smith scheme of Fig.1(b) to a step input demand $r(t) = \beta$, $t \geq 0$,

(b) $\xi(t)$ is the $\ell x1$ vector computed by the convolution

$$\xi(t) = -(\int_o^t W_A(t-t')H)t')dt')\beta \qquad \text{...(11)}$$

where $H(t)$ is the impulse response matrix of $(I_\ell+KG_A)^{-1}K$, and

(c) $$\varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \vdots \\ \varepsilon_\ell(t) \end{bmatrix} \triangleq (I-P_t)^{-1}P_t \sup_{0\leq t'\leq t} \|\xi(t')\|_P \qquad \text{...(12)}$$

where $P_t = N_t^P(W_A)$ is the matrix total variation of W_A on the interval $[0,t]$ (see [5]).

Then, the real input response $u(t)$ of the Smith scheme of Fig.1(a) to the step demand $r(t)$ from zero initial conditions satisfies the bound

$$|u_j(t) - u_j^{(1)}(t)| \leq \varepsilon_j(t) \quad , \quad 1\leq j\leq \ell \quad , \quad t\geq 0 \qquad \text{...(13)}$$

where $u^{(1)}(t) = u_A(t)+\xi(t)$ is the first order correction to the approximate response $u_A(t)$.

The graphical intepretation of (13) is simply that $u_j(t)$ lies in the region between the curves $u_j^{(1)}(t)\pm\varepsilon_j(t)$. Both $u^{(1)}$ and ε are easily evaluated by simulation of low order feedback systems generated by K and G_A.

Theorem 3.4 (Output performance assessment result due to mismatch (see [2])

With the above result holding and also that TG is stable, then

$$\|y(t)-y^{(1)}(t)\|_P \leq N_t^P(Y)\varepsilon(t)+N_t^P(E) \sup_{0\leq t'\leq t} \|u^{(1)}(t')\|_P \qquad \text{...(14)}$$

where $y^{(1)}$ is the response of T_AG_A from zero initial conditions to $u^{(1)}(t)$.

A better bound is however available in the case of scalar systems ($m = \ell = 1$) and is stated below:

Theorem 3.5

Suppose that $m = \ell = 1$, and that the time domain stability result is satisfied, then the controller K will stabilise the Smith scheme of Fig.1(a) and the response $y(t)$ from zero initial conditions to a unit step demand $r(t)$ satisfies the bound

$$|y(t) - y^{(1)}(t)| \leq \varepsilon(t) \quad , \quad t \geq 0 \qquad \ldots(15)$$

where

$$\varepsilon(t) \triangleq \frac{N_t(W_A)}{1-N_t(W_A)} \max_{0 \leq t' \leq t} |\eta(t')| \quad , \quad t \geq 0 \qquad \ldots(16)$$

$$y^{(1)}(t) \triangleq y_A(t) + \eta(t) \quad , \quad t \geq 0 \qquad \ldots(17)$$

and $\eta(t)$ is the response from zero initial conditions of the system $I-(I+G_AK)^{-1}G_AKT_A$ to the input $W_A(t)$.

4. An Illustrative Scalar Example

Consider the single-input/single-output system TG described by

$$G(s) = \frac{1}{(s+1)^3} \quad \text{and} \quad T(s) = e^{-2.0s} \qquad \ldots(18)$$

The model T_AG_A of the form

$$G_A(s) = \frac{1}{1+2.4s} \quad \text{and} \quad T_A(s) = e^{-2.7s} \qquad \ldots(19)$$

was fitted. The open-loop step responses of the plant TG and the model T_AG_A are shown in Fig.3 together with modelling error $E(t)$. The total variation of E is obtained graphically as

$$N_\infty^P(E) = N_\infty(E) = 0.36 \qquad \ldots(20)$$

Using the frequency domain technique and a P+I controller of the form

$$K(s) = k_1 + k_2 s^{-1} \qquad \ldots(21)$$

with $k_1 = 1.5$, $k_2 = 1$, we can check the stability of the mismatched Smith predictor control scheme of Fig.1(a) by checking conditions (7) and (8). Condition (7) boils down to

$$|k_1| < 2.78 \qquad \ldots(22)$$

which is clearly satisfied, whilst condition (8) is checked by plotting the confidence band as shown in Fig.4. As the (-1,0) point does not lie in or on the confidence band at any frequency, the stability of mis-match Smith scheme is guaranteed. This is verified by examination of the closed-loop step responses illustrated in Fig.5.

Turning our attention now to the problem of prediction of the error $y-y_A$ using simulation-based technique. Applying theorem 3.2, to the above scalar example with the same model and the controller, the response $W_A(t)$ was computed to be as in Fig.6. Graphical analysis of this response leads to the conclusion that $N_\infty(W_A) = 0.59<1$, hence verifying the stability prediction. The deterioration in output characteristics is obtained by the use of theorem 3.5 in the scalar case. The correction term $\eta(t)$ (the response from zero initial conditions of the system $I-(I+G_AK)^{-1}G_AKT_A$ to the input $W_A(t)$) is shown in Fig.7 and the bounds $y^{(1)}\pm\varepsilon(t)$ together with the responses y and y_A are illustrated in Fig.8. Note that the performance predicted by the ideal Smith scheme was a reasonable indicator of the performance to be expected of the mismatched Smith scheme.

5. Conclusions

The paper has outlined the principles underlying a new approach for the design of multivariable Smith predictors using plant step data in the present of the plant/model mismatch. The paper has concentrated how the step response modelling error $E = Y-Y_A$ can be used in producing both graphical frequency-domain and simulation-based time-domain methods to assess the stability and closed-loop performance for mismatch predictor control scheme. The details of theory are given in refs [2] and [5] and, at no stage in the design process, is an accurate plant model required. The techniques have been illustrated with one simple scalar example and more examples are given in ref [6].

Acknowledgement

This work was supported by SERC under grant No. GR/B/23250.

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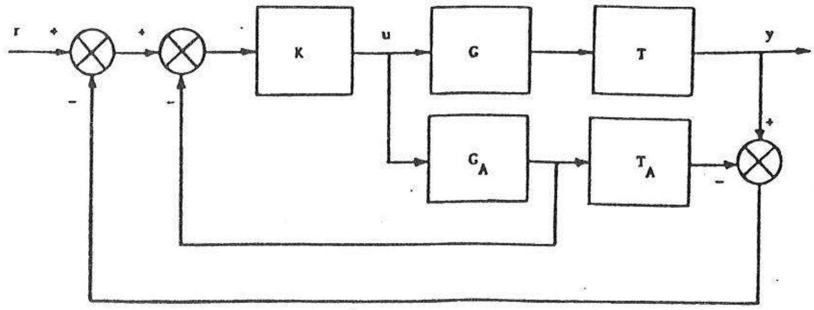

Fig.1(a) Smith Predictor Scheme

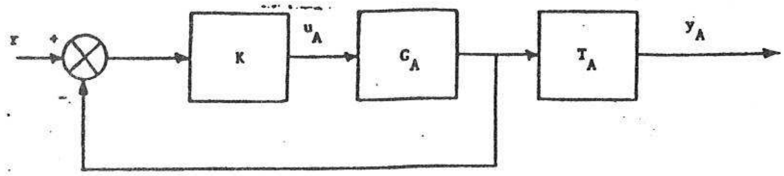

Fig.1(b) Smith Scheme with Zero Mismatch

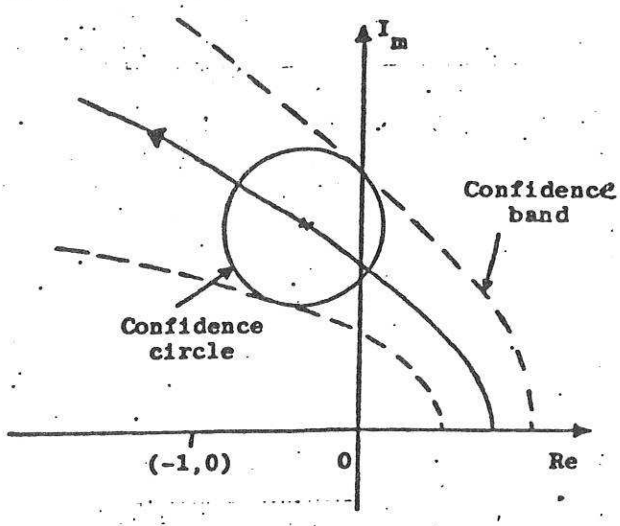

Fig.2 Confidence Band and Confidence Circle

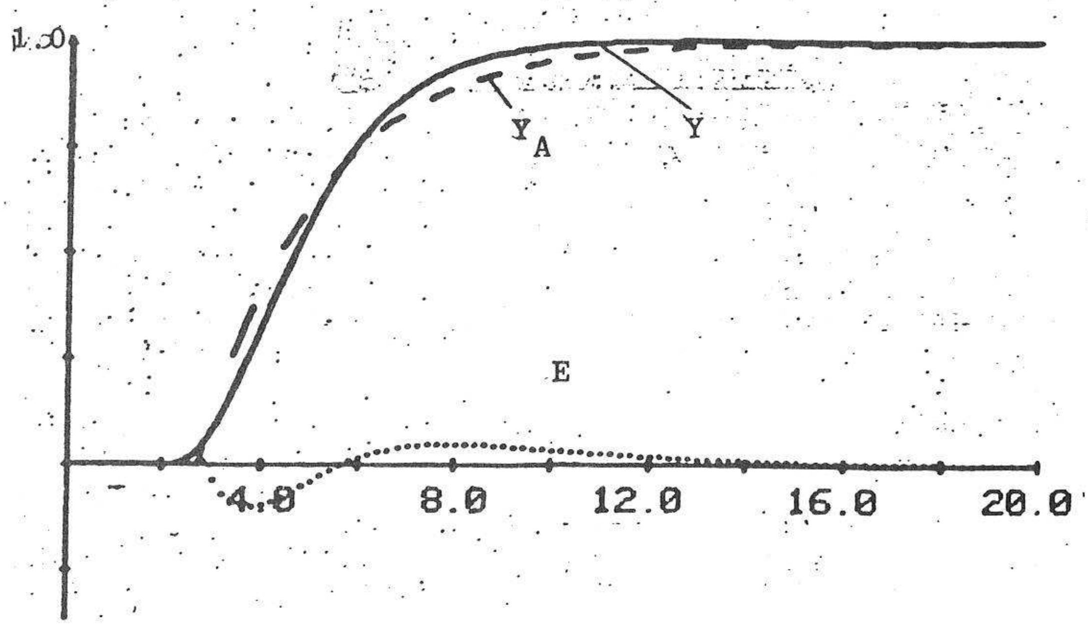

Fig.3 Plant and Model Open-loop Step Responses

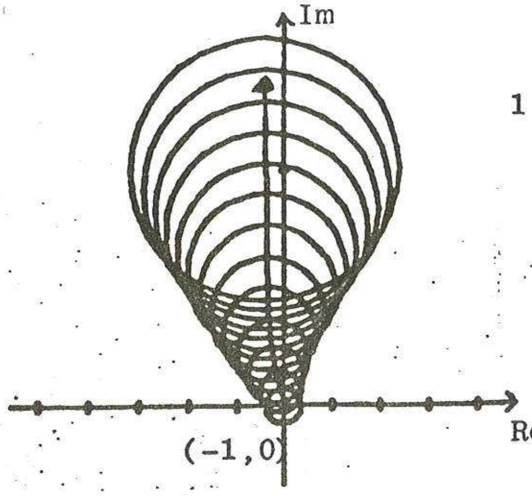

Fig.4 Stability Check using Confidence Band

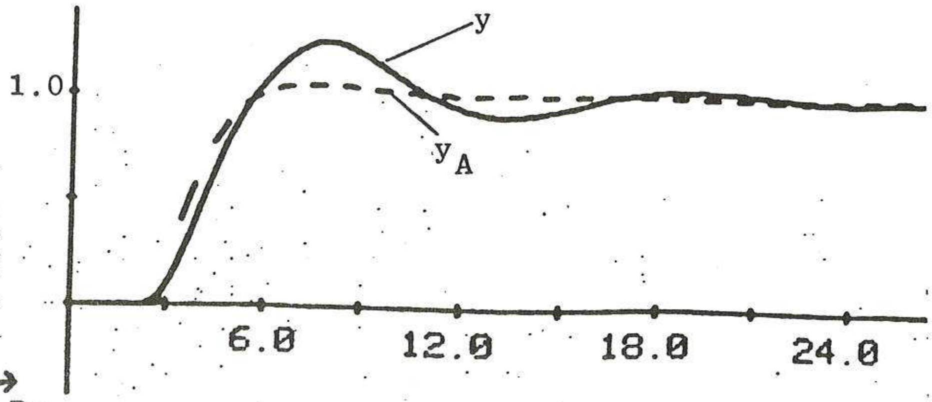

Fig.5 Closed-loop Step Responses

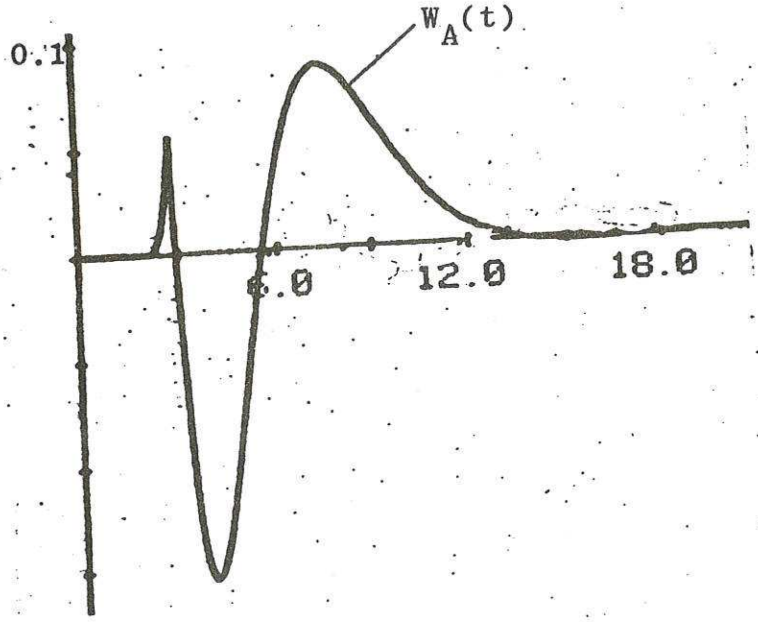

Fig.6 The Function $W_A(t)$

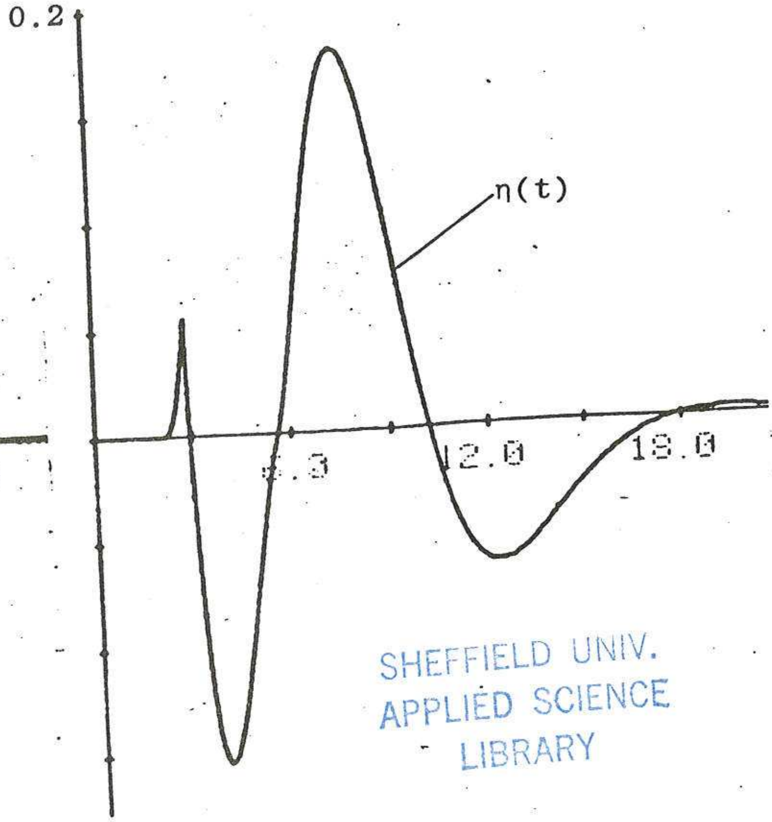

Fig.7 The Function $\eta(t)$

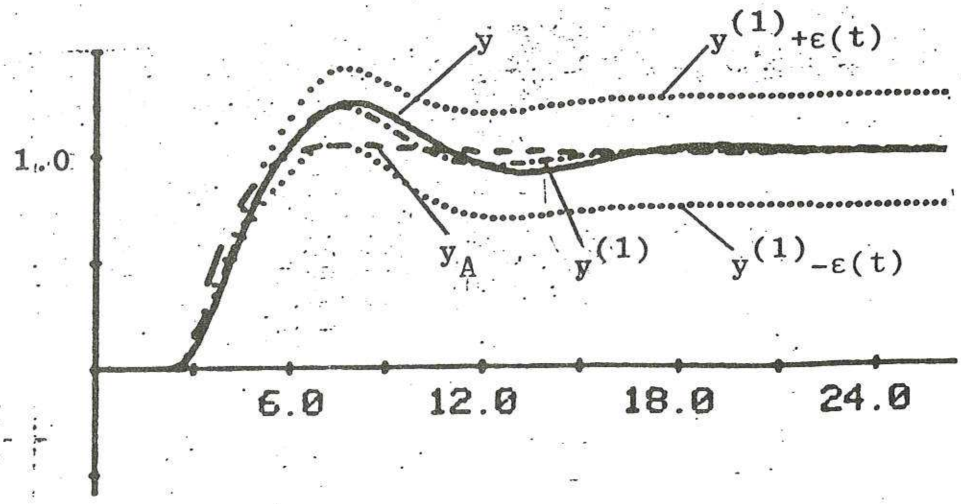

Fig.8 Closed-loop Responses with Bounds on Performance Deterioration