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A SIMULATION AID TO GAIN ESTIMATES FOR

ROBUST TUNING REGULATORS

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ABSTRACT

Gain estimates for Davison's robust tuning regulator are shown to be directly computable from plant open-loop step data using low-order simulation techniques. At no stage of the procedure is a detailed model of plant dynamics required.

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1. INTRODUCTION

The robust tuning regulator originally proposed by Davison [1] provides a simple but effective way of generating integrating process controllers without the need to have a detailed model of the plant available. More precisely, given the m-input/m-output stable multivariable plant described by the state-space model

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \qquad (1)$$

in R^n, the integrating unity negative output feedback controller with transfer function matrix

$$K(s) = G^{-1}(o)\,\frac{\varepsilon}{s} \qquad (2)$$

will stabilize the plant, reject constant disturbances and track step set-point changes for gains ε in some (non-empty) range $o < \varepsilon < \varepsilon^*$ provided that $G(o)$ is nonsingular. The matrix $G(o)$ can be computed directly from the plant transfer function matrix $G(s) = C(sI-A)^{-1}B$ (if available) or estimated from plant step data by $G(o) = Y(\infty)$ where $Y(t)$ is the mxm plant step response matrix with elements $Y_{ij}(t)$ equal to the response from zero initial conditions of the output $y_i(t)$ to a unit step input in $u_j(t)$.

Davison's original technique used only minimal plant information in the form of the steady state data $G(o) = Y(\infty)$ and as a consequence the actual choice of ε (and estimation of ε^*) must be left to the on-line tuning stage. It is impossible to estimate ε^* without incorporating more plant information! It is the purpose of this note to extend Davison's original procedure to generate a lower bound $\varepsilon_o^* > o$ of ε^* by the application of simulation based data analysis techniques to the plant step data $Y(t)$ similar to those described by the authors in ref [2] using approximate models of the plant step response characteristics. Although models of arbitrary desired complexity could be used [2], attention is focussed entirely on the simplest case to retain the spirit of Davison's work and to produce computational algorithms that can be implemented successfully on only limited computing facilities. The availability of an estimate $\varepsilon_o^* < \varepsilon^*$ could be of considerable value at the tuning stage in providing an initial gain range $o < \varepsilon < \varepsilon_o^*$ in which stability is ensured and in putting the commissioning engineer 'in the right ball-park'.

2. GAIN ESTIMATES FROM SIMULATION DATA

The control design philosophy described in ref [2] is to design the controls on the basis of a simple, low-order approximate stable plant model $G_A(s)$ in such a manner that the resultant controller is guaranteed to stabilize the real plant. The basic result is summarized as follows:

Lemma 1 (Theorem 4 in [2]): Let G_A have step response matrix $Y_A(t)$ and define the 'modelling error', $t \geq o$,

$$E(t) = Y(t) - Y_A(t) \triangleq [E^{(1)}(t), \ldots, E^{(m)}(t)] \qquad (3)$$

with columns $E^{(j)}(t)$, $1 \leq j \leq m$. Suppose that K stabilizes G_A and that simulations are undertaken to reliably calculate the matrix

$$W_A(t) \triangleq \left[W_A^{(1)}(t), \ldots., W_A^{(m)}(t)\right] \quad , t \geq o \qquad (4)$$

where $W_A^{(j)}(t)$ is the response from zero initial conditions of the system $(I + K\, G_A)^{-1}K$ to the input vector $E^{(j)}(t)$. Then the controller K will stabilize the real plant G if

(i) GK is both stabilizable and detectable and

(ii) $\gamma < 1$ (5)

where γ is any available upper bound for $r(N_\infty^P(W_A))$.

(Remarks: (a) $r(M)$ denotes the spectral radius of the matrix M,

(b) $N_\infty^P(W_A)$ denotes the matrix with $(i,j)^{th}$ element equal to the total variation [2] of $(W_A^{(j)}(t))_i$, which is easily computed [2] by graphical analysis of $W_A(t)$ and

(c) note that the stability criterion depends only upon the step error data $E(t)$ and hence a detailed plant model is not required for application of the result).

To apply this result to the robust tuning regulator problem, choose an approximate model of plant dynamics of the form

$$G_A(s) \triangleq G_o \text{ diag } \left\{\frac{1}{1+sT_j}\right\}_{1\leq j\leq m} \qquad (6)$$

where G_o is a (conveniently structured) nonsingular estimate of the plant steady state characteristic $G(o)$ and the time constant $T_j > 0$ is chosen to be representative of the open-loop responses to a unit step input in channel j. It follows that

$$Y_A(t) = G_o \text{ diag } \{(1 - e^{-t/T_j})\}_{1\leq j\leq m} \qquad (7)$$

Given a choice of G_A, knowledge of $Y(t)$ enables the computation of $E(t)$ by direct differencing. The following result can then be applied to characterize a set of permissible tuning regulators:

Theorem 1: With the above notation, let

$$h_j^{\varepsilon}(s) = \frac{\varepsilon(1+sT_j)}{s(1+sT_j)+\varepsilon} \quad , \quad 1 \leq j \leq m \tag{8}$$

and

$$E^{(o)}(t) = G_o^{-1}E(t) \quad , \quad t \geq o \tag{9}$$

Then the unity negative feedback integrating control scheme with controller

$$K(s) = \frac{\varepsilon}{s} G_o^{-1} \tag{10}$$

will stabilize the plant G provided that G(o) is nonsingular, $\varepsilon > o$ and

$$\gamma_{\varepsilon} < 1 \tag{11}$$

where γ_{ε} is any conveniently computed upper bound for the spectral radius of $N_{\infty}^{P}(W_A^{\varepsilon})$ and $W_A^{\varepsilon}(t)$ is the mxm matrix with $(i,j)^{th}$ element equal to the response of the system h_i^{ε} from zero initial conditions to the 'normalized' error data $E_{ij}^{(o)}(t)$, $t \geq o$. (Remark: the assumed controller coincides with Davisons control of equation (2) if we set $G_o = G(o)$).

Proof: We simply have to verify that the conditions of lemma 1 are satisfied. It is easily verified that K stabilizes G_A for all choices of $\varepsilon > o$. Also GK is stabilizable and detectable from the assumption that the plant is stable and G(o) is nonsingular and the arguments to be found in Davison [1]. Finally, note that

$$(I_m + K(s)G_A(s))^{-1}K(s) = \text{diag}\,\{h_j(s)\}_{1\leq j\leq m}G_o^{-1} \tag{12}$$

and hence that W_A is just the response from zero initial conditions of the system diag $\{h_j^{\varepsilon}(s)\}_{1\leq j\leq m}$ to the normalized error $E^{(o)}(t)$. This completes the proof.

In practical terms the result provides, for a given value of $\varepsilon > o$, an off-line means of checking the stability of the implemented scheme by using the step data directly and without the need to use or construct a detailed plant model. The actual computations involved consist of the normalization operation (9) and m^2 simulations of the second order systems (8). Both operations can easily be undertaken even with limited computing facilities. The final requirement is to check the spectral radius condition (11). The best answers obtainable from the theory are obtained by choosing $\gamma_\varepsilon = r(N_\infty^P(W_A))$ but more conservative estimates can be used to eliminate the need for eigenvalue calculations e.g., estimates obtained from eigenvalue estimation theorems such as Gershgorin's theorem.

For any given value of ε, the computations described will either predict stability or will be inconclusive (when $\gamma_\varepsilon \geq 1$). Note however that condition (11) is automatically satisfied for all small enough positive gains ε provided that $E(\infty)$ is small enough and hence that the theory predicts stability in a non-empty range $o < \varepsilon < \varepsilon_o^*$ where

$$\varepsilon_o^* = \min \{\varepsilon : \varepsilon > o, \gamma_\varepsilon = 1\} \tag{13}$$

The actual computation of ε_o^* can be undertaken rapidly and efficiently by elementary search techniques. The implicit requirement that $E(\infty)$ is 'small' is easily guaranteed by choosing, for example, $G_o = G(o)$ when $E(\infty) = 0$.

3. ILLUSTRATIVE EXAMPLE

Consider the boiler-furnace system described by Rosenbrock [3] with transfer function matrix

$$G(s) = \begin{pmatrix} \frac{1}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1}{1+4s} \end{pmatrix} \tag{14}$$

Note that $G(o)$ is nonsingular with

$$G^{-1}(o) = \begin{pmatrix} 1.75 & -1.21 & -0.16 & 0.17 \\ -0.98 & 1.87 & -0.23 & -0.32 \\ -0.32 & -0.23 & 1.87 & -0.98 \\ 0.17 & -0.16 & -1.21 & 1.75 \end{pmatrix} \tag{15}$$

Examination of equation (14) (or its step response matrix) suggests the use of the approximate model $G_A(s) = \frac{1}{1+4s}\, G(o)$ or $G_o = G(o)$ and $T_j = 4.0,\ 1 \leq j \leq 4$. The corresponding step response error is given explicitly by

$$E(t) = (G(o) - I_4)(e^{-t/4} - e^{-t/5}) \tag{16}$$

or, after normalization,

$$E^{(o)}(t) = (G^{-1}(o) - I_4)(e^{-t/5} - e^{-t/4}) \tag{17}$$

Clearly, we have

$$h_j^{\varepsilon}(s) = \frac{\varepsilon(1+4s)}{4s^2 + s + \varepsilon}\quad,\ j = 1,2,3,4 \tag{18}$$

and hence

$$W_A^{\varepsilon}(t) = (G^{-1}(o) - I_4)\ \psi_{\varepsilon}(t)$$

where

$$\psi_{\varepsilon}(t) = \mathcal{L}^{-1}\left(\frac{\varepsilon}{(4s^2+s+\varepsilon)(1+5s)}\right) \tag{19}$$

Choosing

$$\gamma'_\varepsilon = N_\infty(\psi_\varepsilon) \; ||G^{-1}(o) - I_4|| \tag{20}$$

where $N_\infty(\psi_\varepsilon)$ is the total variation of ψ_ε on $[o,\infty]$ and $||M||$ denotes the norm $\max\limits_{1 \le j \le m} \sum\limits_i |M_{ji}|$ of an mxm matrix M, the condition $\gamma_\varepsilon < 1$ corresponds to the condition $N_\infty(\psi_\varepsilon) < 1/||G^{-1}(o) - I_4|| = 0.416$, and ε_o^* can be computed by finding the smallest $\varepsilon > o$ such that $N_\infty(\psi_\varepsilon) = 0.416$. The results of a simple numerical search are summarized in Fig. 1, which shows that $\varepsilon_o^* \simeq 0.78$. Choosing $\varepsilon = 0.1$, the plant closed-loop responses to a unit step demand in y_1 as shown in Fig. 2 and are seen to be highly acceptable with notably small interaction effects.

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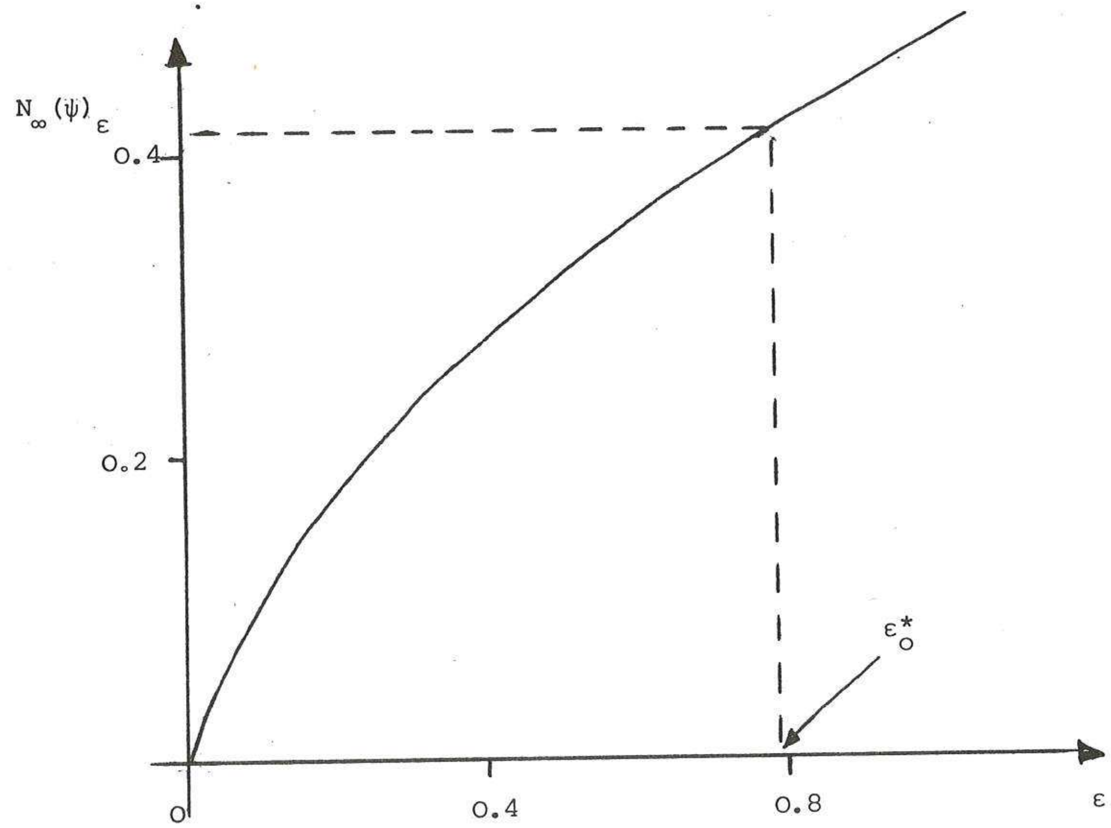

Fig. 1 Estimating ε_o^*

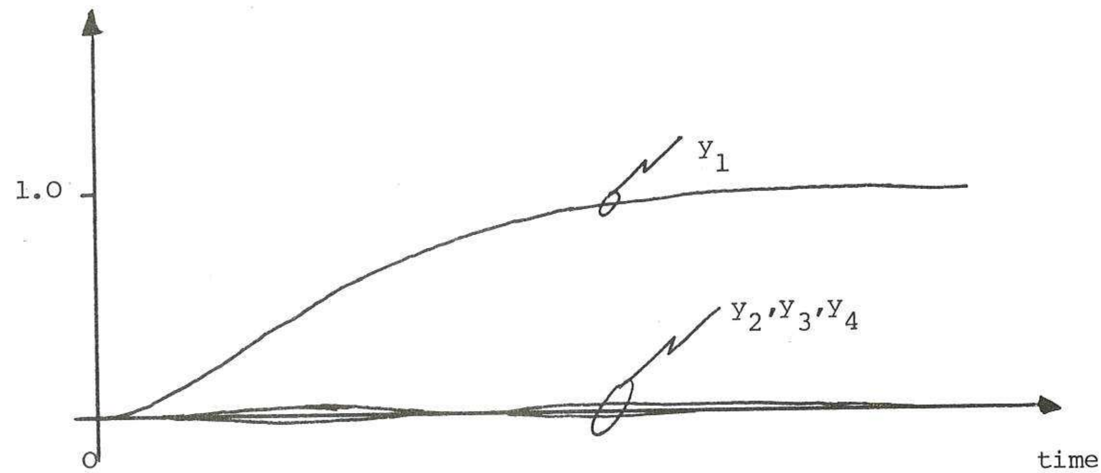

Fig. 2 Closed-loop response to a unit step demand in y_1

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