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IDENTIFICATION OF A NONLINEAR DIFFERENCE EQUATION MODEL OF AN INDUSTRIAL DIESEL GENERATOR

BY

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1. INTRODUCTION

This report describes the identification of a nonlinear difference equation model of a 6996 bhp 12 cylinder industrial diesel generator. The analysis of the data, recorded from experimental trials on the diesel generator, is initially pursued assuming that the relationship between the input (rack position) and the output (engine torque) are linearly related. As the analysis proceeds this assumption is shown to be in error and nonlinear terms are introduced into the model representation. The inclusion of nonlinear terms in the model is shown to provide a representative model of the diesel generator which fits the available data well and which is shown to produce accurate predictions of the system output response over a different data set.

The report begins with a brief description of the nonlinear identification and structure detection techniques which are used in the analysis. This is followed in Section 3.1 with a description of the raw data set. A summary of the linear analysis and results is given in Section 3.2 and this is followed by a detailed description of the nonlinear identification and the final model.

2. IDENTIFICATION TECHNIQUES FOR NONLINEAR SYSTEMS

There are basically three distinct approaches to the identification of nonlinear systems[1] and these are based on functional series methods[1], block structured systems[2] and parameter estimation methods applied to differential or difference equation models[4,5]. Each of the three approaches has distinct features and characteristics which arise largely from the mathematical description which is used to characterise the system rather than the method of identification which is applied. Exhaustive surveys and descriptions of the algorithms which have been developed for these three categories of system description can be found in the literature[1,2,4,5]. The practicalities of the methods can however be judged by considering the identification of a simple non-linear system consisting of a linear dynamic subsystem in cascade with a static nonlinearity. Identification of this system using functional series methods would require the estimation of at least 500 and possibly many more parameters, or typically 40 parameters using a block structured algorithm. In comparison estimation of typically less than 10 parameters in a nonlinear difference equation model would be adequate. It is largely for these reasons together with the fact that difference equation models often arise naturally from physical and chemical laws that these models will be used as the basis for the identification of the diesel generator.

2.1 Testing for Nonlinearities

Before nonlinear identification routines are applied the experimenter should attempt to determine if the process under test exhibits nonlinear characteristics which warrant a nonlinear model[3]. Whenever the input $u(t) + b$, $\overline{u(t)} = 0$, $b \neq 0$ is applied to a system, the system cannot be linear if $\overline{y_b(t)} \neq \overline{y(t)}$ where $\overline{y_b(t)}$ and $\overline{y(t)}$ are the mean levels of the system output for the inputs b(i.e. $u(t) = 0$ and $u(t)+b$ respectively. Alternatively, if the third order moments of the input are zero $[\text{i.e. } E(u(t)u(t+\tau_1)u(t+\tau_2)) = 0 \ \forall \tau_1, \tau_2]$ and all even order moments exist (i.e., sine wave, gaussian, ternary sequence etc) then the process is linear iff[3]

$$\phi_{y'y'^2}(\sigma) = E[y'(t+\sigma)(y'(t))^2] = 0 \ \forall \ \sigma \qquad (1)$$

where the dash indicates that the mean level has been subtracted The test will distinguish between additive noise corruption of the measurements and distortion due to nonlinear effects providing the input and noise are independent.

2.2 Parameter estimation based on the NARMAX model

If the system is linear then it is finitely realizable and can be represented by the linear difference equation model[7]

$$y(t) = \sum_{i=1}^{n_y} (a_i y(t-i)) + \sum_{i=1}^{n_u} (b_i u(t-i)) \quad (2)$$

if the Hankel matrix of the system has finite rank. When the system is nonlinear a similar representaion can be derived by considering the observability of nonlinear systems and utilizing results from automata theory to yield the nonlinear difference equation model[4,5,6]

$$y(t) = F^*[y(t-1),...y(t-n_y),y(t-1),...u(t-n_u)] \quad (3)$$

where $F^*[\cdot]$ is some nonlinear function of $u(\cdot)$ and $y(\cdot)$.the extension to multivariable systems and conditions for the existence of such a model are rigorously defined elsewhere[6]. The Hammerstein, Wiener, bilinear, Volterra and other well known nonlinear models can be shown to be special cases of equation (3).

A similar representation for nonlinear stochastic systems can be derived by considering input-output maps based on conditional probability density functions to yield the model[5]

$$z(t) = F[z(t-1),...z(t-n_z), u(t-1),...u(t-n_u), \varepsilon(t-1),...\varepsilon(t-n_\varepsilon)] + \varepsilon(t) \quad (4)$$

where $\varepsilon(t)$ is the prediction error. This model will be referred to as a <u>N</u>onlinear <u>A</u>uto<u>R</u>egressive <u>M</u>oving <u>A</u>verage model with e<u>X</u>ogenous inputs or NARMAX model[4,5,6].

A NARMAX model with first order dynamics expanded as a second order polynomial nonlinearity would for example be represented as

$$\begin{aligned} y(t) &= F_2[y(t-1),u(t-1)] \\ &= C_1y(t-1) + C_2u(t-1) + C_{11}y^2(t-1) + C_{12}y(t-1)u(t-1) \\ &\quad + C_{22}u^2(t-1) \end{aligned} \quad (5)$$

Assuming that the output measurements are corrupted by additive noise

$$z(t) = y(t) + e(t) \tag{6}$$

gives the input-output model

$$z(t) = C_1 z(t-1)+C_2 u(t-1)+C_{11} z^2(t-1)+C_{12} z(t-1)u(t-1)$$
$$+ C_{22} u^2(t-1)+e(t)-C_1 e(t-1)-2C_{11} z(t-1)e(t-1)$$
$$+ C_{11} e^2(t-1)-C_{12} e(t-1)u(t-1) \tag{7}$$

Because the NARMAX model maps the past input and output into the present output multiplicative noise terms are induced in the model even though the noise was additive at the output. In general the noise may enter the system internally and because the system is nonlinear it will not always be possible to translate this to be additive at the output. This situation will again result in multiplicative noise terms in the NARMAX model with the added complication that the noise source and prediction error will not in general be equal. Since most of the parameter estimation techniques derived for linear systems assume that the noise is independent of the input, biased estimates result when they are applied to nonlinear systems equation (4).

The recursive extended least squares (RELS) algorithm can however be readily adapted to the NARMAX model, by defining the following vectors

$$Q(t) = [z(t-1), u(t-1), z^2(t-1), z(t-1)u(t-1), u^2(t-1), \varepsilon(t-1),$$
$$\varepsilon(t-1)z(t-1), u(t-1)\varepsilon(t-1), \varepsilon^2(t-1)]^T$$
$$\hat{\theta} = [\hat{C}_1, \hat{C}_2 \ldots \hat{C}_s]^T$$
$$\varepsilon(t+1) = z(t+1) - Q(t+1)^T \hat{\theta}(t) \tag{8}$$

for the model of equation (7) for example. With these definitions the standard RELS algorithm[7] can be applied to yield unbiased parameter estimates. The development of a recursive maximum likelihood algorithm (RML) is more involved and requires a complete derivation by working backwards from known conditions of convergence[5]. The major disadvantage of both these algorithms when applied to nonlinear systems is the need to include prediction error terms in the estimation vector. It can be shown that instrumental variables (RIV) will yield unbiased estimates providing the noise terms in the NARMAX model can be represented as a purely linear map[8]. This restriction can be widened slightly by employing a new suboptimal least squares (SOLS) routine[8] based on the model

$$z(t) = F'\left[\hat{y}(t-1),\ldots\hat{y}(t-n_y),u(t-1),\ldots u(t-n_u)\right]+\varepsilon(t) \qquad (9)$$

where $\hat{y}(t-1)$ represents the predicted output. The algorithm will yield unbiased estimates whenever the noise is additive at the output.

The direct application of a maximum likelihood algorithm is not possible because in general the prediction errors will not have a Gaussian distribution. However, by considering the loss function

$$J(\theta) = \frac{1}{2N} \log_e \det \sum_{t=1}^{N} \varepsilon(t;\theta)\varepsilon(t;\theta)^T \qquad (10)$$

it can be shown that the prediction error estimates obtained by minimising equation (10) have very similar asymptotic properties to the maximum likelihood estimates even when $\varepsilon(t)$ is non-gaussian[7]. A prediction error algorithm has been developed for the NARMAX model based on this result. This together with the RELS, RML, RIV and SOLS routines have been augmented with a stepwise regression algorithm, a likelihood ratio test and Akaike tests to detect the correct model structure prior to final estimation.

2.3 Model Validity Tests

Whichever model formulation or identification algorithm is implemented it is important to test that the identified model does adequately describe the data set[3]. When the system is nonlinear the residuals $\xi(k)$ should be unpredictable from all linear and nonlinear combinations of past inputs and outputs and this condition will hold iff

$$\phi_{\xi\xi}(\tau) = \delta(\tau)$$
$$\phi_{u\xi}(\tau) = 0 \;\forall\; \tau \qquad (11)$$
$$\phi_{\xi\xi u}(\tau) = E[\xi(t)\xi(t-1-\tau)u(t-1-\tau)] = 0 \;\forall\; \tau \geq 0$$

Notice that for nonlinear systems the traditional linear tests $\phi_{\xi\xi}(\cdot)$ and $\phi_{u\xi}(\cdot)$ are not sufficient.

If RIV and SOLS are used the residuals may be coloured and specific tests which determine if the process model is correct without testing the whiteness of the residuals are required. It can be shown that models estimated using RIV or SOLS will be unbiased iff[8]

$$\phi_{u^{2'}\xi^2}(\tau) = 0 \;\forall\; \tau$$
$$\phi_{u^{2'}\xi}(\tau) = 0 \;\forall\; \tau \qquad (12)$$
$$\phi_{u\xi}(\tau) = 0 \;\forall\; \tau$$

In equation (12) $\phi_{u^{2'}\xi}(\tau)$ and $\phi_{u\xi}(\tau)$ simply indicate that the neglected terms are either even or odd. Although $\phi_{u^{2'}\xi^2}(\tau)$ should contain all the relevant information sometimes $u^{2'}(t)$ and $\xi^2(t)$ are small and the correlation $\phi_{u^{2'}\xi^2}(\tau)$ may incorrectly appear insignificant. Computation of $\phi_{u^{2'}\xi^2}(\tau)$ and $\phi_{u\xi}(\tau)$ is therefore worthwhile and often detects this latter condition.

3. IDENTIFICATION OF THE DIESEL GENERATOR

3.1 The Data Set

The data which will be used throughout this analysis was supplied as files TESTA5 and TESTA6. Because the raw sampling interval Δt = 5msecs was found to be too small the data on these two files was decimated by retaining only every one hundredth point. The sampling rate of the data used in the analysis is therefore $100\Delta t = 0.5$ secs. The decimated data is stored on files TEST6A.DTA and TEST5A.DTA, and these are illustrated on Figs. 1 and 2 respectively. The input sequence which represents rack position and the output sequence which represents engine torque are plotted in the units of the original data. Apart from the truncation of the data sets at the end of the tests the raw data in Figs. 1, 2 will be used throughout the following analysis. Unless stated otherwise TEST6A.DTA will be used to estimate all the models and TEST5A.DTA will be used as a prediction or testing set.

3.2 Linear Analysis

The initial analysis of the data in TEST6A.DTA involved the application of the correlation test for nonlinear detection described in section 2.1, equation 1. Although $\phi_{y'y'^2}(\sigma)$ was found to have a large positive value further analysis revealed that the third order moments of the input;

$$\phi_{uuu}(k_1,k_2) = {}^1/N \sum_{i=1}^{N} u(i)u(i-k_1)u(i-k_2),\ k_1 = 0,1,...m, k_2 = 0,1,...n,$$

were outside the 95% confidence limits of $\pm\ 1.96/\sqrt{N}$, and consequently the conditions under which the test was defined to operate were violated. Interpretation of the results from this test were therefore indeterminable and the analysis proceeded assuming initially that the input and output are related linearly.

Extensive preliminary analysis of the data in file TEST6A.DTA involved estimating the coefficients in linear models of varying order (m=1,2,3) and with various time delays (k=0,1,2) computing the loss function (or sum of squared residuals) and analysing the residuals.

The results clearly indicated that there was no significant time delay between the system input and output and that a first order dynamic model appeared to be appropriate. Analysis of the data in TEST5A.DTA produced comparable results confirming the validity of the procedures used and the time invariance of the model relating to the data in TEST5A.DTA and TEST6A.DTA.

The estimated first order (m=1) linear dynamic models with various noise model orders are summarised in Table 1 for the model

$$z(t) = \gamma + \alpha z(t-1) + \beta u(t-1) + \varepsilon(t) + \delta_1 \varepsilon(t-1) + \delta_2 \varepsilon(t-2) + \delta_3 \varepsilon(t-3) + \delta_4 \varepsilon(t-4) + \delta_5 \varepsilon(t-5) \quad (13)$$

coeff γ	α	β	δ_1	δ_2	δ_3	δ_4	δ_5
-50.6	0.286	13.27	0.3401				
-54.08	0.2361	14.19	0.3916	0.0504			
-37.91	0.4698	9.909	0.139	-0.1704	-0.2782		
-50.37	0.2935	13.18	0.2758	-0.1176	-0.2242	-0.2105	
-50.89	0.2887	13.3	0.282	-0.09474	-0.2826	-0.2328	-0.3382

Table 1 LINEAR ANALYSIS OF TEST6A.DTA

All the estimates were obtained using a maximum likelihood/ prediction error algorithm. Inspection of Table 1 shows that the coefficients of the process model (γ, α, β) vary widely depending on the order of the noise model. This is quite normal and suggests that the estimates of the process model are biased when a low order noise model is used. Note that the coefficients associated with the models using a fourth and fifth order noise model are very similar. A detailed statistical comparison of the models in Table 1 indicated that the model with a noise model order of five

$$z(t) = -50.89 + 0.2887 z(t-1) + 13.3 u(t-1) + \varepsilon(t) + 0.282\,\varepsilon(t-1) - 0.09474\,\varepsilon(t-2) - 0.2826\,\varepsilon(t-3) - 0.2328\,\varepsilon(t-4) - 0.3382\,\varepsilon(t-5) \quad (14)$$

was a good fit to the data. The parameter estimation results for this model are summarised in Fig. 3. The cross-correlation function between the input and the residuals $\phi_{u\xi}(\tau)$ is within the confidence limits whilst the autocorrelation of the residuals $\phi_{\xi\xi}(\tau)$ indicates that the residuals are white except for the outsiders between lags six and ten. If the approximate 95% confidence intervals at $\pm$ 1.96$\sqrt{N}$ were plotted accurately they would bell out away from the axis for increasing τ and the offending points in $\phi_{\xi\xi}(\tau)$ would probably be marginally inside the confidence bands. Various modifications and additions to the model failed to improve $\phi_{\xi\xi}(\tau)$ and consequently equation (14) can be regarded as the best linear model.

Normally the analysis would be complete at this point since the traditional linear covariance tests $\phi_{\xi\xi}(\tau)$ and $\phi_{u\xi}(\tau)$ in Fig. 3 indicate that the linear model is adequate. However, the nonlinear model validity tests equations (11) and (12) which are illustrated in Fig. 3 suggest that nonlinear terms have been omitted from the model. This is indicated clearly by $\phi_{u^{2'}\xi}(\tau)$, in Fig. 3, which is well outside the confidence limits, and to a much lesser extent by $\phi_{\xi\xi u}(\tau)$. The effects of introducing nonlinearities into the model were therefore investigated.

3.3 Nonlinear Analysis

A prediction error estimation algorithm coupled with a stepwise regression procedure was used to estimate the coefficients in the nonlinear models starting with an initial specification of:-

maximum number of lagged outputs = 2

" " " " inputs = 2

maximum polynomial order of z(1) and u(1) = 3

maximum polynomial order of prediction errors = 2

With this initial specification the total number of possible terms in the model was forty-eight. Allowing the stepwise regression algorithm to sort through all the possible terms using a Fisher F-ratio test operating with 95% confidence bounds produced the following model

$$\begin{aligned} z(t) = & -6.482 + 1.015z(t-1) - 0.1688z(t-2)u(t-1) \\ & + 0.06466z(t-2)u(t-2) + 0.1430u^3(t-1) \\ & + \varepsilon(t) - 0.01202z(t-2)\varepsilon(t-1) \end{aligned} \qquad (15)$$

A summary of the identification results for this model are illustrated in Fig. 4. A comparison with the linear results in Fig. 3 reveals that although $\phi_{\xi\xi}(\tau)$ and $\phi_{u\xi}(\tau)$ remain almost unchanged, $\phi_{u^{2'}\xi}(\tau)$ has now been reduced to be within the confidence band but $\phi_{\xi\xi u}(\tau)$ is now unacceptable. One possible reason for this may be that the model in equation (15) only contains one noise term $0.01202z(t-2)\,\varepsilon(t-1)$ whereas it was shown in the linear analysis that a fifth order linear noise model was appropriate. Additional linear noise terms were therefore included in the nonlinear model to give

$$
\begin{aligned}
z(t) = {} & -6.827+1.079z(t-1) - 0.1229z(t-2)u(t-1) \\
& +0.01527z(t-2)u(t-2) + 0.137u^3(t-1) \\
& +\varepsilon(t) -0.6633\varepsilon(t-1) + 0.01303\varepsilon(t-2) \\
& - 0.01229\varepsilon(t-3) - 0.01824\varepsilon(t-4) + 0.01663\varepsilon(t-5) \qquad (16)
\end{aligned}
$$

Restrictions within the program made it impossible to include the noise cross product term $z(t-2)\varepsilon(t-1)$ which appeared in the model of equation (15). The estimation results associated with the model in equation (16) are illustrated in Fig. 5. The model validity tests indicate an improved fit to the data with $\phi_{\xi\xi}u(\tau)$ within the confidence bands but $\phi_{u^{2'}\xi}(\tau)$ lies on the boundary of the confidence limits for $\tau \geq 5$. In an attempt to improve this situation a term $u(t-1)$ which appeared in the linear model was forced into the nonlinear model of equation (16) to give

$$
\begin{aligned}
z(t) = {} & 4.024+1.278z(t-1)-3.107u(t-1) \\
& - 0.1367z(t-2)u(t-1)+0.01357z(t-2)u(t-2) \\
& + 0.1532u^3(t-1)+\varepsilon(t)-0.8741\varepsilon(t-1) \\
& + 0.02\varepsilon(t-2)+0.0525\varepsilon(t-3)+0.03186\varepsilon(t-4) \\
& + 0.0622\varepsilon(t-5) \qquad (17)
\end{aligned}
$$

The estimation results for this model are summarised in Fig. 6. A comparison with Fig. 5 clearly shows that the model validity tests have been improved and the model in equation (17) is therefore unbiased and provides an accurate representation of the data in file TEST6A.DTA.

To provide a final test on the adequacy of the model in equation (17) the 650 data pairs in TEST6A.DTA were augmented with 450 points from the data file TEST5A.DTA. This gave a combined data file of length 1000 data pairs, and this was used to test the invariance of the model over a different data set. The results are summarised in Fig. 7.

The first 650 data points in this file (from TEST6A.DTA) can be regarded as an estimation set and the last 450 points (from TEST5A.DTA) can be considered as a testing or prediction set. The nonlinear model was estimted using just the first 650 data pairs or the estimation set and is therefore equal to the model in equation (17). This model was then used to predict the output over the whole data set and the deterministic errors and residuals were computed over this range. The model validity tests illustrated in Fig. 7 clearly show that the prediction of the model over the testing set, file TEST5A.DTA is excellent. Application of the model validity tests over the prediction and testing set (Fig. 7) is a severe test and the satisfactory results obtained adds a great deal of confidence to the results and indicates that the model adequately represents the data set.

The noise or prediction error terms in the model equation (17) are only included to ensure that the estimates of the process parameters are unbiased. These terms represent noise induced on the measurements from whatever source. From equation (17) therefore the model which represents the diesel generator can be expressed as

$$z(t) = 4.024+1.278z(t-1)-3.107u(t-1)-0.1367z(t-2)u(t-1) + 0.01357z(t-2)u(t-2)+0.1532u^3(t-1) \quad (18)$$

As a final check on the model of equation (18) the mean of the output $\overline{z(t)}$ was computed about the operating point of the input $\overline{u(t)}$ = 5.962. The mean $\overline{z(t)}$ computed from the estimated model equation (18) was found to be 39.3915 and this compares well with a mean of 39.97 computed from the raw data.

4. DISCUSSION OF RESULTS

The estimated model equation (18) has been shown to provide a good description of the data set. The model cannot however be expected to predict effects which have not been excited by the input sequence. It is difficult to assess the effectiveness of the input excitation in TEST6A.DTA or TEST5A.DTA without further experimentation on the plant. Whenever possible the input signal chosen for an identification experiment should be persistently exciting. This means that the input should excite all the frequencies of interest in the system and if the process is non-linear the input should also excite the process over the whole amplitude range of operation of the process. The design of inputs for nonlinear systems is as yet an unsolved problem but we would recommend the use of inputs similar to that illustrated in Fig. 8.

In Fig. 8 the operating point is changed in steps over the amplitude range of interest and superimposed on these levels is a uniformly distributed or similar signal which has a bandwidth sufficient to excite all the modes of the dynamic subsystems within the plant. Alternatively, the uniformly distributed signal could be superimposed on a sawtooth waveform or sine wave etc.

Inputs such as pure sine waves are useful when applying the structure detection test $\phi_{y'y'^2}(\tau)$ but should not be used as the only basis for parameter estimation. Pseudo-random binary inputs, although excellent for linear systems, are usually totally inappropriate as inputs for nonlinear systems. This can be proved mathematically but is probably best illustrated using the simple nonlinear system illustrated in Fig. 9.

If the input to the system in Fig. 9 is a prbs sequence, identification of a model relating the input u(t) and output y(t) will incorrectly yield a linear model. This problem arises because the probability density function of a prbs is discontinuous, so that the prbs only excites the nonlinearity at the amplitudes +a and -a. The application of parameter estimation routines to the input/output data therefore effectively models the nonlinear block as a linear gain element. Consequently, the estimated model may fit the recorded data adequately but it is totally incorrect and does not represent the underlying process.

5. CONCLUSIONS

The estimated model given in equation (18) has been shown to fit the data in TEST6A.DTA adequately and to provide good prediction over the data set TEST5A.DTA. Providing the input data contained in these files can be regarded as a sufficiently exciting input over both the amplitude and frequency range of operation of the generator then the model should be an adequate mathematical representation of the system which can be used for both simulation and controller design studies. If the input data is not considered to be persistently exciting the model should be regarded only as representative of the system over the range of inputs for which it has been fitted. Further experimentation would then be required to tune the parameters and structure of the model to adequately describe the process over its total operating range.

REFERENCES

1. BILLINGS, S.A.: 'Identification of nonlinear systems - a survey', Proc. IEE, Part D, 1980, 127, pp. 272-275.

2. BILLINGS S.A., FAKHOURI, S.Y.: 'Identification of systems containg linear dynamic and static nonlinear elements', Automatica; 1982, 18, pp. 15-26.

3. BILLINGS, S.A., VOON, W.S.F.: 'Structure detection and model validity tests in the identification of nonlinear systems', Proc. IEE, Part D, 130, 1983, pp. 193-199.

4. BILLINGS S.A., LEONTARITIS I.J.: 'Identification on nonlinear systems using parameter estimation techniques'; IEE Conf. Control and its Applications, Warwick, 1981, pp. 183-187.

5. BILLINGS S.A., LEONTARITIS I.J.: 'Parameter estimation techniques for nonlinear systems', 6th IFAC Symp. Ident. And Syst. Par. Est., Washington D.C. 1982, pp. 427-432.

6. LEONTARITIS I.J., BILLINGS S.A.; 'Input-output models for nonlinear systems', Res. Report 244, University of Sheffield, Jan 1984.

7. GOODWIN G.C., PAYNE R.L.: 'Dynamic System Identification Experiment Design and Data Analysis', Academic Press, N.Y., 1977.

8. BILLINGS S.A., VOON W.S.F.: "Least squares parameter estimation algorithms for nonlinear systems', Int. J. Systems Science (to appear).

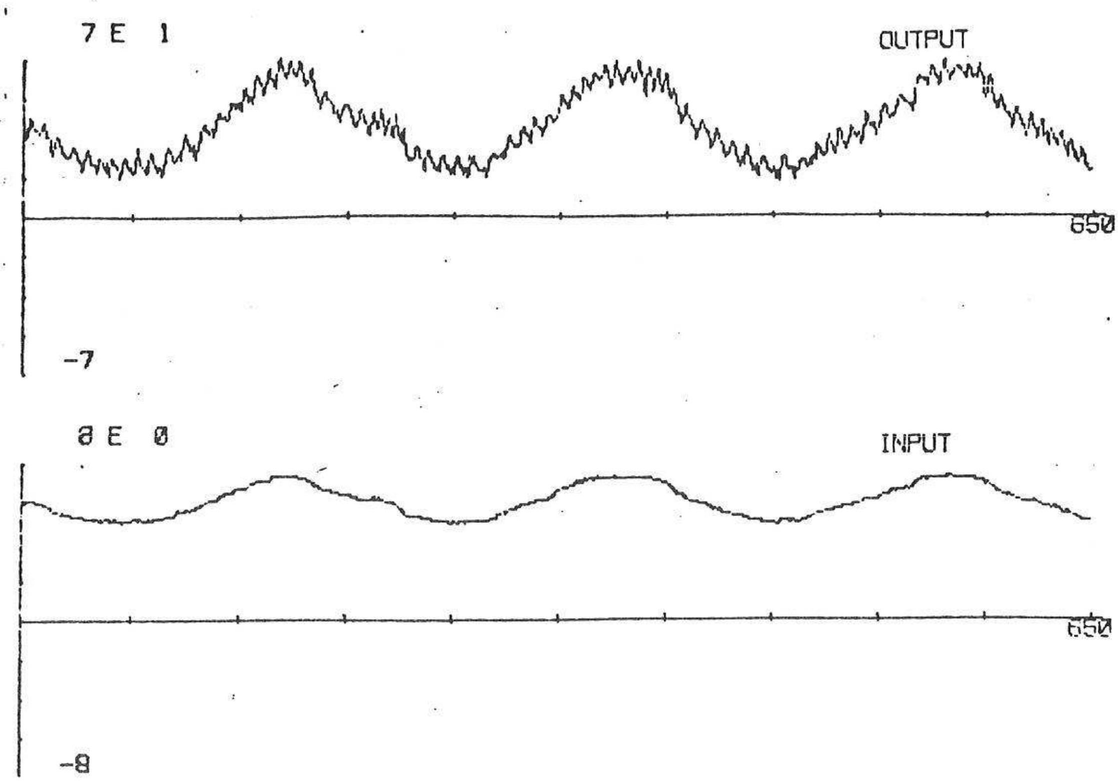


Fig. 1 Raw data TEST6A.DTA

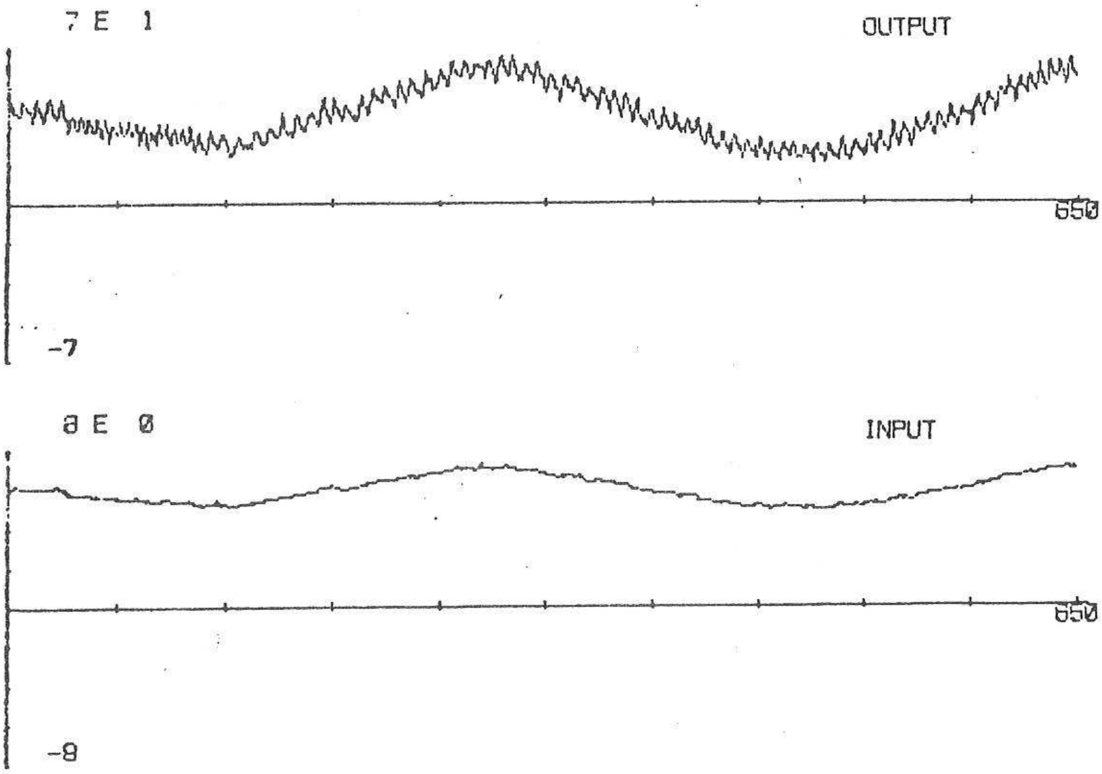
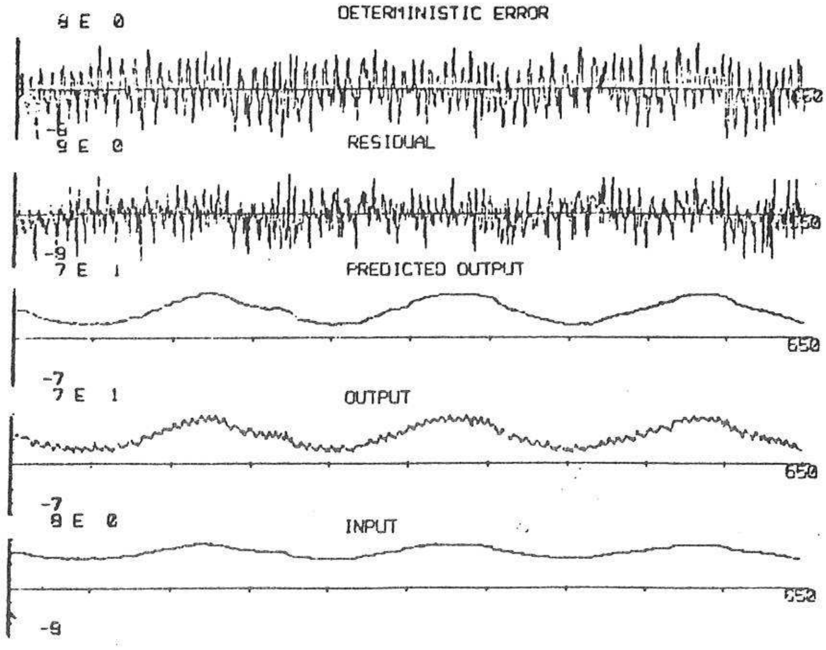


Fig. 2 Raw Data TEST5A.DTA



FINAL PARAMETER ESTIMATION

THE LOSS FUNCTION = 0.245128E+01
AS ESTIMATED BY THE

MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

(1)	THE CONSTANT TERM-	-0.5089E+02
(2)	Z(T-1)**1-	0.2997E+00
(3)	U(T-1)**1-	0.1330E+02
(4)	E(T-1)**1-	0.2920E+00
(5)	E(T-2)**1-	-0.9474E-01
(6)	E(T-3)**1-	-0.2926E+00
(7)	E(T-4)**1-	-0.2329E+00
(9)	E(T-5)**1-	-0.3392E-01

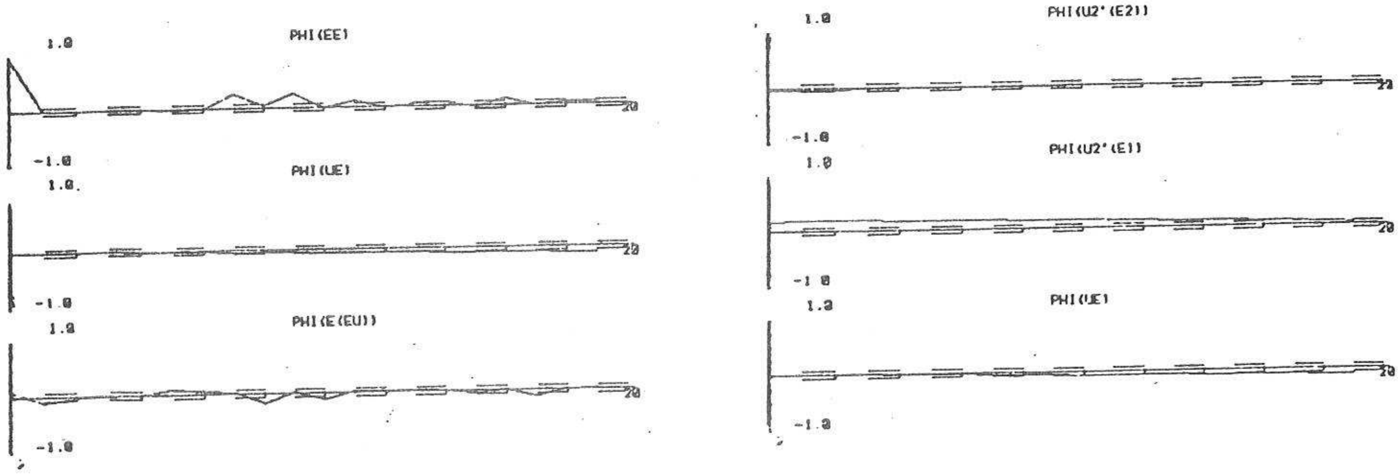
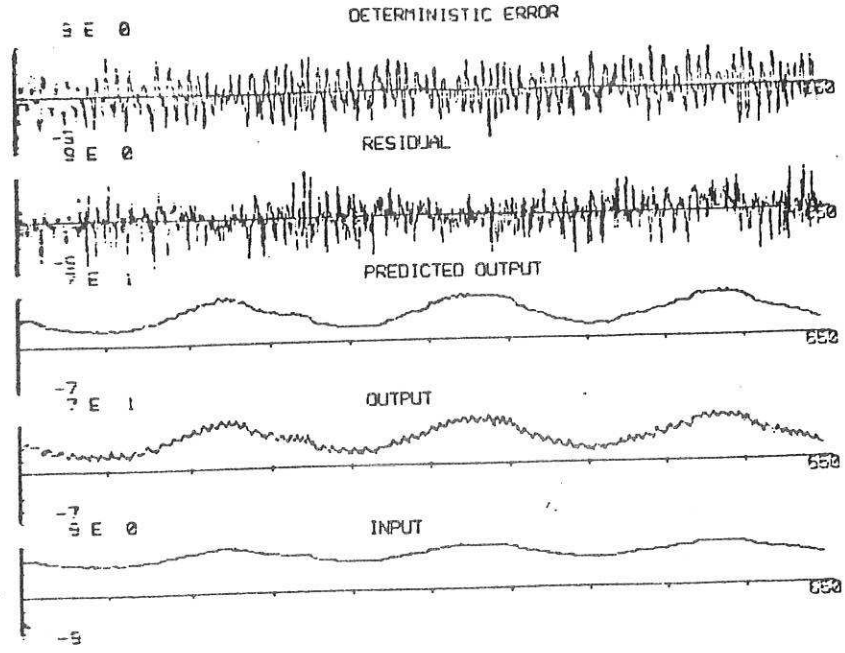


Figure 3 LINEAR MODEL ESTIMATION RESULTS



FARM-ERR

FINAL PARAMETER ESTIMATION

THE LOSS FUNCTION = 0.231732E+01
AS ESTIMATED BY THE

MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

(1)	THE CONSTANT TERM-	-0.6482E+01
(2)	Z(T-1)**1-	0.1015E+01
(15)	Z(T-2)**1*U(T-1)**1-	-0.1699E+00
(16)	Z(T-2)**1*U(T-2)**1-	0.6466E-01
(17)	Z(T-2)**1*E(T-1)**1-	-0.1202E-01
(65)	U(T-1)**3-	0.1430E+00

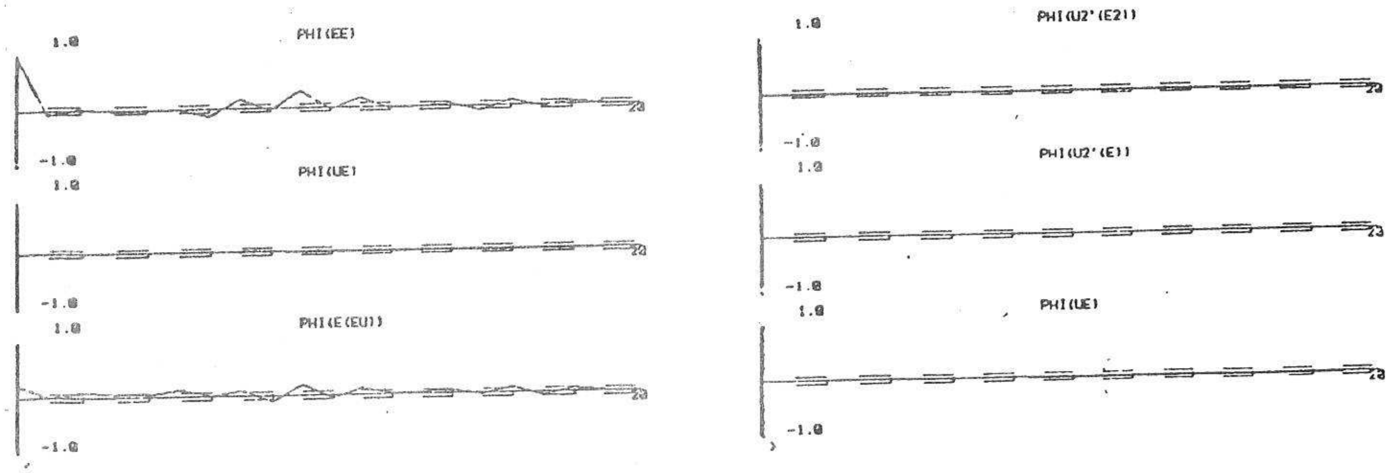
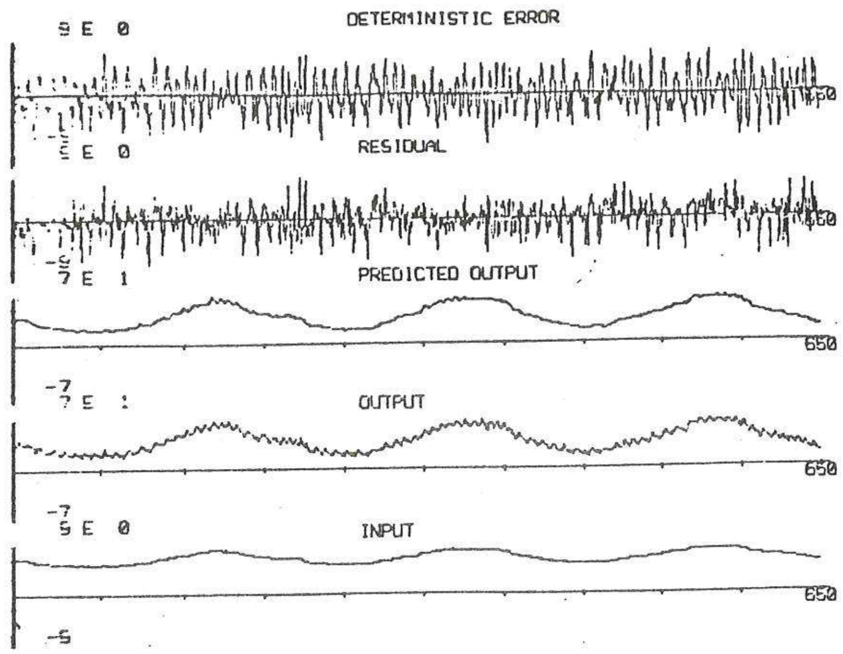


FIGURE 4 Initial nonlinear model estimation results



FINAL PARAMETER ESTIMATION

THE LOSS FUNCTION = 0.221107E+01
AS ESTIMATED BY THE

MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

(1)	THE CONSTANT TERM=	-0.6827E+01
(2)	Z(T-1)**1=	0.1079E+01
(6)	E(T-1)**1=	-0.6639E+00
(7)	E(T-2)**1=	0.1303E-01
(8)	E(T-3)**1=	-0.1229E-01
(9)	E(T-4)**1=	-0.1824E-01
(10)	E(T-5)**1=	0.1663E-01
(21)	Z(T-2)**1*U(T-1)**1=	-0.1229E+00
(22)	Z(T-2)**1*U(T-2)**1=	0.1527E-01
(137)	U(T-1)**3=	0.1370E+00

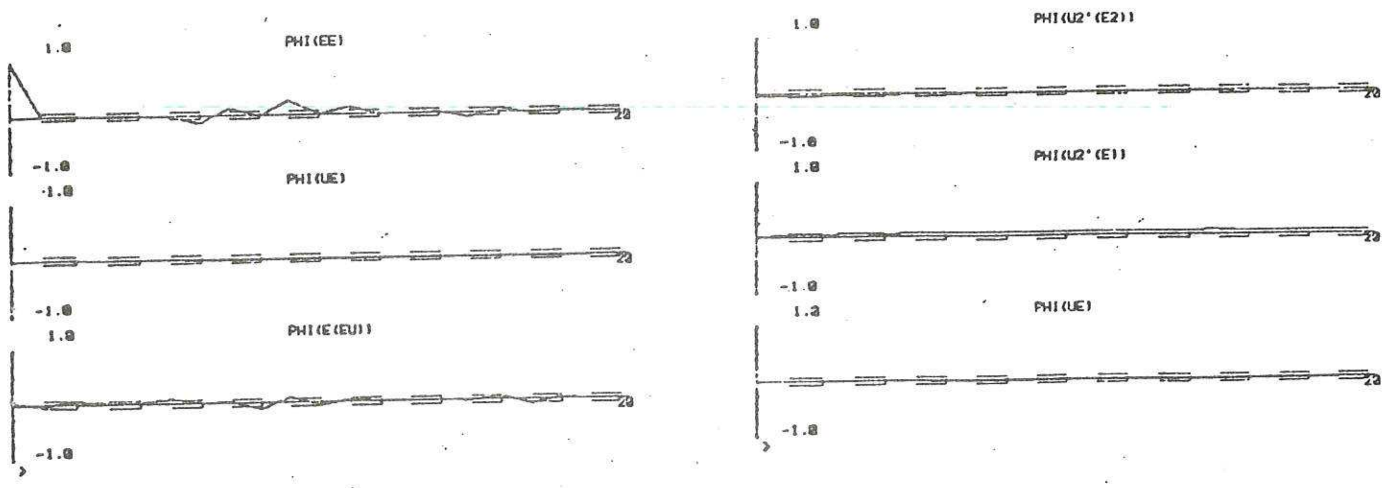
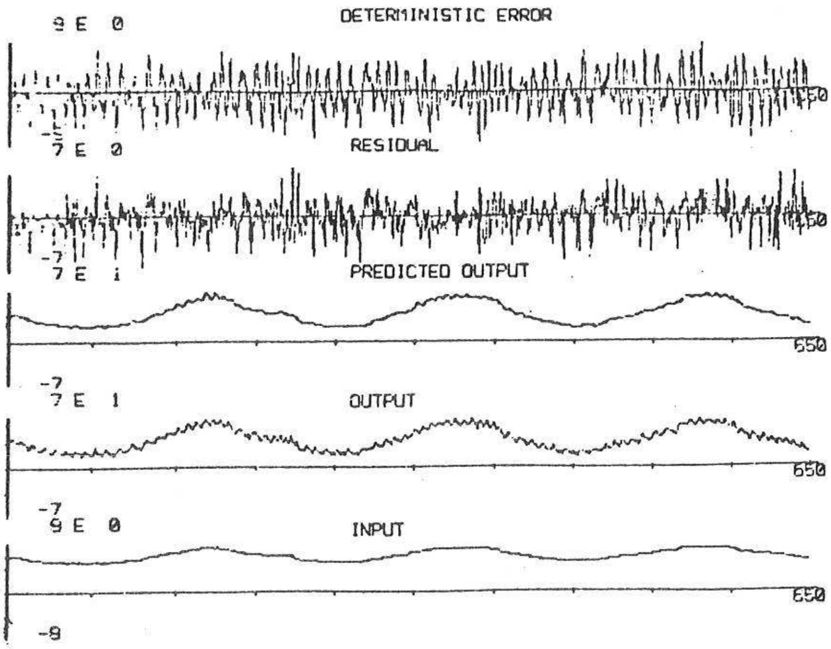


FIGURE 5 Nonlinear model estimation results



FINAL PARAMETER ESTIMATION

THE LOSS FUNCTION = 0.218213E+01
AS ESTIMATED BY THE

MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

(1)	THE CONSTANT TERM-	0.4024E+01
(2)	Z(T-1)**1-	0.1279E+01
(4)	U(T-1)**1-	-0.3107E+01
(6)	E(T-1)**1-	-0.9741E+00
(7)	E(T-2)**1-	0.2009E-01
(8)	E(T-3)**1-	0.5250E-01
(9)	E(T-4)**1-	0.3186E-01
(10)	E(T-5)**1-	0.6229E-01
(21)	Z(T-2)**1*U(T-1)**1-	-0.1367E+00
(22)	Z(T-2)**1*U(T-2)**1-	0.1357E-01
(137)	U(T-1)**3-	0.1532E+00

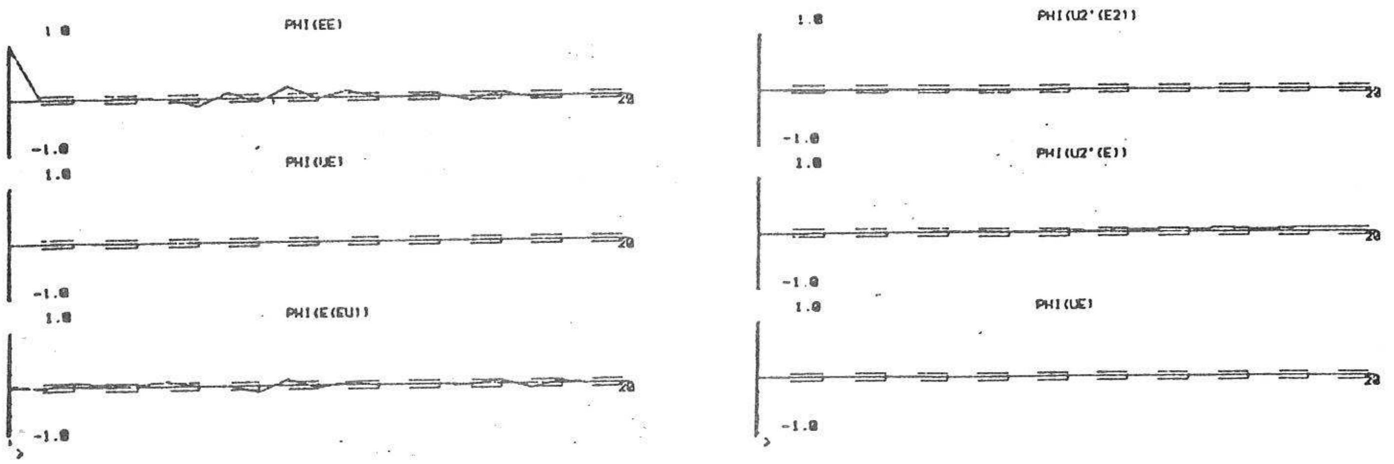


FIGURE 6 Final nonlinear model estimation results

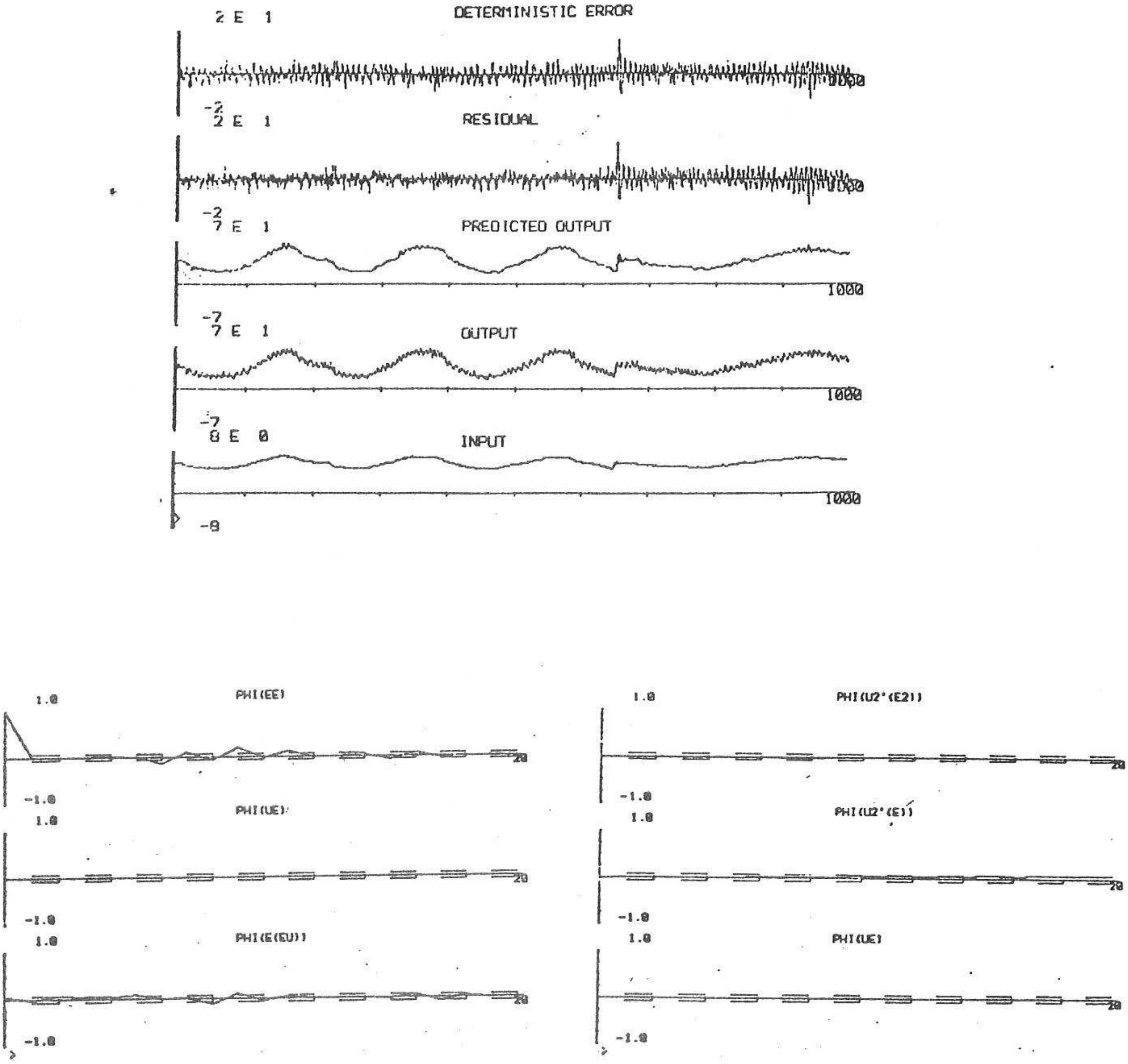


FIGURE 7 Estimation results for the prediction and testing set TEST6A and TEST5A

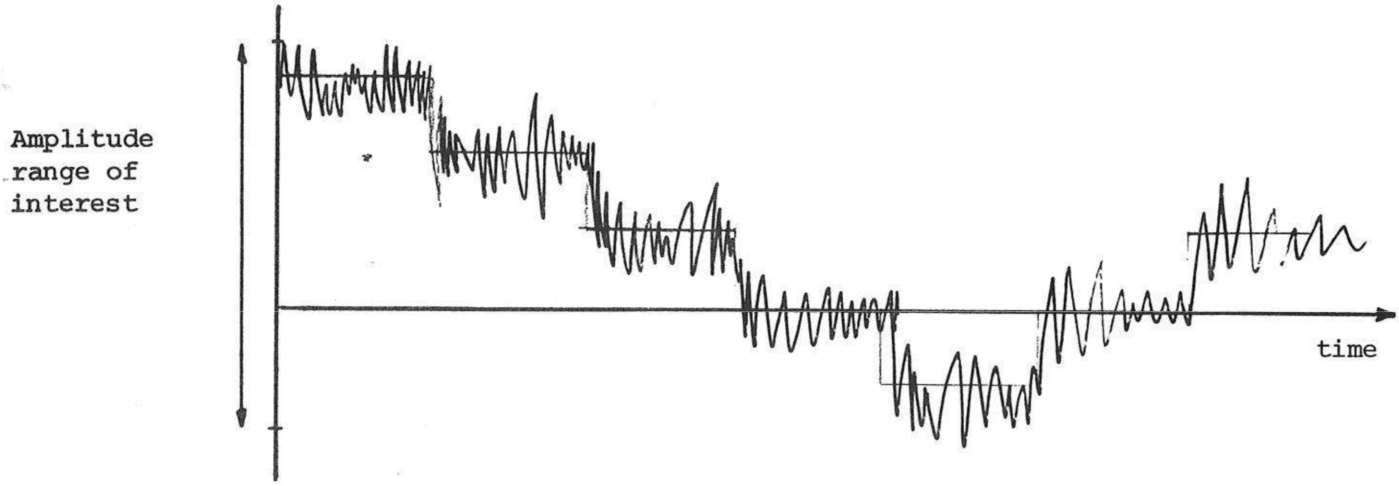


FIGURE 8. Input design for nonlinear systems

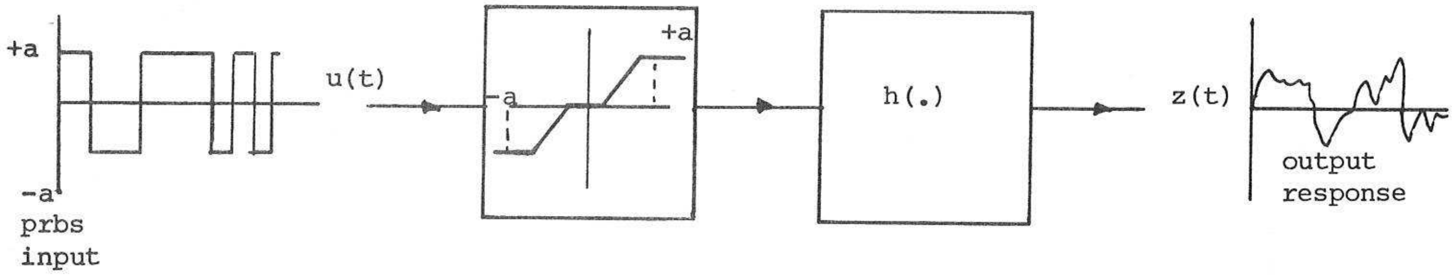


FIGURE 9 Simple Nonlinear System