This is an author produced version of a paper published in;

**Signal and Information Processing (ChinaSIP), 2013 IEEE China Summit & International Conference on.**

White Rose Research Online URL for this paper:  
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**Published paper**

[http://dx.doi.org/10.1109/ChinaSIP.2013.6625363](http://dx.doi.org/10.1109/ChinaSIP.2013.6625363)
BLIND INTERFERENCE SUPPRESSION FOR SATELLITE NAVIGATION SIGNALS BASED ON ANTENNA ARRAYS

Bo Qiu a, Wei Liu a, Renbiao Wu b

aCommunications Research Group
Department of Electronic and Electrical Engineering
University of Sheffield, UK
{b.qiu, w.liu}@sheffield.ac.uk

bTianjin Key Laboratory for Advanced Signal Processing
Civil Aviation University of China, China
rbwu@cauc.edu.cn

ABSTRACT

Power minimization is a traditional method for interference suppression in satellite navigation systems. It works when the interferences are much stronger than the desired signals and noise, and its main advantage is that the direction of arrival (DOA) information of the desired signals is not required. In this paper, a new method for suppressing strong interferences for satellite navigation based on antenna arrays is proposed. The idea is to replace the auxiliary antenna outputs in the traditional power minimization method by the principal components of received array signals, leading to an improved performance, as verified by our simulations.

Index Terms— Power minimization, satellite navigation, principal component analysis, interference suppression.

1. INTRODUCTION

Nowadays, satellite-based navigation systems are playing a more and more important role in every aspect of our life and the most widely used system based on the satellite navigation technology is the global positioning system (GPS) [1]. It has a continuous worldwide coverage and provides users with a wide range of information, such as position, velocity and time. However, the performance of the GPS or a general satellite navigation system is significantly susceptible to interferences from either intentional or unintentional sources, due to that the satellite signals arrive at the receiver with a very low power, typically 20-30 dB below the received thermal noise level [2].

Interference suppression has been studied by many researchers in this field and many methods have been proposed based on either the specific structure (such as cyclostationarity) or the direction of arrival (DOA) information of the signals or both, employing traditional adaptive beamforming algorithms or some blind signal processing techniques [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Unlike the traditional beamforming methods, the power minimization method can form deep nulls in the interference directions without knowledge of the desired satellite signals and it also has a rather low computational complexity due to its simple structure [3, 6].

In this paper, built on the success of the power minimization method, we propose a new method by incorporating principal component analysis (PCA) into the structure and replacing the auxiliary antenna outputs by the principal components of received array signals [13, 14]. The principle components extracted by the PCA operation can be considered as a better representation of the very strong interfering signals and therefore can cancel the interfering signals present in the reference antenna more effectively, leading to an improved performance.

This paper is structured as follows. The signal model is introduced and the power minimization method is reviewed briefly in Section 2. The proposed method based PCA is provided in Section 3. Simulations results are given in Section 4 and conclusions drawn in Section 5.

2. SIGNAL MODEL AND THE POWER MINIMIZATION METHOD

Consider a uniform linear array with M antennas. The array receives a narrowband desired signal s1 from the direction θ1, measured from the broadside of the array, and K − 1 narrowband interferences sk, k = 2, . . . , K from directions θk, k = 2, . . . , K, respectively. The signals are uncorrelated with each other and of zero mean, and the sensor noise is temporally and spatially white. Then the received array data can be expressed as

\[
X_{M \times L} = A_{M \times K}S_{K \times L} + N_{M \times L}
\]  

(1)
where $L$ is the data length, and $X, S$ and $N$ are data matrices denoting the received signals, source signals and noise, respectively, which are defined as

$$S = [s[1], s[2], \ldots, s[L]]$$

$$X = [x[1], x[2], \ldots, x[L]]$$

with

$$x[l] = [x_1[l], x_2[l], \ldots, x_M[l]]^T,$$

$$s[l] = [s_1[l], s_2[l], \ldots, s_K[l]]^T$$

for $l = 1, \ldots, L$. The matrix $A$ is the steering matrix of the signals, given by

$$A = [a_1, a_2, \ldots, a_K]$$

with $a_i$, being the steering vector of the $i$-th impinging signal. Suppose the adjacent sensor spacing is half wavelength of the center frequency $\omega_0$. Then we have

$$a_i = [1, e^{-j\pi \sin(\theta_i)}, e^{-j(M-1)\pi \sin(\theta_i)}]^T$$

Fig. 1. Structure of the power minimization method.

When the interferences are much stronger than the desired signal, the power minimization method can be employed to suppress the interferences effectively and its structure is shown in Fig. 1, where the first received signal $x_1[n]$ is the reference and $w_k$, $k = 2, \ldots, M$ is the adaptive coefficients applied to the remaining received array signals. Since the interferences dominate the received signals, when we minimize the power of the output $y[n]$, ideally they will be canceled effectively, with mainly noise and the desired signals left at the output. The problem can be formulated as

$$\text{min}_w \ w^H R_{xx} w \quad \text{s.t.} \quad c^H w = 1$$

where $c = [1, 0, \ldots]^T$ is an $M \times 1$ vector, $w = [w_1, \ldots, w_M]^T$ is the coefficient vector with $w_1 = 1$, and $R_{xx}$ is the $M \times M$ auto-correlation matrix of the received array data and in practice will be replaced by its finite sample estimate

$$R_{xx} = E\{x[n]x[n]^H\} \approx \frac{1}{L} XX^H$$

Using the method of Lagrange multipliers, the optimal solution is obtained as

$$w_{opt} = (c^H R_{xx}^{-1} c)^{-1} R_{xx}^{-1} c$$

Fig. 2. Structure of the power minimization method.

3. THE PROPOSED APPROACH

In the power minimization method, the key is to minimize the difference between the outputs of the reference antenna and the auxiliary elements so that it can cancel the interference components at the reference antenna effectively. Since principal component analysis can provide good estimation of strong signals, when the interferences are much stronger than the desired navigation signals, we can apply PCA to the received data and replace the auxiliary path of the traditional power minimization method by the extracted principal components. As the principal components are a better representation of the strong interferences than the original signals, it is expected that the interferences at the reference antenna will be canceled more effectively in this way. The new structure is shown in Fig. 2.

Since the interferences, the desired signals and the noise are not correlated, the correlation matrix of the received array data can also be expressed as

$$R_{xx} = R_s + R_i + \sigma^2 I$$

where $R_s$ is the correlation matrix of the desired signals, $R_i$ is that of the interferences, and $\sigma^2$ is the noise power. Since the desired signal is very weak, we further have

$$R_{xx} \approx R_s + \sigma^2 I$$

With eigendecomposition, we have

$$R_{xx} = B \Lambda_r B^H$$

$$R_i = B \Lambda_i B_i^H$$

where

$$B = [b_1, b_2, \ldots, b_M]$$

is the eigenvector matrix of $R_{xx}$, $\Lambda_r$ is a diagonal matrix with all its eigenvalues arranged in a descending order, and $B_i$ and $\Lambda_i$ are defined in a similar way for $R_i$.

Suppose we have $K - 1$ uncorrelated strong interfering signals. Then we have

$$\Lambda_i = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{K-1}, 0, \ldots, 0\} ,$$

where $\lambda_1, \lambda_2, \ldots, \lambda_{K-1}$ are the corresponding eigenvalues. Obviously we have

$$\Lambda_x = \text{diag}\{\lambda_1 + \sigma^2, \lambda_2 + \sigma^2, \ldots, \lambda_{K-1} + \sigma^2, \sigma^2, \ldots, \sigma^2\}$$
We denote the $K - 1$ eigenvectors in $B$ corresponding to the $K - 1$ largest eigenvalues by $b_1, b_2, \ldots, b_{K-1}$ and they form an $M \times (K - 1)$ transformation matrix $B$. Applying $B$ to $X$ then gives the $K - 1$ principal components, which are considered as the outputs of $K - 1$ virtual auxiliary antennas. Taking the first original antenna output as the reference signal, and applying a new set of coefficients $\hat{w}_k$, $k = 2, 3, \ldots, K$ to the new set of auxiliary output signals, we then have the new structure shown in Fig. 2.

To formulate the new problem, we first construct an $M \times K$ transformation matrix $T$ as follows

$$T = [\hat{c}, b_1, \ldots, b_{K-1}]$$

where $\hat{c} = [1, 0, 0, \ldots]^T$ is an $M \times 1$ vector. Applying $T$ to $X$, we have a new set of data

$$\bar{X} = T^T X,$$

where

$$\bar{X} = [\bar{x}[1], \bar{x}[2], \bar{x}[3], \ldots, \bar{x}[L]]$$

Now applying the standard power minimization method to $\bar{X}$, we have

$$\min_{\bar{w}} \bar{w}^T \bar{R}_{\bar{y}} \bar{w} \quad \text{s.t.} \quad \bar{c}^T \bar{w} = 1$$

where

$$\bar{R}_{\bar{y}} = E\{\bar{x}[n]\bar{x}[n]^H\} = T^T E\{x[n]x[n]^H\} T,$$

$\bar{w} = [1, \hat{w}_2, \ldots, \hat{w}_K]^T$, and $\bar{c}_1 = [1, 0, 0, \ldots]^T$ is an $K \times 1$ vector. Following the solution in (8), we can obtain the optimum solution for the new weight vector $\bar{w}_{opt}$

$$\bar{w}_{opt} = (\bar{c}_1^H (T^T \bar{R}_{\bar{y}} T)^{-1} \bar{c}_1)^{-1} (T^T \bar{R}_{\bar{y}} T)^{-1} \bar{c}_1$$

As a comparison with our above method, we can use the remaining $M - K + 1$ eigenvectors of $B$ to form a new transformation matrix and apply it to the original data, which then gives the remaining $M - K + 1$ components of the data. By taking the average of the $M - K + 1$ components, we arrive at a new output where the strong interfering signals have been canceled to a great degree. We denote the $K - 1$ eigenvectors in $B$ corresponding to the $K - 1$ largest eigenvalues by $b_1, b_2, \ldots, b_{K-1}$ and they form an $M \times (K - 1)$ transformation matrix $B$. Applying $B$ to $X$ then gives the $K - 1$ principal components, which are considered as the outputs of $K - 1$ virtual auxiliary antennas. Taking the first original antenna output as the reference signal, and applying a new set of coefficients $\hat{w}_k$, $k = 2, 3, \ldots, K$ to the new set of auxiliary output signals, we then have the new structure shown in Fig. 2.

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where

$$\bar{R}_{\bar{y}} = E\{\bar{x}[n]\bar{x}[n]^H\} = T^T E\{x[n]x[n]^H\} T,$$

$\bar{w} = [1, \hat{w}_2, \ldots, \hat{w}_K]^T$, and $\bar{c}_1 = [1, 0, 0, \ldots]^T$ is an $K \times 1$ vector. Following the solution in (8), we can obtain the optimum solution for the new weight vector $\bar{w}_{opt}$

$$\bar{w}_{opt} = (\bar{c}_1^H (T^T \bar{R}_{\bar{y}} T)^{-1} \bar{c}_1)^{-1} (T^T \bar{R}_{\bar{y}} T)^{-1} \bar{c}_1$$

As a comparison with our above method, we can use the remaining $M - K + 1$ eigenvectors of $B$ to form a new transformation matrix and apply it to the original data, which then gives the remaining $M - K + 1$ components of the data. By taking the average of the $M - K + 1$ components, we arrive at a new output where the strong interfering signals have been canceled to a great degree. We refer to this method as the PCA method in our simulations part and the coefficient vector is given by

$$w_{\text{pca}} = \frac{1}{M - K + 1} \sum_{i=K}^{M} b_i$$

4. SIMULATIONS AND RESULTS

In this section, the proposed method is compared with the power minimization method and the PCA method using computer simulations. The data length for all simulations are set to $L = 1000$.

The first set of simulations is based on a uniform linear array with 5 antennas and half-wavelength spacing. The SOI arrives from the broadside ($\theta = 0^\circ$) and one interference comes from $20^\circ$. The signal to noise ratio (SNR) of the SOI varies from $-30$ dB to $-5$ dB, and the interference to noise ratio (INR) is fixed at $10$ dB. The output SINR versus the input SNR, averaged over 1000 simulation runs, is shown in Fig. 3. It can be seen that the performance of the power minimization method declines when the input SNR increases, while the proposed method has always achieved a better performance and the improvement becomes significant for larger SNR values. For the PCA method, when the input SNR is less than about $-10$ dB, it gives the worst performance, but it outperforms the power minimization method when SNR is larger than about $-10$ dB, but still not as good as the proposed method.

In the second set of simulations, we have one more interference impinging upon the array from an angle of $-60^\circ$ with an INR of $10$ dB. Fig. 4 shows the result. We have a similar observation as in the first set of simulations. The proposed method always gives the best performance among the three solutions. However, when the input SNR is larger than about $-10$ dB, the PCA method has reached almost the same performance as the proposed one.

In the third set of simulations, we increase the antenna number to 10 and the remaining parameters are the same as in the first set of simulations. The output SINR versus input SNR is shown in Fig. 5.
with a similar result as in Fig. 3. The main difference is that now the turning point is not \(-10\) dB, but about \(-13\)dB. Now we increase the number of interferences to two, like the settings in the second set of simulations. The result is shown in Fig. 6. A major difference between Fig. 4 and Fig. 6 is that when the input SNR is larger than about \(-10\) dB, the PCA method even outperforms the proposed one, although the difference is extremely small and can be ignored.

5. CONCLUSIONS

By incorporating the principal component analysis into the power minimization method, a novel method for strong interference suppression has been proposed for satellite navigation applications, where the auxiliary antenna outputs in the traditional power minimization method are replaced by the principal components of the received array signals. Simulation results have shown that a much improved performance has been achieved by the proposed method, especially for the range of larger input SNR values.

6. REFERENCES