Monograph:
Owens, D.H., Chotai, A. and Wang, H.M. (1983) Plant Step Data and Gain Estimates in the Robust Feedback Tuning Regulator Problem. Research Report. ACSE Report 211 . Department of Control Engineering, University of Sheffield, Mappin Street, Sheffield

PLANT STEP DATA AND GAIN ESTIMATES IN THE ROBUST FEEDBACK TUNING REGULATOR PROBLEM

by

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Research Report No. 211

January 1983

This research is supported by the Science and Engineering Research Council under grant GR/B/23250

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Abstract

Estimates of the maximum gain ε^* required to retain stability of Davison's robust feedback regulator are derived based on simple graphical operations on plant open-loop step response data. Both proportional plus integral and integral control are considered.

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1. Introduction

Given a stable, linear, m-input/m-output, time-invariant process whose model is either known or unknown, the work of Davison [1] has shown how plant step response data obtained from model simulations or plant tests can be used as the basis of an on-line tuning procedure capable of ensuring plant stability, asymptotic tracking of step set-point changes and rejection of constant disturbances on the output. More precisely, if G(o) is the matrix of open-loop d.c. gains obtained from plant step responses, the unity, negative output feedback scheme with controller transfer function matrix

$$K(s) \quad = \quad \varepsilon\, G^{-1}(o)\, \frac{1}{s} \qquad \ldots(1)$$

is capable of achieving these objectives for gains ε in a non-empty range

$$0 < \varepsilon < \varepsilon^* \qquad \ldots(2)$$

provided that the required inverse of G(o) exists. The possibility of including proportional control action has been noted in [1], [2], [3] and [4] but, in all cases, the upper gain bound ε^* is unknown, being revealed only at the <u>on-line</u> tuning stage.

This paper considers the problem of <u>off-line</u> estimation of lower bounds $\varepsilon_o^* > 0$ for ε^*. Given such a lower bound, stability is then known to be guaranteed in the gain range

$$0 < \varepsilon < \varepsilon_o^* \qquad \ldots(3)$$

Such information could be of great value in practice by putting the commissioning engineer 'in the right ballpark' and hence speeding up the on-line tuning exercize. In particular, a large value of ε_o^* indicates large stability margins whilst a small value suggests that gains could be severely limited in practice.

Exact evaluation of ε^* requires, in principle, an exact plant model to be available. If a model is not available then the accuracy to which ε_o^* approximates ε^* will depend upon the available plant information. Following Davison [1], this paper assumes that plant open-loop step response data is available and that the control engineer wishes to estimate a value of ε_o^* using only the simplest graphical operations on the transient step data.

The fundamental techniques introduced by the authors in [5] are outlined in section 2 and used in sections 3 and 4 to estimate a ε_o^* using a simple plant model and the total variation of the modelling error and integrated modelling error respectively. For generality, the results are frequently stated for a more general proportional plus integral form of control, controller (1) being a special case. An illustrative example is given in section 5.

2. Approximate Models in Control Design

In a recent paper [5], the authors introduced basic theoretical results to enable the successful design of output feedback control systems for linear, multivariable, convolution plant described by the strictly proper, mxm transfer function matrix G(s) using a simple, approximate model $G_A(s)$. Both time and frequency domain approaches are given in [5] but, for this paper, we need only consider the frequency domain stabilization result given below:

Lemma 1: If the controller $K(s)$ stabilizes the model $G_A(s)$ under unity negative output feedback, then it will also stabilize the real plant $G(s)$ under unity negative feedback if

(a) both plant and model are stable,

(b) the composite system GK is both controllable and observable, and

(c) $$\sup_{s\in D} \gamma(s) < 1 \qquad \ldots(4)$$

where D is the usual Nyquist 'infinite' semi-circle in the closed, right-half, complex plane and $\gamma(s)$ is any available real-valued function on D satisfying

$$\gamma(s) \geq r(\tilde{L}(s)) \qquad \forall \quad s \in D \qquad \ldots(5)$$

with

$$\tilde{L}(s) \triangleq \|(I_m+K(s)G_A(s))^{-1}K(s)\|_p \, \Delta(s) \qquad \ldots(6)$$

and $\Delta(s)$ any available matrix valued function satisfying

$$\Delta(s) \geq \|(G(s)-G_A(s))\|_p \quad \forall \, s \in D \qquad \ldots(7)$$

(Note: (i) $r(M)$ denotes the spectral radius of the mxm matrix M.

(ii) the 'absolute value' $\|M\|_p$ of the mxm matrix $M = [M_{ij}]$ is the matrix $\|M\|_p \triangleq [|M_{ij}|]$ of moduli of the elements of M.

(iii) if A and B are two real mxm matrices, the relation $A \leq B$ denotes the element inequality $A_{ij} \leq B_{ij}$, $1 \leq i, j \leq m$).

The result follows from theorem 1 in [5] assuming square plant and unity feedback. In the following sections, it is applied to the robust tuning regulator problem by choice of model G_A and bound $\Delta(s)$. (Remark: condition (b) can be relaxed to that of stabilizability and detectability of GK provided that stable uncontrollable and unobservable modes are not regarded as unacceptable).

3. ε_o^* and the Modelling Error

Following Davison [1], it is assumed that plant open-loop step responses are available in the form of the mxm step response matrix

$$Y(t) = \begin{pmatrix} Y_{11}(t) & \cdots & Y_{1m}(t) \\ \vdots & & \vdots \\ Y_{m1}(t) & \cdots & Y_{mm}(t) \end{pmatrix} \qquad \ldots(8)$$

where $Y_{ij}(t)$ is the response of the plant output $y_i(t)$ from zero initial conditions to a unit step input in $u_j(t)$ with $u_k(t) \equiv 0$, $k \neq j$. If the plant is stable, then clearly [1] the steady-state matrix

$$G(o) = \lim_{t\to+\infty} Y(t) \qquad \ldots(9)$$

can be estimated graphically, its nonsingularity can be checked and the controller (1) constructed. Intuitively, the time variation of $Y(t)$ is sufficient data to calculate ε^* and hence a suitable ε_o^*. Consider therefore the approximate plant model

$$G_A(s) = \tilde{G}(o)\,\frac{1}{1+sT} \qquad \ldots(10)$$

where $T>0$ is a representative time constant of the plant G deduced by inspection of $Y(t)$ and $\tilde{G}(o)$ is a nonsingular estimate or convenient approximation of the d.c. gain $G(o)$. The model G_A has step response matrix

$$Y_A(t) = \tilde{G}(o)\,(1 - e^{-t/T}) \qquad \ldots(11)$$

with modelling error [5]

$$E(t) \triangleq Y(t) - Y_A(t) = Y(t) - \tilde{G}(o)(1 - e^{-t/T}) \qquad \ldots(12)$$

Both G and G_A have identical steady state characteristics if $G(o) = \tilde{G}(o)$ when

$$\lim_{t\to\infty} E(t) = 0 \qquad \ldots(13)$$

The importance of the modelling error E is that it can be used to provide a simple bound for $\|G(s)-G_A(s)\|_p$ as follows ([5], lemma 2):

Lemma 2:

$$\|G(s)-G_A(s)\|_p \leq N_\infty^{\,p}(E) \quad \forall \quad \mathrm{Re}\,s \geq 0 \qquad \ldots(14)$$

The constant matrix $N_\infty^{\,p}(E)$ has (i,j)th element $N_\infty(E_{ij})$ defined to be the total variation of the (i,j)th element of $E(t)$. It has been noted [5] that $N_\infty(E_{ij})$ can be evaluated graphically from $E_{ij}(t)$ by estimating its local maxima and minima $\{t_{ijk}\}_{k\geq o}$ on the extended half-axis $t\geq 0$, ordering the data in increasing form $0 = t_{ijo} \leq t_{ij1} \leq \cdots$ and setting

$$N_\infty(E_{ij}) = \sum_{k\geq 1} \left|E_{ij}(t_{ijk}) - E_{ij}(t_{ijk-1})\right| \qquad \ldots(15)$$

It is known [5] that graphical estimation of these parameters is insensitive to noise provided that the signal to noise ratio is fairly high.

The main result of this section can now be stated as follows:

<u>Theorem 1</u>: If the plant G(s) is stable with G(o) nonsingular and it is possible to choose a representative time constant T>0 so that

$$r_\infty \triangleq r(\|\tilde{G}^{-1}(o)\|_p N_\infty^{\,p}(E)) < \min(1, k_1^{-1}) \qquad \ldots(16)$$

then the controller $K(s) = \tilde{G}^{-1}(o)(k_1+k_2s^{-1})$ will stabilize the plant G(s) for any gains $k_1 \geq 0$, $k_2 > 0$ satisfying

$$\gamma_\infty \sup_{s\in D} \left| \frac{(1+s)(k_1s+k_2T)}{s(s+1)+(k_1s+k_2T)} \right| < 1 \qquad \ldots(17)$$

where $\gamma_\infty < \min(1, k_1^{-1})$ is any convenient upper bound for r_∞.

<u>Proof</u>: Using the defined forms of K(s) and $G_A(s)$ in lemma 1 with $\Delta(s) = N_\infty^{\,p}(E)$ indicates that K stabilizes G_A and also stabilizes G if

$$\sup_{s\in D} r\left(\left| \frac{(1+sT)(k_1s+k_2)}{s(1+sT)+(k_1s+k_2)} \right| \|\tilde{G}^{-1}(o)\|_p N_\infty^{\,p}(E)\right) < 1 \qquad \ldots(18)$$

as the nonsingularity of G(o) ensures the stabilizability and detectability of GK. Inequality (18) is satisfied if

$$\gamma_\infty \sup_{s\in D} \left| \frac{(1+sT)(k_1s+k_2)}{s(1+sT)+(k_1s+k_2)} \right| < 1 \qquad \ldots(19)$$

(17) follows by replacing s by s' = Ts. The necessity of (16) in satisfying (17) is seen by noting that (17) must be satisfied at s = 0 and $|s| \to \infty$.

In practice, the result indicates that a range of gains k_1, k_2 can be obtained by analysis of (17) provided that the plant G can be modelled by the 'first-order lag' (10) with error small enough to satisfy (16). In many situations (16) is not a severe constraint as, for example, in the case of $m = 1$ and $k_1 < 1$, it reduces to the requirement that the total variation of the modelling error is strictly less than the plant d.c. gain. In the case of plant G with oscillatory characteristics or badly conditioned d.c. gain matrix however, (16) can be violated as the first order model G_A is then not sufficiently representative of plant dynamics.

The power of the result lies in the fact that a detailed plant model is not necessary to check the stability conditions. The designer simply follows the procedure:

Step 1: Obtain the plant step response matrix $Y(t)$.

Step 2: Choose a representative time constant $T>0$ and evaluate the modelling error $E(t)$.

Step 3: Evaluate $N_\infty^P(E)$ by graphical analysis of the error $E(t)$ [5].

Step 4: Evaluate $M \triangleq \|\tilde{G}^{-1}(o)\|_p \, N_\infty^P(E)$ and evaluate $\gamma_\infty = r_\infty$ as given by (16) or any conveniently calculated upper bound $\gamma_\infty \geq r_\infty$ such as the vector induced matrix norm

$$\|M\|_m = \max_i \sum_{j=1}^{m} |M_{ij}| \qquad \ldots(20)$$

or any other norm such as the maximum singular value of M.

Step 5: (a) If $\gamma_\infty \geq \min(1, k_1^{-1})$, the approach fails and the designer must either compute a better upper bound γ_∞ if possible or return to step 2 with an 'improved' value of T or apply the technique outlined in section 4.

(b) If $\gamma_\infty < \min(1, k_1^{-1})$, move to step 6.

Step 6: Use condition (17) to estimate suitable values for k_1 and k_2.

There are several ways of approaching step 6 apart from the obvious numerical search procedures. The following corollaries to theorem 1 provide simple graphical and algebraic techniques respectively:

Corollary 1.1: The conclusions of theorem 1 hold if (17) is replaced by the equivalent condition that the point $(-(k_2T)^{-1},0)$ of the complex plane does not lie in or on the 'band' generated by plotting the Nyquist locus of the transfer function $g(s,k_1,k_2T) \triangleq \frac{(k_1 s+k_2T)}{s(s+1)k_2T}$ and superimposing at each frequency a circle of radius

$$r(s,k_1,k_2T) \triangleq \gamma_\infty |k_1 s + k_2T|/(|s|k_2T) \qquad \ldots(21)$$

Proof: Write (17) in the form

$$|(k_2T)^{-1} + g(s,k_1,k_2T)| > r(s,k_1,k_2T), \forall s \in D \qquad \ldots(22)$$

and interpret in graphical form as shown in Fig.1 for $s = i\omega$, $\omega \geq 0$. It is automatically satisfied for $|s| \to \infty$ by (16).

Corollary 1.2: The conclusions of theorem 1 hold if $k_1 = 0$ and (17) is replaced by the algebraic condition

$$0 < k_2T < \sqrt{2} - 1 \qquad \ldots(23)$$

Proof: Elementary calculus indicates that the supremum in (17) is achieved by the frequencies ω satisfying $\omega \triangleq 0$ or, if $\alpha = k_2T$,

$$\omega^2 = -1 + \sqrt{\alpha(\alpha+2)} \qquad \ldots(24)$$

Equation (23) ensures that (24) has no real solution and hence that (17) reduces to $\gamma_\infty<1$ which is satisfied by assumption.

The case of $k_1 = 0$ reduces to Davisons controller of equation (1) with $k_2 \triangleq \varepsilon$. Corollary 1.2 then reduces to the estimate

$$\varepsilon_o^* \triangleq (\sqrt{2} - 1)/T \qquad \ldots(25)$$

which is very easy to compute but more conservative than the corresponding estimate resulting from Corollary 1.1 which yields an estimate for ε_o^* by reading off the point $(-\mu,0)$ where the trailing edge of the uncertainty band cuts the negative real axis as illustrated in Fig.1. The corresponding value of ε_o^* is obtained from the formula,

$$\varepsilon_o^* = 1/\mu T \qquad \ldots(26)$$

This formula can also be used for $k_1 \neq 0$ provided that the ratio k_1/k_2T is fixed and only $\varepsilon \triangleq k_2$ regarded as a design variable, as is easily proved by noting that $g(s,k_1,k_2T) \equiv g(s,k_1/k_2T,1)$ and $r(s,k_1,k_2T) \equiv r(s,k_1/k_2T,1)$ are then independent of ε.

4. ε_o^* and the Integrated Modelling Error

Although the techniques of section 3 yield values of ε_o^*, the modelling error $E(t)$ must be small enough to satisfy (16). This constraint can be removed if the integrated modelling error

$$E^o(t) \triangleq \int_o^t E(t')dt' \qquad \ldots(27)$$

is computed. There are no numerical problems with this operation and it contains the implicit bonus of filtering (in part) high frequency noise in the error signal. Clearly

$$\frac{1}{s}\,(G(s) - G_A(s)) = \int_0^\infty e^{-st}\,E(t)dt \qquad \ldots(28)$$

and hence, in a similar manner to that of [5], for Res ≥ 0,

$$|s^{-1}(G(s)-G_A(s))_{ij}| \leq \int_0^\infty |E_{ij}(t)|dt = N_\infty(E_{ij}^{\,o}) \qquad \ldots(29)$$

which is finite by (13) provided that $\tilde{G}(o) = G(o)$. The main result of this section can now be stated:

Theorem 2: If the plant $G(s)$ is stable with $G(o)$ nonsingular, $\tilde{G}(o) = G(o)$ and $T>0$, then the controller $K(s) = \varepsilon G^{-1}(o)s^{-1}$ will stabilize the plant $G(s)$ for any $\varepsilon>0$ satisfying

$$\gamma_\infty^{\,o}\,\varepsilon \sup_{\substack{s=i\omega\\ \omega\geq 0}} \left|\frac{s(1+s)}{\varepsilon T+s(1+s)}\right| < 1 \qquad \ldots(30)$$

where $\gamma_\infty^{\,o}$ is any convenient upper bound for

$$r_\infty^{\,o} \triangleq r(\,\|G^{-1}(o)\|_p N_\infty^{\,p}(E^o)) \qquad \ldots(31)$$

Proof: Follows in a similar manner to that of theorem 1 using the model (11) with $\tilde{G}(o) = G(o)$ and the upper bound $\Delta(s) \triangleq |s|N_\infty^{\,p}(E^o)$ for $G(s)-G_A(s)$ obtained from (29).

The result has a similar structure to theorem 1 but note the absence of any constraint on modelling error, that only integral action is considered and the requirement that $\tilde{G}(o) = G(o)$. Condition (30) can be checked by numerical or algebraic means but the easiest technique is graphical in nature as stated in the following corollary:

Corollary 2.1: The conclusions of theorem 2 hold if (30) is replaced by the equivalent condition that the point $(-(\varepsilon T)^{-1},0)$ of the complex plane does not lie in or on the band generated by plotting the Nyquist locus of the transfer function $g(s) \triangleq 1/s(s+1)$ and superimposing at each frequency a circle of constant radius

$$r_{\infty}^{o}(s) \triangleq \gamma_{\infty}^{o}/T \qquad \ldots(32)$$

If the trailing edge of the band cuts the negative real axis at the point $(-\mu^{o},0)$, then we can choose

$$\varepsilon_{o}^{*} = 1/\mu^{o}T \qquad \ldots(33)$$

Proof: Follows by writing (30) in the form

$$\left| \frac{1}{s(s+1)} + (\varepsilon T)^{-1} \right| > \gamma_{\infty}^{o}/T \quad , \quad s = i\omega, \ \omega>0 \qquad \ldots(34)$$

and interpreting the relation in a similar manner to Fig.1.

(Remark: the limitations on gain implicit in the result are revealed by noting that (34) requires $\varepsilon<1/\gamma_{\infty}^{o}$ by letting $\omega\to\infty$ and hence $\varepsilon_{o}^{*}<1/\gamma_{\infty}^{o}$).

5. Illustrative Example

Consider the boiler-furnace system described by Rosenbrock [6] with transfer function matrix

$$G(s) = \begin{pmatrix} \frac{1}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1}{1+4s} \end{pmatrix} \qquad \ldots(35)$$

and the problem of design of an integral robust regulator of the form of (1) using open-loop step data only. Choose the case of $\tilde{G}(o) = G(o)$ with regulator given by (1) where $G(o)$ is nonsingular with

$$G^{-1}(o) = \begin{pmatrix} 1.75 & -1.21 & -0.16 & 0.17 \\ -0.98 & 1.87 & -0.23 & -0.32 \\ -0.32 & -0.23 & 1.87 & -0.98 \\ 0.17 & -0.16 & -1.21 & 1.75 \end{pmatrix} \qquad \ldots(36)$$

and the model time constant $T = 4.0$. The corresponding modelling error can be represented in graphical array form or, for the purposes of this paper, in the equation form

$$E(t) = (G(o) - I_4)(e^{-t/4} - e^{-t/5}) \qquad \ldots(37)$$

from which we deduce the total variation

$$N_\infty^{\ p}(E) = (G(o) - I_4)0.164 \qquad \ldots(38)$$

Choosing the easily computed bound $\gamma_\infty = ||G^{-1}(o)||_m ||N_\infty^{\ p}(E)||_m$ for r_∞ yields $\gamma_\infty = 3.4x(1.35x0.164) = 0.753 < 1$ and hence theorem 1 can be applied to estimate ε_o^*. Using corollary 1.2 and equation (25) yields

$$\varepsilon_o^* = 0.1 \qquad \ldots(39)$$

In contrast, corollary 1.1 and equation (26) yields the improved estimate

$$\varepsilon_o^* \simeq 1/(1.2 \times 4.0) = 0.21 \qquad \ldots(40)$$

by plotting the Nyquist plot of $1/s(s+1)$ with circles of radius γ_∞/ω as given in Fig.1 to obtain $\mu \simeq 1.2$. Finally, noting that the assumptions of theorem 2 are satisfied it is easily verified that the integrated error has total variation

$$N_\infty^P(E^o) = G(o) - I_4$$

and we can take, for simplicity, $\gamma_\infty^o = \|G^{-1}(o)\|_m \|G(o) - I_4\| = 4.59$ and $r_\infty^o(s) = \gamma_\infty^o/T = 1.15$. Plotting the Nyquist diagram of $1/s(s+1)$ with circles of radius $r_\infty^o(s)$ then yields $\mu^o \simeq 1.55$ as shown in Fig.2 and hence

$$\varepsilon_o^* \simeq 1/(1.55 \times 4.0) = 0.16 \qquad \ldots(41)$$

by corollary 2.1.

Note that the three approaches reveal different choices for ε_o^*, corollary 1.2 being the most pessimistic with corollary 1.1 being least pessimistic. All of the results could beimproved by better choices for γ_∞ and γ_∞^o but this is unnecessary in this case as the closed-loop responses to a unit step demand in the first output indicate when $\varepsilon = 0.1$ (Fig.3). Note that interaction effects are low whilst y_1 responds with small overshoot and zero steady state error. The response speed is slow but this can be improved by inclusion of proportional action.

6. Conclusions

Lower bounds for the maximum gain required to retain stability of Davisons robust feedback regulator can be derived based upon the total variation of the error in modelling available plant step responses by a simple first order model. The total variation is easily evaluated by graphical inspection of the error. If the modelling error is small enough in the sense of theorem 1, the gain bound can be estimated rapidly by algebraic means or by a Nyquist diagram with superimposed circles representing the modelling error. In the case of larger modelling errors, theorem 2 provides a similar Nyquist-type criterion based on the total variation of the integrated modelling error, but the plant and model steady-state characteristics must be the same for its application.

The illustrative example indicates that the method can work well in practice. It is however based upon sufficient conditions for stabilization [5] and hence can be conservative in its predictions. The effects of this conservatism can be off-set by the use of a more complex approximate model [5] to represent plant dynamics with the associated increase in complexity of the design exercize. This possibility was not considered, nor was the choice of other forms of $\Delta(s)$ considered as, in general, they will tend to increase computational requirements beyond those in the spirit of Davisons work. The possibilities can be illustrated however by combining (14) and (29) and setting

$$\Delta(s) = \begin{cases} N_\infty(E) & \text{whenever } |s| N_\infty(E^O) \geq N_\infty(E) \\ |s|\, N_\infty(E^O) & \text{otherwise} \end{cases} \qquad \ldots(42)$$

Acknowledgements

This work is supported by the UK Science and Engineering Research Council under grant GR/B/23250.

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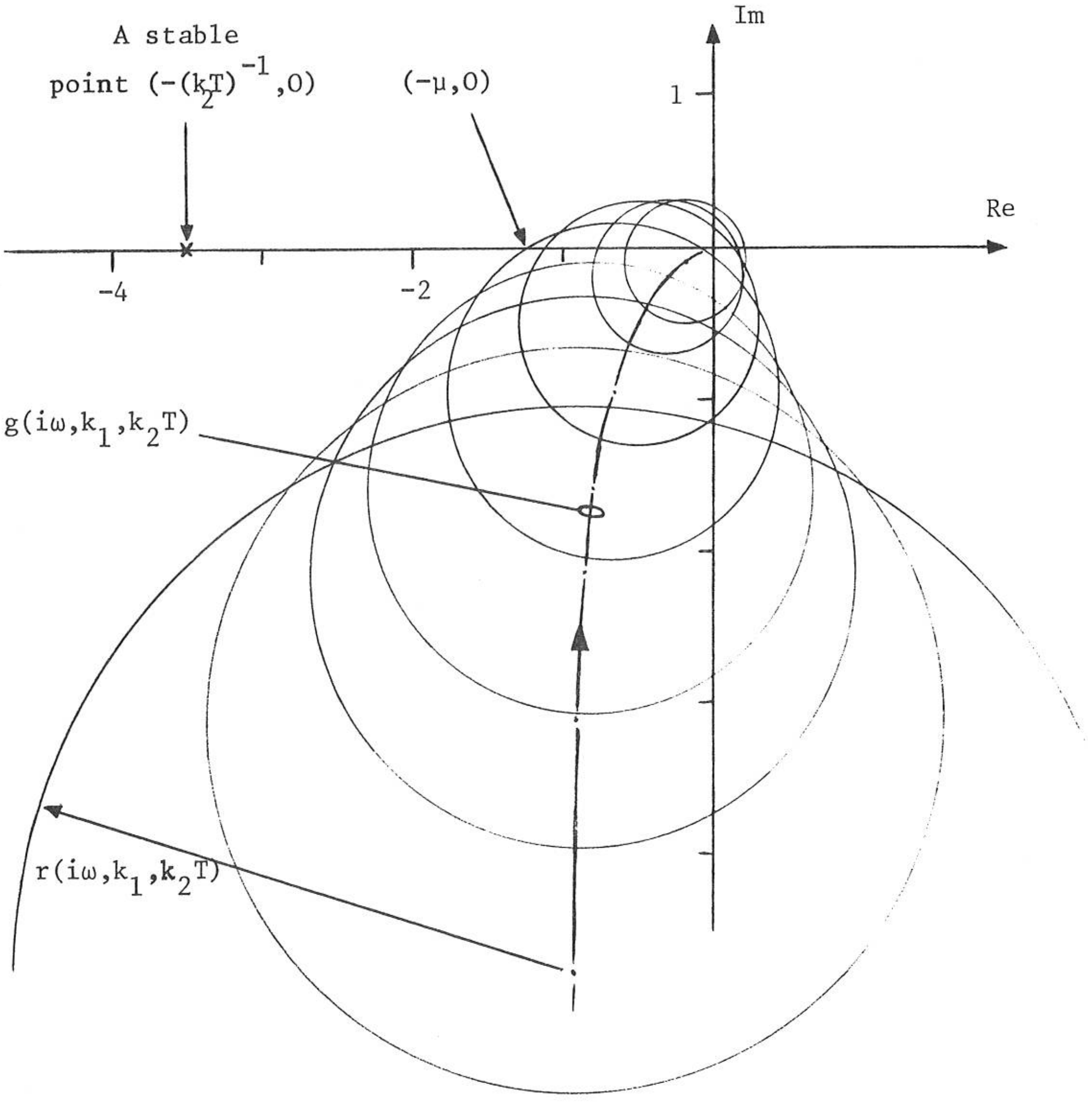

Fig. 1. Graphical Stability Criterion and Evaluation of μ

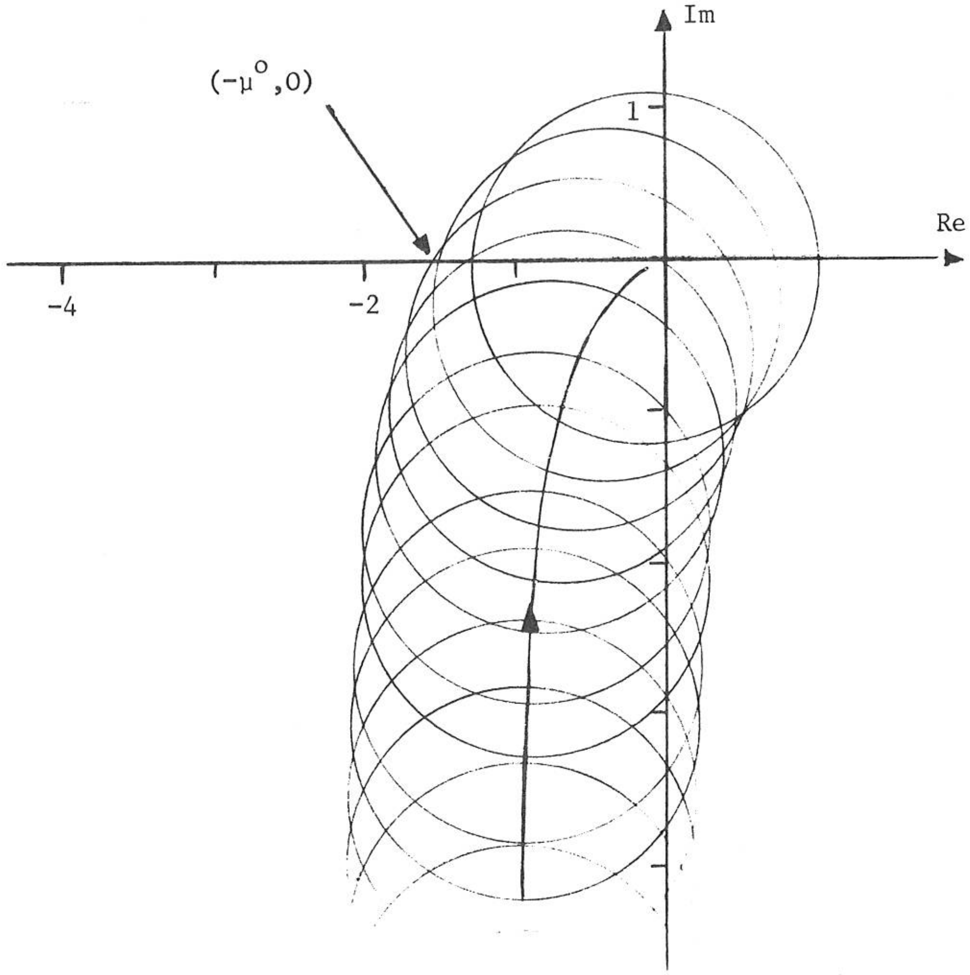

Fig. 2. Evaluation of μ^o

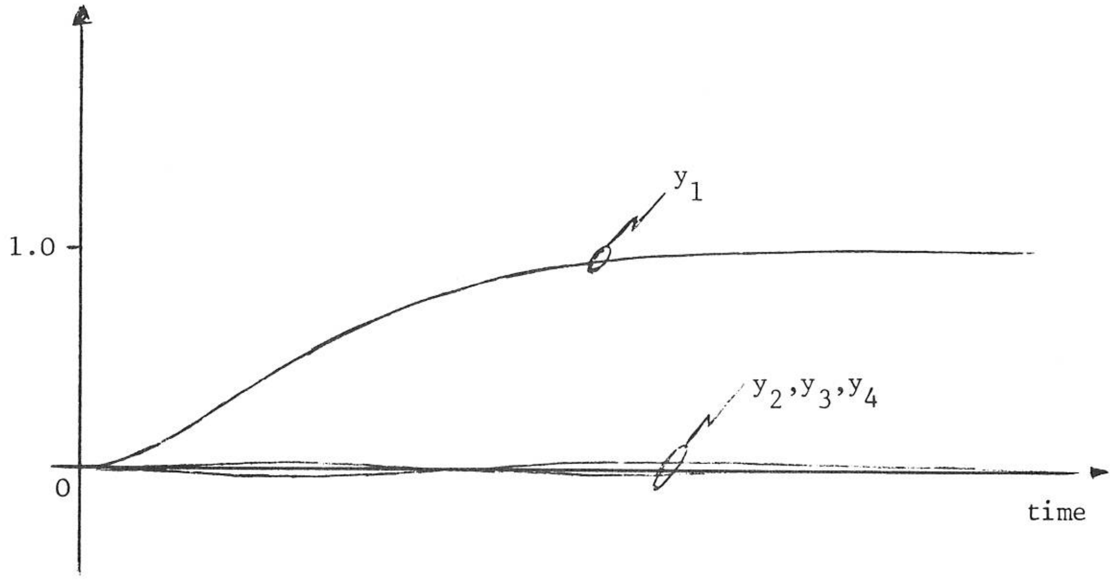

Fig. 3. Closed-loop responses to a unit demand in y_1