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Preliminary Analyses and Simulations for the Application of Modern Control Methods to Mine Ventilation

by

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ABSTRACT

Linear dynamic analysis is carried out of some simple mine ventilation networks to assess the likely instability and interaction problems to be encountered in the comprehensive multivariable control of air flow around a mine. It is shown that distributed mine capacitance alone does not generate serious instability problems under closed loop control but interaction elimination requires either tailored diagonalising control structures or high gains in multiple diagonal controllers. The interaction of roadway and fan dynamics yields an overdamped system so permitting the use of high controller gains. The report is written also to provide an analytical check on computer simulations of mine ventilation control systems.
Acknowledgement

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1 Introduction

1.1 Background

Most underground mines involve an interconnected network of roadways and working areas that evolves in size and complexity with time. Natural ventilation is usually insufficient either for human respiration, the removal of vapours created by the mining process or for the maintenance of acceptable working temperatures or humidity levels for men or machinery. Ventilation requirements for the sustenance of human life is about 20 cfm (0.01 m³/s) per person. In the control of both the chemical and physical quantity of the air, clean fresh air must be supplied and contaminants (gas, dust, heat and moisture) must be removed by the ventilation system. Considering the entire, mine ventilation requirements far outstrip the minimal 20 cfm (0.01 m³/s) per person, usually exceeding 200 cfm (0.1 m³/s) and on occasion, 2000 cfm (1 m³/s) per person. Circulation of 10 to 20 tons of air per ton of mineral mined is not unusual today under adverse condition. Fresh air must therefore be forced (or more usually induced) around the network by a main ventilating fan at the top of one of the two main mine-shafts. Working areas of the mine except development headings (i.e. extending closed-end tunnels) are usually connected to both the main intake and return airways of the mine (via ventilation control doors) so that the network comprises numerous parallel paths. Control of
distribution to the different areas of the mine is thus effected by bleeding off a proportion of the main intake airstream via preset apertures in the air-lock doors assessing these areas. The apertures are adjusted from time to time as the hydraulic resistances of the different roadways changes with time. When such measures are inadequate, either because of insufficient main fan pressure or because of the short-circuiting effect of other roadway branches, booster fans may be installed to ensure the provision of sufficient air to a particular area of the mine. Similar fans supply blind heading.
1.2 Control Possibilities

Although hydraulically or electrically actuated apertures are conceivable and thyristor-controlled, variable-speed fans equally possible, little has been done to-date to exercise continuous remote control of mine air distribution in a feedback manner from flow-measurements made by pitot-tube or rotating-vane anemometers. Such schemes are potentially attractive nowadays with the advent of control computers ruggedised for central or distributed mine control. However the ventilation network is a highly interactive system containing considerable dynamic elements, thus necessitating (probably) the need for modern multivariable control techniques for the design of comprehensive control schemes that will behave in a stable manner with the air supply to one working area relatively unaffected by changes of ventilation to another. The case for such an approach becomes more acute when emergency conditions such as mine fires are to be controlled.

1.3 Simplifications for Analytical Studies

In this preliminary appraisal of the potential for modern control theory applied to ventilation we consider analytically a few simple networks described by models of minimum complexity under diagonal proportional control. The purpose is two fold

(i) To assess the likely instability, interaction and sensitivity problems that might be encountered in full scale computer based investigations and implementations,

(ii) To provide analytical checks on simulations of these simple networks (using established and novel simulation packages) to generate the confidence needed before proceedings to more detailed and extensive models.
In this report the following simplifications are made in the interests of clarity and analytical tractability.

(a) Mine roadways are represented by two hydraulic (aerodynamic) resistances either side the lumped roadway volume rather than by a continuously spatially distributed process.

(b) Resistance is here defined as the ratio of pressure drop across/air flow along the roadway and is treated as constant (rather than using the more realistic quadratic relationship).

(c) Expansion and contraction of the air is regarded as isothermal so that the effective bulk modulus of the air can be regarded as equal to atmospheric pressure, (pressure changes being small in comparison).

(d) Control is exercised by fan pressure manipulation (rather than adjustable apertures i.e. adjustable series resistances) and except in Section 5, it is assumed that this can be changed instantaneously.

(e) Fan pressure is assumed to be invariant with air flow, again except for Section 5 where internal fan resistance is included.

(f) Heat generation and temperature control problems are not considered.

It is fully appreciated that these approximations are unacceptable for real-life investigations where such simplifications would generate unacceptable steady state errors. For dynamic control studies as this, however, it is maintained that the simplification will assist, not hinder, the acquisition of the insight needed for initial control system synthesis. Control systems designed on the basis of such simple models should, of course, be tested and
tuned on full system simulations that include all the above-mentioned secondary effects.

1.4 Report Layout

The following Sections of the report give formulae derived for the dynamic impedance of a single roadway (Section 2) a matrix representation in Sections 3 and 4 of a mine comprising a main trunk road and two parallel working areas with control from individual booster fans and the sensitivity of the air flow to various disturbances under diagonal control conditions. Simultaneous control of main fan and one booster is examined in Section 5. These foregoing studies assume instant response of fan pressure and, despite interaction, fail to generate serious instability problems. In Section 6 therefore a fan of substantial inertia is investigated to see whether or not the oscillation of stored energy between fan rotor and roadway air-compression can yield problems of oscillation. Throughout Sections 2 to 5 formulae are stated rather than derived. Their derivations are given in Appendices 1 to 4. Conclusions are given in Section 7. Appendix 5 gives a basis for realistic parameter estimation. Appendix 6 gives simulation results based on this preliminary analyses using software TUTSIM.

2 Model for Roadway Impedance

If a roadway is represented by a single compartment of volume $V =$ that of the roadway inserted between two resistances $R = $ half the roadway resistance as shown in Fig.1 on next page.
then, as shown in Appendix 1, the dynamic impedance \( Z(D) \) of the roadway is given by

\[
Z(D) = \frac{P_i}{q} \frac{2R(1 + \frac{TD}{2})}{1 + TD}
\]

where time constant \( T \) is given by

\[
T = \frac{VR}{P}
\]

assuming isothermal conditions, where \( P \) is atmospheric pressure if applied pressure \( p_i << \bar{p} \)

Note that \( Z(D) \) relates applied pressure to the flow \( q \) adjacent to the pressure source, Relating \( P_i \) to the flow \( q \) at the remote end of the roadway requires the impedance \( Z'(D) \) given by

\[
Z'(D) = \frac{P_i}{q'} \frac{2R(1 + \frac{TD}{2})}{2}
\]

2.1 Step-Responses

From the above expressions for \( Z(D) \) and \( Z'(D) \) it follows that the predicted responses of \( q \) and \( q_1 \) to a step-change in \( P_i \) will be exponential as illustrated in Fig.2.
In reality the roadway should be described by multiple \( N \) compartments of volume \( V/N \) alternating with resistance elements \( 2R/N \) where \( N \to \infty \). This yields the partial differential equation model

\[
A \frac{\partial P}{\partial t} = -P \frac{\partial q}{\partial X} \tag{4}
\]

\[
\rho \frac{\partial P}{\partial t} = P \frac{\partial^2 P}{\partial X^2} \tag{5}
\]

Where \( A \) is the cross-sectional area of the roadway, \( \rho \) the roadway resistivity and \( x \) the distance measured along the roadway so that

\[
2R = \frac{\rho L}{A} \tag{6}
\]

where \( L \) is the length. The full model is difficult to solve analytically or computationally (with accuracy) however but it can be shown that \( q' \) behaves in an impulsive manner whilst \( q \) has infinite initial slope after a step in \( P_t \). Nevertheless, once other system dynamics (e.g. the fan response) are included, the detailed and lumped parameter models produce similar overall behaviour.
The important conclusion from this section is that a mine network can be modelled first in terms of only its roadway resistances $2R_1, 2R_2, 2R_3, \ldots$ (i.e. in steady state only). Thereafter it is merely necessary to substitute the appropriate impedance $Z_i$ (or $Z_i^*$) etc. to obtain the dynamic model for the system network. This method is followed in subsequent Sections but results may be double checked from the first principles as demonstrated in Appendix 1.

3 Twin Branch System with Main Fan and Two Boosters Control

3.1 Steady-State Model

The system is shown in Fig.3 omitting roadway volumes since these can be included (using the Z(D) substitution) later as stated above.

![Network diagram](image)

**Fig.3 Network with two parallel paths and booster fans**

As shown in Appendix 2, the two air flows through fans (a) and (b) (in steady state) are given by:
\[ q_1 = \frac{P_a(R_1 + 2R_2) + P_2R_2 - P_bR_d}{2[R_d(R_1 + R_2) + 2R_1R_2]} \]  
(7) 

and

\[ q_2 = \frac{P_b(R_1 + 2R_1) + P_2R_1 - P_aR_d}{2[R_d(R_1 + R_2) + 2R_1R_2]} \]  
(8)

Note that, as would be expected, \( P_a(P_b) \) affects \( q_1(q_2) \) more powerfully than it affects \( q_2(q_1) \) and that interaction would be zero if \( R_d = 0 \) (i.e. if the two subsystems were physically uncoupled). Note also however that the interaction is significant if

\[ R_d > 2R_1, 2R_2 \]  
(9)

which would be the case if the main roadway were long.

3.2 Special Case: Symmetrical System

For analytical simplicity we now consider the special case of

\[ R_1 = R_2 = R \]  
(10)

\[ V_1 = V_2 = V \]  
(11)

where the \( V \)'s again denote volumes. Equations (7) and (8) thus reduce to

\[ q_1 = \frac{(P_a - P_b)R_d + 2(P_a + P_b)R}{4R(R_d + R)} \]  
(12)

and

\[ q_2 = \frac{(P_b - P_a)R_d + 2(P_a + P_b)R}{4R(R_d + R)} \]  
(13)

\( P_a - P_b \) effectively drives the circulating flow \( q_1, q_2 \) around the parallel loop of Fig.3 whilst

\( P_1 + P_a + P_b \) drives total mine flow \( q_1 + q_2 \) as would be expected.
3.3 Matrix Representation of the Process

Equations (12) and (13) can be combined as shown in Appendix 3 into a single matrix equation of the form

\[
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = G \begin{bmatrix}
P_a \\
P_b
\end{bmatrix} + G_f P_f
\]  \hspace{1cm} (14)

where, in this symmetrical case

\[
G = \begin{bmatrix}
g_1 & -g_2 \\
-g_2 & g_1
\end{bmatrix}
\]  \hspace{1cm} (15)

where

\[
g_1 = \frac{2R+R_d}{4R(R_d+R)}
\]  \hspace{1cm} (16)

and

\[
g_2 = \frac{R_d}{4R(R_d+R)}
\]  \hspace{1cm} (17)

whereas

\[
G_f = g_f I
\]  \hspace{1cm} (18)

in which

\[
I = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]  \hspace{1cm} (19)

and

\[
g_f = \frac{1}{2(R_d+R)}
\]  \hspace{1cm} (20)

For the dynamic process model it is merely necessary to replace 2R by Z(D) as previously defined (and \(R_d\) by its equivalent impedance \(Z_d(D)\) if the volume of the main road is also significant). The asymmetric process would yield asymmetric matrices of course.
3.4 Application of Diagonal Proportional Control

If we wish to regulate flow $q_1$ close to reference value $q_{1r}$ and $q_2$ close to $q_{2r}$ simultaneously then the simplest closed-loop control strategy would take the form

$$
\begin{bmatrix}
P_a \\
P_b
\end{bmatrix} = K
\begin{bmatrix}
q_{1r} - q_1 \\
q_{2r} - q_2
\end{bmatrix}
$$

(21)

where

$$K = k I$$

(22)

i.e. a simple (proportional) diagonal controller where $k$ is the proportional gain of either control loop. Because the process is interactive however the (closed-loop) controlled process will remain interactive (i.e. non diagonal) with such a controller. Inter alia the following results indicate the extent of the remaining interaction and the degree of stability of this two-input, two-output system.

3.5 Closed-Loop System Equations and Sensitivities

Combining process equation (14) with control equation (21) to eliminate control vector $[P_a, P_b]^{T}$ yields the closed-loop system equation (23) relating output vector $[q_1, q_2]^{T}$ to reference vector $[q_{1r}, q_{2r}]$ and disturbance $P_r$:

$(P_r$ is a disturbance" rather than a "control" with this control system which acts on $P_a$ and $P_b$, not on $P_r$)

$$
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = [I + GK]^{-1} \left( GK \begin{bmatrix} q_{1r} \\
q_{2r}
\end{bmatrix} + G_r P_r \right)
$$

(23)
where, from equations (15) and (22), as shown in Appendix 3:

$$[I + GK]^{-1} GK = \frac{1}{[1 + 2g_1K + K^2(g_1^2 - g_2^2)]} \begin{bmatrix} q_1 + K(g_1^2 - g_2^2) & -g_2 \\ -g_2 & g_1 + K(g_1^2 - g_2^2) \end{bmatrix}$$

(24)

Thus, substituting for $g_1$ and $g_2$ using equations 16 and 17 we obtain the sensitivities of $q_1(q_0)$ to variations in $q_{1r}(q_{2r})$, $q_{2r}$ ($q_{1r}$) $P_r$, shown in Appendix 3. These sensitivities work out to be

$$\frac{\partial q_1}{\partial q_{1r}} = \frac{\partial q_2}{\partial q_{2r}} = \frac{K(2R + R_d + k)}{(2R + K)(2(R_d + R) + K)}$$

(25)

$$\frac{\partial q_1}{\partial q_{2r}} = \frac{\partial q_2}{\partial q_{1r}} = \frac{-KR_d}{(2R + K)(2(R_d + R) + K)}$$

(26)

$$\frac{\partial q_1}{\partial P_r} = \frac{\partial q_2}{\partial P_r} = \frac{1}{2(R_d + R) + K}$$

(27)

The results make sense for several reasons:

(a) If $R_d$ is set to zero (eliminating process interaction) we get

$$\frac{\partial q_1}{\partial q_{1r}} = \frac{\partial q_2}{\partial q_{2r}} = \frac{K}{2R + K}$$

(28)

Which is a simple result quickly derivable for a single branch network, and also as expected:

$$\frac{\partial q_1}{\partial q_{2r}} = \frac{\partial q_2}{\partial q_{1r}} = 0$$

(29)

(b) Considering simultaneous identical steps in $q_{1r}$ and $q_{2r} = \Delta q_r$ then:

$$\Delta q_1 = \Delta q_2 = \left[ \frac{\partial q_1}{\partial q_{1r}} + \frac{\partial q_1}{\partial q_{2r}} \right] \Delta q_r + \frac{\partial q_1}{\partial P_r} \Delta P_r$$

(30)
and substituting for the three sensitivities in equation (30) using equations (25) through (27) we get

$$
\Delta q_1 = \frac{K\Delta q_r + \Delta P_f}{2(R_d + R) + K}
$$

which agrees with the direct solution for the symmetrical closed loop system

$$
R_2 = R, \text{ so that } q_1 = q_2 = q \text{ if } q_{1r} = q_{2r} = q_r
$$

$$
\Delta q = k (q_r - q)
$$

$$
\Delta P_f = 2q R_d + q_2R
$$

re-writing \( q = P_f + Kq_r / 2(R_d + R) + k \)

and that again:

$$
\Delta q = \frac{\Delta P_f + K\Delta q_r}{2(R_d + R) + K}
$$

This agreement is reassuring that the derivation of the more general results (25), (26) and (27) are correct.

Discussion and a Numerical Example

From equation (25) for closed loop sensitivity \( \delta q_1 / \delta q_{1r} \), it is clear that \( q_1 \neq q_{1r} \), in general. Increasing \( k \gg \gg 2(R_d + R) \) will make the actual flow approach the reference value closely whilst from equation (26) and (27) for \( \delta q_1 / \delta q_2 \) and \( \delta q_1 / \delta P_f \) the effects of changes in \( q_2 \) and \( P_f \) become much less significant. Likewise \( q_2 \) approaches \( q_{2r} \) more closely with dependence on \( q_{1r} \) and \( P_f \) if \( k \) is made \( \gg 2(R_d + R) \). For instance setting \( k = 10 \), \( R_1 = 1, R_d = 2, q_{1r} = q_{2r} = 4 \) and \( P_f = 10 \) results in \( q_1 (= q_2) = 3.125 \) i.e. close but not
numerical check on the calculation validity, \( P_n \) works out to be 10(4-3.125) = 8.75 from the control law and 2(3.125) + 2(6.25) - 10 also = 8.75 from the pressure/flow equation.]

3.7 Conclusion

From the foregoing steady-state analysis static interaction between the air-flow in the two parallel roadways of Fig.3 remains strong while ever \( R_d \gg 2R+k \) (i.e. if two working area are ventilated via a long shaft and or trunk roadway). Nevertheless the system is diagonally dominant on open loop control and more-so on closed loop control in steady state. This characteristic is a hopeful sign that dynamic instability due to interaction will be avoided also but a dynamic analysis, on the lines of Section 4, is necessary to ensure this.

4 Dynamic Analysis of Two-Booster Fan System: Including Branch-Road Capacitance

Modifying the steady state sensitivity formula (25) for dynamic analysis merely involves the substitution of

\[
Z(D) = \frac{2R(1-TD)}{2(1+TD)}
\]  

(33)

for parameter 2R if the volume \( V \) of each branch roadway is to be taken into account. This is assuming that the feedback flow measurements are of \( q_1 \) and \( q_2 \) in Fig.3 (not \( q_1' \) and \( q_2' \), in which case impedance \( Z'(D) = 2R(1+TD/2) \) should be used instead). In this analysis volume \( V_d \) of the main roadway is neglected for simplicity of analysis but could
Concluded if necessary for simulation purposes by replacing \( R_d \) by \( Z_d(D) = T_d D/(1 + T_d D) \) where \( T_d = R_d V_d / P \). This simplification \( Z_d(D) = R_d \) is valid if the dominant volume of the mine is concentrated in the two parallel working areas of the mine, not in the main access roadways. The substitution of \( Z(D) \) for \( R \) in (25) yields

\[
\frac{\partial q_1}{\partial q_{1r}} = \frac{K[2R(1 + \frac{TD}{2}) + (R_d + K)(1 + TD)](1 + TD)}{[2R(1 + \frac{TD}{2}) + K(1 + TD)][2(R + R_d) + K + (R + K + 2R_d)TD]}
\]

(34)

The closed-loop transfer function between \( q_1 \) and its reference \( q_{1r} \) (as between \( q_2 \) and \( q_{2r} \)) is therefore second order with two (real) denominator time constants

\[
T_{c1} = \frac{\pi(R + K)}{(2R + K)}
\]

(35)

\[
T_{c2} = \frac{\pi(R + 2R_d + K)}{(2R + 2R_d + K)}
\]

(36)

The system therefore reduces essentially to two first order systems in cascade (since \( T_{c1} \) and \( T_{c2} \) are real for damping ratios \( R_d/R \) and \( k/R \)) and therefore the damping ratio \( \zeta = (T_{c2}/\sqrt{T_{c1}T_{c2}}) \geq 1 \) i.e. overdamped and cannot oscillate. Furthermore, since \( T_{c1} \) and \( T_{c2} \) are both positive the system is incapable of exponential instability (provided of course the controller is connected in the correct sense i.e. \( k > 0 \)).

Conclusion
correctly connected despite their being two interconnected energy storage reservoirs (the 2 pressurised volumes V) in the system and despite the use of high-gain control. We have of course assumed instant-response fans and since energy is stored in kinetic form in the fan and rotor it is possible that energy could oscillate between these and the potential energy stores (i.e. the roadway volumes). We therefore examine the effect of including fan dynamics (which have been so far neglected) in Section 6. Before that we examine an alternative control strategy acting on one booster fan and the main fan to see whether or not the asymmetry thus introduced can potentially cause instability.

5 Control of the Main Fan and One Booster

5.1 Process and Control Model

The system considered in this Section is shown in Fig.4

![Diagram of parallel system with air-flows controlled by the main fan and a single booster](image)

Fig.4 Parallel system with air-flows controlled by the main fan and a single booster

It is readily shown that the open-loop process may be described (in steady-state) by the
matrix equation

\[
\begin{bmatrix}
P_d \\
P_f
\end{bmatrix}
= \begin{bmatrix}
R_1 & -R_2 \\
R_3 & R_2 + R_3
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix}
\]  
(37)

in terms of the two parallel flows \( q_1 \) and \( q_2 \) through the working areas of the mine.

One possible proportional control strategy is:

\[
\begin{bmatrix}
P_d \\
P_f
\end{bmatrix}
= \begin{bmatrix}
K_d & 0 \\
0 & K_i
\end{bmatrix}
\begin{bmatrix}
q_1 \_r \\
q_2 \_r
\end{bmatrix}
\]  
(38)

so that combining the process and control equations yields the closed-loop system equation

\[
\begin{bmatrix}
Z_1 + K_d & -Z_2 \\
R_3 & Z_2 + R_3 + K_i
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix}
= \begin{bmatrix}
K_d & 0 \\
0 & K_i
\end{bmatrix}
\begin{bmatrix}
q_1 \_r \\
q_2 \_r
\end{bmatrix}
\]  
(39)

if we set

\[
Z_1 = \frac{R_1 (1 + \frac{T_1 D}{2})}{1 + T_1 D}
\]  
(40)

and

\[
Z_2 = \frac{R_2 (1 + \frac{T_2 D}{2})}{1 + T_2 D}
\]  
(41)

where \( T_1 \) and \( T_2 \) are the time constants \( R_1 V_1 / P \) and \( R_2 V_2 / P \) of roadways 1 and 2 respectively.

5.2 Closed-Loop Characteristic Equation

Equation (39) is thus a dynamic model of the closed loop system whose stability can be checked by examination of the roots of the characteristic equation:

\[
\begin{vmatrix}
Z_1 + K_d & -Z_2 \\
R_3 & Z_2 + R_3 + K_i
\end{vmatrix} = 0
\]  
(42)
\[ (Z_1 + K_2)(Z_2 + R_3 + K_2) + R_3 Z_2 = 0 \]  \hspace{1cm} (43)

Complete analytical solution for the roots of (43) is difficult and we therefore resort to
numerical example:

5.3 Numerical Example

Noting that the open-loop system is non-diagonally dominant (and therefore more prone
to closed-loop instability) if

\[ R_1 < \frac{R_2 R_3}{(R_2 + R_3)} \]  \hspace{1cm} (44)

we first choose an example satisfying this condition in an attempt to "force" instability. We
also chose relatively high controller gains compared to \( R_1, R_2 \) and \( R_3 \). In particular we set
\( R_1 = 0.1, R_2 = R_3 = 1, k_d = 10, k_i = 2 \) and \( T_1 = T_2 = 2 \). These parameters substituted
into equations (40), (41) and characteristic eqn(43) yield

\[ 142.7 \ D^2 + 154.1 \ D + 41.4 = 0 \]  \hspace{1cm} (45)

i.e. a quadratic equation in \( D \) of the form

\[ aD^2 + bD + c = 0 \]  \hspace{1cm} (46)

Solutions for \( D \) will be real and negative since \( \sqrt{b^2 - 4ac} \) is real and \( < b \) so that, as in the
example of Section 4, the closed loop system here is still overdamped despite the
asymmetry and practically reasonable attempts to force the system into instability.

5.4 Numerical Example 2

Since the high gains of example 1 may have forced the dynamic closed loop system into
diagonal dominance (despite satisfying condition 44) we here consider a high resistance,
low gain system. In particular we set \( R_1 = 1, R_2 = R_3 = 4, k_i = k_d = 2 \) and \( T_1 = T_2 = 2 \)
yielding characteristic equation:

\[ 122 \, D^2 + 152 \, D + 46 = 0 \]  \tag{47} 

Again \( \sqrt{b^2 - 4ac} \) is real and \(< b \) so yielding two negative real roots and hence overdamped, stable behaviour of the closed loop system. Further reduction of gain would not be sensible due to the loss of static accuracy by the system.

5.5 An Alternative Control Structure

The control structure (38) in the previous example used \( q_1 \) to set booster pressure \( P_d \) and \( q_2 \) to set main fan pressure \( p_t \). This "remote control" feature of the second loop was incorporated deliberately to try to force the system into instability. A more likely-control to be adopted would use \((q_3 = q_1 + q_2)\) to set \( p_t \), so necessitating use of the open loop equation

\[
\begin{bmatrix}
Z_1 + Z_2 & -Z_2 \\
-Z_2 & Z_2 + R_3
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_3
\end{bmatrix} = \begin{bmatrix}
P_d \\
P_t
\end{bmatrix}
\]  \tag{48}

This open loop system is always diagonally dominant since

\[ Z_1 Z_2 + \frac{Z_2^2}{R_3} + Z_2 R_3 > \frac{Z_2^2}{R_3} \]  \tag{49}

so that using this control structure is indeed less likely to cause closed loop instability.

5.6 Conclusions

In this Section it has been demonstrated (though not comprehensively proved) that very asymmetric ventilation control systems remain difficult to destabilise when all dynamics other than those of roadway capacitance are neglected. Counter examples may well be possible but determined efforts to set up a practically-justifiable unstable network plus
control structure have so far failed. This is practically reassuring of course. In Section 6 we examine the possible destabilising effect of variable-speed fan inertia.

6 Interaction of Fan and Roadway Dynamics

The kinetic energy of a variable speed fan and the potential energy of the air compressed within the volume (capacitance) of a single mine roadway again constitute a second order system potentially capable of oscillation. We here develop a linear model for such a combination and assess its damping ratio.

6.1 Process Model

The fan is now ascribed an internal resistance $R_f$ to allow for some reduction in output pressure $P_f$ from its no-load value $P_{f0}$ due to increased flow $q$. Specifically we assume that:

$$ P_f = P_{f0} - R_f q $$  \hspace{1cm} (50)

whilst $P_{f0}$ is assumed to be linearly speed dependent thus

$$ P_{f0} = k_f \Omega $$  \hspace{1cm} (51)

where $\Omega$ = angular speed of rotation and $k_f$ is a fan constant. Now the fan motor will exhibit some slow-down from no-load speed $\Omega$ to $\Omega_0$ with increasing torque $\tau_m$ developed by the motor so that

$$ \Omega = \Omega_0 - k_m \tau_m $$

or

$$ \tau_m = \frac{(\Omega_0 - \Omega)}{K_m} $$  \hspace{1cm} (52)
where $K_m$ can be taken as constant to a first approximation. Now $\tau_m$ will only equal load torque $\tau_L$ in steady state and $\tau_L$ will depend on flow rate $q$ through the fan thus
\[
\tau_L = Kq \tag{53}
\]
so that, in steady state, equations (51) and (53) satisfy the requirements of energy conservation, viz:
\[
\Omega \tau_L = P_k q \tag{54}
\]
(in steady state)

In the dynamic situation the torque difference $\tau_m - \tau_L$ will produce acceleration $D\Omega$ of the fan and motor inertia $J$ so that
\[
\tau_m - \tau_L = J D \Omega \tag{55}
\]
and the model is completed by the roadway impedance relationship
\[
q Z(D) = P_l \tag{56}
\]
as before, where again,
\[
Z(D) = \frac{2R(1+TD)}{2} \tag{57}
\]

6.2 System Block Diagram and Transfer-Function

The equations (50) through (58) can be represented in the block diagram form of Fig.5 on next page and, as shown in Appendix 4, the diagram can be reduced in stages to produce the overall open loop transfer function between the set fan speed $\Omega_0$ and air flow $q$ in the following form:
\[
\frac{q}{\Omega_0} = \frac{K'\omega_0^2(1+TD)}{\omega_0^2 + 2\zeta \omega_0 D + D^2} \tag{58}
\]
Undamped natural frequency $\omega_0$ is given by:

$$\omega_0^2 = \frac{(A+1)}{T_m T'}$$

(59)

where parameter

$$A = \frac{K_m K_f^2}{(2R + R_d)}$$

(60)

and time constants $T_m$ and $T'$ by

$$T_m = K_m J$$

(61)

and

$$T' = \frac{\tau(R + R_d)}{(2R + R_d)}$$

(62)

Open-loop process gain $K'$ is given by

$$K' = \frac{KA}{(A+1)}$$

(63)
where

\[ K = \frac{1}{K_m K_i} \]  

(64)

and damping ratio \( \zeta \) by:

\[ \zeta = \frac{1}{2} \frac{T_m + T' + AT}{\sqrt{T_m T'(A+1)}} \]

(65)

6.3 Degree of Damping of the Open-Loop System

Now as also shown in Appendix 4

\[ A = \frac{(\Omega_o - \Omega)}{\Omega} \]  

(66)

(in steady-state)

and hence

\[ A < 1.0 \]  

(67)

for a well designed motor so that

\[ \zeta = \frac{1}{2} \frac{T_m + T'}{\sqrt{T_m T'}} \]  

(68)

Thus the damping ratio is almost equal to that produced by two cascaded first-order lags of time constant \( T_m \) and \( T' \) respectively. Thus, although the open-loop system is potentially capable of oscillation (i.e. \( \zeta < 1.0 \)) this condition will not occur in practice where fan drives of small slip are used. This condition i.e. (66) and (67) again ensures overdamped behaviour (\( \zeta > 1.0 \)).

Of course now closing a control loop between a measurement of \( q \) and set speed \( \Omega_o \) could generate underdamping (as can occur with all second order systems with any open loop \( \zeta \)) if sufficient controller gain is used.
6.4 Conclusion

Under practical circumstances the linear model developed for the fan/roadway combination should not oscillate under open loop conditions although it could do so under closed loop operation. For simplicity of modelling, the fact that parameter $A << 1.0$, it is allowable to represent the fan/roadway combination as a cascade of first-order roadway and fan dynamics thus

$$\frac{q}{Q_0} = \frac{K(1+TD)}{(2R+R)(1+T_mD)(1+T'D)} \tag{69}$$

or

$$\frac{q}{F_{\phi}} = \frac{1}{(2R+R)(1+TD)} \left[ \frac{1+TD}{1+T'D} \right] \left[ \frac{1}{1+T_mD} \right] \tag{70}$$

The absence of oscillation (an open loop) comes about despite the two dissimilar sources of energy storage in the system (i.e. the kinetic energy of the fan and motor and the potential energy of the compressed mine air) because of the heavy dissipation of energy resulting from the roadway resistance. It should be investigated in future however as to whether parabolic fan and roadway pressure-flow characteristics might introduce oscillation. The increased local droop of the fan characteristic would increase the effective value of parameter $A$, whilst the upturned characteristic of the roadway would increase the linearised resistance $R$ locally; i.e. two conflicting effects. A simulation study would be necessary or an analytical small perturbation study.
7. Overall Conclusions

Analyses have been carried out to assess the extent of the likely instability and interaction problems to be encountered in attempting comprehensive feedback control of mine ventilation networks. The investigations have been confined to linear models of simple networks in the interests of simplicity of analysis at this preliminary stage pending full scale simulation studies.

The analyses have demonstrated that closed-loop instability of multiple loop control systems is unlikely to come about as a result of mere network complexity although considerable interaction will occur unless control loop gains can be increased sufficiently above roadway resistance values. Restriction on these gain settings could arise through the combination of mine and actuator (fan or powered aperture) time lags. In this connection however a study of the combined effect of roadway and fan inertial dynamics has shown that the open-loop combination is overdamped and therefore should permit relatively high controller gain settings.

The investigation has therefore been encouraging in that serious instability problems have not been revealed and it has been demonstrated how modern multivariable control techniques are applicable to the analysis and synthesis of the necessary control structures for the mine ventilation application.

A fuller simulation study should now be undertaken that includes higher order (i.e. mult-capacitance) models of the mine roadways and work areas, their nonlinear characteristics (i.e. turbulent not laminar flow) and all actuator dynamics.

It should be stressed that the study has focused on the control of air distribution not a
thermodynamic effects nor directly on temperature, humidity or pollutant control. Suitably linearised however these important additional aspects could nevertheless be encompassed by the techniques demonstrated in this report.

It is believed that this study and those recommended as a follow-up to it are novel in the field of mine ventilation. Research studies in this area previously have concentrated on detailed nonlinear but essentially steady-state control with little regard to control strategy. The techniques shown here have the potential to complete this missing link.
Appendix I

Impedance of Roadway Modelled by Two Resistances and a Lumped capacitance

Model Diagram

Symbols

$2R =$ total roadway resistance

$V =$ total volume of roadway

$\bar{P} =$ atmospheric pressure

$P =$ pressure in lumped volume compartment $V$

$P_f =$ applied pressure at L.H. end

$q =$ flow through pressure source

$q'$ = flow at remote (R.H.) end of roadway

Model Equations

$q_1 = \frac{(P_f-P)}{R} \quad , \quad q' = \frac{P}{R}$

Rate of accumulation of air in

$V = \frac{VDP}{\bar{P}} = q - q'$
Balance Derivation

Balance \( Z = \frac{P_f}{q} \), .'. eliminate unwanted variables \( P \) & \( q' \) from eqns. above:

\[
q'(1 + \frac{VR}{P} D) = q \quad \text{or} \quad q' = \frac{q}{1 + TD}
\]

\[
T = \frac{VR}{P} \quad \text{and} \quad q = \frac{P_f}{R} - q' = \frac{P_f}{R} - \frac{q}{1 + TD}
\]

\[
\therefore q(2 + TD) = \frac{P_f}{1 + TD}
\]

\[
\therefore Z = \frac{P_f}{q} = \frac{2R[1 + (\frac{T}{2})D]}{1 + TD}
\]

Where \( T = \frac{VR}{P} \)

If \( Z' \) denotes \( \frac{P_f}{q'} \) then since \( q' = q/(1+TD) \) it follows: that

\[
Z' = \frac{P_f}{q'} = 2R \left[1 + (\frac{T}{2})D\right]
\]
Appendix 2

Twin Branch System with Main Fan and Two Boosters-Derivation of Steady-State Model

Model Diagram

\[ q_1 = \frac{- (P_f - P_p)}{2R_1} \]
\[ q_2 = \frac{- (P_f - P_b)}{2R_2} \]
\[ q_1 + q_2 = (P_f - P_l)R_d \]
\[ p_d = (q_1 + q_2)R_d + P_f \]
\[ 2R_1 q_1 = -(q_1 + q_2)R_d + P_f + P_p \]
\[ 2R_2 q_2 = -(q_1 + q_2)R_d - P_f + P_b \]

\[ \begin{bmatrix} 2R_1 + R_d & R_d \\ R_d & 2R_2 + R_d \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} P_a \\ P_b \end{bmatrix} + \begin{bmatrix} P_f \\ P_f \end{bmatrix} \]

\[ \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \frac{1}{4R_1R_2 + 2R_d(R_1 + R_2)} \begin{bmatrix} 2R_2 + R_d & -R_d \\ -R_d & 2R_1 + R_d \end{bmatrix} \begin{bmatrix} P_a + P_f \\ P_b + P_f \end{bmatrix} \]
\[ q_1 = \frac{P_a(R_d+2R_2)+P_2R_2-P_0R_d}{2[R_d(R_1+R_2)+2R_1R_2]} \]

\[ q_2 = \frac{P_a(R_d+2R_1)+P_2R_1-P_0R_d}{[2R_d(R_1+R_2)+2R_1R_2]} \]
Appendix 3

Derivation of Closed Loop Matrix Model for Twin Booster Fan System under Diagonal Proportional Control

The symmetrical Process Model is represented by

\[
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = G \begin{bmatrix}
P_a \\
P_b
\end{bmatrix} + G_f P_f,
\]

Where

\[
G = \begin{bmatrix}
g_1 & -g_2 \\
-g_2 & g_1
\end{bmatrix}
\]

and

\[
g_1 = \frac{2R+R_d}{4R(R_d+R)} , \quad g_2 = \frac{R_d}{4R(R_d+R)}
\]

in steady state, while

\[
G_f = g_f I , \quad g_f = \frac{1}{2(R_d+R)} , \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

{For the dynamic version of the model we merely replace 2R by \(Z(D)\),

\[
Z(D) = \frac{2R(1+TD)}{2} \left(1+TD\right)
\]

The Control Law examined is

\[
\begin{bmatrix}
P_a \\
P_b
\end{bmatrix} = K \begin{bmatrix}
q_{1r}-q_1 \\
q_{2r}-q_2
\end{bmatrix} , \quad K = kl
\]
For the Closed Loop Model, combining process and control equations we get

\[
\begin{bmatrix}
q_1 \\
q_2 \\
\end{bmatrix} = (1+GK)^{-1} \begin{bmatrix}
GK & q_{1r} \\
q_{2r} & \\
\end{bmatrix} + G_F P_I
\]

where

\[
(1+GK)^{-1} = \begin{bmatrix}
1+g_1 k & -g_2 k \\
-g_2 k & 1+g_1 k \\
\end{bmatrix}^{-1}
\]

\[
\therefore \quad (1+GK)^{-1} GK = \frac{k}{1+2g_1 k+k^2 (g_1^2-g_2^2)} \begin{bmatrix}
1+g_1 k & g_2 k \\
g_2 k & 1+g_1 k \\
\end{bmatrix} \begin{bmatrix}
g_1 & -g_2 \\
g_2 & g_1 \\
\end{bmatrix}
\]

\[
\therefore \quad (1+GK)^{-1} GK = \frac{k}{1+2g_1 k+k^2 (g_1^2-g_2^2)} \begin{bmatrix}
g_1+k(g_1^2-g_2^2) & -g_2 \\
-g_2 & g_1+k(g_1^2-g_2^2) \\
\end{bmatrix}
\]

**Control Sensitivities**

Now, considering the leading diagonal terms

\[
g_1^2 - g_2^2 = \frac{1}{4R(R_d + R)}
\]

so that

\[
\frac{\partial q_1}{\partial q_{1r}} = \frac{k(g_1+k(g_1^2-g_2^2))}{1+2g_1 k+k^2 (g_1^2-g_2^2)}
\]

\[
= \frac{k(2R+R_d+k)}{4R(R_d + R) + 2k(2R+R_d) + k^2}
\]

\[
\therefore \quad \frac{\partial q_1}{\partial q_{1r}} = \frac{k(2R+R_d+k)}{(2R+k)(2(R_d + R)+k)}
\]

\[
\frac{\partial q_2}{\partial q_{2r}} = \frac{k(2R+R_d+k)}{(2R+k)(2(R_d + R)+k)}
\]

For the off-diagonal (i.e. loop interaction terms) we deduce that:
\[
\frac{\partial q_1}{\partial q_{2r}} = - \frac{g_2 k}{1 + 2g_1 k + k^2 (g_1^2 - g_2^2)}
\]

\[
\frac{\partial q_1}{\partial q_{2r}} = \frac{\partial q_2}{\partial q_{1r}} = - \frac{kR_d}{(2R + k)[2(R + R_d) + k]}
\]

Finally, the Sensitivity to Disturbance, \(P_t\) is obtained thus:

\[
g_t = \frac{1}{2(R_d + R)}
\]

\[
1 + g_1 k + g_2 k = 1 + \frac{2(R + R_d)k}{4R(R + R_d)} = \frac{1 + k}{2R}
\]

\[
\therefore g_t(1 + g_1 k + g_2 k) = \frac{2R + k}{4R(R + R_d)}
\]

and since

\[
1 + 2g_1 k + k^2 (g_1^2 - g_2^2) = \frac{(2R + k)[2(R + R_d) + k]}{4R(R + R_d)}
\]

it follows that:

\[
\frac{\partial q_1}{\partial P_t} = \frac{\partial q_2}{\partial P_t} = \frac{1}{2(R_d + R) + k}
\]
Appendix 4

Derivation of Second-Order Transfer Function between Demanded Fan Speed and Airflow in the Presence of Fan Drive Inertia

The block diagram of the system (Fig.) reduces in two stages as shown below:

(The top Fig. is convenient for simulation purposes). Two further stages of reduction can be
performed as shown below:

which can be modified to give

where

\[ T' = \frac{\pi(R_f + R)}{(R_f + 2R)} \]

The final version (above) thus takes the form
where $K = 1 / k_m$, $k_i$ and $A = k_i$ and $A = (k_i)^2k_m / (2R + R_i)$

Hence
\[
\frac{\Omega_0}{q} = \frac{KA(1+TD)}{(1+T_mD) (1+T'D) - A(1+TD)}
\]
\[
= \frac{KA(1+TD)}{A+1+D(T_m+T'+AT)+T_mT'D^2}
\]
\[
= \frac{k_i\omega_0^2}{\omega_0^2+2\zeta\omega_0D+D^2}
\]

Where
\[
k_i' = \frac{KA}{(A+1)}
\]
\[
\omega_0^2 = \frac{(A+1)}{T_mT'}
\]
\[
2\zeta\omega_0 = \frac{(T_m+T'+AT)}{T_mT'}
\]

\[
\zeta = \frac{1}{2} \left[ \frac{T_m+T'+AT}{\sqrt{T_mT'(A+1)}} \right]
\]

**Relative Magnitude of Parameter A**

In steady state, $k_i = P_{m0} / \Omega$ Whilst $k_m = (\Omega_0 - \Omega) / \tau$

and $2R + R_i = P_{m0} / q$
\[ A = \frac{P_{\tau0}^2}{\Omega^2} \left( \frac{\Omega_0 - \Omega}{\tau} \right) \frac{q}{P_{\tau0}} = \frac{P_{\tau0}q}{\tau \Omega} \left( \frac{\Omega_0 - \Omega}{\Omega} \right) \]

\[ A = \frac{(\Omega_0 - \Omega)}{\Omega} \quad , \quad P_{\tau0}q = \tau \Omega \]

For efficiency the fan drive will be designed to ensure only a small relative droop in speed on application of full load. So \( A \ll 1.0 \) for normal applications.
Appendix 5

Estimation of Resistance and Time Constant Values for Mine Workings

Basis

A typical mine requires 0.25 m water gauge of pressure between its upcast and downcast shafts to provide the total ventilation needs of the mine.

Suppose the mine comprises 20 parallel paths and that the average air velocity in each is 7 m/s. Thus, if the average cross sectional area, A, per path = 7 m² (i.e. 3 m dia approximately) the total mine air flow = 20 x 7 x 7 = 980 m³/s (i.e. 58,800 m³/min).

Resistances

\[ R_{\text{mine}} = \frac{0.25}{980} = 25 \times 10^{-5} \text{ s/m}^2 \]

\[ R_{\text{roadway}} = 20 \times 25 \times 10^{-5} = 500 \times 10^{-5} = 5 \times 10^{-3} \text{ s/m}^2 \]

Capacitances

Roadway volume = A x length = 7x500 m (say) = 3500 m³

Mine volume = 20 x 3500 = 70 \times 10^3 \text{ m}^3

Atmospheric Pressure P = 12 m water gauge

Mine time constant =

\[ \frac{V_{\text{mine}} R_{\text{mine}}}{P} = \frac{V_{\text{roadway}} R_{\text{roadway}}}{P} \]

\[ = \frac{70 \times 10^3 \times 25 \times 10^{-5}}{12} = 1.46 \text{ s} \]
Comment

The foregoing analysis provides initial priming values for simulation scaling and testing. Obviously site data is needed for any specific mine study but the foregoing illustrates the order of numerical values to be expected and a basis for parameter estimation where a complete set of pressure and air flow data around the network is unavailable.

Fan Dynamics

Note that the time constant $T_m$ of the main fan and motor is probably also around 1.5 s to 2.0 s bearing in mind that its starting time is usually 5-10 secs, a 3000 kw motor has a time constant around 0.25 to 0.5 seconds and a ventilating fan inertia may be 4 x the motor inertia. Obviously manufacturers data or a speed-change test is needed in a real case-study. What is important is that the mine and fan time constants are comparable in magnitude so that both need to be considered in dynamic control studies.
Appendix 6

A fuller simulation study is done with the help of a software TUTSIM. The study starts with simple one roadway mine structure and goes up to feedback control of mine roadways included the higher order (i.e. multi-capacitance) models of roadways and work areas. The responses are checked against predicted or calculated behaviour and time constants.

For the sake of simplicity at this preliminary stage the values of mine volume, mine resistance and fan resistance is assumed as follows:

\[ V = 70,000 \text{ m}^3 \]

\[ P = 12 \text{ m water gauge} \]

\[ R_{\text{mine}} = 25 \cdot 10^5 \text{ s/m}^2 \]

\[ R_{\text{fan}} = R_f = 1/10 \text{ of a roadway resistance} = 2.5 \cdot 10^5 \text{ s/m}^2 \]

for simulation pressure outside mine is also considered as zero.

Mine as a single road-way.

\[ \dot{P}_1 = 0 \quad q_1 \quad R_1 \quad V_1 \quad P \quad q_1' \quad \frac{P_f}{R'} \]

Volume outflow \[ q'_1 = \frac{P_f}{(R_1 + R_f)} \]
Pressure inside mine = $P_i = \left( \frac{P_1 R_1}{R_1 + R_f} \right)$

Block diagram is as follows

Time constant = $\frac{V}{P} \left[ \frac{R_1 R_f}{R_1 + R_f} \right]$

by putting values = $0.133$ approximately

$q_i$ (steady state) = $3636.3636\ldots \frac{m^3}{s}$

The resulting graph is on page G.1
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Mine as a two roadways in series.

\[ q_1 = -(P_1 + P)/R_1 = -P/R_1 \]

\[ q_2 = -(P_2 + P_1)/R_2 \]

\[ q_3 = (P_1 + P_2)/R_1 \]

\[ V_1 = V_2 = 35000 \text{ m}^3 \]

\[ R_1 = R_2 = 2R = 5 \times 10^{-4} \text{ m}^2/\text{s} \]

\[ P_1 - \int \frac{\bar{P}}{V_1}(q_1 + q_2 - q_3) \]

Similarly

\[ P_2 - \int \frac{\bar{P}}{V_2}(q_1 + q_2 - q_3) \]

Block diagram is as follows
Simulation result is on page G.2

It can be seen from simulation result that $P_2$ falls more suddenly than $P_1$ since volume $V_2$ is closer to the fan. Because $V_1$, is a large vol. and $P_1$ is initially zero, flow $q_2$ overshoot before setting to its final predicted value 3636 m$^3$/sec. $q_1$ rises more gradually because $P_1$ falls only gradually but $q_1 = q_2$ in steady state and time response are of similar duration to those of the single (lumped) roadway model (c.f. graph G.1.) as equated, but the variation of flow through the mine is now more closely related to the equated behaviour of a spatially distributed (i.e. non lumped) mine model. i.e. changes occur more slowly at long distances from the source of disturbance.

**Simulation considering fan inertia**

Instead of considering fan as a step change in pressure, now we will consider some time delay $T_f$ in changing fan pressure to a demanded value. The following block diagram is a circuit with Transfer-function

$$\frac{1}{1 + T_f D}$$

can do that

Results are shown on page G.3 with $T_f = .1$ and on G.4 with $T_f = .5$
# EGIATE+ VERSION OF TUTSIM

Model File: ser1.sim
Date: 11 / 23 / 1991
Time: 18: 8
Timing: 0.0010000, DELTA: 2.00000, RANGE

Plot Blocks and Scales:

<table>
<thead>
<tr>
<th>BlockNo</th>
<th>Plot-MINimum</th>
<th>Plot-MAXimum</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horz</td>
<td>0</td>
<td>0.00000</td>
<td>1.50000; Time</td>
</tr>
<tr>
<td>Y1</td>
<td>4</td>
<td>-2.00000</td>
<td>2.00000; pressure P2</td>
</tr>
<tr>
<td>Y2</td>
<td>7</td>
<td>-2.00000</td>
<td>2.00000; pressure P1</td>
</tr>
<tr>
<td>Y3</td>
<td>5</td>
<td>-5.000E+03</td>
<td>5.000E+03; vol. flow q2</td>
</tr>
<tr>
<td>Y4</td>
<td>6</td>
<td>-5.000E+03</td>
<td>8.000E+03; vol. flow q1</td>
</tr>
</tbody>
</table>

![Graph](image-url)
COLLEGIATE+ VERSION OF TUTSIM

File: ser3.sim
Date: 11 / 23 / 1991
Time: 18 : 13
Timing: 0.0100000 , DELTA ; 1.5000 , RANGE

Plot Blocks and Scales:

Format:
Block No., Plot-MINimum, Plot-MAXimum, Comment

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<th>Block</th>
<th>Plot-MINimum</th>
<th>Plot-MAXimum</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>4, -2.0000</td>
<td>2.0000</td>
<td>pressure P2</td>
</tr>
<tr>
<td>Y2</td>
<td>7, -2.0000</td>
<td>2.0000</td>
<td>pressure P1</td>
</tr>
<tr>
<td>Y3</td>
<td>5, -5.000E+03</td>
<td>5.000E+03</td>
<td>vol. flow q2</td>
</tr>
<tr>
<td>Y4</td>
<td>6, -5.000E+03</td>
<td>5.000E+03</td>
<td>vol. flow q1</td>
</tr>
</tbody>
</table>

$T_f = 0.1$
COLLEGIATE+ VERSION OF TUTSIM

File: ser3.sim
Date: 11 / 23 / 1991
Time: 18 : 19
Timing: 0.0100000 ,DELTA ; 1.50000 ,RANGE
PlotBlocks and Scales:

Format:
BlockNo, Plot-MINimum, Plot-MAXimum; Comment
Horz: 0 ; 0.0000 ; 1.5000 ; Time
Y1: 4 ; -2.0000 ; 2.0000 ; pressure P2
Y2: 7 ; -2.0000 ; 2.0000 ; pressure P1
Y3: 5 ; -5.000E+03 ; 5.000E+03 ; vol. flow q2
Y4: 6 ; -5.000E+03 ; 5.000E+03 ; vol. flow q1

\[ \tau = 0.5 \]

Y1: pressure P2

TUTSIM

0.0000 Time

1.5000

-2.0000

-1.6000

-1.2000

-1.0000

-0.8000000

-0.6000000

-0.4000000

0.0000

0.4000000

0.8000000

1.20000

1.60000
Mine as a two parallel Roadways

\[ q_1 = \frac{-P_3 + P_1}{R_1} \]
\[ q_2 = \frac{-P_3 + P_2}{R_2} \]
\[ q_3 = \frac{P_1 + P_3}{R_f} \]

\[ P_1 = \int \frac{P}{V_1} (q_1 + q_2 - q_3) \]
\[ P_2 = \int \frac{P}{V_2} (q_1 + q_2 - q_3) \]

The block diagram is as follows
Taking

\[ R_1 = R_2 = 5 \times 10^{-4} \text{ m}^2/\text{s} \]

\[ V_1 = V_2 = 35,000 \text{ m}^3 \]

\[ R_f = 2.5 \times 10^5 \text{ m}^2/\text{s} \]

Simulation result is on page G.5
COLLEGIATE+ VERSION OF TUTSIM

File: para3.sim
Date: 11/23/1991
Time: 18:28
Timing: 0.0100000, DELTA: 1.50000, RANGE

Plot Blocks and Scales:
Format:
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<th>Plot-MAXimum</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horz</td>
<td>0, 0.0000</td>
<td>1.50000</td>
<td>Time</td>
</tr>
<tr>
<td>Y1:</td>
<td>4, -2.0000</td>
<td>2.00000</td>
<td>pressure P2</td>
</tr>
<tr>
<td>Y2:</td>
<td>6, -2.0000</td>
<td>2.00000</td>
<td>pressure P3</td>
</tr>
<tr>
<td>Y3:</td>
<td>5, -5.000E+03</td>
<td>5.000E+03</td>
<td>vol. inflow q2</td>
</tr>
<tr>
<td>Y4:</td>
<td>2, -5.000E+03</td>
<td>5.000E+03</td>
<td>vol. outflow q3</td>
</tr>
</tbody>
</table>

Y1: pressure P2

\[ 2.0000 \]

1.6000

1.2000

0.8000000

0.4000000

0.0000

-0.4000000

-0.8000000

-1.2000

-1.6000

-2.0000

0.0000 Time 1.5000
Main fan with two booster fans in each parallel roadway

From the previous discussion in section 3 and 4.

The block diagram of this system is as follows.
Where

\[ T_1 = \frac{V_1 R_1}{P} \]

\[ T_2 = \frac{V_2 R_2}{P} \]

\[ T'_1 = \frac{T_1(R_1+R_d)}{(2R_1+R_d)} \]

\[ T'_2 = \frac{T_2(R_2+R_d)}{2(R_2+R_d)} \]

Simulation is done with two options

(1) Two booster fans are working with same pressure.

(2) Only one booster fan is working while other is stopped.

Page G.6 is showing both booster fans are working with same pressure.

Page G.7 is showing only one booster is working. By giving different values to this fan different curves are with same scales are superimpose on same graph. So that it is easy to compare the volumetric flow in both roadways under different conditions.
Main fan and one booster:

\[ P_{a1} = 0.7 \]
\[ P_f = 1 \]
\[ P_{a2} = 0 \]
Mine as a three Roadways

(Two parallel roadways in series with one roadway)

\[ q = -\frac{P_a + P}{R_a} \]
\[ q_1 = -\frac{P_3 + P_a}{R_b} \]
\[ q_2 = -\frac{P_3 + P_a}{R_c} \]
\[ q_3 = -\frac{P + P_3}{R_f} \]

\[ P_a = \int \frac{P}{V_1} (q - (q_1 + q_2)) \]
\[ P_b = \int \frac{P}{V_2} (q_1 + q_2 - q_3) \]
\[ P_c = \int \frac{P}{V_3} (q_1 + q_2 - q_3) \]
block diagram is as follows

Taking

\[ R_a = R_b = R_c = 16.67 \cdot 10^{-5} \text{ m}^2/\text{s} \]

\[ V_1 = V_2 = V_3 = 23333.33 \text{ m}^3 \]

\[ R_f = 2.5 \cdot 10^{-5} \text{ m}^2/\text{s} \]

simulation result is on page G.8
EGIATE+ VERSION OF TUTSIM

Model File: serpar2.sim
Date: 11 / 23 / 1991
Time: 18 : 34
Timing: 0.010000 ,DELTA : 1.5000 ,RANGE
PlotBlocks and Scales:
Format:
BlockNo, Plot-MINimum, Plot-MAXimum; Comment
Horz: 0, 0.0000 , 1.5000 ; Time
Y1: 10, -2.0000 , 2.0000 ; pressure Pa
Y2: 7, -2.0000 , 2.0000 ; pressure Pb
Y3: 12, -5.000E+03 , 5.000E+03 ; vol. inflow q
Y4: 9, -5.000E+03 , 5.000E+03 ; vol. flow q1

Y1: pressure Pa

2.0000

1.6000

1.2000

0.8000000

0.4000000

0.0000

-0.4000000

-0.8000000

-1.2000

-1.6000

-2.0000

0.0000 Time

1.5000

TUTSIM
Application of feedback control

(Main fan with two booster fans in each parallel roadway)

By applying diagonal proportional controller to both booster fan in each parallel roadway, the close loop response can be produce by adding following circuit to each booster.

$q_r = \text{Reference value of } q.$
$q = \text{vol. flow.}$
$K = \text{Proportional gain of controller.}$
$T_f = \text{Fan time constant.}$
$P_d = \text{Demanded value of pressure.}$
Where

\[ P_d = K(q_r - q) \]
\[ P_a = P_d / (1 + T_f) \]

Giving the output of above block diagram to following block diagram.

and considering numerical example of section 3.6, we will take the symmetrical values

\[ R_1 = R_2 = R = 1 \]
\[ q_{1r} = q_{2r} = 4 \]
\[ R_d = 2 \]
\[ P_f = 10 \]
\[ K = 10 \]
The result is shown on page G.9

The steady state value of \( q \) is very close to the numerically calculated value 3.125

By increasing \( K \gg R_d + R \) makes the actual flow approach the reference value i.e. 4

This can be seen on page G.10. Where different responses for different value of \( K \) is shown.

**Sperate Control of Two Parallel Roadways**

Now we can show that if we take different reference value of volume flow for each roadway and apply different feedback gain to each roadway, the control of each roadway can be handled separately. Now setting

\[
q_{1r} = 3 \\
q_{2r} = 5 \\
K = K_1 = K_2
\]

While keeping all the other parameters value same as previous case.

The response of \( q_1 \) and \( q_2 \) in steady-state is almost same as numerically calculated values

i.e. \( q_1 = 2.5 \) and \( q_2 = 3.75 \) at \( K = 10 \)

The responses can be seen on page G.11

Now increasing \( K_2 \gg 2( R_d + R \) while keeping \( K_1 \) unchanged, the \( q_2 \) will increase towards its reference value i.e 5. But steady-state value of \( q_1 \) is not effected by increasing \( K_2 \), inspite of interaction between two parallel roadways. It can be seen that flow in one branch can be controlled without effecting rest of mine. This can be seen on page G.12

By increasing \( K_1 \gg 2( R_d + R \) the reverse effect of above mentioned description can be
seen on page G.13

Page G.14 shows $q_1$ and $q_2$ (in steady-state) goes to their corresponding reference values as $K_1$ and $K_2$ increased simultaneously.
COLLEGIATE+ VERSION OF TUTSIM

Model File: cont3.sim
Date: 12 / 3 / 1991
Time: 14 : 5
Timing: 1.0000 , DELTA : 2.500E+03 , RANGE
Plot Blocks and Scales:
Format:
Horz. BlockNo. Plot-MINimum, Plot-MAXimum; Comment
Y1: 17, 0.0000, 5.0000; vol. flow q1
Y2: 16, 0.0000, 5.0000; vol. flow q2
Y3: 
Y4: 

Y1: vol. flow q1

TUTSIM

0.0000 5.0000
4.5000
4.0000
3.5000
3.0000
2.5000
2.0000
1.5000
1.0000
0.5000
0.0000

0.0000 Time 2.500E+03
COLLEGIATE+ VERSION OF TUTSIM

Model File: cont3sim
Date: 12/3/1991
Time: 14:21
Timing: 1.0000, DELTA: 2.500E+03, RANGE
Plot Blocks and Scales:
Format:
Block No., Plot-MINimum, Plot-MAXimum; Comment
Horz: 0, 0.0000, 2.500E+03; Time
Y1: 17, 0.0000, 5.0000; vol. flow q1
Y2: 16, 0.0000, 5.0000; vol. flow q2
Y3: , , ;
Y4: , , ;

Y1: vol. flow q1
0.0000 0.5000000
0.0000 1.0000000
0.0000 1.5000000
0.0000 2.0000000
0.0000 2.5000000
0.0000 3.0000000
0.0000 3.5000000
0.0000 4.0000000
0.0000 4.5000000
0.0000 5.0000000

0.0000 2.500E+03

TUTSIM

0.0000 Time

v=500
v=300
v=200
v=100
v=50
v=10
v=5

COLLEGIATE+ VERSION OF TUTSIM

Model File: cont3f.sim
Date: 12 / 19 / 1991
Time: 16 : 27
Timing: 1.0000 , DELTA : 2.500E+03 , RANGE

Plot Blocks and Scales:
Format:
BlockNo, Plot-MINimum, Plot-MAXimum; Comment
Horiz: 0 , 0.0000 , 2.500E+03 ; Time
Y1: 17 , 0.0000 , 5.0000 ; vol. flow q1
Y2: 16 , 0.0000 , 5.0000 ; vol. flow q2
Y3: , , ;
Y4: , , ;

Y1: vol. flow q1
5.0000
4.5000
4.0000
3.5000
3.0000
2.5000
2.0000
1.5000
1.0000
0.5000
0.0000
0.0000 Time 2.500E+03

$q_{y1}$

$k_2 = 2.0$

$q_{y2}$

$k_1 = 2.0$

$q_{y1} = 3$

$q_{y2} = 5$
COLLEGIATE+ VERSION OF TUTSIM

Model File: cont3.sim
Date: 12 / 16 / 1991
Time: 14 : 31
Timing: 1.0000, DELTA: 2.500E+03, RANGE

Plot Blocks and Scales:
Format:

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<th>Plot-MAXimum</th>
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<tr>
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<td>0.0000</td>
<td>2.500E+03</td>
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<tr>
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<td>17</td>
<td>0.0000</td>
<td>5.0000</td>
</tr>
<tr>
<td>Y2:</td>
<td>16</td>
<td>0.0000</td>
<td>5.0000</td>
</tr>
<tr>
<td>Y3:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y4:</td>
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</table>

![Graph showing Y1: vol. flow q1 over time]
Model File: cont3.sim
Date: 12 / 16 / 1991
Time: 14: 47
Timing: 1.0000, DELTA: 2.500E+03, RANGE
Plot Blocks and Scales:
Format:
BlockNo, Plot-MINimum, Plot-MAXimum: Comment
Horz: 0, 0.0000, 2.500E+03: Time
Y1: 17, 0.0000, 5.0000: vol. flow q1
Y2: 16, 0.0000, 5.0000: vol. flow q2
Y3: ...
Y4: ...

Y1: vol. flow q1
5.0000
4.5000
4.0000
3.5000
3.0000
2.5000
2.0000
1.5000
1.0000
0.5000
0.0000

0.0000 Time 2.500E+03

TUTSIM

q_2 at k_2 50, 100, 200
q_1

k_1

k_2 50
COLLEGIATE+ VERSION OF TUTSIM

- Model File: cont3.sim
- Date: 12/16/1991
- Time: 14:37
- Timing: 1.0000, DELTA: 2.500E+03, RANGE

Plot Blocks and Scales:

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<td></td>
<td>Time</td>
</tr>
<tr>
<td>Y1: 17</td>
<td>0.0000</td>
<td>5.0000</td>
<td>vol. flow q1</td>
</tr>
<tr>
<td>Y2: 16</td>
<td>0.0000</td>
<td>5.0000</td>
<td>vol. flow q2</td>
</tr>
<tr>
<td>Y3:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y4:</td>
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<td></td>
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</tbody>
</table>

Diagram:

- Y1: vol. flow q1
- TUTSIM
- Time range: 0.0000 to 2.500E+03
References

G.A. Evans and J.B Edwards 1987 "Investigation into control problems in large scale mining system " Dept. of control engineering, University of Sheffield, U.K.

