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The Impact of Inflation Uncertainty on Economic Growth: A MRS-IV Approach

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Abstract

We empirically investigate inflation uncertainty effects on output growth for the US by implementing a Markov regime switching model as we account for endogeneity problems. We show that inflation uncertainty—obtained from a Markov regime switching GARCH model—has a negative and regime dependent impact on output growth. Moreover, we find that the smooth probability of high growth regime falls long before the recent financial crisis was imminent. This might be driven by a regime dependent causality, an issue which has been left unexplored.

Keywords: Growth; inflation uncertainty; Markov-switching modeling; Markov-switching GARCH.
JEL classification: E31, E32

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1 Introduction

A vast literature in monetary economics claims that the main challenge for a central bank is to achieve sustainable economic growth while maintaining low and stable inflation. To that end, Friedman (1977) points out at two issues which later led researchers to explore the theoretical and empirical linkages between output growth and the first and the second moments of inflation. The key implication of Friedman (1977) is that inflation uncertainty exerts a negative impact on economic growth. First, he argues that there is a positive correlation between the level of inflation and its second moment.[1] Second, he suggests that inflation uncertainty distorts the efficient allocation of resources.[2] It is also well acknowledged that during periods of high uncertainty, external funds become prohibitively expensive due to heightened asymmetric information problems causing managers to delay or cancel fixed investment projects. Lower investment, in return, hinders output growth.

When we inspect the theoretical literature, we find several studies that suggest optimal policy rules to achieve low inflation while maintaining economic growth. There is also a deep empirical literature which examines the impact of inflation uncertainty on economic growth; yet the results are rather mixed. While some empirical studies provide evidence that inflation uncertainty has a negative impact on economic growth some others show that the effect can be positive or insignificant. A key element in this line of research is the method that one uses to generate a measure of inflation uncertainty. A review of the empirical literature shows that some researchers construct a proxy for inflation uncertainty based on the dispersion of expected inflation forecasts gathered from survey data. Alternatively, some researchers use the standard deviation of inflation while others implement ARCH/GARCH models to

[1] A vast empirical literature provides support for this hypothesis. See for instance Caglayan et al. (2008) and the references therein.

[2] Beaudry et al. (2001) claim that in a volatile economic environment those managers who cannot differentiate profit opportunities across different projects either reduce investment projects or channel available resources towards safe options.

generate a measure of inflation uncertainty. As a consequence it is not be too surprising to find that results depend on the chosen proxy.[3]

We generally observe that empirical studies which use a survey based uncertainty proxy support the view that an increase in inflation uncertainty will dampen economic growth.[4] In contrast, those studies which implement a standard deviation based uncertainty measure fail to provide evidence of a significant impact of inflation uncertainty on economic activity.[5] However, those studies that are based on survey data are criticized on various grounds. For instance, Cukierman and Wachtel (1979) and Cukierman (1983) show that measures of inflation uncertainty constructed from survey data are highly correlated with the actual standard deviation of inflation. Similarly, Jansen (1989) and Grier and Perry (2000) show that an uncertainty based on simple standard deviation induces a positive bias of inflation uncertainty and argue against its use.

As an alternative several researchers resort to exploiting the time series characteristics of the inflation and output using variants of ARCH/GARCH models.[6] Yet, despite its attractiveness, if macroeconomic and financial series were to exhibit structural breaks, simple ARCH/GARCH models would not be appropriate. In this context, for instance, Lamoureux and Lastrapes (1990), Hamilton and Susmel (1994), and Gray (1996) point out that standard GARCH models may overstate the persistence in conditional variance and understate the level of uncertainty when regime shifts in the underlying series are overlooked. In particular, Evans and Wachtel (1993) infer that those models which do not account for regime changes in the inflation process will underestimate not only the level of uncertainty but also its impact

[3]Mitchell, Mouratidis and Weale (2007) show that the correlation between a standard deviation based uncertainty proxy driven from survey data and that based on a parametric time series modeling is low. Their work suggests that robustness tests using different proxies of uncertainty to quantify the impact of inflation uncertainty on growth might not be much useful in obtaining a clearer view.

[4]See for instance, among others, Davis and Kanago (1996), Judson and Orphanides (1999) and Hayford (2000).

[5]See Barro (1996) and Clark (1997).

[6]For instance see Grier et al. (2004), Fountas, Karanasos and Kim (2006) and Mallik and Chowdury (2011).

on economic growth.

A more general shortcoming of the literature is that the vast majority of the empirical research on the link between inflation uncertainty and economic growth use reduced form models which are based on empirical regularities rather than an analytical framework. One reason why researchers follow this approach can be explained by noting that the certainty equivalence principle holds when the policy makers are assumed to minimize a quadratic loss function (QLF) which is subject to linear demand and supply curves.[7] In such a framework the central bank follows linear optimal policy rules and ignores the role of uncertainty. To overcome these shortcomings, the recent literature in monetary economics has relaxed the assumptions of QLF and the linearity of the state variables. For instance, Cukierman and Gerlach (2003) suggest that a central bank is more reactive to inflation deviation form its target when the economy is in expansion than in recession. Several researchers, including Nobay and Peel (2003), Rurge-Murcia (2000; 2003), Dolado et al. (2004) and Surico (2007; 2008), assume that central banks use a linear exponential (linex) loss function which allows one to explore the role of second moments of the relevant variables in an empirical framework.

In this study we take advantage of these developments in the literature. First of all we present a theoretical model to guide us in our empirical investigation on the sensitivity of output growth to inflation uncertainty. To do so we construct a theoretical framework assuming that the central bank uses an asymmetric linex loss function while we allow inflation and output growth to follow a Markov process. This assumption entertains the possibility that the policy maker (i.e. the central bank) weighs positive and negative deviation of inflation and output-gap from their respective targets differently. Using this framework, we show that inflation volatility affects the optimal reaction function of the central bank and the demand curve. As we estimate our empirical model, we account for various econometric

[7] Rotemberg and Woodford (1999) and Woodford (2003) provide a formal definition for the argument that the objective of monetary policy is to minimize the squared deviation of output and inflation from their respective targets.

issues including the generated regressor problem and the possibility of endogeneity between inflation, inflation uncertainty and growth within a Markov switching framework.[8]

To achieve our goals, we employ a two step approach. In the first step we generate a proxy for inflation uncertainty using a Markov switching GARCH model as suggested in Gray (1996). In the second step, we estimate the impact of inflation uncertainty on economic growth following Spagnolo et al. (2005) to overcome endogeneity problems between economic growth and inflation. We should stress that different from the literature, we allow for the unobserved states of inflation and output growth to be independent of each other and test for time varying causality as in Psaradakis et al. (2005).[9] We do so to exploit a wider information set to instrument the inflation series in the second step.

We carry out our investigation using quarterly US real GDP and inflation series for the period between 1960:I–2009:IV. Our results can be summarized as follows. We show that in recessions both inflation and inflation uncertainty have a negative and significant impact on output growth. Our investigation also provides evidence on the approaching rough economic conditions ahead of time. In particular we show that the smooth probability of high growth regime falls to low levels long before the 2007/08 financial crisis was imminent. Last, we observe that the first and the second moments of inflation follow an increasing trend as of 2000 while the 'Taylor principle' is satisfied. This is an interesting phenomena as heightened inflation and its volatility since 2000 along with the movements in smooth probability of high growth regime provide support for the suggestion that the policy makers benefited from good luck during the great moderation rather than the good policies. Yet, more investigation along these lines are warranted.

The paper is organized as follows. Section 2 lays out a theoretical framework to guide

[8]Some researchers, see for example Wilson (2006), have used bivariate GARCH modeling approach to guard against the generated regressor problem. However, Harvey, Ruiz and Shephard (1994) argue that this approach is subject to identification problem and the results are difficult to interpret.

[9]Earlier empirical studies such as Phillips (1991) and Sola et al. (2002; 2007) also question the assumption of perfect correlation between the unobserved states of inflation and output growth.

us in our empirical investigation. Section 3 provides information about the data and the empirical methodology. In this section we present our empirical model, discus problems associated with estimation and clarify the causality tests that we implement in investigating the association between output growth and the first and second moments of inflation. Section 4 discusses our findings while Section 5 concludes the study.

2 The Model

This section presents an analytical model to guide us in our empirical investigation on the effects of inflation volatility on output growth. To obtain such an association, we follow Svensson (1997) in describing the dynamics of the supply and demand curves. To derive this relationship between output growth and inflation uncertainty, we assume that the central bank has an asymmetric loss function with respect to inflation where output growth and inflation are subject to regime switches.[10] Hence state dependent inflation and output gap equations takes the following form:

$$
\begin{aligned}
\pi_{t+1}(S_{t+1}) \quad &= \pi_t + a_1(S_t)y_t + \sigma^{\pi}_{(S_t)}\varepsilon_{t+1} && (1)\\
y_{t+1}(S_{t+1}) \quad &= \beta_1(S)y_t + \beta_2(S_t)(i_t - \pi_t) + \sigma^{y}_{(S_t)}\eta_{t+1} && (2)\\
\text{with} \quad & \varepsilon_{t+1} \sim i.i.d.N(0,1), \text{ and } \eta_{t+1} \sim i.i.d.N(0,1)
\end{aligned}
$$

where π_t denotes inflation at time t, y_t is the output-gap, i is the nominal interest rate, ϵ_t and η_t denote supply and demand shocks, respectively. S_j, $j \in \{1, 2, ..N\}$ is a vector of unobserved-state variable which follows a Markov process with transition probability $\{P\}_{ij}$. Defining the probability that the unobserved state at time t is in regime j given the in-

[10] A similar framework used by Patton and Timmermann (2007) to prove properties of optimal forecast under the assumption that a forecaster has a linex loss function and the target variable follows a Markov regime switching process.

formation available at time $t-1$ as $\widehat{\xi}_{t|t-1} = P(S_t = j|\Psi_{t-1})$ then it follows that $E(\xi_{t+1}| \xi_t, \Psi_t) = P\xi_{t|t}$. For a given starting value $\xi_{1|0}$, Hamilton (1989) shows that an optimal estimate of the unknown state probability can be derived by iterating the following two equations

$$\widehat{\xi}_{t|t} = \frac{\widehat{\xi}_{t|t-1} \odot \eta_t}{1'(\widehat{\xi}_{t|t-1} \odot \eta_t)} \tag{3}$$

$$\widehat{\xi}_{t+1|t} = \mathbf{P}\widehat{\xi}_{t|t} \tag{4}$$

An important aspect of the monetary policy is that the central bank chooses the interest rate before observing the demand and supply shocks based on the information which is available at the end of the previous period. This is captured by an intertemporal loss function given by

$$\min E_t \sum_{\tau=1}^{\infty} \delta^{\tau} L_{t+\tau} \tag{5}$$

Here we follow, Nobay and Peel (2003), Ruge-Murcia (2003) and Surico (2007; 2008) and assume that the central bank has an asymmetric linex loss function with respect to inflation

$$E_t L(\pi_{t+1}(S_{t+1})) = E_t \exp\{\mu[\pi_{t+1}(S_{t+1}) - \pi^*]\} - \mu E_t[\pi_{t+1}(S_{t+1}) - \pi^*] - 1 \tag{6}$$

where μ is the asymmetry parameter. When this parameter is positive, $\mu > 0$, then the exponential component $(\exp\{\mu[\pi_{t+1}(S_{t+1}) - \pi^*]\})$ will rule over the linear component. In this case the central bank will be more concerned about inflation exceeding the set target level π^* since the cost of high inflation exceeds that of low inflation. Thus, positive deviations from the inflation target will dominate negative deviations. Under such circumstances, forecasters will systematically overpredict the target variable. The converse is true for $\mu < 0$. When $\mu = 0$ the loss function becomes quadratic. We use (4) to rewrite Equation (6) as

$$L_t = \widehat{\xi}'_{t+1|t}\mathbf{P} E_t \exp\{\mu[\pi_{t+1}(S_{t+1}) - \pi^*]\} - \mu\widehat{\xi}'_{t+1|t}\mathbf{P} E_t[\pi_{t+1}(S_{t+1})] + \mu\pi^* - 1 \tag{7}$$

After taking the expected value of (7) and using the result that if $\pi_{t+1} \sim N(\pi_{t+1|t}, \sigma_{\pi}^2)$ then $E_t exp(\mu\pi_{t+1}) = \exp(\mu\pi_{t+1|t} + \frac{\mu^2}{2}\sigma_{\pi}^2)$, one obtains:

$$L_t = \widehat{\xi}'_{t+1|t}\mathbf{P}\exp\left\{\mu[\pi_{t+1|t}(S_{t+1}) - \pi^*] + \frac{\mu^2}{2}\sigma^2_{\pi_t(S_{t+1})}\right\} - \mu\widehat{\xi}'_{t+1|t}\mathbf{P}[\pi_{t+1|t}(S_{t+1})] + \mu\pi^* - 1 \quad (8)$$

Next one can show that the first order condition of (8) with respect to $\pi_{t+1|t}(S_{t+1})$ takes the following form:

$$\pi_{t+1|t}(S_{t+1|t}) \quad = \quad \pi^* - \frac{\mu}{2}\sigma^2_{\pi_t(S_t)} \quad (9)$$

We compute $\pi_{t+1|t}(S_{t+1|t})$ using the expected value (1) with respect to the information set available at time t: If we forward (1) and (2) one period, we can show that the left-hand side of Equation (9) can be written as:

$$\pi_{t+1|t}(S_{t+1|t}) = \pi_t(S_t) + a_1(S_t)y_t(S_t) \quad (10)$$

Substituting (10) into (9) and solving the resulting equation with respect to output-gap we obtain

$$y_t(S_t) = \pi^* - \frac{\mu}{2}\sigma^2_{\pi_t(S_t)} - \left(\frac{1}{a_1(S_t)}\right)\pi_t(S_t) \quad (11)$$

This equation implies that output gap is negatively related to the first and the second moments of inflation. That is both the level and variability of inflation exerts a negative impact on the output gap.

3 Data and Econometric Methodology

3.1 Data

To empirically investigate the link between inflation uncertainty and output, we use quarterly consumer price index (CPI) and GDP for the United States. Data are obtained from the International Financial Statistics of the International Monetary Fund and span the period 1960:I–2009:IV.

We measure output growth (y_t) by the first difference of the log real GDP $\left[y_t = log\left(\frac{IPI_t}{IPI_{t-1}}\right)\right]$. Similarly, we compute the inflation rate (π_t) as the first difference of the log of consumer price index $\left[\pi_t = log\left(\frac{CPI_t}{CPI_{t-1}}\right)\right]$. We check for the presence of GARCH effects in the inflation series by applying the Lagrange Multiplier test. This test reveals significant GARCH effects. We then estimate a simple GARCH(1,1) model for inflation where the conditional variance follows $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 h_{t-1}$. As the sum of ARCH coefficients and GARCH terms ($\alpha_1 + \alpha_2$) from this model is very close to one, we suspect that the effects of past shocks on current variance is very strong; i.e. the persistence of volatility shocks is strong. In this context, Lamoureux and Lastrapes (1990) and Gray (1996) point out that the high volatility persistence may be due to regime shifts in the conditional variance. Hence, we use the generalized regime switching (GRS) GARCH model introduced by Gray (1996) to model inflation and inflation uncertainty. The likelihood function for this generalized regime switching model takes the form:

$$L = \sum_{t=1}^{T} log\left[p_{1t}\frac{1}{\sqrt{2\Pi h_{1t}}}exp\left\{-\frac{(\pi_t - \mu_{1t})^2}{2h_{1t}}\right\} + (1-p_{1t})\frac{1}{\sqrt{2\Pi h_{2t}}}exp\left\{-\frac{(\pi_t - \mu_{2t})^2}{2h_{2t}}\right\}\right],$$

where μ_{it} and h_{it} are the conditional mean and variance of inflation for regime $i = 1, 2$ and are given by:

$$h_{it} = \alpha_{0i} + \alpha_{1i}\varepsilon_{t-1}^2 + \alpha_{2i}h_{t-1} \tag{12}$$

$$\mu_{it-1} = \theta_{0i} + \sum_{j=1}^{p} \theta_{ji} \pi_{t-j-1} \tag{13}$$

where $\varepsilon_{t-1} = \pi_{t-1} - [p_{1t-1}\mu_{1t-1} + (1 - p_{1t-1})\mu_{2t-1}]$, and $h_{t-1} = p_{1t-1}\left(\mu_{1t-1}^2 + h_{1t-1}\right) + (1 - p_{1t-1})\left(\mu_{2t-1}^2 + h_{2t-1}\right) - [p_{1t-1}\mu_{1t-1} + (1 - p_{1t-1})\mu_{2t-1}]^2$.

The regime probability p_{1t} follows a simple nonlinear recursive system such that $p_{1t} = P_{11}\left[\frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1}+f_{2t-1}(1-p_{1t-1})}\right] + (1 - P_{22})\left[\frac{f_{2t-1}(1-p_{1t-1})}{f_{1t-1}p_{1t-1}+f_{2t-1}(1-p_{1t-1})}\right]$. Assuming conditional normality, the conditional distribution of inflation, f_{it}, where $i = 1, 2$, can be written as $f_{it} = f\left(\pi_t \mid S_t = i, \Omega_{t-1}\right) = \frac{1}{\sqrt{2\Pi h_{it}}} exp\left\{-\frac{(\pi_t-\mu_{it})^2}{2h_{it}}\right\}$.

3.2 Empirical Modeling

We examine the impact of inflation uncertainty on economic growth by augmenting Equation (11) with several lags of output-growth and inflation. We do so to guard against the possibility of the mispecification of demand and supply curves given by Equations (1) and (2). For example, Canova (2007) shows that an omitted variable will induce a highly autocorrelated residuals. The specification for our baseline model takes the following form:

$$y_t = \phi_i + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=0}^{k} \varphi_{ji} \pi_{t-j} + \delta_i \sigma_{\pi_t} + \xi_t, \tag{14}$$

$$\xi_t \mid \Omega_{t-1} \sim N\left(0, \sigma_i^2\right), \text{for } i = 1, 2 \text{ regimes},$$

where y_t is the growth rate of output at time t and σ_{π_t} captures the effect of inflation uncertainty on economic growth.

3.3 Econometric Issues

We use the conditional variance of the inflation process, which is generated by the GRS-GARCH(1,1), as a proxy of inflation uncertainty in Equation (14). However, its presence in (14) may lead to biased coefficient estimates and standard deviation as the series is a generated regressor. According to Pagan (1984) although one may overcome these problems by using an instrumental variables approach, the use of lagged observations as instruments may not be possible when an endogenous variable is a function of the entire history of the available data. Under such circumstances, Pagan and Ullah (1988) suggest testing the validity of the underlying assumptions of the model that generates the proxy. For instance, Ruge-Murcia (2003) follows these suggestions and uses lagged conditional volatility of unemployment obtained from a GARCH(1,1) model as an own instrument after checking for any remaining heteroscedasticity in the standardized residuals. Here, we too follow a similar route. We generate our inflation volatility measure implementing a GARCH(1,1) model and carefully check whether the model is well specified and whether there is any neglected heteroscedasticity. We then use the lags of this proxy as an instrument when we investigate the association between inflation uncertainty and output growth.

Another potential problem is that the inflation uncertainty measure used in Equation (14) could respond to an exogenous shock to either inflation or to output growth where the causation between inflation uncertainty and economic growth is not totally clear. This is so because a negative demand or supply shock would increase uncertainty, reduce output while the behavior of the level of inflation depends on the type of the shock. So an unobservable shock can increase the correlation between output growth and inflation uncertainty due to the presence of endogeneity between inflation uncertainty, inflation and output-growth. In this context, although, lags of a proper inflation uncertainty proxy can be used as an own instrument there is still the endogeneity problem between output growth and inflation which one has to account for. Kim (2004) and Spagnolo et al. (2005) note that the maximum likelihood estimation of a Markov-switching model based on Hamilton filter yields inconsistent parameter estimates in the presence of endogenous variables. These two studies get around the endogeneity problem by implementing a two-step model. In the first step both studies use an instrumenting equation to generate a proxy of the endogenous variables and in the second step they estimate a Markov switching model using the proxy generated from the first stage.[11]

Last but not the least, we should note that the causal relationship between output and money supply has been shown to be unstable.[12] In particular, unless there is a priori information when and why causality between money and output change, it is very difficult to account for such changes. Psaradakis et al. (2005) overcome these difficulties by proposing a framework based on vector autoregressive (VAR) models with time-varying parameters. The crucial element of this approach is to model the time variation in parameters such that it would be possible to capture the changes in causality between target variables. More concretely, the authors assume that changes in causality is driven by an unobserved state which follows a hidden Markov process. We implement this approach so as to exploit a wider information set to use better instruments which might be difficult to obtain by using other linear and even non-linear models.

3.3.1 MRS with Instrumental Variables

We employ the instrumental variables (IV) approach proposed by Spagnolo et al. (2005) where the reduced-form equations for the endogenous regressors also have state-dependent parameters. In particular, we estimate the following system of equations for output growth and the instrumenting equation for inflation:

$$\widehat{\pi}_t = \theta_{0i} + \sum_{j=1}^{L} \theta_{ji} y_{t-j} + \sum_{j=1}^{N} \eta_{ji} \pi_{t-j} + \varepsilon_t \tag{15}$$

[11] Kim (2004) uses a linear OLS regression to model the endogenous variables while Spagnolo et al. (2005) allow the instrumenting equation to have state-dependent parameters.

[12] An example of this instability can be traced observing the contradictory results obtained from causality tests implemented by Stock and Watson (1989) and Friedman Knutter (1993). It should also be noted that researchers including Swanson (1998) and Psaradakis et al. (2005) show that the causal relationship between output and money is sample specific.

$$y_t = \phi_i + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=0}^{k} \varphi_{ji} \widehat{\pi}_{t-j} + \delta_i \widehat{\sigma}_{\pi_t} + \xi_t \tag{16}$$

where $i = 1, 2$ indicates the regime. To estimate this type of model within the Markov regime switching framework while using lags of inflation and output growth as instruments, Spagnolo et al. (2005) suggest implementing a recursive algorithm as in Hamilton (1994). This process yields a likelihood function which can be maximized with respect to $\theta_i = (\phi_i, \beta_{ji}, \varphi_{ji}, \delta_i, \theta_{ji}, \eta_{ji})$. The conditional probability density function of the data $w_t = (y_t, \pi_t)$ given the state S_t and the history of the system can be written as follows:

$$\begin{aligned} pdf(w_t \mid w_{t-1}, ..., w_1; \eta) = {} & \frac{1}{\sqrt{2\pi}\sigma_{s_t}} exp \\ & \left[-\frac{1}{2} \left(\frac{y_t - \phi_i - \sum_{j=1}^{m} \beta_{ji} y_{t-j} - \sum_{j=1}^{k} \varphi_{ji} \widehat{\pi}_{t-j} - \delta_i \widehat{\sigma}_{\pi_t}}{\sigma_{s_t}} \right)^2 \right] \\ & \times \frac{1}{\sqrt{2\pi}\theta_{s_t}} exp \\ & \left[-\frac{1}{2} \left(\frac{\widehat{\pi}_t - \theta_{0i} - \sum_{j=1}^{L} \theta_{ji} y_{t-j} - \sum_{j=1}^{N} \eta_{ji} \pi_{t-j}}{\theta_{s_t}} \right)^2 \right] \end{aligned} \tag{17}$$

4 Empirical Results

In this section we discuss the empirical evidence. The approach we follow here is based on Psaradakis et. al. (2005) as we exploit the information gathered from time-varying causal linkages between output on inflation. For the purpose of brevity we do not provide parameter estimates obtained for Gray's (1996) model which we use to generate inflation uncertainty.[13]

[13]Details are available upon request from the authors.

4.1 Discussion

Table 1 presents the coefficient estimates for Equation (16). The implied unconditional mean growth $\frac{\phi_i}{1-\sum_{j=1}^{m}\beta_{ji}}$ in State 1 and State 2 are approximately equal to zero and 0.018%, respectively. Thus, State 1 is identified as a low growth or recessionary regime and State 2 is identified as the high growth or expansionary regime. Estimates of the transition probabilities q and p are 0.813 and 0.959, respectively, which imply high persistence for both regimes. Results shown in Table 1 suggest that inflation uncertainty have a negative and significant impact on output growth during recessions (δ_1) while this effect is negative but insignificant during expansions (δ_2). The size of the negative impact of inflation uncertainty in recessions is five times higher than the corresponding effect in expansions. When we inspect the impact of inflation we find that its effect is positive and insignificant in recessions (φ_{11}) but negative and significant in expansions (φ_{12}). This might be due to asymmetric preferences of the FED with respect to both inflation and output growth.[14] Our findings suggest that monetary authorities focus on controlling inflation in expansions leading to low inflation variation which does not exert any impact on output growth. In contrast, during periods of recession the central bank will be more concerned about negative output-gap. Hence, in recessions monetary policy authorities will reduce the interest rate to boost the demand side of the economy putting less weight on inflation. Under such circumstance inflation variability can be expected to be higher in recessions as inflation is allowed to rise.

Figure 1 shows the smooth probability of high growth regime. It is worth noting that the smooth probability drops to low levels between the third quarter of 1980 and the third quarter of 1983 as well as the third quarter of 2006 and the third quarter of 2009. The first period, (i.e., 1980Q3-1983:Q3) coincides with the change of monetary policy framework adopted by the FED as documented by a large number of studies. For example, Clarida et

[14] Caglayan et al. (2012) using an open economy New-Keynesian model and asymmetric preferences on the part of central show that the FED has asymmetric preferences as reflected by the impact of inflation and output-gap volatility on optimal reaction function.

al. (2000) and Lubik and Shorfeide (2004) demonstrate that the way monetary policy was conducted changed significantly with the appointment of Paul Volcker as the FED Chairman at the third quarter of 1979. Providing further support for the above argument, Bernanke and Mihov (1998) show that the operating procedures of the FED shifted during the period 1979:Q4-1982:Q3 from Federal Funds rate to non-borrow reserves targeting.

The smooth probability of high growth regime drops down to low levels a second time in the third quarter of 2006, long before the 2007/2008 financial crisis, and stays low until the end of the third quarter of 2009. This observation, which has not been reported earlier, simply points at the approaching rough economic conditions ahead. To understand the reasons behind this observation, we estimate the model with different sets of instruments. As a result of our experiments we are convinced that the results are driven by the use of good instruments which proxy inflation in the growth equation. For example, when the instrument set includes only the lags of inflation or output, the smooth probability obtained from our model drops to low values at the onset of financial crisis and remains low only for few quarters.[15] This observation reflects the fact that linear (and even non-linear) models might not properly capture the causal effects between inflation and output growth in the sense explained by Psaradakis et al. (2005).

The parameter estimates for the instrumenting equation which we present in the second column of Table 1 show that in regime 1 the first two lags of output growth and the first lag of inflation are significant. Alternatively, in regime 2 only the first two lags of inflation are significant. Thus, the instrumenting equation suggest for the presence of a complex causal relationship between output and inflation.[16]

So far we have shown that the effects of inflation uncertainty on output growth is negative and regime dependent. We also show that this effect is stronger during periods of recession.

[15] These results are available from the authors upon request.

[16] We briefly present the time-varying causality methodology of Psaradakis et al. (2005) and the regression results for the model in the Appendix.

We further demonstrate that the smooth probability of high growth regime drops to low levels between the third quarter of 1980 and third quarter of 1983 as well as in 2006 long before the recent financial crisis was imminent. These observations complement the information given in Figure 2 which plots the movements in our measure of inflation volatility. This figure demonstrates that volatility in the 70s was quite high and it falls to lower levels in mid nineties which then increases to a higher level following the turn of the century peaking in 2006-2009 period. It is generally well accepted that the high level of inflation of the 1970s and to some extent the high inflation volatility and output volatility were driven by passive monetary policies pursued by the FED. Similarly, many economists believe that after 1979 the FED pursue a monetary policy which satisfied the so called 'Taylor principle' and reacted to expected inflation more than proportionally. Hence, several economists including Clarida et al. (2000) and Benati and Surico (2009) argue that the great moderation was due to good policy. However, the visual information provided in Figures 1 and 2 that inflation and its volatility are increasing during the post-Volker period, a period when the Fed was pursuing active monetary policy, appears to provide support for the advocates of good-luck hypothesis.[17] However, this issue, although important and relevant, is beyond the scope of our investigation and we cannot pursue it fully. Yet, our findings may motivate researchers to theoretically or empirically examine the good-luck *versus* good-policy hypothesis to explain the behavior of the US economy during the great moderation.

5 Conclusion

In this paper we examine the impact of inflation uncertainty on economic growth for the US economy. Our contribution to the literature is twofold. We initially provide an analytical framework to guide us in our empirical investigation as we examine the link between output

[17]See Stock and Watson (2002), Sims and Zha (2006) and Gambetti et al. (2008) along these lines.

growth and inflation volatility. To do so, we assume that the central bank has an asymmetric loss function with respect to inflation and we allow policy constraints to follow a Markov regime switching process. In this set up certainty equivalence does not hold. Hence, we show that inflation uncertainty affects the output growth dynamics.

Second, different from the literature, our empirical model allows regime shifts in inflation, inflation uncertainty and output as we account for endogeneity by using a Markov regime-switching model with instrumental variables. To do so we adopt a two stage modeling approach. In the first step we construct an inflation uncertainty proxy by implementing a Markov-switching GARCH(1,1) model. Then, in the second step, we use a Markov switching model to estimate the impact of inflation uncertainty on economic growth while the economy transits between expansions and recessions. This approach provides a set-up where we can examine whether inflation uncertainty effects differ across the business cycle. It should be noted that in the second step, we overcome the endogeneity problem between output growth and inflation by instrumenting the endogenous variables as suggested by Spagnolo et al. (2005).

We carry out our investigation using quarterly data for the US over 1960:Q4-2009:Q4, and obtain the following results. We find that the first and the second moments (i.e. the level and volatility) of inflation have a negative and regime dependent impact on output growth: uncertainty effects are significant during the recessionary state, but not in the expansionary regime. This finding might reflect that the FED has asymmetric preferences concerning inflation.

Our investigation also provides evidence that the economy was heading towards a recession long before the onset of the recent financial crisis. We find that the smooth probability of high growth regime has dropped to a low value at the third quarter of 2006. To understand the reasons behind this observation, we estimate our model with different sets of instruments. As a result of our experiments we are convinced that the results are driven by the information set from which we drive our instruments. Last, we comment on the role that good luck may have played during the great moderation. In particular we point out at the complementary evidence provided by the smooth probability and the inflation volatility: inflation volatility appears to follow an increasing trend as of 2000 and the smooth probability drops to low levels in 2006 long before the 2007/08 financial crises despite the FED was pursuing active monetary policies. Perhaps these observations motivate researchers to provide theoretical and empirical studies that explore the role of good luck *versus* good policy over the great moderation.

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Figure 1: The Smooth Probabilities of State 1

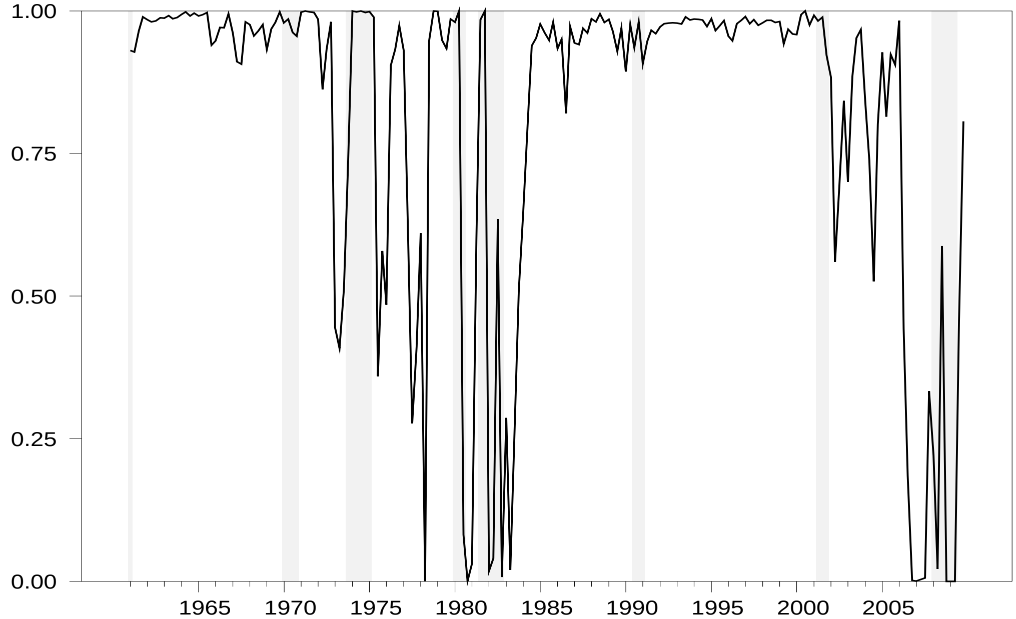

Figure 2: Uncertainty Proxy

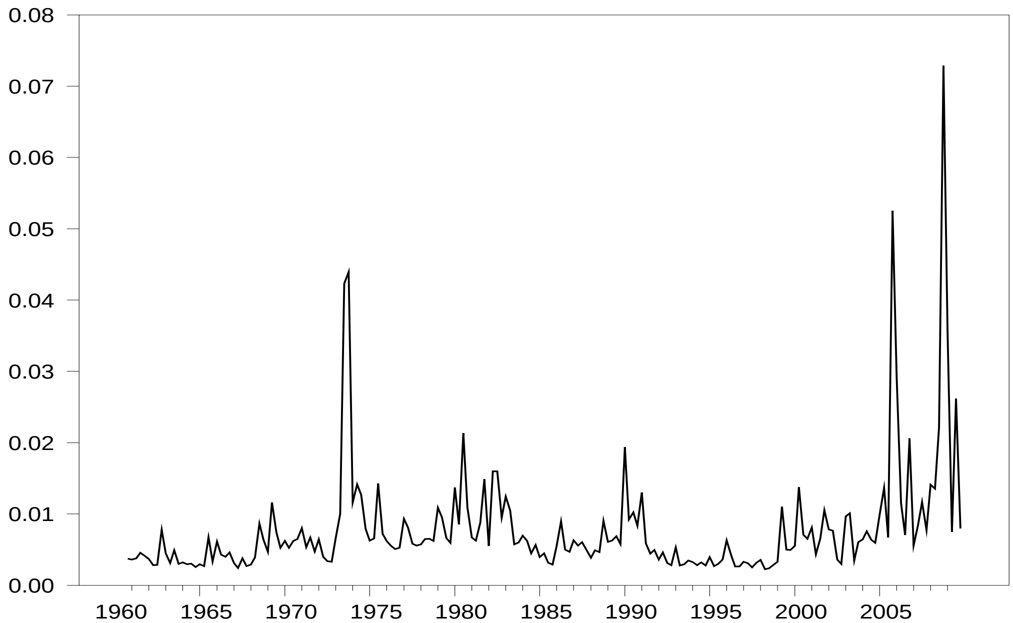

Table 1: Estimates of Parameters of the Model for Output Growth and Inflation

Parameter	Estimate	Std. error	Parameter	Estimate	Std. error
ϕ_1	0.004	0.005	θ_{01}	0.006**	0.002
β_{11}	0.470**	0.211	θ_{11}	0.274*	0.161
β_{21}	-0.366*	0.219	θ_{21}	-0.424**	0.191
β_{31}	0.296	0.198	θ_{31}	0.198	0.182
φ_{11}	0.404	0.344	θ_{41}	0.189	0.213
δ_1	-0.258**	0.121	η_{11}	0.504***	0.156
ϕ_2	0.011***	0.001	η_{21}	-0.156	0.141
β_{12}	0.105	0.091	θ_{02}	0.000	0.001
β_{22}	-0.119	0.078	θ_{12}	0.037	0.043
β_{32}	0.404***	0.086	θ_{22}	0.052	0.045
φ_{12}	-0.465***	0.098	θ_{32}	-0.026	0.039
δ_2	-0.049	0.091	θ_{42}	0.063	0.041
$\sigma_{\xi 1}$	0.008***	0.001	η_{12}	0.716***	0.075
$\sigma_{\xi 2}$	0.006***	0.000	η_{22}	0.205**	0.081
q	0.813***	0.088	σ_{η_1}	0.009***	0.001
p	0.959***	0.024	σ_{η_2}	0.004***	0.000
Log likelihood =	1463.300				

Notes: The estimates on the left hand side of the table are for the model $y_t = \phi_i + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=0}^{k} \varphi_{ji} \widehat{\pi}_{t-j} + \delta_i \widehat{\sigma}_{\pi_t} + \xi_t$. The estimates on the right hand side of the table are for the model $\widehat{\pi}_t = \theta_{0i} + \sum_{j=1}^{L} \theta_{ji} y_{t-j} + \sum_{j=1}^{N} \eta_{ji} \pi_{t-j} + \varepsilon_t$, where $\xi_t \mid \Omega_{t-1} \sim N\left(0, \sigma_{\xi i}^2\right)$. and $\eta_t \mid \Omega_{t-1} \sim N\left(0, \sigma_{\eta i}^2\right)$. Regimes are indexed by $i = 1, 2$.

Significance at the 10%, 5% and 1% are denoted by *, **, ***.

Appendix: Time-Varying Causality Test

This section presents Psaradakis et al. (2006) Markov switching VAR model that we estimate to analyze the casualty between output (y_t) and inflation (π_t) conditional on inflation volatility $\widehat{\sigma}_{\pi_t}$.

$$\begin{aligned}\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} \mu_{10}\left(1-S_{1,t}\right)+\mu_{11}S_{1,t} \\ \mu_{20}\left(1-S_{2,t}\right)+\mu_{21}S_{2,t} \end{bmatrix} \\ &+\sum_{k=1}^{h_1}\begin{bmatrix} \phi_{10}^{(k)}\left(1-S_{1,t}\right)+\phi_{11}^{(k)}S_{1,t} & \gamma_1^{(k)}S_{1,t} \\ \gamma_2^{(k)}S_{2,t} & \phi_{20}^{(k)}\left(1-S_{2,t}\right)+\phi_{21}^{(k)}S_{2,t} \end{bmatrix}\begin{bmatrix} y_{t-k} \\ \pi_{t-k} \end{bmatrix} \\ &+\sum_{k=1}^{h_2}\begin{bmatrix} \theta_{10}^{(k)}\left(1-S_{1,t}\right)+\theta_{11}^{(k)}S_{1,t} \\ \theta_{20}^{(k)}\left(1-S_{2,t}\right)+\theta_{21}^{(k)}S_{2,t} \end{bmatrix}\widehat{\sigma}_{\pi_{t-k}}+\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, t=1,2,...,T \end{aligned} \tag{18}$$

The latent state variables, denoted by $S_{1,t}$ and $S_{2,t}$, take the values of 0 or 1 at time t depending on the prevailing regime. The error terms, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, are white noise processes which are independent of $S_{1,t}$ and $S_{2,t}$ with mean zero and covariance matrix which depends on $S_{1,t}$ and $S_{2,t}$. Finally, assuming that $S_{1,t}$ and $S_{2,t}$ are independent of each other, we obtain the following four different states which are indexed by S_t:

$$S_t=\begin{cases} 1 \text{ if } S_{1,t}=1 \text{ and } S_{2,t}=1 \\ 2 \text{ if } S_{1,t}=0 \text{ and } S_{2,t}=1 \\ 3 \text{ if } S_{1,t}=1 \text{ and } S_{2,t}=0 \\ 4 \text{ if } S_{1,t}=0 \text{ and } S_{2,t}=0 \end{cases}$$

Using this indexation, Equation (18) can be written as follows:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix}=\begin{bmatrix} \mu_{11} \\ \mu_{21} \end{bmatrix}+\sum_{k=1}^{h_1}\begin{bmatrix} \phi_{11}^{(k)} & \gamma_1^{(k)} \\ \gamma_2^{(k)} & \phi_{21}^{(k)} \end{bmatrix}\begin{bmatrix} y_{t-k} \\ \pi_{t-k} \end{bmatrix}+\sum_{k=1}^{h_2}\begin{bmatrix} \theta_{11}^{(k)} \\ \theta_{21}^{(k)} \end{bmatrix}\widehat{\sigma}_{\pi_{t-k}}+\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \qquad \text{if } S_t=1 \tag{19}$$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix}=\begin{bmatrix} \mu_{10} \\ \mu_{21}+\mu_{21} \end{bmatrix}+\sum_{k=1}^{h_1}\begin{bmatrix} \phi_{10}^{(k)} & 0 \\ \gamma_2^{(k)} & \phi_{21}^{(k)} \end{bmatrix}\begin{bmatrix} y_{t-k} \\ \pi_{t-k} \end{bmatrix}+\sum_{k=1}^{h_2}\begin{bmatrix} \theta_{10}^{(k)} \\ \theta_{21}^{(k)} \end{bmatrix}\widehat{\sigma}_{\pi_{t-k}}+\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \qquad \text{if } S_t=2 \tag{20}$$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \mu_{10} + \mu_{11} \\ \mu_{20} \end{bmatrix} + \sum_{k=1}^{h_1} \begin{bmatrix} \phi_{11}^{(k)} & \gamma_1^{(k)} \\ 0 & \phi_{20}^{(k)} \end{bmatrix} \begin{bmatrix} y_{t-k} \\ \pi_{t-k} \end{bmatrix} + \sum_{k=1}^{h_2} \begin{bmatrix} \theta_{11}^{(k)} \\ \theta_{20}^{(k)} \end{bmatrix} \widehat{\sigma}_{\pi_{t-k}} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \qquad \text{if } S_t = 3 \quad (21)$$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \end{bmatrix} + \sum_{k=1}^{h_1} \begin{bmatrix} \phi_{10}^{(k)} & 0 \\ 0 & \phi_{20}^{(k)} \end{bmatrix} \begin{bmatrix} y_{t-k} \\ \pi_{t-k} \end{bmatrix} + \sum_{k=1}^{h_2} \begin{bmatrix} \theta_{10}^{(k)} \\ \theta_{20}^{(k)} \end{bmatrix} \widehat{\sigma}_{\pi_{t-k}} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \qquad \text{if } S_t = 4 \quad (22)$$

As we can observe, the state variables $S_{1,t}$ and $S_{2,t}$ reflect the causality patterns. Provided that at least one of the $\gamma_1^{(1)}, ..., \gamma_1^{(h1)}$ is not equal to zero, π_t is Granger causal for y_t when $S_{1,t} = 1$ ($S_t = 1$ or $S_t = 3$) and it is not Granger causal for y_t when $S_{1,t} = 0$ ($S_t = 2$ or $S_t = 4$). In a similar manner, given that at least one of the $\gamma_2^{(1)}, ..., \gamma_2^{(h1)}$ is not equal to zero y_t is Granger causal for π_t when $S_{2,t} = 1$ ($S_t = 1$ or $S_t = 2$) and it is not Granger causal for π_t when $S_{2,t} = 0$ ($S_t = 3$ or $S_t = 4$).

Table 2 presents the results for this model. The first column of the Table shows the estimation results of the output growth equation where we use a fourth order reduced form VAR. Here the Granger causality test does not reflect any structural interaction among output, inflation and output. We observe that the coefficient of time-varying causality (i.e. $\gamma_i^{(j)}$, $i = 1, 2$ and $j = 1, 2, 3, 4$) are significant in states 1 and 3.[18] More specifically, the significance of $\gamma_1^{(1)}$ shows that inflation Granger cause output in the sense that inflation has forecasting power in predicting output. Alternatively, $\gamma_2^{(1)}$ and $\gamma_2^{(3)}$ indicate that output provide significant information for forecasting inflation. We also observe that inflation uncertainty is significant in both equation.

[18]Note that i and j denote the regime and the order of lagged value respectively.

Table 2: Estimates of the Markov Switching Granger Causality Model for Output Growth and Inflation

Parameter	**Estimate**	**Std. error**	**Parameter**	**Estimate**	**Std. error**
$p_{11}^{(1)}$	0.848***	0.077	$p_{11}^{(2)}$	0.919***	0.036
$p_{00}^{(1)}$	0.792***	0.081	$p_{00}^{(2)}$	0.770***	0.084
μ_{10}	0.006***	0.002	μ_{20}	0.008***	0.002
μ_{11}	0.013***	0.002	μ_{21}	-0.001	0.001
$\phi_{10}^{(1)}$	0.065	0.086	$\phi_{20}^{(1)}$	0.531***	0.091
$\phi_{10}^{(2)}$	0.114	0.079	$\phi_{20}^{(2)}$	-0.559***	0.091
$\phi_{10}^{(3)}$	-0.054	0.067	$\phi_{20}^{(2)}$	0.536***	0.092
$\phi_{10}^{(4)}$	0.003	0.117	$\phi_{20}^{(2)}$	0.142	0.090
$\phi_{11}^{(1)}$	0.247**	0.098	$\phi_{21}^{(1)}$	0.552***	0.053
$\phi_{11}^{(2)}$	0.228**	0.096	$\phi_{21}^{(2)}$	0.049	0.063
$\phi_{11}^{(3)}$	-0.208	0.093	$\phi_{21}^{(3)}$	0.263***	0.049
$\phi_{11}^{(4)}$	0.010	0.122	$\phi_{21}^{(4)}$	0.122***	0.044
$\gamma_{1}^{(1)}$	-0.309*	0.158	$\gamma_{2}^{(1)}$	0.082**	0.041
$\gamma_{1}^{(2)}$	0.071	0.147	$\gamma_{2}^{(2)}$	-0.032	0.040
$\gamma_{1}^{(3)}$	-0.085	0.155	$\gamma_{2}^{(3)}$	-0.065*	0.037
$\gamma_{1}^{(4)}$	-0.201	0.137	$\gamma_{2}^{(4)}$	0.031	0.034
$\theta_{10}^{(1)}$	0.116**	0.056	$\theta_{20}^{(1)}$	-0.290**	0.111
$\theta_{10}^{(2)}$	-0.164**	0.069	$\theta_{20}^{(2)}$	-0.379***	0.136
$\theta_{11}^{(1)}$	-0.412***	0.135	$\theta_{21}^{(1)}$	0.034	0.041
$\theta_{11}^{(2)}$	0.149	0.108	$\theta_{21}^{(2)}$	0.119***	0.040
$\sigma_{11}^{(1)}$	0.005***	0.001	$\sigma_{22}^{(1)}$	0.002***	0.000
$\sigma_{11}^{(2)}$	0.003***	0.000	$\sigma_{22}^{(2)}$	0.005***	0.001
$\sigma_{11}^{(3)}$	0.009***	0.002	$\sigma_{22}^{(3)}$	0.001***	0.000
$\sigma_{11}^{(4)}$	0.015***	0.003	$\sigma_{22}^{(4)}$	0.009***	0.002
Log likelihood =	1511.400				

Notes: *, **, *** denote significance at the 10%, 5% and 1% levels.