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DETECTION OF FATIGUE CRACKS IN A BEAM USING A
SPATIO-TEMPORAL DYNAMICAL SYSTEM
IDENTIFICATION METHOD

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Detection of fatigue cracks in a beam using a spatio-temporal dynamical system identification method

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Abstract

The identification of fatigue cracks in a beam is investigated in this paper. It is shown that due to the influence of the elastic non-linearity of fatigue cracks, the homogeneity, along the length of the beam, of the spatio-temporal dynamics of the vibrating beam is destroyed. By using spatio-temporal dynamical system identification techniques, a new approach is developed to detect this nonhomogeneity. The cracked beam is divided into several spatial regions and a coupled map lattice (CML) model is identified and verified in one of the regions using an Orthogonal Forward Regression (OFR) least-squares algorithm. This CML model is then used to predict the dynamical behaviour of the other regions and in this way to detect the nonhomogeneity of the overall system.

1 Introduction

The detection and characterisation of fatigue cracks in a beam have been investigated over many years. Most of the techniques developed for this problem are based on vibration measurements and spectral analysis because it has been shown that sub- and super-harmonic resonance regimes, of a vibrating beam, are highly sensitive to the presence of fatigue cracks in structures (Bovsunovsky and Surace 2005 and references therein). However, to apply the vibration based methods, a mathematical model is generally needed. There have been considerable attempts to understand and model the dynamics of a vibrating cracked beam. Well known models include the one-dimensional cracked beam model (Christides and Barr 1984), breathing crack model (Shen and Chu 1992), and bilinear oscillator model (Chu and Shen 1992) etc. But all of the above models arise from mainly theoretical consideration and are derived by analytic modelling methods where often a large number of assumptions have to be made in order to obtain such models.

For instance, there are several assumptions on the displacement, velocity and stress fields in the Christides and Barr (1984) model. A pair of symmetric cracks is always assumed to remain open as the beam is vibrating so as to avoid the non-linear characteristics of an opening and closing crack. It should be stressed that although some information about the physical properties for many of these systems might be available, normally not all the dynamical structures and parameters are known, therefore, the resulting theoretical models are often over-simplified, or even in error. These problems can result in large discrepancies between simulated and observed patterns both qualitatively and quantitatively. Therefore, there is a need to use identification methods to refine, update, validate, or even replace these theoretical models.

From a spatio-temporal dynamics point of view, the opening and closing of a crack not only causes the dynamical behaviour of the vibrating beam to be significantly nonlinear but also destroys the homogeneity, along the length of the beam, of the spatio-temporal dynamics because of the local decrease of the flexural rigidity of the damaged cross-section of the beam. The identification of homogeneous spatio-temporal dynamical systems has been studied by several researchers including Coca and Billings (2001), Billings and Coca (2002) and Billings, Guo and Wei (2005). The identification algorithms developed are mainly used to obtain a spatio-temporal model with both discrete space and time. These kind of models are generally called coupled map lattice (CML) models of spatio-temporal systems. It is well known that computer simulations have emerged as an effective and powerful tool to study complex spatio-temporal systems. In such cases the spatio-temporal dynamical systems by necessity are discretised in space as well as in time. This was one of the main motivations for the introduction of CML models of spatio-temporal systems. CML models were developed in the late 80's and can exhibit surprisingly rich dynamical behaviours, including spatio-temporal chaos, intermittency, traveling waves and pattern formation (Kaneko (1985, 1986, 1989a, 1989b and 1993). CML's have been used to model convected temperature fluctuations in the atmosphere (Platt and Hammel 1997), boiling processes (Yanagita 1992), spatio-temporal chaos in fluid flows (He, Cao and Li 1995) and cloud dynamics (Yanagita and Kaneko 1997). In this paper, a CML model will be used to model the dynamics of a fatigue cracked beam in vibration using a nonhomogeneous spatio-temporal system identification method. The idea behind the proposed identification method is that the cracked beam is divided into several spatial regions and a CML model is identified and verified in one of the regions using an Orthogonal Forward Regression (OFR) least-squares algorithm (Billings, Chen, and Korenberg 1989). This CML model is then used to predict the dynamical behaviour of the other spatial regions and in this way to detect the nonhomogeneity of the overall system. The advantages of the proposed method are that an analytic mathematical model is not necessary and the method can be used in association with classical spectral analysis techniques which can provide more reliable crack detection results.

The paper is organised as follows. Section 2 presents a dynamic model of a fatigue cracked beam which is used to generate data for this simulation study. The basic concept of CML models is introduced in section 3. The nonhomogeneous spatio-temporal system identification algorithm for detecting the correct terms and determining the associated parameter estimates is presented in section 4. Section 5 applies the proposed identification approach to the crack detection problem, and finally conclusions are given in section 6.

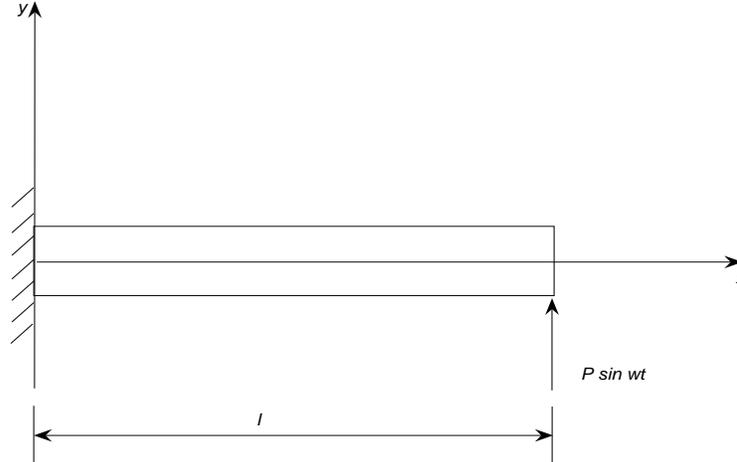


Figure 1: Schematic diagram of the beam

2 Dynamic model of a fatigue cracked beam

For the purpose of this numerical simulation study, a simplified dynamic model of the cracked beam is adapted from Tsyfansky and Beresnevich (2000) to generate data. The beam under consideration, which is shown in Fig. (1), is a solid-spar homogeneous viscoelastic beam which resists the greater part of an external load.

The dynamic model is defined by using a viscoelastic cantilever beam performing bending vibrations excited by a harmonic test force, $P \sin \omega t$

$$\frac{\partial^2}{\partial x^2} [EI(x) \frac{\partial^2 y}{\partial x^2}] + \mu(x) \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2} [bEI(x) \frac{\partial^3 y}{\partial t \partial x^2}] = q(x, t) \quad (1)$$

where $EI(x)$ and $\mu(x)$ are the flexural rigidity in bending and the distributed mass, respectively, of the beam cross-section with co-ordinate x , y is the lateral displacement of the beam cross-section measured from the static equilibrium position, and b is the coefficient of internal friction. In this study, for the sake of simplicity b is set to be zero, that is internal friction is ignored, therefore eqn. (1) is simplified to be

$$\frac{\partial^2}{\partial x^2} [EI(x) \frac{\partial^2 y}{\partial x^2}] + \mu(x) \frac{\partial^2 y}{\partial t^2} = q(x, t) \quad (2)$$

The boundary conditions are

$$y(0, t) = 0; \frac{\partial y}{\partial x}(0, t) = 0; \frac{\partial^2 y}{\partial x^2}(l, t) = 0; \frac{\partial^3 y}{\partial x^3}(l, t) = 0 \quad (3)$$

The first two boundary conditions mean the end 0 is restrained so that the displacement and slope vanish always. The last two boundary conditions indicate that the end l is free so that both curvature and shear are zero.

To model the fatigue crack in the beam, Tsyfansky and Beresnevich (2000) consider a fatigue crack as an additional elastic non-linearity describing the cyclical process of opening and closing of the crack edges during vibration and use a piecewise linear crack model. More specifically, this cyclical process is simulated using a crack parameter, σ . The parameter σ is a step function describing the relationship between the flexural rigidity of the damaged cross-section and the bend angle $\phi = \partial y / \partial x$. The mathematical form of this function is dependent on the location of the fatigue crack above or below the centreline along which x is measured.

In the case of an upper-half crack location the crack edges for the static equilibrium position of the system are initially opened due to the action of the beams own weight. The crack parameter σ is described by the expression

$$\sigma = \begin{cases} 0, & \frac{\partial y}{\partial x} > \phi_0 \\ \sigma_c, & \frac{\partial y}{\partial x} \leq \phi_0 \end{cases} \quad (4)$$

where $\sigma_c = (1 - I_d/I_0)$ is a measure of the relative crack value, I_d being the second moment of the damaged cross-section, and I_0 is the second moment of the undamaged cross-section, ϕ_0 is the threshold value of the bend angle, ϕ , corresponding to the instant of closing (or opening) of the crack edges. Similarly, for a crack located in the lower-half of the beam, the crack parameter σ is calculated as

$$\sigma = \begin{cases} 0, & \frac{\partial y}{\partial x} \leq \phi_0 \\ \sigma_c, & \frac{\partial y}{\partial x} > \phi_0 \end{cases} \quad (5)$$

The crack parameter, σ , defines the change in flexural rigidity of the damaged cross-section $EI(x - x_d)$ in accordance with the following mathematical expression

$$EI(x - x_d) = EI_0[1 - \sigma\delta(x - x_d)] \quad (6)$$

where EI_0 is the flexural rigidity in bending of the undamaged cross-section, and x_d is the co-ordinate of the damaged cross-section of the beam.

The intensity of the distributed load $q(x, t)$ is given by

$$q(x, t) = P \sin \omega t \delta(x - x_p) k(y - y_{st}) \delta(x - x_k) \quad (7)$$

where P and ω are the amplitude and the frequency of the external harmonic excitation, $\delta(x - x_p)$ is a Dirac delta function, and $x_p = l$ is the co-ordinate of the cross-section to which the external harmonic force is applied. l is the length of the beam. Note that the extra term inserted in (7)

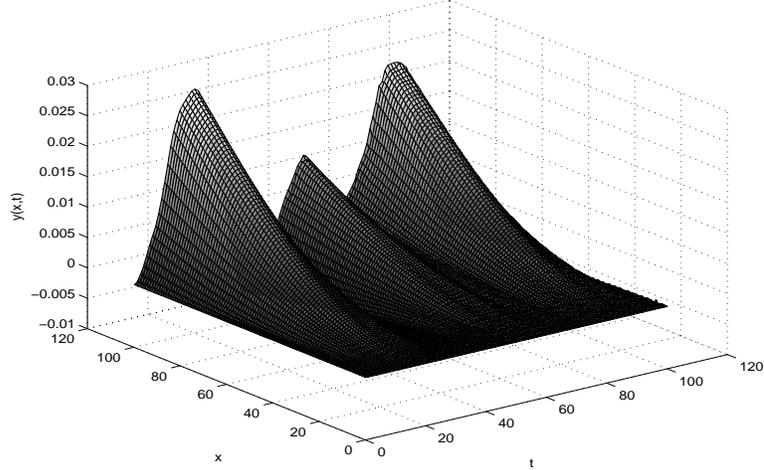


Figure 2: The numerical solution of the fatigue cracked beam

is considered as introducing an additional elastic support in the beam in order to balance the influence of the beams own weight with the position of the bend point, $\phi = \phi_0$, of the damaged cross-section such that ϕ_0 can be set to be zero. k and y_{st} are the stiffness coefficient and the initial static deformation in compression of the additional elastic support, and $x_k = 0.9l$ is the co-ordinate of the cross-section when interacting with the additional elastic support.

The partial differential equation (2), subject to the conditions (3)-(7) was numerically solved using a fourth-order Runge-Kutta method with spatial sampling interval 0.12 and time sampling interval 0.01 and it was assumed that the crack is located in the upper-half of the beam and the location of the crack was taken as $x_d = l/20$ with $l = 12\text{m}$. The parameters used in the numerical algorithm were $EI(x) = 3.3903e + 07 \text{ Nm}^2$, $\mu(x) = 1.9933e + 02 \text{ kg/m}$, $P = 0.2G$, where G is the weight of the beam, $\omega = 10$, $\sigma_c = 0.17$, $k = 1.0e + 04$, $ky_{st}/G = 0.34$, and $\phi_0 = 0$. The data is shown in Fig. (2). A detailed analysis about the effects of a fatigue crack on the excitation of superharmonic responses in vibrating beam can be found in Tsyfansky and Beresnevich (2000). In this paper, a spatio-temporal identification method will be presented as an alternative method of crack detection problem.

3 The CML model

A CML model is a discrete space and time representation of spatio-temporal dynamic systems which is defined in a lattice as follows. Let I denote a d -dimensional lattice consisting of the set of all integer coordinate vectors $i = (i_1, \dots, i_d) \in \mathbf{Z}^d$. The deterministic CML model of spatio-temporal dynamical systems defined over I is of the following form

$$x(t) = f(x(t-1), u(t-1)) \quad (8)$$

$$y(t) = h(x(t))$$

where $x(t) = \{x_i(t)\}_{i \in I} \in X = \prod_{i \in I} X_i, X_i \subset \mathbf{R}^n, y(t) = \{y_i(t)\}_{i \in I} \in Y = \prod_{i \in I} Y_i, Y_i \subset \mathbf{R}^l$, and $u(t) = \{u_i(t)\}_{i \in I} \in U = \prod_{i \in I} U_i \subset \mathbf{R}^m$, are the global state, output, and input, respectively. $f : X \times U \rightarrow X$ is a sequence of differentiable maps $f = \{f_i\}_{i \in I}$ and $h : X \rightarrow Y$ is a sequence of differentiable maps sequence $h = \{h_i\}_{i \in I}$.

The CML model (8) can also be written, in terms of the index of nodes, as follows

$$\begin{aligned} x_i(t) &= f_i(x_i(t-1), u_i(t-1), \mathbf{s}^m x_i(t-1), \mathbf{s}^m u_i(t-1)) \\ y_i(t) &= h_i(x_i(t), \mathbf{s}^m x_i(t-1)), i \in I \end{aligned} \quad (9)$$

where $x_i(t) \in X_i \subset \mathbf{R}^n, y_i(t) \in Y_i \subset \mathbf{R}^l$, and $u_i(t) \in U_i \subset \mathbf{R}^m, X_i, Y_i$, and U_i are open sets, n, l , and m -dimensional vectors representing the local state, output, and input variables respectively at the i th node in I . \mathbf{s}^m is a spatial shift operator, which is defined as

$$\mathbf{s}^m = (s^{p_1}, s^{p_2}, \dots, s^{p_m}) \quad (10)$$

such that

$$\mathbf{s}^m x_i(t) = (x_{i+p_1}(t), x_{i+p_2}(t), \dots, x_{i+p_m}(t)) \quad (11)$$

where p_1, p_2, \dots, p_m are the indices of the neighbours of the i th node - that is the region in I around the i th node, which influences the dynamics of that particular node.

Generally it is also assumed that the following input-output representation

$$y_i(t) = g_i(\mathbf{q}^{n_y} y_i(t), \mathbf{q}^{n_u} u_i(t), \mathbf{s}^{m'} \mathbf{q}^{n_y} y_i(t), \mathbf{s}^{m'} \mathbf{q}^{n_u} u_i(t)) \quad (12)$$

can be derived for any site from (9). In (12), \mathbf{q} is a backward shift operator such that

$$\mathbf{q}^{n_y} y_i(t) = (y_i(t-1), y_i(t-2), \dots, y_i(t-n_y)) \quad (13)$$

Remark 1 In eqn. (12) $g_i, i \in I$ are generally nonlinear differentiable maps depending on the history of the local input and output variables, and on the variables at some neighbouring nodes. If $g_i, i \in I$ are unknown then the nonlinearity of g_i makes it difficult to apply traditional identification techniques. It is common practice to approximate nonlinear input-output equations from the available data using a known set of basis functions or regressors. Typical classes of regressors used in nonlinear identification include polynomial and rational functions, Gaussian

radial basis functions and wavelets. In this paper, polynomials are chosen as basis functions to approximate the CML model (12).

Remark 2 It is worth noting that if $g_i = g_j$ for all $i, j \in I$, the CML model is considered to be spatially homogeneous and nonhomogeneous otherwise. As mentioned earlier, the identification problem of homogeneous spatio-temporal dynamic systems has been studied by several researchers. In this paper, the identification problem of nonhomogeneous spatio-temporal dynamic systems is considered and applied to the crack detection problem in structures.

Generally, a spatio-temporal dynamic system in the form of a partial differential equation can be discretised as a CML model. For instance, reconsider the partial differential equation (2), by approximating the spatial and time derivatives using a finite difference method and letting $x^{(1)}(x, t) = y(x, t)$, $x^{(2)}(x, t) = \partial y(x, t)/\partial t$, and $u(x, t) = q(x, t)$ the system (2) can then be described as the following CML model

$$\begin{aligned}
 x_i^{(1)}(t) &= x_i^{(1)}(t-1) + Tx_i^{(2)}(t) \\
 x_i^{(2)}(t) &= x_i^{(2)}(t-1) + T\frac{1}{\mu_i}u_i(t) - \frac{T}{h^4\mu_i}(EI_{i+1}x_{i+2}^{(1)}(t) - 2(EI_{i+1} + EI_i)x_{i+1}^{(1)}(t) \\
 &\quad + (EI_{i+1} + 4EI_i + EI_{i-1})x_i^{(1)}(t) - 2(EI_i + EI_{i-1})x_{i-1}^{(1)}(t) - 2(EI_i - EI_{i-1})x_{i-2}^{(1)}(t)) \\
 y_i(t) &= x_i^{(1)}(t)
 \end{aligned} \tag{14}$$

where $x_i^{(1)} = y(x_i, t)$, $x_i^{(2)} = \dot{y}(x_i, t)$, $h = x_i - x_{i-1}$, $i = 1, \dots, M$, and T is the time sampling interval.

Obviously this is a nonhomogeneous spatio-temporal system because the damaged cross-section makes the flexural rigidity $EI(x)$ non-uniform along the length of the beam.

4 Identification approach

The identification of CML models of homogeneous spatio-temporal dynamic systems can generally be obtained using data from one single node and its neighbours. This is because the underlying system is assumed to be homogeneous and all the dynamics at all the nodes are the same. Although the model structures or parameters are different in the case when the underlying system is nonhomogeneous, in principle, the identification of such nonhomogeneous spatio-temporal dynamic systems can be conducted by applying an identification algorithm to every single node in the overall lattice. However, this will often be a formidable task because the size of the lattice maybe very high. For instance, a two dimensional lattice of size 100×100 contains 10,000 nodes. This means that a total of 10,000 CML models would need to be identified. On the other hand, there may only be a few nodes that appear to be nonhomogeneous such as for example in a uniform beam with one or several fatigue cracks. In this case it is not necessary to identify a single CML model for each single node in the lattice. In this paper, a

strategy is proposed to solve this problem. The basic idea behind the approach is that the lattice is initially divided into several regions, say two or three, and a CML model is then identified and validated using the observations from one of the regions. This identified CML model is then used to predict the dynamic response of the other regions and the predicted results may be good or bad depending on which region is used. If the dynamics in a region is the same as that in the region used for identification, then the identified and validated model should be able to produce good predictions. If the prediction results are not acceptable in some region, this may indicate that nonhomogeneity may have occurred in this particular region. This bad region can then be refined, that is, divided into smaller subregions before the strategy is repeated until all the dynamics of this nonhomogeneous system are obtained.

Assume that the maps $g_i, i \in I$ in (12) are all unknown and to be identified. Let $\Omega \subset I$ be a subarea of the lattice I . The identification problem of $g_i, i \in \Omega$ can be stated as follows.

Given observations $y_i(t-1), \dots, y_i(t-n_y), u_i(t-1), \dots, u_i(t-n_u)$, and $\mathbf{s}^{m'} \mathbf{q}^{n_y} y_i(t), \mathbf{s}^{m'} \mathbf{q}^{n_u} u_i(t), i \in \Omega$, the objective of CML identification is to approximate the input-output relationship function g_i from these observations. In this paper, the algorithm and results for CML identification using polynomials are presented.

Let $\alpha = (\alpha_1, \dots, \alpha_n)$ be a multi-index, that is an n -tuple of nonnegative integers α_k , and denote by \mathbf{z}^α the monomial $z_1^{\alpha_1} \dots z_n^{\alpha_n}$, which has degree $|\alpha| = \sum_{k=1}^n \alpha_k$. Let s be a positive integer, and let $\Lambda = \{\alpha \mid |\alpha| < s\}$ a set of multi-indices, then the set of polynomials of total order s is $\Sigma_s = \text{span}\{\mathbf{z}^\alpha \mid |\alpha| < s\}$. Note that Σ_s is a L -dimensional space, where $L = 1 + n + (n+1)n/2! + \dots + (n+s-1) \dots (n+1)n/s!$. Approximating nonlinear function g_i in (12) using the polynomial approximation space Σ_s yields the following representation

$$y_i(t) = \theta_0 + \sum_{i_1=1}^n \theta_{i_1} z_{i_1}(t) + \sum_{i_1=1}^n \sum_{i_2=i_1}^n \theta_{i_1 i_2} z_{i_1}(t) z_{i_2}(t) + \dots + \sum_{i_1=1}^n \dots \sum_{i_s=i_s-1}^n \theta_{i_1 \dots i_s} z_{i_1} \dots z_{i_s} + \varepsilon(t) \quad (15)$$

where all θ represent polynomial coefficients and all $z(t)$ represent lagged terms in y_i, u_i , and their corresponding neighbours, and $\varepsilon(t)$ denotes the error of this representation. Given the representation (15), the objective of the identification algorithm is to select the significant terms from this set while estimating the corresponding monomial coefficients. In this paper, an Orthogonal Forward Regression algorithm (OFR) (Chen, Billings, and Luo 1989) is employed. The OFR algorithm involves a stepwise orthogonalisation of the regressors and a forward selection of the relevant terms based on the Error Reduction Ratio (ERR) criterion (Billings, Chen, and Kronenberg 1988). The algorithm provides the optimal least-squares estimate of the polynomial coefficients θ .

For a given candidate regressor set $G = \{\varphi_m\}_{m=1}^M$, the OFR algorithm can be summarised as follows

Step 1

$$I_1 = I_M = \{1, \dots, M\}$$

$$w_m(t) = \varphi_m(t), \hat{b}_m = \frac{w_m^T y}{w_m^T w_m} \quad (16)$$

$$l_1 = \arg \max_{m \in I_1} (\hat{b}_m^2 \frac{w_m^T y}{y^T y}) = \arg \max_{m \in I_1} (err_m) \quad (17)$$

$$w_1^0 = w_{l_1}, c_1^0 = \frac{w_1^{0T} y}{w_1^{0T} w_1^0} \quad (18)$$

$$a_{1,1} = 1 \quad (19)$$

Step $j, j > 1$

$$I_j = I_{j-1} \setminus l_j - 1 \quad (20)$$

$$w_m(t) = \varphi_m(t) - \sum_{k=1}^{j-1} \frac{w_k^{0T} y}{w_k^{0T} w_k^0} w_k^0, \hat{b}_m = \frac{w_m^T y}{w_m^T w_m} \quad (21)$$

$$l_j = \arg \max_{m \in I_j} (\hat{b}_m^2 \frac{w_m^T y}{y^T y}) = \arg \max_{m \in I_j} (err_m) \quad (22)$$

$$w_j^0 = w_{l_j}, c_j^0 = \frac{w_j^{0T} y}{w_j^{0T} w_j^0} \quad (23)$$

$$a_{k,j} = \frac{w_k^{0T} \varphi_{l_j}}{w_k^{0T} w_k^0}, k = 1, \dots, j-1. \quad (24)$$

The procedure is terminated at the M_s -th step when the termination criterion

$$1 - \sum_{m=1}^{M_s} err_m < \rho \quad (25)$$

is met, where ρ is a designated error tolerance, or when a given number of terms in the final model is reached.

The estimated coefficients are calculated from the following equation

$$\begin{pmatrix} \theta_{l_1} \\ \theta_{l_2} \\ \vdots \\ \theta_{l_{M_s}} \end{pmatrix} = \begin{pmatrix} 1 & a_{1,2} & \cdots & a_{1,M_s} \\ 0 & 1 & \vdots & a_{2,M_s} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} c_1^0 \\ c_2^0 \\ \vdots \\ c_{M_s}^0 \end{pmatrix} \quad (26)$$

and the selected terms are $\varphi_{l_1}, \dots, \varphi_{l_{M_s}}$.

Based on the algorithm, the iterative identification procedure for nonhomogeneous spatio-temporal systems can be outlined as follows

Step 1 Divide the spatial domain into several subregions and choose one of the regions for CML identification.

Step 2 Determine the spatial neighbourhood sites (represented by $\mathbf{s}^{m'}$) of the i th site in the selected subregion.

Step 3 Select the time lags n_y and n_u , then the process variables involved in the identification are

$$\{\mathbf{s}^{m'} y_i(t-1), \dots, \mathbf{s}^{m'} y_i(t-n_y), \mathbf{s}^{m'} u_i(t-1), \dots, \mathbf{s}^{m'} u_i(t-n_u)\} \quad (27)$$

Step 4 Apply the orthogonal least squares algorithm to obtain the terms and parameters of the CML model.

Step 5 Apply model validity tests to evaluate the model. If no valid models are found, then the set of candidate terms is refined with a higher polynomial degree to the set of candidate terms.

Step 6 Apply the obtained and validated CML model to other subregions to predict the dynamic responses in these subregions.

Step 7 Check the predicted errors for each subregion to detect the nonhomogeneous subregions. Select one of these subregions, go back to Step 1 and repeat the above procedure until all the dynamics have been identified.

Remark 3 The final model and parameters need to be assessed. A commonly used approach to check the validity of the identified model is to use higher order statistical correlation analysis (Billings and Voon 1986, Billings and Zhu 1994).

Remark 4 Note that in the above identification procedure, the spatial neighbourhood sites (represented by $\mathbf{s}^{m'}$) of the identified site and the time lags (n_y, n_u) need to be known *a priori*. In other words, the neighbourhood of the identified site, that is, the region around that site which influences the dynamics of that site in the spatial domain and the time domain need to be known before starting the identification. In practice, these two factors are important in determining the spatio-temporal dynamics of the underlying system. This problem is related to the embedding dimension problem in system reconstruction theory (Casdagli, 1992).

5 Applications to crack detection and analysis of results

To apply the proposed nonhomogeneous spatio-temporal system identification method, the spatial domain was sampled with a sampling interval 0.12 so that the number of nodes was 101.

Terms	Estimates	ERR
$y_i(t - 1)$	2.2067e+00	9.8740e-01
$y_i(t - 2)$	-1.2335e+00	1.1585e-02
constant	2.2280e-04	1.4261e-04
$y_{i+2}(t - 1)$	-3.3404e-01	1.3888e-04
$y_{i+2}(t - 2)$	1.4812e-01	2.0796e-05
$u_i(t - 1)$	1.0685e-07	1.7473e-05
$y_{i+1}(t - 1)$	2.0214e-01	1.3456e-05
$y_{i+2}(t - 3)$	7.9581e-02	1.2724e-05
$y_{i+1}(t - 2)$	-1.3099e-01	6.9377e-06
$y_{i+1}(t - 3)$	4.0859e-02	1.5818e-06
$u_i(t - 2)$	-7.9296e-08	1.0959e-06

Table 1: The terms and parameters of the final CML model

These 101 nodes were divided into two parts along the length of the beam corresponding to the two sections from 0 to 6m and 6m to 12m for a total length of the beam $l = 12\text{m}$, respectively. Note that the location of the damaged cross-section is $l/20 = 0.6\text{m}$ which corresponds the node $i = 6$ and lies in the first part. Considering that an external force was applied at the far end of the beam which is located in the second area, the second area was used to identify a CML model for this spatio-temporal dynamics in this region. The data for identification was generated randomly in the subregion. Because the external force is only applied to the node $i = 99$ (here the node $i = 100$ and $i = 101$ are considered as boundary nodes), the identification data must include the data from the node $i = 99$. With the output and input lags set to be 3 and the neighbourhood set to be $i - 2$, $i - 1$, $i + 1$, and $i + 2$ it can be recognised that the considered input variables for the algorithm are polynomial combinations of $y_i(t - 1)$, $y_i(t - 2)$, $y_i(t - 3)$, $u_i(t - 1)$, $u_i(t - 2)$, $u_i(t - 3)$, $y_{i-1}(t - 1)$, $y_{i-1}(t - 2)$, $y_{i-1}(t - 3)$, $y_{i+1}(t - 1)$, $y_{i+1}(t - 2)$, $y_{i+1}(t - 3)$, $y_{i-2}(t - 1)$, $y_{i-2}(t - 2)$, $y_{i-2}(t - 3)$, $y_{i+2}(t - 1)$, $y_{i+2}(t - 2)$, $y_{i+2}(t - 3)$. The identified model is listed in Table (1), where ERR denotes the Error Reduction Ratio.

Table (1) shows that the obtained input-output representation at node i is

$$\begin{aligned}
y_i(t) = & 0.0002228 + 2.2067y_i(t - 1) - 1.2335y_i(t - 2) + 0.00000010685u_i(t - 1) \\
& -0.000000079296u_i(t - 2) + 0.20214y_{i+1}(t - 1) - 0.13099y_{i+1}(t - 2) + 0.040859y_{i+1}(t - 3) \\
& -0.33404y_{i+2}(t - 1) + 0.14812y_{i+2}(t - 2) + 0.079581y_{i+2}(t - 3)
\end{aligned} \tag{28}$$

The one-step-ahead predicted output and associated error for all the beam, that is both regions of the beam are shown in Figs. (3) and (4). A two-dimensional plane view (see Figs. (5) and (6)) of the errors clearly indicates there are large predicted errors between nodes 3 and 10. It can be observed that the one-step-ahead predicted error falls between 0.001 and -0.0015 for the first subregion and 0.001 and -0.001 for the second subregion. Figs. (7-10) show the mean

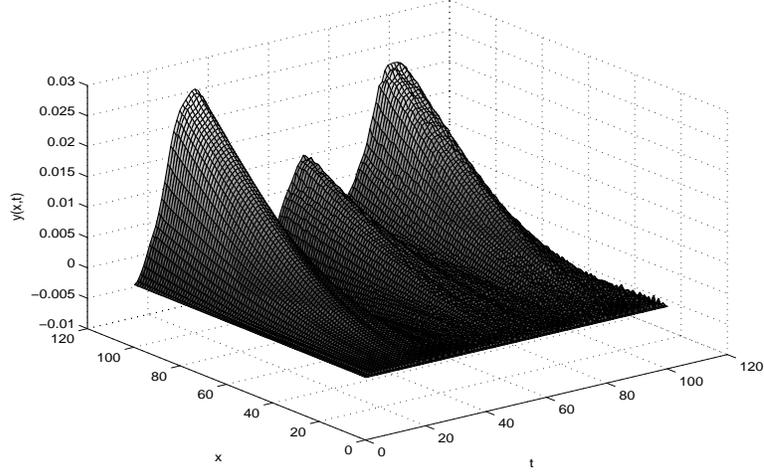


Figure 3: One-step-ahead predicted output of the fatigue cracked beam using the identified CML model over all the beam

values, the ratio between the standard deviations of the one-step-ahead predicted error and the mean values of the output, the standard deviations of the output, and the ratio between the standard deviations of the one-step-ahead predicted error and the standard deviations of the output. From these figures, it can be seen that from node 10 to node 99 the standard deviations of the one-step-ahead predicted error are very small and are close to zero whilst they are quite large for the nodes between 3 and 10. All these observations confirm the existence of fatigue cracks in this spatio-temporal dynamic system and that this lies just between nodes $i = 3$ and $i = 10$ which corresponds to the section between $0.36m$ and $1.12m$ along the length of the beam.

In order to obtain a CML model for the damaged area, that is the area between $0m$ and $1.12m$, the proposed identification algorithm was applied again to the data from this area. The obtained CML model is listed in Table (2). The one-step-ahead predicted error of this CML model for the crack-affected area, which falls in between 0.0004 and -0.0004 , is plotted in Fig.(11).

From Table (2), it can be seen that the obtained input-output representation at node i in the crack-affected area is

$$\begin{aligned}
y_i(t) = & 0.40331y_{i+1}(t-1) + 0.56505y_{i-1}(t-1) - 0.41632y_i(t-3)y_{i-2}(t-2) \\
& + 0.61025y_i(t-1)y_{i+2}(t-1) - 0.57826y_i(t-3)y_{i+2}(t-3) + 0.53877y_{i-2}(t-1)y_{i+1}(t-1) \\
& - 0.52157y_i(t-3)y_{i-2}(t-3) + 0.58352y_{i-1}(t-2)y_{i+1}(t-2) + 0.17437y_{i+2}(t-2) \\
& + 0.22286y_{i-1}(t-3)y_{i+1}(t-3)
\end{aligned} \tag{29}$$

The above results show that the identified CML models (28) and (29) together provide an excellent dynamical representation for the cracked beam in vibration.

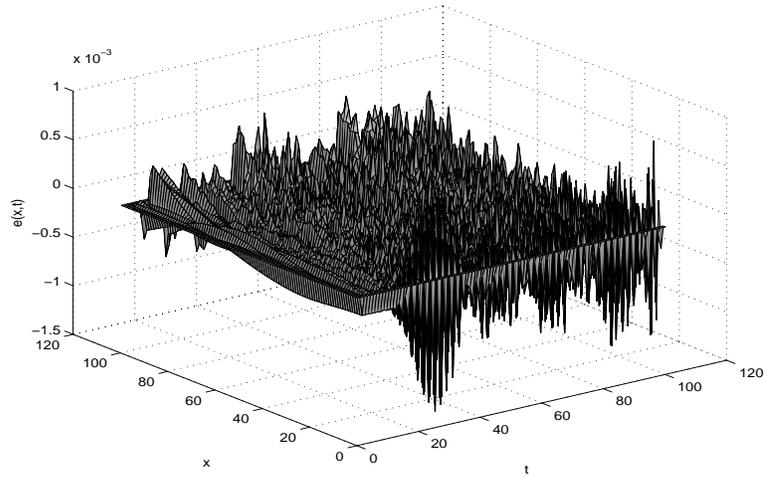


Figure 4: One-step-ahead predicted error for model (28) over all the beam

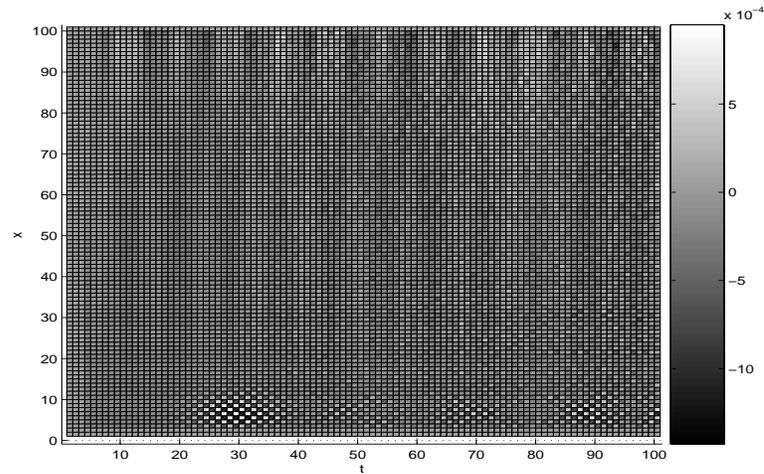


Figure 5: Two dimensional plane view of one-step-ahead predicted error for model (28) over all the beam

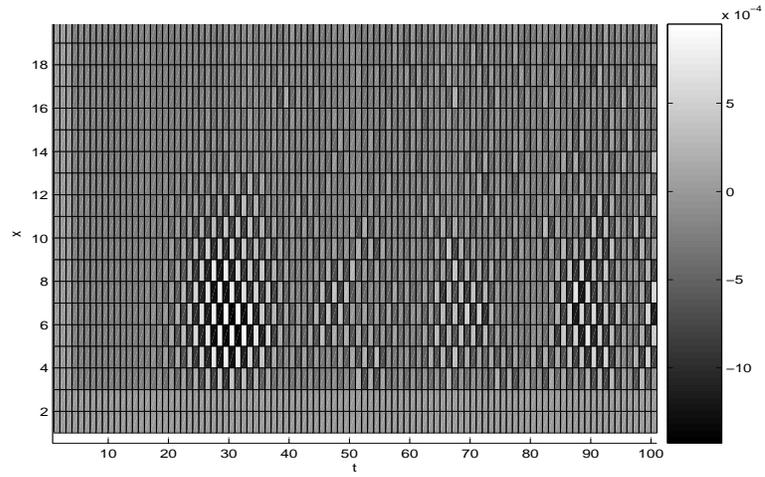


Figure 6: Two dimensional plane view of the one-step-ahead predicted error around the damaged cross-section $x_d = 0.6m$

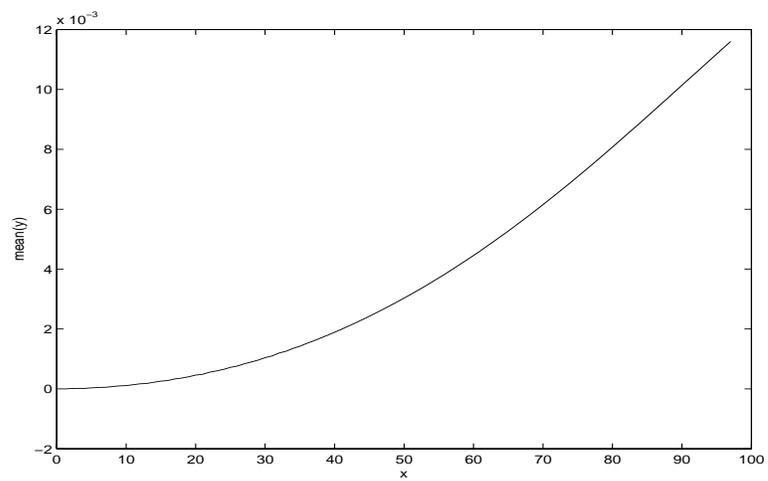


Figure 7: Mean values of the output vs. the length of the beam

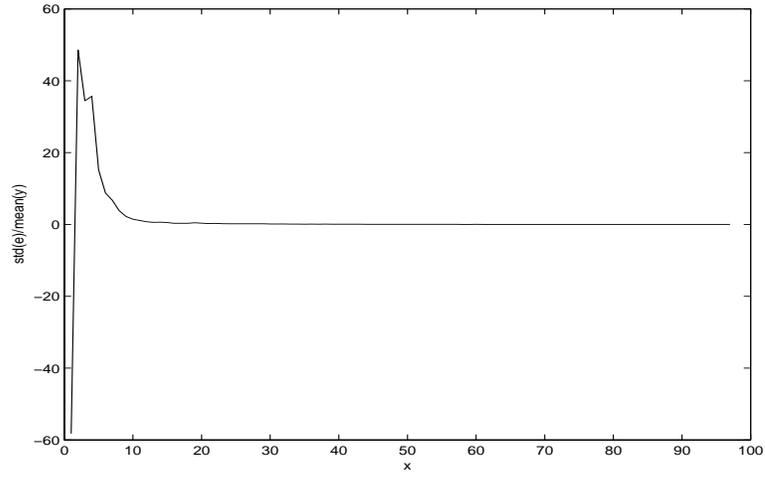


Figure 8: The ratio between the standard deviations of the one-step-ahead predicted error and the mean values of the output vs. the length of the beam

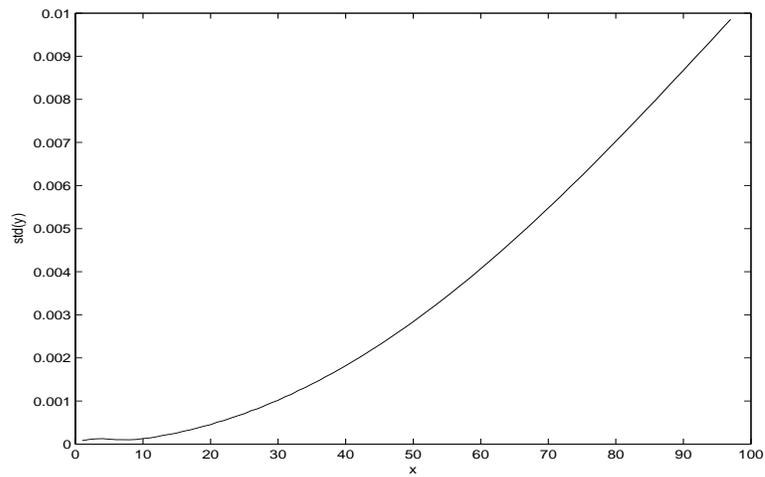


Figure 9: The standard deviations of the output vs. the length of the beam

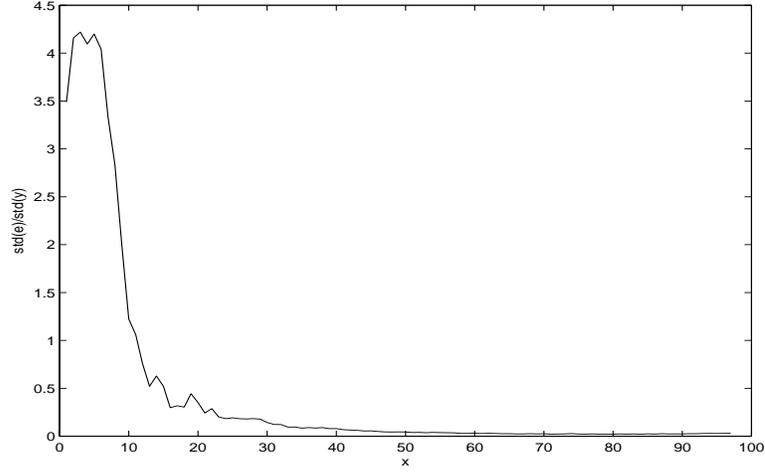


Figure 10: The ratio between the standard deviations of the one-step-ahead predicted error and the standard deviations of the output vs. the length of the beam

Terms	Estimates	ERR
$y_{i+1}(t-1)$	4.0331e-01	9.9935e-01
$y_{i-1}(t-1)$	5.6505e-01	2.6809e-04
$y_i(t-3)y_{i-2}(t-2)$	-4.1632e-01	3.4129e-05
$y_i(t-1)y_{i+2}(t-1)$	6.1025e-01	3.1586e-05
$y_i(t-3)y_{i+2}(t-3)$	-5.7826e-01	1.9373e-05
$y_{i-2}(t-1)y_{i+1}(t-1)$	5.3877e-01	9.5245e-06
$y_i(t-3)y_{i-2}(t-3)$	-5.2157e-01	9.4620e-06
$y_{i-1}(t-2)y_{i+1}(t-2)$	5.8352e-01	8.4744e-06
$y_{i+2}(t-2)$	1.7437e-01	7.4490e-06
$y_{i-1}(t-3)y_{i+1}(t-3)$	2.2286e-01	3.2115e-06

Table 2: The terms and parameters of the final CML model for the crack-affected area

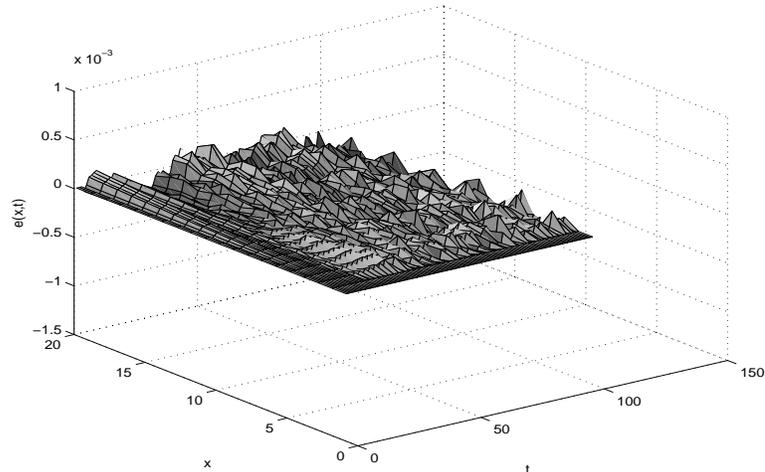


Figure 11: One-step-ahead predicted error of the CML model for the crack-affected area

6 Conclusions

An identification method for CML models of nonhomogeneous spatio-temporal dynamic systems has been proposed and applied to the crack detection problem in structures. The new method extends identification techniques for homogeneous systems in such a way that the identification algorithm can be applied in a coarse-to-fine manner so that the nonhomogeneity of the system can be detected. It has been shown by simulation that the proposed method can not only effectively detect cracks in a beam but also generates an excellent CML model representation for the spatio-temporal dynamics of the vibrating beam.

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