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Improved Determination of Q-Factor and Resonant Frequency by a Quadratic Curve-Fitting Method

M. P. Robinson and J. Clegg

***Abstract*—The Q-factor and peak frequency of resonant phenomena give useful information about the propagation and storage of energy in an electronic system and therefore its electromagnetic compatibility performance. However, the calculation of Q by linear interpolation of a discrete frequency response to obtain the half-power bandwidth can give inaccurate results, particularly if the data are noisy or the frequency resolution is low. We describe a more accurate method that makes use of the Lorentzian shape of the resonant peaks and involves fitting a second-order polynomial to the reciprocal power plotted against angular frequency. We demonstrate that this new method requires less than one quarter the number of frequency points as the linear method to give comparable accuracy in Q. The new method also gives comparable accuracy for signal-to-noise ratios that are approximately 8 dB greater. It is also more accurate for determination of peak frequency. Examples are given both from measured frequency responses and from simulated data obtained by the transmission line matrix method.**

***Index Terms*—Electromagnetic compatibility (EMC) measurements, interpolation, Q-factor, resonance, resonant frequency.**

I. Introduction

Resonant phenomena are encountered in the field of electromagnetic compatibility (EMC) when the dimensions of circuit boards, cables, screened enclosures, and other structures are large compared to the frequencies of interest. Although the Q-factors of these resonances are often neglected, they are actually of great significance because they describe the energy absorption and hence the height of the peaks in the frequency response. These are often more important than the exact frequencies of the resonances. Q is important in the energy-balance approach that Hill *et al.* take to characterizing the shielding effectiveness of large enclosures [1], while Dawson *et al.* have extracted peak parameters from frequency responses in order to validate computational electromagnetic (CEM) models [2]. The Q-factors of the individual modes are key parameters in the design of stirred-mode chambers and other reverberant environments [3]. Measurements of the changes in Q-factors and resonant frequencies are used to characterize the contents of shielded enclosures by means of the resonant perturbation technique [4]. In many cases, the data are obtained by either simulation or computer-controlled instrumentation, and consist of scalar values of voltage, electric field, etc. at discrete frequency points.

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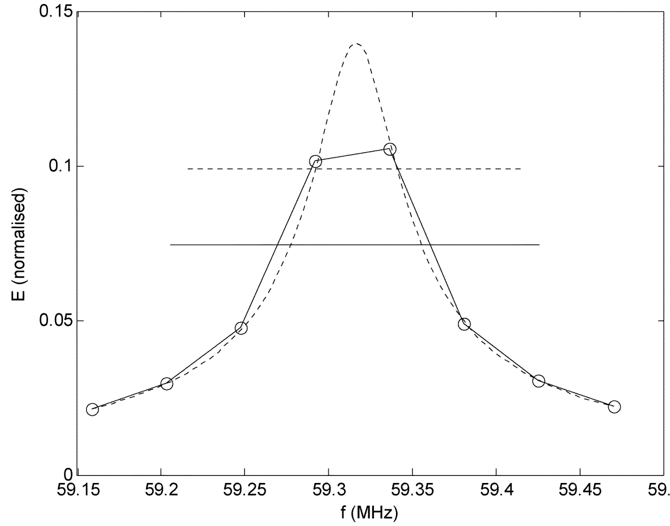

Fig. 1. Simulated frequency response of electric field strength in a screened room, showing how a linear interpolation leads to an overestimate of half-power bandwidth. (Solid line) Interpolated response. (Dotted line) "True" response.

A simple and well-known method of calculating the Q from a peak in a frequency response is to find the maximum power, divide it by two, find the bandwidth at half-power, and divide this into the resonant frequency. This "traditional" method was well suited to analogue instrumentation that gives a continuous curve on a display as an output, and to graphical calculation techniques. However, with discrete frequency points and numerical calculations it can lead to errors in the resonant frequency, and more so in Q-factor, particularly if the sampled frequency points are sparse. This is because it is unlikely that a frequency point will lie exactly on the peak, so the peak power is underestimated, the bandwidth overestimated, and the Q is too low. Further errors come from linear interpolation between points—the method used in many automated network analyzers (ANAs). This is illustrated in Fig. 1, which shows how the bandwidth is overestimated owing to the poor frequency resolution. In this example, it is about 85% too high, and the peak frequency is also in error by 21 kHz.

A better approach is to use more of the points near the peak to improve accuracy. A technique that applies this idea to transmission (S_{21}) measurements of the Q of a cavity is described admirably by Leong and Mazierska [5]. Their method involves fitting a circle to complex S_{21} values plotted on a Smith Chart, and removes the effects of cables, connectors, and mismatches to give an accurate determination of Q-factors in the range 10^3–10^7. It is well-suited to precision metrology, in a setup where phase information is available. In the field of EMC, however, we often have to use scalar instruments or deal with data which could have been recorded alongside phase information but was not. There are often practical limits to the smallness of the frequency step. In computational electromagnetics, results from time-domain simulations are converted to the frequency domain by Fourier transforms giving discrete points. To improve the resolution means running the model for longer, which can often take several hours. So we need a method of improving on the linear-interpolation method without needing more points.

In this correspondence, we describe a quadratic curve-fitting method of obtaining both Q and resonant frequency, and we compare it with linear interpolation for measurements and numerical simulation. We consider the effects of sparse data and poor signal-to-noise ratio (SNR) on each method.

II. Calculation of Peak Parameters

A. Frequency Response of an Oscillator

The standard theory of an oscillator shows that the power P developed in a resonant system such as a tuned circuit or shielded enclosure is given by

$$P \propto \frac{\omega_0}{Q} \frac{1}{\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}} \tag{1}$$

where ω is the angular frequency and ω_0 the peak angular frequency. This is the Lorentzian line shape familiar to spectroscopists. Hence

$$\frac{1}{P} = C\frac{Q}{\omega_0}\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}\right] \tag{2}$$

where C is a constant. At frequencies near resonance

$$\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right) \approx 2\left(1 - \frac{\omega}{\omega_0}\right)$$

so

$$\begin{aligned} \frac{1}{P} &= C\frac{Q}{\omega_0}\left[4\left(1 - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}\right] \\ &= C\frac{Q}{\omega_0}\left[4 + 4\frac{\omega^2}{\omega_0^2} - 8\frac{\omega}{\omega_0} + \frac{1}{Q^2}\right] \\ &= a\omega^2 + b\omega + c \end{aligned} \tag{3}$$

which is a quadratic in ω with coefficients given by

$$a = \frac{4QC}{\omega_0^3} \quad b = -\frac{8QC}{\omega_0^2} \quad c = \frac{4QC}{\omega_0} + \frac{C}{Q\omega_0}. \tag{4}$$

Algebraic manipulation of these expressions gives the required resonant angular frequency and Q, and also the peak power P_0

$$\omega_0 = -\frac{b}{2a} \quad Q = \frac{1}{2}\left(\frac{4ac}{b^2} - 1\right)^{-\frac{1}{2}} \quad P_0 = \left(c - \frac{b^2}{4a}\right)^{-1}. \tag{5}$$

A plot of $1/P$ against ω should therefore be a parabola, and by fitting a second-order polynomial to values of $1/P$ and ω we can determine the Q-factor and the resonant frequency $f_{\mathrm{res}} = \omega_0/2\pi$ from (5). For a frequency response describing voltage or electric field, we should plot the reciprocal of the square against ω.

Suitable routines for the curve-fitting are provided by many data analysis programs such as Matlab [6]. Algorithms are also available for those who prefer a do-it-yourself approach, such as the linear least squares method described by Press *et al.* [7]. This method uses singular value decomposition of the matrix before solving the linear set of equations for the coefficients of the fitting curve. This is because quite often the matrix can be close to singular and by using the singular value decomposition this problem can be overcome.

The question arises of how many points to include in the curve fitting stage. Empirically we have found that the most effective algorithm is to start at the maximum power P_m, then in the positive direction continue to include points until a point is reached where $P < 0.5P_m$, then repeat for the negative direction. This guarantees that there will be at least three data points, which is necessary for fitting a quadratic.

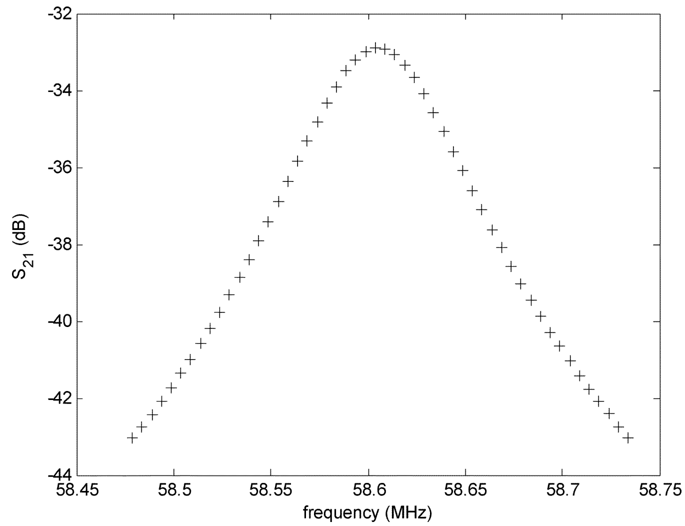

Fig. 2. Data created by selecting every fourth point from the full dataset.

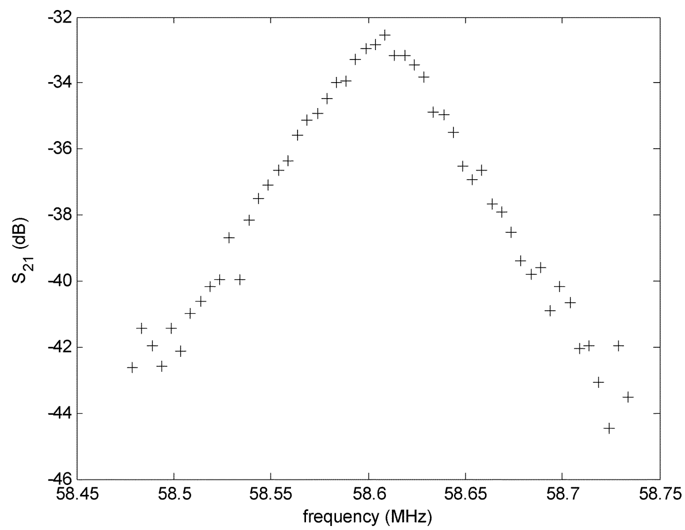

Fig. 3. Frequency response modified by the addition of Gaussian noise at −30 dB relative to the peak power.

The new method takes about 12 times longer to compute the parameters than does the linear-interpolation method, with the exact difference depending on the number of points between the half-power limits. However in most situations the run-time of either method is likely to be insignificant compared to the time taken acquiring the data or performing the numerical simulations.

B. Measured Data

To evaluate the quadratic-fit technique we used measured data from the frequency response of a screened room, loaded with “contents” (actually a human subject) in order to reduce the Q to a value typical of many EMC situations. The data were obtained with an ANA coupled to small monopole antennas in the roof of the chamber, giving an S_{21} measurement. We extracted 205 points close to the fundamental resonance of the chamber at approximately 59 MHz. The bandwidth is

TABLE I
MEAN AND RANGE OF Q-FACTOR AND RESONANT FREQUENCY AS CALCULATED BY THE LINEAR-INTERPOLATION AND QUADRATIC CURVE-FIT METHODS, AS A FUNCTION OF FREQUENCY STEP SIZE

		Q-factor				Resonant frequency (MHz)			
		linear interp.		quadratic curve-fit		linear interp.		quadratic curve-fit	
Freq. step (kHz)	no. of data sets	mean	range	mean	range	mean	range	mean	range
1.25	1	700.2	-	700.9	-	58.60500	-	58.60508	-
2.5	2	700.2	0.96	700.7	1.3	58.60563	0.0012	58.60508	0.00002
5.0	4	699.8	1.5	700.8	2.6	58.60563	0.0037	58.60508	0.00002
10	8	697.3	8.9	701.1	3.6	58.60563	0.0087	58.60508	0.00007
20	16	688.3	34	701.2	4.8	58.60563	0.0187	58.60509	0.00013
40	11	659.2	110	701.9	11	58.60659	0.0362	58.60509	0.00015
80	11	626.2	420	705.8	27	58.60050	0.0725	58.60535	0.00050

TABLE II
MEAN AND RANGE OF Q-FACTOR AND RESONANT FREQUENCY AS CALCULATED BY THE LINEAR-INTERPOLATION AND QUADRATIC CURVE-FIT METHODS, AS A FUNCTION OF SIGNAL-TO-NOISE RATIO (REFERENCED TO PEAK POWER)

		Q-factor				Resonant frequency (MHz)			
		linear interp.		quadratic curve-fit		linear interp.		quadratic curve-fit	
SNR (dB)	no. of data sets	mean	range	mean	range	mean	range	mean	range
-60	20	699.8	8.2	701.8	4.7	58.60375	0.0000	58.60507	0.00014
-50	20	700.6	14	702.0	13	58.60525	0.0050	58.60507	0.00049
-40	20	707.3	59	701.9	51	58.60500	0.0150	58.60520	0.00135
-30	20	746.2	360	705.9	220	58.60550	0.0250	58.60511	0.00458
-20	20	1418	2400	724.6	460	58.60650	0.0350	58.60480	0.01512

approximately 84 kHz, and the Q is therefore 700. The linear-interpolation and quadratic-fit methods were implemented in Matlab [6], using the function "polyfit." This gives the coefficients of the quadratic by means of a least mean squares algorithm. With a small frequency step of 1.25 kHz and high SNR, this initial data yields similar values of peak frequency and Q for the two methods.

To investigate the effect of increasing the frequency step, we "depopulated" the initial data by picking every nth value. An example is shown in Fig. 2, in which every fourth point has been chosen, thus increasing the step to 5 kHz. Clearly there are n ways of doing this for each value of n. We selected $n = 1, 2, 4, 8, 16, 32$, and 64, and created up to 16 new datasets for each value. We then used the linear-interpolation and quadratic-fit methods to calculate peak frequency and Q.

To explore the effect of poor SNR, we took the above datasets, and added Gaussian noise to each data point, at levels of -60 to -20 dB referenced to the power at the peak. An example is shown in Fig. 3 where the SNR is -30 dB. We repeated the procedure 20 times for each SNR value. As before we compared the values of peak frequency and Q as obtained by the two methods.

C. Numerical Modeling

To compare the two methods further, we performed a numerical simulation of the screened room using the transmission line matrix (TLM) method. This is a time-domain simulation which essentially gives the impulse response of the room; the frequency response may then be obtained by a Fourier transform. For our simulations we modeled an empty room, and increased the losses by making the reflection coefficients of the walls equal to -0.999 rather than -1. This gives a Q of approximately 1200, a bandwidth of 49 kHz, and a resonant frequency of 59.315 MHz.

With a grid size of 50 mm, it was necessary to run the model for 1.05×10^6 time steps in order to get a frequency resolution of 11.4 kHz. This took 42 h on a computer with an Athlon 2100XP processor. To produce datasets with larger frequency steps, we took the time response and truncated it, to give durations of 1/2, 1/4, 1/8, 1/16, and 1/32 of the original response, and correspondingly larger frequency steps of up to 370 kHz. We then calculated the peak frequency and Q of each frequency response using the linear-interpolation and quadratic-fit methods.

III. RESULTS

Table I shows the values of Q-factor as calculated by the linear-interpolation and quadratic-fit methods, with the frequency step ranging from 1.25–80 kHz. For each method the spread of Q values increases with frequency step. However the range of values for the quadratic-fit method is much less than for the linear-interpolation method. At a frequency step of 80 kHz, which is similar to the bandwidth of the resonance (84 kHz), the range is 27 for the improved method but 420 for linear interpolation. The quadratic-fit method gives comparable accuracy for frequency step sizes that are four to five times greater.

Table I also shows a similar comparison for the peak frequency. At a frequency step of 80 kHz, the ranges of f_{res} are 73 and 0.5 kHz for the linear and quadratic methods, respectively. The linear method can only locate the peak to a precision of plus or minus half the step size, while the quadratic-fit method, which uses more points, can locate the peak much more precisely.

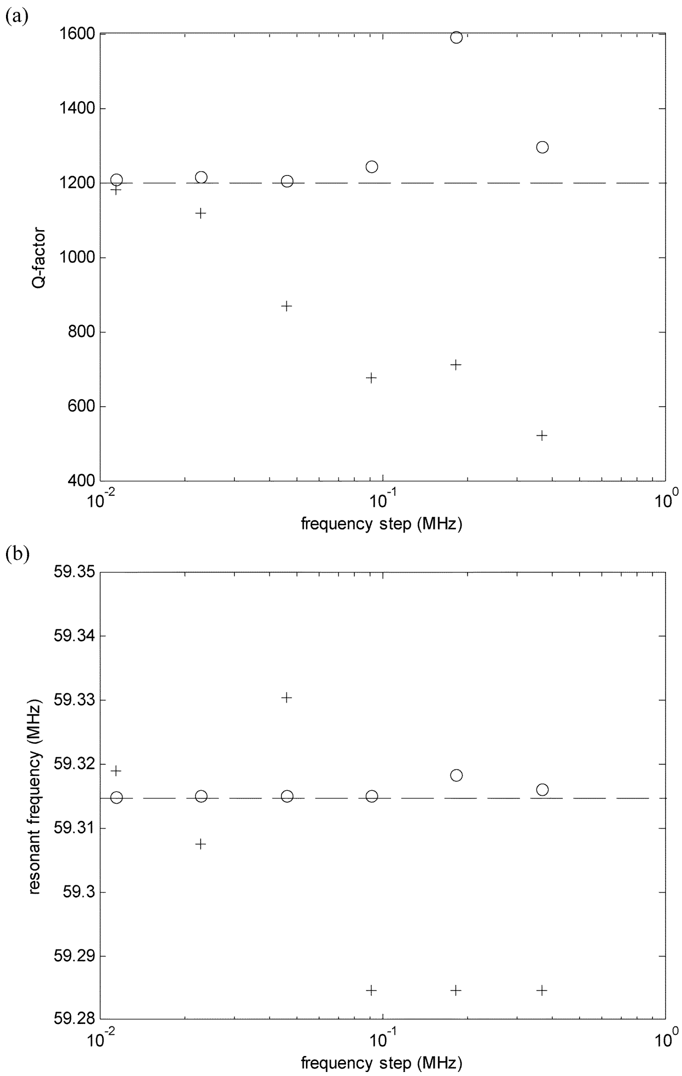

Fig. 4. ***Q***-factor and resonant frequency of TLM simulation, calculated by the quadratic-fit (o) and linear-interpolation (+) methods, as a function of frequency step size. (a) ***Q***-factor. (b) Resonant frequency.

Table II shows the values of Q-factor as calculated by the linear-interpolation and quadratic-fit methods, as the SNR is increased from -60 to -20 dB relative to the peak power. As might be expected, the spread of Q values increases with SNR for each method. However, the quadratic-fit method is superior in both precision and accuracy. At an SNR of -30 dB the range of Q values is 220 for the quadratic method and 360 for linear interpolation. We estimate that the quadratic-fit method gives comparable ranges in Q-factor for SNR values that are 8 dB higher. Furthermore the linear method underestimates the bandwidth of the resonance, giving Q values that are too high (mean 746), while the mean of the Q values at an SNR of -30 dB is 706, i.e., still close to the "true" value of 701.

Table II also shows the effect of increasing SNR on the peak frequency. The quadratic-fit method again shows superior performance. The range of values of f_{res} at an SNR of -20 dB is 15 kHz for the quadratic method, which is similar to the range of values for the linear method at an SNR of -40 dB.

In Fig. 4(a), the Q-factors obtained from the TLM simulation using the two methods are plotted against frequency step. Fig. 4(b) shows the corresponding plots for resonant frequency. It can be seen that for the linear-interpolation method the values of Q and peak frequency begin to deviate from their "true" values at a frequency step of approximately 15 kHz, while the quadratic-fit method maintains accuracy up to a step size of about 100 kHz. Note that this is approximately twice the half-power bandwidth of 49 kHz. As the frequency step for TLM is inversely proportional to the run time, these results show that a simulation followed by linear-interpolation would need to be run for six to seven times as long as one using the quadratic method.

IV. Conclusion

The quadratic curve-fitting method of obtaining peak parameters is better than the simple linear method, because it is less sensitive to poor frequency resolution and to the effects of Gaussian noise. The need to apply a polynomial-fitting algorithm is not a disadvantage because this can be done very quickly on a modern computer. The quadratic-fit method will give more accurate values of Q-factor and resonant frequency from existing data, and will enable new measurements and simulations to be performed with less stringent requirements on frequency resolution and signal-to-noise.

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