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Measurements of Equivalent Circuit Parameters for Modelling the Impact of Iron Losses in SR Machine

J. Corda and S. M. Jamil
School of Electronic and Electrical Engineering
University of Leeds,
Leeds, LS2 9JT, UK
E-mail: j.corda@leeds.ac.uk

Abstract—The paper outlines a methodology for determining the nonlinear parameters in the equivalent circuit representing a switched reluctance machine in the model which includes iron losses. The parameters are derived from measurements carried out under static conditions. The methodology is validated by comparison of the results for the machine's pull-out force obtained by measurements with results obtained by predictions using the dynamic model based on the equivalent circuit.

I. INTRODUCTION

Phase windings of switched reluctance (SR) machine are energised from an electronic converter commutated by signals from a rotor-position detector. The converter operates either in a single-pulse or a chopping/p.w.m. mode. The resulting flux waveforms are principally determined by the magnitude of d.c. supply voltage and pulse widths and differ in various parts of the machine magnetic circuit. As an illustration, flux waveforms of a 3-phase SR machine operating in a single-pulse phase excitation mode are shown in Fig.1 [1]. A fair prediction of iron losses under such excitation of magnetic circuit requires a complex computational undertaking.

![Flux-waveforms in a 3-phase SR machine](image1)

Fig. 1. Flux-waveforms in a 3-phase SR machine [1]

From an electromagnetic design perspective, the modelling of iron losses effects on the performance of a hypothetical machine specified with a set of supply and load conditions, should be accomplishable on the basis of magnetic material data and a set of geometric parameters. (The latter are iteratively varied during the design process in searching for an optimum set.) The prediction of iron losses based on such an approach has been dealt with in several publications, e.g. [2-7].

On the other hand, the development of a control algorithm for an existing machine is normally carried out using a computer model which relies on the equivalent circuit. Due to the aforesaid complexity of SR machine flux waveforms, the representation of iron loss effects in the equivalent circuit requires an adequate approach.

II. EQUIVALENT CIRCUIT AND MACHINE CHARACTERISATION

A functional phase equivalent circuit of a SR machine is shown in Fig.2, where $R$ and $L$ are the phase winding resistance and self-inductance, and $r$ represents a lumped resistance associated with iron losses. The latter two parameters ($L$ and $r$) are non-linear multivariable functions, i.e. $L$ is dependent on position and the instantaneous current, while $r$ is dependent on position, the rate of flux-linkage change and the phase switching frequency. (The rate of flux-linkage

![Functional phase equivalent circuit with iron loss “lumped” resistance](image2)

Fig. 2. Functional phase equivalent circuit with iron loss “lumped” resistance ($r$)
change is determined by the phase supply voltage and the instantaneous current, i.e. \( \frac{dy}{dt} = -V_i - R_i - AV_{\text{excess}} \).

The inductance's dependence on current becomes more pronounced at positions leading to a closer alignment between related saliencies, due to an increased impact of magnetic saturation.

In this paper, a linear tubular SR actuator [8] is used as an example of the machine with enhanced iron losses caused by the presence of solid steel parts in the magnetic core. An oscilloscope reading of typical phase excitation current waveform of such an actuator is shown on the upper trace of Fig. 3. The waveform has three distinct regions: (A) the current build-up, (B) the chopping region and (C) the current decay. A significant presence of eddy currents and hysteresis is evident from the decaying region of the induced emf waveform shown on the lower trace of Fig. 3. Namely, when the phase current has reduced to zero there is still a decaying e.m.f. which can only be attributed to the magnetic flux continuing to flow through the magnetic core. Referring to the equivalent circuit of Fig. 2, the fact that the total phase current is zero during this decay means that the magnetizing current continues to flow through the branch represented by the 'equivalent iron loss resistance' during the remaining flux decay.

In order to include the effect of iron losses in the dynamic model of SR machine based on the equivalent circuit, a set of characteristics representing variations of the equivalent iron loss resistance is needed. The method for measurement of these characteristics is described in Section IV.

Regarding the inductance variation with position and current, which is usually expressed in the model through a set of magnetisation characteristics (flux-linkage/position/current) [9], several measurement methodologies have been established so far. However, the necessity to decouple the inductance from the equivalent iron loss resistance has resulted in an improved procedure which is outlined in the following section.

III. MEASUREMENTS OF FLUX-LINKAGE (INDUCTANCE)

The values representing the phase inductance in the equivalent circuit should be decoupled from the iron loss resistance values, i.e. the measurement method should ensure that true inductance values are obtained.

For the convenience of integrating the non-linear voltage differential equations of the SR machine dynamic model, the magnetisation characteristics expressed through the flux-linkage, rather than the inductance, are used. (Inductance and flux-linkage are related through a simple relationship \( L = \phi/I \)). These characteristics are measured for a range of locked mover positions by integrating the emf induced in the search coil while the excitation is raised from zero to the required level. It should be noted that this method assumes that the search coil links all the flux produced by the main phase winding, and hence the most desirable arrangement is a bifilar one, where a search coil formed of a thin wire is wound together with the main coil.

In order to avoid the impact of eddy currents on the inductance (flux-linkage) measurements, i.e. to measure the true value of inductance, the excitation current was increased in an automated slow ramp fashion (1 A/sec) by controlling the output from a variable d.c. power supply unit. The measurement setup is shown in Fig. 4. The current was ramped up to the maximum level of the required range (0 to 4 A) and then ramped back down to zero. Before applying a ramped d.c. excitation, the machine's magnetic core was demagnetized by applying an a.c. (50 Hz) excitation from a variable autotransformer whose variable output voltage was manually increased and then reduced down to zero. This process was repeated at each position in the range covering a full cycle of inductance variation.

![Fig. 4. Setup for measurements of magnetisation characteristics](image-url)

![Fig. 5. Measured magnetisation characteristics of experimental SR machine](image-url)
The automated ramping of the excitation current was achieved using a programmable d.c. supply unit controlled by an output from a data capture card installed in a computer. The input channels of the data capture card collected the signal from a current transducer and the induced emf from the search coil. The flux-linkage was obtained by integrating the induced emf over the interval of ramped rise of the excitation current. The integration was performed numerically after the emf waveform was captured. Such an integration, as a contrast to an analogue integration, eliminates the errors which would otherwise arise by a drift of the integrator circuit. The programming details of the measurement are given in [10].

Fig. 5 shows a set of magnetisation characteristics obtained by applying the above procedure on the experimental machine.

IV. MEASUREMENTS OF IRON LOSS EQUIVALENT RESISTANCE

A. Measurements for the rising and falling flux regions

The aforementioned regions of the phase current waveform (rising, chopping and falling), shown in Fig.1, are associated with flux-linkage variations and these determine three sets of characteristics for the equivalent iron loss resistance.

The measurements for determining sets of characteristics related to the rising and falling regions of the flux-linkage, were performed by locking the mover at a fixed position and switching the phase winding from a controller as shown in Fig.2. When the transistors are switched on, the magnetising current \( i_s = \frac{\mu I}{L} \) rises from zero to the peak value which is determined by the pulse duration at a given supply voltage, and the current \( i \) through the equivalent iron loss resistance branch 'steps' to a positive value \( \left( e/\tau \right) \). When the transistors are switched off, the total current flows through the diodes, the magnetising current (flux) decays, and the direction of emf and current \( i_s \) is changed. From the instant when magnitudes of \( i_s \) and \( i \) become equal, the total current \( i_{tot} \) becomes zero and the diodes stop conducting. Afterwards, both \( i_s \) and \( i \) decay exponentially as the remaining stored magnetic energy is dissipated by eddy currents in the core.

The equivalent iron loss resistance can be expressed as

\[
r = \frac{E_{rms}^2}{P_r}
\]

where \( E_{rms} \) is rms value of the induced emf and \( P_r \) is the iron loss power.

\( E_{rms} \) is obtained from the waveform of induced emf which is captured from the search coil.

\[
E_{rms} = \frac{1}{T_0} \int_0^{T_0} e^2 \, dt
\]

It should be noted that the quantities related to the equivalent circuit are associated with the main winding. The induced emf in the search coil, which is not shown in Fig.2 for the sake of clarity, needs to be referred to the primary side (main winding) by a scaling factor representing the turns ratio, \( N_{main}/N_{search} \).

The iron loss power can be expressed as

\[
P_r = \frac{1}{T_0} \int_0^{T_0} i_s e \, dt = \frac{1}{T_0} \int_0^{T_0} i_s e \, dt - \frac{1}{T_0} \int_0^{T_0} i_s e \, dt
\]

The second term in the above equation is zero, and hence

\[
P_r = \frac{1}{T_0} \int_0^{T_0} i_s e \, dt
\]

So, the iron loss power can be obtained by integrating the product of two directly measurable quantities \( i_s e \), which are captured from the current transducer and the search coil, respectively. (The measurement of iron loss power in this manner, as a contrast to the power separation method, eliminates the errors arising from cumbersome estimation of losses in semiconductor elements of the switching circuit.)
Preliminary measurements have shown that an approximately linear nature of the equivalent iron loss resistance against position. Hence, the measurements are performed only at the fully aligned and misaligned positions, and linear interpolation is used for other positions.

The equivalent iron loss resistance for rising and falling regions of the flux-linkage waveform were measured for a range of peak values of current and various phase switching frequencies at a given phase supply voltage (50 V). The measured characteristics are shown in Fig.6 for the fully aligned position and in Fig.7 for the fully misaligned position.

B. Measurements for the chopping region

In the chopping region, the phase current varies between the upper and lower limit. The amplitude of this nearly triangular variation is usually small relative to the mean value between the two limits and therefore the resulting frequency of chopping is high. It was found in [6] that the iron loss at high frequency of excitation did not vary significantly with the level of d.c. offset of the excitation flux, and that the iron loss for both sinusoidal and triangular excitation was similar. For this reason the measurements of equivalent iron loss resistance for representation of the chopping region were performed with sinusoidal high frequency excitation without any d.c. offset.

The measurement setup with the equivalent phase circuit diagram is shown in Fig.8. The variable high frequency sinusoidal supply was provided from a power amplifier. The sinusoidal waveforms of the excitation current ($I_{exc}$) and the induced emf in the search coil were captured in the oscilloscope. The latter needs to be scaled by the turns ratio ($N_{search}/N_{power}$), in order to obtain the induced emf in the main coil ($E$). The phase shift $\theta$ between phasors $E$ and $I_{exc}$ is found from the measured waveforms, and the value of equivalent iron loss resistance is then obtained from the basis the phasor diagram (Fig.9) as

$$r = \frac{E}{I_{exc} \cos \theta}$$  \hspace{1cm} (5)

(It is worth mentioning that this method is invalid for measurements of the inductance values under saturated magnetic core conditions.)

The measured characteristics of the equivalent iron loss resistance for the chopping region are shown in Fig. 10. As in the previous section, the measurements were performed at fully aligned and misaligned positions, and linear interpolation is used for other positions.

V. ADAPTATION OF THE MACHINE MODEL

The inclusion of equivalent iron loss resistance in the basic model of SR machine [9] requires splitting the phase current into two components: magnetising current ($I_m$) and iron loss current ($I_L$).

The phase circuit equations are now

$$\frac{d \psi(x, i_p)}{dt} = v - R \cdot i_{exc} - N \cdot \Delta \psi_{magnet}$$  \hspace{1cm} (6)

$$i_{exc} = i_p + I_L = i_p + \frac{1}{r} \frac{d \psi(x, i_p)}{dt}$$  \hspace{1cm} (7)

where $x$ denotes the position and

$\begin{align*}
  v &= +V \quad \text{when both transistors are ON} \\
  v &= 0 \quad \text{when one transistor is ON and the other is OFF} \\
  v &= -V \quad \text{when both transistors are OFF}
\end{align*}$
The dynamic model based on the above equations uses stably measured magnetisation and equivalent iron loss resistance characteristics in the form of look-up tables as input data which describe the machine in the equivalent circuit. The machine thrust is computed on the basis of instantaneous values of flux-linkage and magnetising current.

The measured equivalent circuit parameter characteristics were used in the dynamic model of the experimental linear SR machine. The enhanced presence of iron losses in this machine causes a reduction of the machine pull-out force. (The term "pull-out force is used for the maximum running force that machine can develop at a given phase switching frequency.)

Table I gives a comparison between measured predicted results for the pull-out force in the constant current chopping mode at the rated level of 3.2 A. For the change of phase frequency from 6 to 20 Hz, which is approximately threefold increase of speed, the discrepancy varies between -3.2% and +2.3%. (Frequency of 20Hz corresponds to the speed of 0.2 m/s which has been taken as the speed limit for the experimental rig which is gravitationally loaded.)

<table>
<thead>
<tr>
<th>FREQUENCY [Hz]</th>
<th>MEASURED FORCE [N]</th>
<th>PREDICTED FORCE [N]</th>
<th>DISCREPANCY [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>158</td>
<td>153</td>
<td>-3.2</td>
</tr>
<tr>
<td>10</td>
<td>153</td>
<td>149</td>
<td>-2.6</td>
</tr>
<tr>
<td>20</td>
<td>131</td>
<td>124</td>
<td>+2.3</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

The development of a control algorithm for an existing SR machine normally relies on a dynamic model based on the machine's equivalent circuit. This paper described the measurement methodology for determining the parameters of the equivalent circuit which accounts for the presence of iron losses. The parameters were determined through static measurements which were then used in the machine's dynamic model to predict instantaneous current, flux and force waveforms, and subsequently the pull-out force at various speeds.

REFERENCES