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# A SENSITIVITY ANALYSIS OF FACTORS INFLUENCING THERMAL CONDUCTIVITY OF NANOFLUIDS

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## ABSTRACT

Numerical investigations are conducted to study the effect of particle clustering and interfacial layer thickness on thermal conductivity of nanofluids. The interplay between these two factors is examined through Kapitza radius, which hasn't often been rigorously investigated in literature. Degree of enhancement is analysed for increasing aggregate size, particle concentration and interfacial resistance. The numerical work is validated with experimental data for water-based nanofluids of Alumina, CuO and Titania nanoparticles. Particle concentrations are varied up to 4vol%. Aspect ratio (radius of gyration of aggregate to radius of primary particle,  $Rg/a$ ) varied between 2 to 60. It can be confirmed that the enhancement decreases with interfacial layer thickness. The rate of decrease takes a significant turn after a particular aggregate size. For a given interfacial resistance, enhancement ratio is mostly sensitive to  $vol\% < 4$  and  $Rg/a < 20$ . A good agreement is found between predicted and experimental data. In summary, the present work provides important information on the interval of aggregate sizes, particle concentrations and interfacial resistances that will be useful in manufacturing high thermal conductivity nanofluids. For future work, efforts are underway to refine the model by incorporating cluster evolution dynamics as a function of particle-scale properties.

## INTRODUCTION

Nanofluids are solid-liquid composites where metallic or non-metallic nanometre-sized particles are stably suspended in common liquids such as water, engine oil and ethylene glycol. It was almost a decade ago Eastman et al (1997) first reported that nanofluids possess anomalously enhanced thermal conductivities compared to their respective base liquids. Their nanofluid was made up of Al<sub>2</sub>O<sub>3</sub> nanoparticles suspended in water. At particle volume concentration of 5%, they had observed 29% enhancement in thermal conductivity. In another instance when 8vol% of Al<sub>2</sub>O<sub>3</sub> was suspended in Ethylene Glycol, a massive 40% enhancement was reported (Wang et al., 1999). These numbers were warmly welcomed by research quarters in the backdrop of conventional heat transfer liquids already having reached their upper limits. As a consequence, a boom in nanofluids research occurred. Since then, majority of Investigators reported enhancement in thermal conductivity (Das et al., 2003; Murshed et al., 2005; Wen & Ding, 2004; Zhu et al., 2007). Nevertheless, some groups reported otherwise (Kim et al., 2007; Putnam et al., 2006). Long-established effective medium theories such as Maxwell (1881) and Hamilton-Crosser (Hamilton & Crosser, 1962) seemingly failed to predict experimental data. This situation led to investigation of mechanisms behind the thermal conductivity behaviour of nanofluids.

Keblinski et al (2002) postulated four possible mechanisms that could have caused the experimentally observed thermal conductivity enhancement. These are; Brownian movement of nanoparticles, liquid layering around nanoparticles, mode of heat transport within nanoparticles, and particle clustering. Based on one or more of these postulates, more than a dozen models have been developed since then. For example the work of Jang and Choi (2004), Koo and Kleinstreuer (2004) and Prasher et al (2006a) can be illustrated. Moreover, the importance of two mechanisms is pronounced often; interfacial layer, and, particle clustering. However, none of these models are capable of fully capturing the thermal conductivity behaviour of nanofluids.

In the present work, numerical investigations are conducted to study the effect of particle clustering and interfacial layer thickness on thermal conductivity of nanofluids. The interplay between these two factors is examined through Kapitza radius (Every et al., 1992). The degree of enhancement is analysed for increasing aggregate size and particle concentration. The numerical work is validated with experimental data on water-based nanofluids of metals and metal oxide nanoparticles.

## EXPERIMENTAL METHOD

A particle aggregate is assumed to be comprising of some particles arranged like chains (known as backbones) and others stay as individual (dead end) particles (Prasher et al., 2006b). For modelling purposes, an aggregate of this nature is then embedded in an imaginary sphere of radius R<sub>g</sub>. The numerical investigations are conducted based on a set of equations which are explained below.

The effective thermal conductivity k<sub>eff</sub> of nanofluid can be written as outlined by Prasher et al (2006b);

$$\frac{k_{eff}}{k_f} = \frac{([k_a + 2k_f] + 2\phi_a[k_a - k_f])}{([k_a + 2k_f] - 2\phi_a[k_a - k_f])} \quad (1)$$

Where k<sub>f</sub>, k<sub>a</sub>, φ<sub>p</sub> are thermal conductivities of the base fluid and particle aggregate, and, particle volume fraction respectively. The term k<sub>a</sub> is determined following Nan et al's 1997 model for randomly oriented cylindrical particles;

$$k_a = k_{nc} \frac{3 + \phi_c [2\beta_{11}(1 - L_{11}) + \beta_{33}(1 - L_{33})]}{3 - \phi_c [2\beta_{11}L_{11} + \beta_{33}L_{33}]} \quad (2)$$

Where k<sub>nc</sub> is the thermal conductivity of the imaginary medium in which the backbones are embedded in. φ<sub>c</sub> is the vol fraction of backbone particles. Other parameters are defined in equations 3 to 6.

$$L_{11} = 0.5p^2 / ([p^2 - 1] - 0.5p \cosh^{-1} p / [p^2 - 1]) \quad (3)$$

$$L_{33} = 1 - 2L_{11} \quad (4)$$

$$\beta_{ii} = \frac{(k_{ii}^c - k_{nc})}{[k_{nc} + L_{ii}(k_{ii}^c - k_{nc})]} \quad (5)$$

$$k_{ii}^c = \frac{k_p}{(1 + \gamma L_{ii} k_p / k_f)} \quad (6)$$

Where *p* is the aspect ratio for the aggregate defined by R<sub>g</sub>/a. What remains is to calculate the term k<sub>nc</sub>. From Prasher 2006b it can be written;

$$(1 - \phi_{nc})(k_f - k_{nc}) / \left( (k_f + 2k_{nc}) + \phi_{nc}(k_p - k_{nc}) / (k_f + 2k_{nc}) \right) = 0 \quad (7)$$

In order to correlate interfacial thermal resistance to particle radius ( $a$ ), a non-dimensional parameter  $\alpha$  is introduced;  $\alpha = a_k/a$  where  $a_k$  is Kapitza radius Every et al., 1992 defined using interfacial thermal resistance ( $R_{Bd}$ ) and fluid thermal conductivity ( $k_f$ ). Hence  $a_k = R_{Bd} \cdot k_f$

## RESULTS

To demonstrate the accuracy of present simulation technique, our data is compared with literature data for water based nanofluids of Alumina, CuO and Titania, shown in Fig 1. The aspect ratio for nanoparticles ( $R_g/a$ ) was fixed at a value of 2 in these plots. The value of 2 was chosen assuming almost no aggregation (i.e.  $R_g = 2a$ ), in line with many literature.

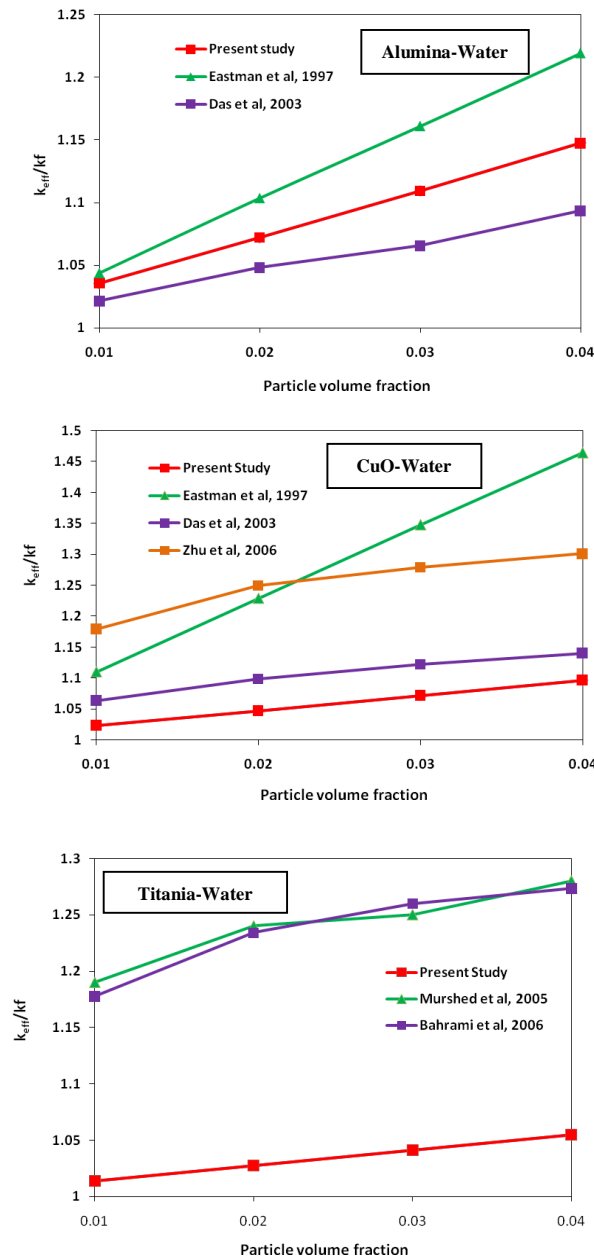


Figure 1: Comparison of enhancement ratio ( $k_{eff}/k_f$ ) obtained from present work with literature data for water-based Alumina, CuO and Titania nanofluids

It can be seen that our models are in good agreement with literature data for aqueous Alumina and copper oxide nanofluids. For the Titania nanofluid, the model predictions follow the same trend as the experimental data. Then the enhancement ratio was calculated against  $Rg/a$  in a range of  $\alpha$  values;  $0.01 \leq \alpha \leq 10$ , for two particle volume fractions. These are shown in Fig 2 for typical cases of volume fractions 0.5% and 4%.

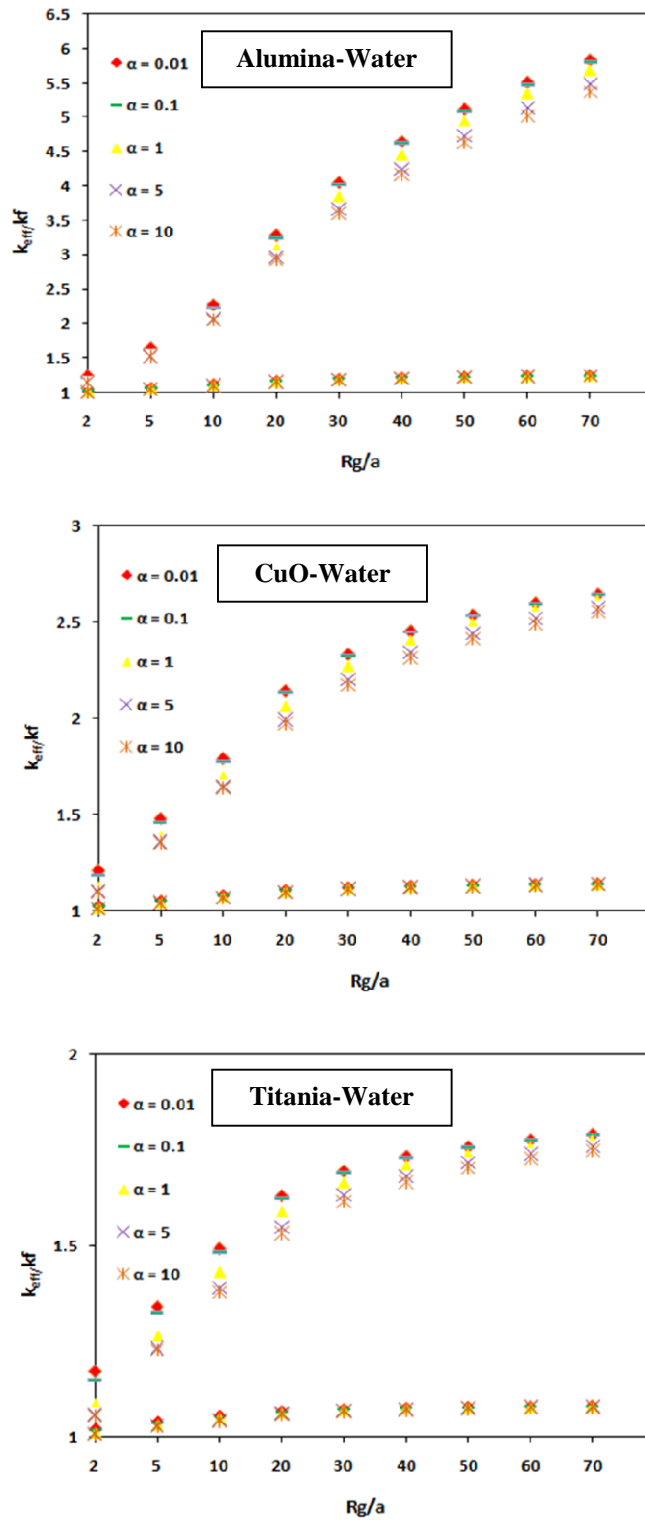


Figure 2: Dependence of enhancement ratio ( $k_{eff}/k_f$ ) with aspect ratio and the interfacial resistance. On each plot the bottom set of lines represent 0.5vol%, the top set is for 4vol%

At 0.5vol% concentration, the enhancement ratio is very weakly sensitive to aspect ratio as well as  $\alpha$ . moreover, as  $\phi$  increases to 4vol% ,  $\alpha < 1$  does not make impact on enhancement ratio; i.e.,  $\alpha = 1$  appears like a turning point for its influence. These observations are consistent for all nanofluids shown in Fig 2. Upon this observation, the value for  $\alpha$  was fixed to 1 in the following simulations.

Presented in Fig 3 are the relationship between the enhancement ratio, aspect ratio and particle volume fraction.

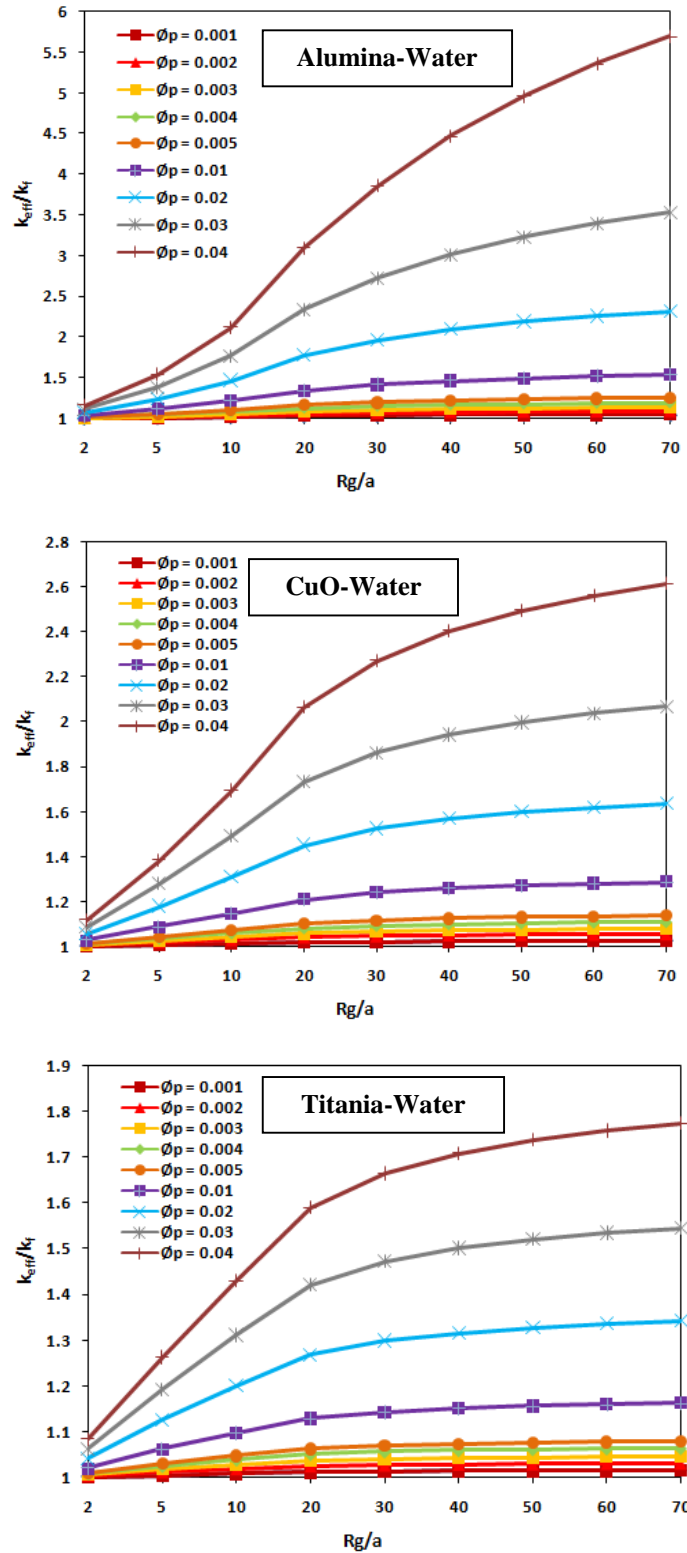


Figure 3: The effect of aspect ratio and particle volume fraction on enhancement ratio

The volume fractions chosen in Fig 3 were chosen to have an upper limit of 4%. In most practical applications, this is thought to be the limiting cases, considering the clogging problem and pumping costs. It can be noted that a significant impact on enhancement ratio begins when the vol% >1. Moreover, there is a steep increase in enhancement ratio up to  $Rg/a=20$ , after which the lines gradually flatten out. Interesting for stable nanofluids formulated from spherical primary particles, it has been known that  $Rg/a$  is often less than 10.

## CONCLUSIONS

The present work clearly suggests that particle aggregation (reflected by  $Rg/a$ ) has influence on thermal conductivity enhancement. This influence is significant when  $Rg/a < 20$ . The influence of interfacial thermal resistance, which has been accounted for by  $\alpha$  which reflects Kapitza radius, plays important role beyond  $\alpha = 1$ . Moreover, to achieve a sensible enhancement in thermal conductivity, particle concentration in the nanofluid is suggested to be above 1vol%. In summary, the present work provides important information on the interval of aggregate sizes, particle concentrations and interfacial resistances that will be useful in manufacturing high thermal conductivity nanofluids. Further work is being carried out on the influence of the type of interaction on self-assembly and mechanical strength of nanoparticles using theories of contact mechanics.

## NOMENCLATURE

$a_k$	Kapitza radius
$a$	radius of primary nanoparticles
$d_l$	chemical dimension
$d_f$	fractal dimension
$k$	thermal conductivity
$N$	number of particles
$R_g$	radius of gyration

### Greek symbols

$\phi$	volume fraction
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### Subscripts

$a$	aggregate
$c$	backbone
$eff$	effective
$f$	fluid
$nc$	dead ends
$p$	particle

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